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On $\mathcal{F}$-Contractions for Weak α-Admissible Mappings in Metric-Like Spaces

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Abstract: In the paper, we consider some fixed point results of $\mathcal{F}$-contractions for triangular α-admissible and triangular weak α-admissible mappings in metric-like spaces. The results on $\mathcal{F}$-contraction type mappings in the context of metric-like spaces are generalized, improved, unified, and enriched. We prove the main result but using only the property $(\mathcal{F}1)$ of the strictly increasing mapping $\mathcal{F} : (0, +\infty) \to (-\infty, +\infty)$. Our approach gives a proper generalization of several results given in current literature.

Keywords: Banach principle; metric-like space; fixed point theorem; Wardowski type contraction; triangular α-admissible mapping; triangular weak α-admissible mapping

MSC: 47H10; 54H25

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1. Introduction and Preliminaries

First, we recall some notions introduced recently in several papers.

In 2012, Samet et al. [1] introduced the concept of α-admissible mappings as follows.

Definition 1. *Let $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ and $\alpha : \mathcal{X}^2 \to [0, +\infty)$. Then, $\mathcal{T}$ is called α-admissible if for all $\xi, \zeta \in \mathcal{X}$ with $\alpha(\xi, \zeta) \geq 1$ implies $\alpha(\mathcal{T}\xi, \mathcal{T}\zeta) \geq 1$.*

Furthermore, one says that $\mathcal{T}$ is a triangular α-admissible mapping if it is α-admissible and if

$$\alpha(\xi, \eta) \geq 1 \text{ and } \alpha(\eta, \zeta) \geq 1 \text{ implies } \alpha(\xi, \zeta) \geq 1, \xi, \zeta, \eta \in \mathcal{X}.$$

For triangular α-admissible mapping, the following result is known ([2], Lemma 7):

Lemma 1. *Let $\mathcal{T}$ be a triangular α-admissible mapping. Assume that there exists $\xi_0 \in \mathcal{X}$ such that $\alpha(\xi_0, \mathcal{T}\xi_0) \geq 1$. Define sequence $\{\xi_n\}$ by $\xi_n = \mathcal{T}^n \xi_0$. Then,*

$$\alpha(\xi_m, \xi_n) \geq 1 \text{ for all } m, n \in \mathbb{N} \cup \{0\} \text{ with } m < n.$$

In [3], the author presented the notion of weak α-admissible mappings as follows:

Definition 2. *Let $\mathcal{X}$ be a nonempty set and let $\alpha : \mathcal{X}^2 \to [0, +\infty)$ be a given mapping. A mapping $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ is said to be a weak α-admissible one if the following condition holds:*

$$\text{for } \xi \in \mathcal{X} \quad \text{with } \alpha(\xi, \mathcal{T}\xi) \geq 1 \text{ implies } \alpha\left(\mathcal{T}\xi, \mathcal{T}^2\xi\right) \geq 1. \tag{1}$$

Remark 1. *It is customary to write $A(\mathcal{X}, \alpha)$ and $WA(\mathcal{X}, \alpha)$ as the collection of all (triangular) α-admissible mappings on $\mathcal{X}$ and the collection of all (triangular) weak α-admissible mappings on $\mathcal{X}$ (see[3]) . One can verify that $A(\mathcal{X}, \alpha) \subseteq WA(\mathcal{X}, \alpha)$.*

Now, we recall some basic concepts, notations, and known results from partial metric and metric-like spaces. In 1994 Matthews ([4]) introduced notion of partial metric space as follows.

Definition 3. *Let $\mathcal{X}$ be a nonempty set. A mapping $d_{pm} : \mathcal{X}^2 \to [0, +\infty)$ is said to be a partial metric on $\mathcal{X}$ if for all $\xi, \zeta, \eta \in \mathcal{X}$ the following four conditions hold:*

(1) $\xi = \zeta$ *if and only if* $d_{pm}(\xi, \xi) = d_{pm}(\xi, \zeta) = d_{pm}(\zeta, \zeta)$;
(2) $d_{pm}(\xi, \xi) \leq d_{pm}(\xi, \zeta)$;
(3) $d_{pm}(\xi, \zeta) = d_{pm}(\zeta, \xi)$;
(4) $d_{pm}(\xi, \eta) \leq d_{pm}(\xi, \zeta) + d_{pm}(\zeta, \eta) - d_{pm}(\zeta, \zeta)$.

In this case, the pair $(\mathcal{X}, d_{pm})$ is called a partial metric space. Obviously, every metric space is a partial metric space. The inverse is not true. Indeed, let $\mathcal{X} = [0, +\infty)$ and $d_{pm}(\xi, \zeta) = \max\{\xi, \zeta\}$. Under these conditions $(\mathcal{X}, d_{pm})$ is a partial metric space but is not a metric space because $d_{pm}(1, 1) = 1 > 0$. For more details, see ([5–11]).

For the following notion see [12].

Definition 4. *Let $\mathcal{X}$ be a nonempty set. A mapping $d_{ml} : \mathcal{X}^2 \to [0, +\infty)$ is said to be a metric-like on $\mathcal{X}$ if for all $\xi, \zeta, \eta \in \mathcal{X}$ the following three conditions hold:*

(1) $d_{ml}(\xi, \zeta) = 0$ *implies* $\xi = \zeta$;
(2) $d_{ml}(\xi, \zeta) = d_{ml}(\zeta, \xi)$;
(3) $d_{ml}(\xi, \eta) \leq d_{ml}(\xi, \zeta) + d_{ml}(\zeta, \eta)$.

The pair $(\mathcal{X}, d_{ml})$ is called a metric-like space or dislocated metric space by some authors. A metric-like mapping d_{ml} on $\mathcal{X}$ satisfies all the conditions of a metric except that $d_{ml}(\xi, \xi)$ may be positive for some $\xi \in \mathcal{X}$. The following is a list of some metric-like spaces:

1. $(\mathbb{R}, d_{ml})$, where $d_{ml}(\xi, \zeta) = \max\{|\xi|, |\zeta|\}$ for all $\xi, \zeta \in \mathbb{R}$.

One can see that $(\mathbb{R}, d_{ml})$ is a metric-like space, but it is not a metric space, due to the fact that $d_{ml}(|-2|, |-2|) = 2 > 0$. On the other hand, $(\mathbb{R}, d_{ml})$ is a partial metric space.

2. $([0, +\infty), d_{ml})$, where $d_{ml}(\xi, \zeta) = \xi + \zeta$ for all $\xi, \zeta \in [0, +\infty)$.

It is clear that $([0, +\infty), d_{ml})$ is a metric-like space where $d_{ml}(\xi, \xi) > 0$ for each $\xi > 0$. Since $d_{ml}(2, 2) = 2 + 2 = 4 > 3 = 2 + 1 = d_{ml}(2, 1)$, it follows that $d_{ml}(\xi, \xi) \leq d_{ml}(\xi, \zeta)$ does not hold. Hence, $([0, +\infty), d_{ml})$ is not a partial metric space.

3. $(\mathcal{X}, d_{ml})$, where $\mathcal{X} = \{0, 1, 2\}$ and $d_{ml}(0, 0) = d_{ml}(1, 1) = 0, d_{ml}(2, 2) = \frac{5}{2}, d_{ml}(0, 2) = d_{ml}(2, 0) = 2$, $d_{ml}(1, 2) = d_{ml}(2, 1) = 3, d_{ml}(0, 1) = d_{ml}(1, 0) = \frac{3}{2}$.

It is clear that $(\mathcal{X}, d_{ml})$ is a metric-like (that is a dislocated metric) space with $d_{ml}(2, 2) > 0$. This means that $(\mathcal{X}, d_{ml})$ is not a standard metric space. However, $(\mathcal{X}, d_{ml})$ is also not a partial metric space because $d_{ml}(2, 2) \nleq d_{ml}(2, 0)$.

4. $(\mathcal{X}, d_{ml})$, where $\mathcal{X} = C([0,1], \mathbb{R})$ is the set of real continuous functions on $[0,1]$ and $d_{ml}(f,g) = \sup_{t\in[0,1]} (|f(t)| + |g(t)|)$ for all $f, g \in C([0,1], \mathbb{R})$.

This is an example of metric-like space that is not a partial metric space. Indeed, for $f(t) = 2t$, we obtain $d_{ml}(f,f) = \sup_{t\in[0,1]} (2t + 2t) = 4 > 0$. Putting $g(t) \equiv 0$ for all $t \in [0,1]$, we obtain that $d_{ml}(f,f) = 4 \nleq d_{ml}(f,g) = d_{ml}(f,0) = 2$.

Note that some of the metric-like spaces given in the list are not partial metric spaces. It is clear that a partial metric space is a metric-like space and the inverse is not true. Now, we give the definitions of convergence and Cauchyiness of the sequences in metric-like space (see [12]).

Definition 5. *Let $\{\xi_n\}$ be a sequence in a metric-like space $(\mathcal{X}, d_{ml})$.*

(i) The sequence $\{\xi_n\}$ is said to be convergent to $\xi \in \mathcal{X}$ if $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi) = d_{ml}(\xi, \xi)$;

(ii) The sequence $\{\xi_n\}$ is said to be $d_{ml}-$Cauchy in $(\mathcal{X}, d_{ml})$ if $\lim_{n,m\to+\infty} d_{ml}(\xi_n, \xi_m)$ exists and is finite;

(iii) A metric-like space $(\mathcal{X}, d_{ml})$ is $d_{ml}-$complete if for every $d_{ml}-$ Cauchy sequence $\{\xi_n\}$ in $\mathcal{X}$ there exists an $\xi \in \mathcal{X}$ such that $\lim_{n,m\to+\infty} d_{ml}(\xi_n, \xi_m) = d_{ml}(\xi, \xi) = \lim_{n\to+\infty} d_{ml}(\xi_n, \xi)$.

More details on partial metric and metric-like spaces can be found in ([5–7,11,13–18]), and information on other classes of generalized metric spaces and contractive mappings can be found in: ([1,3–37]).

Remark 2. *In metric-like space (as in the partial metric space), the limit of a sequence need not be unique and a convergent sequence need not be a $d_{ml}-$Cauchy sequence (see examples in Remark 1.4 (1) and (2) in [10]). However, if the sequence $\{\xi_n\}$ is $d_{ml}-$ Cauchy such that $\lim_{n,m\to+\infty} d_{ml}(\xi_n, \xi_m) = 0$ in $d_{ml}-$ complete metric-like space $(\mathcal{X}, d_{ml})$, then the limit of such sequence is unique. Indeed, in such a case if $\xi_n \to \xi$ as $n \to +\infty$, we get that $d_{ml}(\xi, \xi) = 0$ (by (iii) of Definition 5). Now, if $\xi_n \to \xi, \xi_n \to \zeta$ and $\xi \neq \zeta$, we obtain*

$$d_{ml}(\xi, \zeta) \le d_{ml}(\xi, \xi_n) + d_{ml}(\xi_n, \zeta) \to d_{ml}(\xi, \xi) + d_{ml}(\zeta, \zeta) = 0 + 0 = 0. \tag{2}$$

By (1) from Definition 4, it follows that $\xi = \zeta$, which is a contradiction.

Now, we give the definition of the continuity for self-mapping $\mathcal{T}$ defined on a metric-like space $(\mathcal{X}, d_{ml})$ as follows (see for example [10,11,34]) :

Definition 6. *Let $(\mathcal{X}, d_{ml})$ be a metric-like space and $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ be a self-mapping. We say that $\mathcal{T}$ is $d_{ml}-$ continuous in point $\xi \in \mathcal{X}$ if $\lim_{n\to+\infty} d_{ml}(\mathcal{T}\xi_n, \mathcal{T}\xi) = d_{ml}(\mathcal{T}\xi, \mathcal{T}\xi)$, for each sequence $\{\xi_n\} \subseteq \mathcal{X}$ such that $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi) = d_{ml}(\xi, \xi)$. In other words, the mapping $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ is $d_{ml}-$ continuous if the following holds true:*

$$\xi_n \overset{d_{ml}}{\to} \xi \text{ implies } \mathcal{T}\xi_n \overset{d_{ml}}{\to} \mathcal{T}\xi. \tag{3}$$

Definition 7. *Let $(\mathcal{X}, d_{ml})$ be a metric-like space. A sequence $\{\xi_n\}$ in it is called $0 - d_{ml}-$ Cauchy sequence if $\lim_{n,m\to+\infty} d_{ml}(\xi_n, \xi_m) = 0$. The space $(\mathcal{X}, d_{ml})$ is said to be $0 - d_{ml}-$ complete if every $0 - d_{ml}-$ Cauchy sequence in $\mathcal{X}$ converges to a point $\xi \in \mathcal{X}$ such that $d_{ml}(\xi, \xi) = 0$.*

It is obvious that every $0 - d_{ml}-$ Cauchy sequence is a $d_{ml}-$ Cauchy sequence in $(\mathcal{X}, d_{ml})$ and every $d_{ml}-$ complete metric-like space is a $0 - d_{ml}-$ complete metric-like space. In addition, every $0-$ complete partial metric space $(\mathcal{X}, d_{ml})$ is a $0 - d_{ml}-$ complete metric-like space. In the sequel, some results on metric-like spaces are given. Proofs to most of the results are self-evident.

Proposition 1. *Let $(\mathcal{X}, d_{ml})$ be a metric-like space. Then, we have the following:*

(i) *If the sequence $\{\xi_n\}$ converges to $\xi \in \mathcal{X}$ as $n \to +\infty$ and if $d_{ml}(\xi, \xi) = 0$, then, for all $\zeta \in \mathcal{X}$, it follows that $d_{ml}(\xi_n, \zeta) \to d_{ml}(\xi, \zeta)$;*

(ii) *If $d_{ml}(\xi, \zeta) = 0$, then $d_{ml}(\xi, \xi) = d_{ml}(\zeta, \zeta) = 0$;*

(iii) *If $\{\xi_n\}$ is a sequence such that $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi_{n+1}) = 0$, then $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi_n) = \lim_{n\to+\infty} d_{ml}(\xi_{n+1}, \xi_{n+1}) = 0$;*

(iv) *If $\xi \neq \zeta$, then $d_{ml}(\xi, \zeta) > 0$;*

(v) *$d_{ml}(\xi, \xi) \leq \frac{2}{n} \sum_{i=1}^{n} d_{ml}(\xi, \xi_i)$ holds for all $\xi, \xi_i \in \mathcal{X}$, where $1 \leq i \leq n$;*

(vi) *Let $\{\xi_n\}$ be a sequence such that $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi_{n+1}) = 0$. If $\lim_{n,m\to+\infty} d_{ml}(\xi_n, \xi_m) \neq 0$, then there exists $\varepsilon > 0$ and sequences $\{m(k)\}$ and $\{n(k)\}$ such that $n(k) > m(k) > k$, and the following sequences tend to ε when $k \to +\infty$:*

$$
\begin{gathered}
\left\{d_{ml}\left(\xi_{n(k)}, \xi_{m(k)}\right)\right\}, \left\{d_{ml}\left(\xi_{n(k)+1}, \xi_{m(k)}\right)\right\}, \left\{d_{ml}\left(\xi_{n(k)}, \xi_{m(k)-1}\right)\right\}, \\
\left\{d_{ml}\left(\xi_{n(k)+1}, \xi_{m(k)-1}\right)\right\}, \left\{d_{ml}\left(\xi_{n(k)+1}, \xi_{m(k)+1}\right)\right\}.
\end{gathered} \tag{4}
$$

Notice that, if the condition (vi) is satisfied then the sequences $d_{ml}\left(\xi_{n(k)+q}, \xi_{m(k)}\right)$ and $d_{ml}\left(\xi_{n(k)+q}, \xi_{m(k)+1}\right)$ also converge to ε when $k \to +\infty$, where $q \in \mathbb{N}$. For more details on (i)–(vi), the reader can see in ([26,27,36]). The concept of $\mathcal{F}$-contraction was introduced by Wardowski in [16] (for more details, see also: [5,9,14–18,24,28,31–33]).

Definition 8. *Let $\mathcal{F} : (0, +\infty) \to (-\infty, +\infty)$ be a mapping satisfying the following:*

($\mathcal{F}$1) *$\mathcal{F}$ is a strictly increasing, that is, for $\alpha, \beta \in (0, +\infty)$, $\alpha < \beta$ implies $\mathcal{F}(\alpha) < \mathcal{F}(\beta)$,*

($\mathcal{F}$2) *For each sequence $\{\alpha_n\} \subset (0, +\infty)$, $\lim_{n\to+\infty} \alpha_n = 0$ if and only if $\lim_{n\to+\infty} \mathcal{F}(\alpha_n) = -\infty$,*

($\mathcal{F}$3) *There exists $k \in (0, 1)$ such that $\lim_{\alpha\to0^+} \alpha^k \mathcal{F}(\alpha) = 0$.*

Definition 9. *Let $(\mathcal{X}, d)$ be a metric space. A mapping $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ is said to be an $\mathcal{F}$-contraction if there exist $\mathcal{F} : (0, +\infty) \to (-\infty, +\infty)$ satisfying ($\mathcal{F}$1), ($\mathcal{F}$2) and ($\mathcal{F}$3) and $\tau > 0$ such that*

$$
d(\mathcal{T}\xi, \mathcal{T}\zeta) > 0 \text{ implies } \tau + \mathcal{F}(d(\mathcal{T}\xi, \mathcal{T}\zeta)) \leq \mathcal{F}(d(\xi, \zeta)), \tag{5}
$$

for all $\xi, \zeta \in \mathcal{X}$.

In 2014, Piri and Kumam [32] investigated some fixed point results concerning $\mathcal{F}$ contraction in complete metric spaces by replacing the condition ($\mathcal{F}$3) with the condition:

($\mathcal{F}3'$)$\mathcal{F}$ is continuous on $(0, +\infty)$.

Recently, in 2018, Qawaqueh et al. ([9]) defined and proved the following:

Definition 10. *Let $(\mathcal{X}, d_{ml})$ be a metric-like space and $\alpha : \mathcal{X}^2 \to [0, +\infty)$. A mapping $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ is said to be an $(\alpha, \beta, \mathcal{F})$-Geraghty contraction mapping if there exist $\beta \in \mathcal{G}$ and $\tau > 0$ such that, for all $\xi, \zeta \in \mathcal{X}$ with $d(\mathcal{T}\xi, \mathcal{T}\zeta) > 0$ and $\alpha(\xi, \zeta) \geq 1$,*

$$
\alpha(\xi, \zeta)(\tau + \mathcal{F}(d_{ml}(\mathcal{T}\xi, \mathcal{T}\zeta))) \leq \beta(\mathcal{M}(\xi, \zeta))\mathcal{F}(\mathcal{M}(\xi, \zeta)), \tag{6}
$$

where

$$
\begin{gathered}
\mathcal{M}(\xi, \zeta) = \max\left\{d_{ml}(\xi, \zeta), d_{ml}(\xi, \mathcal{T}\xi), d_{ml}(\zeta, \mathcal{T}\zeta), \frac{d_{ml}(\xi, \mathcal{T}\zeta) + d_{ml}(\mathcal{T}\xi, \zeta)}{4}, \right. \\
\left. \frac{[1 + d_{ml}(\xi, \mathcal{T}\xi)]d_{ml}(\zeta, \mathcal{T}\zeta)}{1 + d_{ml}(\xi, \zeta)}\right\},
\end{gathered} \tag{7}
$$

$\mathcal{F} : (0, +\infty) \to (-\infty, +\infty)$ *is strictly increasing function satisfying* $(\mathcal{F}1)$, $(\mathcal{F}2)$ *and* $(\mathcal{F}3)$ *and* $\mathcal{G}$ *is a family of all functions* $\beta : [0, +\infty) \to [0, 1)$ *which satisfy the condition:* $\beta(t_n) \to 1$ *implies* $t_n \to 0$ *as* $\to +\infty$.

It is worth noticing that authors in [9] denote with $E(\mathcal{X}, \alpha, \beta, \mathcal{F})$ the collection of all almost generalized $(\alpha, \beta, \mathcal{F})$-contractive mappings. However, it is not clear what "almost generalized $(\alpha, \beta, \mathcal{F})$-contractive mappings" mean.

Theorem 1. *Let* $(\mathcal{X}, d_{ml})$ *be a metric-like space and* $\alpha : \mathcal{X}^2 \to [0, +\infty)$. *A mapping* $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ *be an* $(\alpha, \beta, \mathcal{F})$*-Geraghty contraction mapping. Assume that the following conditions are satisfied:*

(i) $\mathcal{T} \in E(\mathcal{X}, \alpha, \beta, \mathcal{F}) \cap WA(\mathcal{X}, \alpha)$.
(ii) *There exists* $\xi_0 \in \mathcal{X}$ *such that* $d_{ml}(\xi_0, \mathcal{T}\xi_0) \geq 1$.
(iii) $\mathcal{T}$ *is* $d_{ml}-$ *continuous.*

Then, $\mathcal{T}$ *has a unique fixed point* $\eta \in \mathcal{X}$ *with* $d_{ml}(\eta, \eta) = 0$.

2. Main Result

In this section, we improve the whole concept by introducing a new definition and new approaches. Firstly, we introduce the following:

Definition 11. *Let* $(\mathcal{X}, d_{ml})$ *be a metric-like space and* $\alpha : \mathcal{X}^2 \to [0, +\infty)$. *A mapping* $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ *is said to be a triangular* $(\alpha, \mathcal{F})$*-contraction one if there exists* $\tau > 0$ *such that, for all* $\xi, \zeta \in \mathcal{X}$ *with* $d_{ml}(\mathcal{T}\xi, \mathcal{T}\zeta) > 0$ *and* $\alpha(\xi, \zeta) \geq 1$ *holds true,*

$$\alpha(\xi, \zeta)(\tau + \mathcal{F}(d_{ml}(\mathcal{T}\xi, \mathcal{T}\zeta))) \leq \mathcal{F}(\mathcal{M}(\xi, \zeta)), \tag{8}$$

where

$$\mathcal{M}(\xi, \zeta) = \max\left\{ d_{ml}(\xi, \zeta), d_{ml}(\xi, \mathcal{T}\xi), d_{ml}(\zeta, \mathcal{T}\zeta), \frac{d_{ml}(\xi, \mathcal{T}\zeta) + d_{ml}(\zeta, \mathcal{T}\xi)}{2}, \frac{[1 + d_{ml}(\xi, \mathcal{T}\xi)] d_{ml}(\zeta, \mathcal{T}\zeta)}{1 + d_{ml}(\xi, \zeta)} \right\}, \tag{9}$$

$\mathcal{F} : (0, +\infty) \to (-\infty, +\infty)$ *is strictly increasing function.*

Example 3 from [9], for instance, illustrates the validity of this definition but without the function $\beta : [0, +\infty) \to [0, 1)$. Definition 11 is an improvement of the definition given in [9] in several directions. Now, we prove the main result of our paper:

Theorem 2. *Let* $(\mathcal{X}, d_{ml})$ *be a* $0 - d_{ml}-$ *complete metric-like space and* $\alpha : \mathcal{X}^2 \to [0, +\infty)$. *Assume that a mapping* $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ *is a triangular* $(\alpha, \mathcal{F})$*-contraction one. Suppose further that the following conditions are satisfied:*

(i) $\mathcal{T} \in WA(\mathcal{X}, \alpha)$;
(ii) *There exists* $\xi_0 \in \mathcal{X}$ *such that* $\alpha(\xi_0, \mathcal{T}\xi_0) \geq 1$;
(iii) $\mathcal{T}$ *is* $d_{ml}-$ *continuous.*

Then, $\mathcal{T}$ *has a unique fixed point* $\widehat{\xi} \in \mathcal{X}$ *with* $d_{ml}\left(\widehat{\xi}, \widehat{\xi}\right) = 0$.

Proof. First of all, we show the following two claims:

I. If $\widehat{\xi}$ is a fixed point of $\mathcal{T}$ then $d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)=0$.

II. The uniqueness of a possible fixed point.

Firstly, we prove I. Indeed, if $\widehat{\xi}$ is a fixed point of $\mathcal{T}$ and if $d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)>0$, then, putting $\xi=\zeta=\widehat{\xi}$ in (8), we get

$$\tau+\mathcal{F}\left(d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\xi}\right)\right)\leq\alpha\left(\widehat{\xi},\widehat{\xi}\right)\left(\tau+\mathcal{F}\left(d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\xi}\right)\right)\right)\leq\mathcal{F}\left(\mathcal{M}\left(\widehat{\xi},\widehat{\xi}\right)\right), \tag{10}$$

where

$$\begin{aligned}\mathcal{M}\left(\widehat{\xi},\widehat{\xi}\right) &= \max\Bigg\{d_{ml}\left(\widehat{\xi},\widehat{\xi}\right),d_{ml}\left(\widehat{\xi},\mathcal{T}\widehat{\xi}\right),d_{ml}\left(\widehat{\xi},\mathcal{T}\widehat{\xi}\right),\frac{d_{ml}\left(\widehat{\xi},\mathcal{T}\widehat{\xi}\right)+d_{ml}\left(\widehat{\xi},\mathcal{T}\widehat{\xi}\right)}{2},\\ &\qquad \frac{\left[1+d_{ml}\left(\widehat{\xi},\mathcal{T}\widehat{\xi}\right)\right]d_{ml}\left(\widehat{\xi},\mathcal{T}\widehat{\xi}\right)}{1+d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)}\Bigg\}\end{aligned}$$

$$=\max\left\{d_{ml}\left(\widehat{\xi},\widehat{\xi}\right),d_{ml}\left(\widehat{\xi},\widehat{\xi}\right),d_{ml}\left(\widehat{\xi},\widehat{\xi}\right),d_{ml}\left(\widehat{\xi},\widehat{\xi}\right),\frac{\left[1+d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)\right]d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)}{1+d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)}\right\}=d_{ml}\left(\widehat{\xi},\widehat{\xi}\right).$$

Then, from (10), it follows

$$\tau+\mathcal{F}\left(d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)\right)\leq\mathcal{F}\left(d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)\right),$$

which is a contradiction. Hence, the assumption that $d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)>0$ is wrong. We proved claim I.

Now, we shall prove II. Suppose that $\mathcal{T}$ has two distinct fixed point $\widehat{\xi}$ and $\widehat{\zeta}$ in $\mathcal{X}$. By (I), we get $d\left(\widehat{\xi},\widehat{\xi}\right)=d_{ml}\left(\widehat{\zeta},\widehat{\zeta}\right)=0$. Since $d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right)=d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\zeta}\right)>0$ and $\alpha\left(\widehat{\xi},\widehat{\zeta}\right)\geq 1$, according to (8), we get:

$$\tau+\mathcal{F}\left(d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\zeta}\right)\right)\leq\alpha\left(\widehat{\xi},\widehat{\zeta}\right)\left(\tau+\mathcal{F}\left(d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\zeta}\right)\right)\right)\leq\mathcal{F}\left(\mathcal{M}\left(\widehat{\xi},\widehat{\zeta}\right)\right), \tag{11}$$

where

$$\mathcal{M}\left(\widehat{\xi},\widehat{\zeta}\right)=\max\left\{d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right),d_{ml}\left(\widehat{\xi},\widehat{\xi}\right),d_{ml}\left(\widehat{\zeta},\widehat{\zeta}\right),\frac{d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right)+d_{ml}\left(\widehat{\zeta},\widehat{\xi}\right)}{2},\frac{\left[1+d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)\right]d_{ml}\left(\widehat{\zeta},\widehat{\zeta}\right)}{1+d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right)}\right\}$$

$$=\max\left\{d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right),0,0,d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right),\frac{[1+0]\cdot 0}{1+0}\right\}=d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right).$$

In other words, taking $\alpha\left(\widehat{\xi},\widehat{\zeta}\right)\geq 1$ into consideration,

$$\tau+\mathcal{F}\left(d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right)\right)\leq\mathcal{F}\left(d_{ml}\left(\widehat{\xi},\widehat{\zeta}\right)\right) \tag{12}$$

is a contradiction. Hence, the uniqueness of fixed point is proved.

In the sequel, we prove the existence of the fixed point of $\mathcal{T}$.

Let $\xi_0\in\mathcal{X}$ be such that $\alpha\left(\xi_0,\mathcal{T}\xi_0\right)\geq 1$. Furthermore, we define the sequence $\{\xi_n\}$ in $\mathcal{X}$ with $\xi_{n+1}=\mathcal{T}\xi_n$ for all $n\in\mathbb{N}\cup\{0\}$. If $\xi_k=\xi_{k+1}$ for some $k\in\mathbb{N}\cup\{0\}$, then by the previous, ξ_k is a unique fixed point of $\mathcal{T}$ and the proof of the theorem is finished. Now, let us suppose that $\xi_n \neq \xi_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Since $\mathcal{T} \in WA(\mathcal{X}, \alpha)$ and $\alpha(\xi_0, \mathcal{T}\xi_0) \geq 1$, we have

$$\alpha(\xi_1, \xi_2) = \alpha(\mathcal{T}\xi_0, \mathcal{T}\mathcal{T}\xi_0) \geq 1, \alpha(\xi_2, \xi_3) = \alpha(\mathcal{T}\xi_1, \mathcal{T}\mathcal{T}\xi_1) \geq 1.$$

Using this process again, we get $\alpha(\xi_n, \xi_{n+1}) \geq 1$.

Because $\mathcal{T} : \mathcal{X} \to \mathcal{X}$ is a triangular $(\alpha, \mathcal{F})$-contraction mapping with $\alpha(\mathcal{T}\xi_{n-1}, \mathcal{T}\mathcal{T}\xi_{n-1}) = \alpha(\xi_n, \xi_{n+1}) \geq 1$, we have according to Lemma 1:

$$\begin{aligned} &0 < \tau + \mathcal{F}(d_{ml}(\xi_n, \xi_{n+1})) \\ &\leq \alpha(\xi_n, \xi_{n+1})(\tau + \mathcal{F}(d_{ml}(\mathcal{T}\xi_{n-1}, \mathcal{T}\xi_n))) \leq \mathcal{F}(\mathcal{M}(\xi_{n-1}, \xi_n)), \end{aligned} \tag{13}$$

where

$$\begin{aligned} & \mathcal{M}(\xi_{n-1}, \xi_n) \\ = & \max\left\{ d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_{n-1}, \mathcal{T}\xi_{n-1}), d_{ml}(\xi_n, \mathcal{T}\xi_n), \frac{d_{ml}(\xi_{n-1}, \mathcal{T}\xi_n) + d_{ml}(\mathcal{T}\xi_{n-1}, \xi_n)}{2}, \right. \\ & \left. \frac{[1 + d_{ml}(\xi_{n-1}, \mathcal{T}\xi_{n-1})] d_{ml}(\xi_n, \mathcal{T}\xi_n)}{1 + d_{ml}(\xi_{n-1}, \xi_n)} \right\} \end{aligned}$$

$$= \max\left\{ d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_n, \xi_{n+1}), \frac{d_{ml}(\xi_{n-1}, \xi_{n+1}) + d_{ml}(\xi_n, \xi_n)}{2}, d_{ml}(\xi_n, \xi_{n+1}) \right\}$$

$$\leq \max\left\{ d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_n, \xi_{n+1}), \frac{d_{ml}(\xi_{n-1}, \xi_n) + d_{ml}(\xi_n, \xi_{n+1}) - d_{ml}(\xi_n, \xi_n) + d_{ml}(\xi_n, \xi_n)}{2} \right\}$$

$$\begin{aligned} = & \max\left\{ d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_n, \xi_{n+1}), \frac{d_{ml}(\xi_{n-1}, \xi_n) + d_{ml}(\xi_n, \xi_{n+1})}{2} \right\} \\ \leq & \max\{ d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_n, \xi_{n+1}) \}. \end{aligned}$$

If $\max\{d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_n, \xi_{n+1})\} = d_{ml}(\xi_n, \xi_{n+1})$, then a contradiction follows from

$$0 < \tau + \mathcal{F}(d_{ml}(\xi_n, \xi_{n+1})) \leq \mathcal{F}(d_{ml}(\xi_n, \xi_{n+1})). \tag{14}$$

Thus, we conclude that $\max\{d_{ml}(\xi_{n-1}, \xi_n), d_{ml}(\xi_n, x_{n+1})\} = d(\xi_{n-1}, \xi_n)$ for all $n \in \mathbb{N}$. Therefore, since $\alpha(\xi_n, \xi_{n+1}) \geq 1$, we have

$$\tau + \mathcal{F}(d_{ml}(\xi_n, \xi_{n+1})) < \mathcal{F}(d_{ml}(\xi_{n-1}, \xi_n)),$$

where from one can conclude that $d_{ml}(\xi_n, \xi_{n+1}) < d_{ml}(\xi_{n-1}, \xi_n)$ for all $n \in \mathbb{N}$. This further means that there exists $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi_{n+1}) = \overline{d_{ml}} \geq 0$. If $\overline{d_{ml}} > 0$, we obtain a contradiction since by $(\mathcal{F}1)$, it follows:

$$\tau + \mathcal{F}\left(\overline{d_{ml}} + 0\right) \leq \mathcal{F}\left(\overline{d_{ml}} + 0\right),$$

where $\mathcal{F}\left(\overline{d_{ml}} + 0\right) = \lim_{n\to+\infty} \mathcal{F}(d_{ml}(\xi_n, \xi_{n+1}))$. We use the fact that strictly increasing function $\mathcal{F} : (0, +\infty) \to (-\infty, +\infty)$ has a left and right limit in every point from $(0, +\infty)$. Hence, we obtain that $\lim_{n\to+\infty} d_{ml}(\xi_n, \xi_{n+1}) = 0$. Now, we prove that the sequence $\{\xi_n\}_{n\in\mathbb{N}\cup\{0\}}$ is a $d_{ml}-$ Cauchy sequence by supposing the contrary. When we put $\xi = \xi_{m(k)}, \zeta = \xi_{n(k)}$ in (8), we get

$$\alpha\left(\xi_{m(k)}, \xi_{n(k)}\right)\left(\tau + \mathcal{F}\left(d_{ml}\left(\xi_{m(k)+1}, \xi_{n(k)+1}\right)\right)\right) \leq \mathcal{F}\left(\mathcal{M}\left(\xi_{m(k)}, \xi_{n(k)}\right)\right), \tag{15}$$

where

$$\mathcal{M}\left(\xi_{m(k)},\xi_{n(k)}\right) = \max\left\{d_{ml}\left(\xi_{m(k)},\xi_{n(k)}\right), d_{ml}\left(\xi_{m(k)},\xi_{m(k)+1}\right), d_{ml}\left(\xi_{n(k)},\xi_{n(k)+1}\right), \frac{d_{ml}\left(\xi_{m(k)},\xi_{n(k)+1}\right)+d_{ml}\left(\xi_{n(k)},\xi_{m(k)+1}\right)}{2}, \frac{\left[1+d_{ml}\left(\xi_{m(k)},\xi_{m(k)+1}\right)\right]d_{ml}\left(\xi_{n(k)},\xi_{n(k)+1}\right)}{1+d_{ml}\left(\xi_{m(k)},\xi_{n(k)}\right)}\right\} \to \max\left\{\varepsilon,0,0,\frac{\varepsilon+\varepsilon}{2},\frac{[1+0]\cdot 0}{1+\varepsilon}\right\}=\varepsilon.$$

Since $\alpha\left(\xi_{m(k)},\xi_{n(k)}\right)\geq 1$ from the previous inequality, we get

$$\tau+\mathcal{F}\left(d_{ml}\left(\xi_{m(k)+1},\xi_{n(k)+1}\right)\right)<\mathcal{F}\left(\mathcal{M}\left(\xi_{m(k)},\xi_{n(k)}\right)\right), \tag{16}$$

that is,

$$\tau+\mathcal{F}(\varepsilon+0)\leq\mathcal{F}(\varepsilon+0). \tag{17}$$

We obtain the contradiction, which means that the sequence $\{\xi_n\}_{n\in\mathbb{N}\cup\{0\}}$ is a $0-d_{ml}-$ Cauchy. This means that there exists a unique (by Remark 2) point $\widehat{\xi}\in\mathcal{X}$ such that

$$d_{ml}\left(\widehat{\xi},\widehat{\xi}\right)=\lim_{n\to+\infty}d_{ml}\left(\xi_n,\widehat{\xi}\right)=\lim_{n,m\to+\infty}d_{ml}\left(\xi_n,\xi_m\right)=0. \tag{18}$$

Since the mapping $\mathcal{T}$ is $d_{ml}-$ continuous, we get that $\lim_{n\to+\infty}d_{ml}\left(\mathcal{T}\xi_n,\mathcal{T}\widehat{\xi}\right)=d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\xi}\right)$, i.e., $\lim_{n\to+\infty}d_{ml}\left(\xi_{n+1},\mathcal{T}\widehat{\xi}\right)=d_{ml}\left(\mathcal{T}\widehat{\xi},\mathcal{T}\widehat{\xi}\right)$. According to Remark 2, it follows that $\mathcal{T}\widehat{\xi}=\widehat{\xi}$, that is, $\widehat{\xi}$ is a fixed point of $\mathcal{T}$. □

Remark 3. *The following results are immediate corollaries of Theorem 2. Indeed, replacing $\mathcal{M}(\xi,\zeta)$ in (8) with one of the following sets:*

$$\max\left\{d_{ml}(\xi,\zeta),d_{ml}(\xi,\mathcal{T}\xi),d_{ml}(\zeta,\mathcal{T}\zeta)\right\},$$

$$\max\left\{d_{ml}(\xi,\zeta),d_{ml}(\xi,\mathcal{T}\xi),d_{ml}(\zeta,\mathcal{T}\zeta),\frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}\right\},$$

$$\text{and }\max\left\{d_{ml}(\xi,\zeta),\frac{d_{ml}(\xi,\mathcal{T}\xi)+d_{ml}(\zeta,\mathcal{T}\zeta)}{2},\frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}\right\},$$

we get that Theorem 2 also holds true.

Immediate consequences of Theorem 2 are the following new contractive conditions that compliment the ones given in [23,35].

Corollary 1. *Let $(\mathcal{X},d_{ml})$ be a $0-d_{ml}-$ complete $0-d_{ml}-$ metric-like space and $\alpha_i:\mathcal{X}^2\to[0,+\infty)$. Assume that a mapping $\mathcal{T}:\mathcal{X}\to\mathcal{X}$ is a triangular $(\alpha_i,\mathcal{F})$- contraction where $\mathcal{F}:(0,+\infty)\to(-\infty,+\infty)$ is the strictly increasing mapping. Suppose further that the following conditions are satisfied:*

(i) $\mathcal{T}\in WA(\mathcal{X},\alpha_i)$;
(ii) *There exists $\xi_0\in\mathcal{X}$ such that $\alpha_i(\xi_0,\mathcal{T}\xi_0)\geq 1, i=\overline{1,6}$;*
(iii) *$\mathcal{T}$ is $d_{ml}-$ continuous.*

In addition, suppose that there exist $\tau_i > 0$, $i = \overline{1,6}$ and, for all $\xi, \zeta \in \mathcal{X}$ with $d_{ml}(\mathcal{T}\xi, \mathcal{T}\zeta) > 0$ and $\alpha_i(\xi,\zeta) \geq 1, i = \overline{1,6}$, the following inequalities hold true:

$$\alpha_1(\xi,\zeta)(\tau_1 + d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)) \leq \mathcal{M}(\xi,\zeta)$$

$$\alpha_2(\xi,\zeta)(\tau_2 + \exp(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta))) \leq \exp(\mathcal{M}(\xi,\zeta))$$

$$\alpha_3(\xi,\zeta)\left(\tau_3 - \frac{1}{d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)}\right) \leq -\frac{1}{\mathcal{M}(\xi,\zeta)}$$

$$\alpha_4(\xi,\zeta)\left(\tau_4 - \frac{1}{d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)} + d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)\right) \leq -\frac{1}{\mathcal{M}(\xi,\zeta)} + \mathcal{M}(\xi,\zeta)$$

$$\alpha_5(\xi,\zeta)\left(\tau_5 + \frac{1}{1-\exp(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta))}\right) \leq \frac{1}{1-\exp(\mathcal{M}(\xi,\zeta))}$$

$$\alpha_6(\xi,\zeta)\left(\tau_6 + \frac{1}{\exp(-d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)) - \exp(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta))}\right) \leq \frac{1}{\exp(-\mathcal{M}(\xi,\zeta)) - \exp(\mathcal{M}(\xi,\zeta))}$$

where $\mathcal{M}(\xi,\zeta)$ is one of the following sets:

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta), \frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}, \frac{[1+d_{ml}(\xi,\mathcal{T}\xi)]\,d_{ml}(\zeta,\mathcal{T}\zeta)}{1+d_{ml}(\xi,\zeta)}\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta), \frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), \frac{d_{ml}(\xi,\mathcal{T}\xi)+d_{ml}(\zeta,\mathcal{T}\zeta)}{2}, \frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta)\}$$

$$\mathcal{M}(\xi,\zeta) = \max\{d_{ml}(\xi,\zeta)\} = d_{ml}(\xi,\zeta).$$

Then, in each of these cases, $\mathcal{T}$ has a unique fixed point in $\mathcal{X}$.

Proof. If we put $\alpha_i(\xi,\zeta) = \alpha(\xi,\zeta)$, $i = \overline{1,6}$ and $\mathcal{F}(\iota) = \iota$, $\mathcal{F}(\iota) = \exp(\iota)$, $\mathcal{F}(\iota) = -\frac{1}{\iota}, \mathcal{F}(\iota) = -\frac{1}{\iota} + \iota, \mathcal{F}(\iota) = \frac{1}{1-\exp(\iota)}, \mathcal{F}(\iota) = \frac{1}{\exp(-\iota)-\exp(\iota)}$ in Theorem 2, respectively, then every of the functions $\iota \mapsto \mathcal{F}(\iota)$ is strictly increasing on $(0,+\infty)$, and the result follows according to Theorem 2. □

Remark 4. *Putting $\alpha_i(\xi,\zeta) = 1$ for all $\xi,\zeta \in \mathcal{X}, i = \overline{1,6}$ in the previous corollary, we get the following six new contractive conditions:*

$$\tau_1 + d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta) \leq \mathcal{M}(\xi,\zeta)$$

$$\tau_2 + \exp(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)) \leq \exp(\mathcal{M}(\xi,\zeta))$$

$$\tau_3 - \frac{1}{d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)} \leq -\frac{1}{\mathcal{M}(\xi,\zeta)}$$

$$\tau_4 - \frac{1}{d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)} + d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta) \leq -\frac{1}{\mathcal{M}(\xi,\zeta)} + \mathcal{M}(\xi,\zeta)$$

$$\tau_5 + \frac{1}{1-\exp(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta))} \leq \frac{1}{1-\exp(\mathcal{M}(\xi,\zeta))}$$

$$\tau_6 + \frac{1}{\exp(-d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)) - \exp(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta))} \leq \frac{1}{\exp(-\mathcal{M}(\xi,\zeta)) - \exp(\mathcal{M}(\xi,\zeta))}$$

where $\mathcal{M}(\xi,\zeta)$ *is one of the following sets:*

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta), \frac{d(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}, \frac{[1+d_{ml}(\xi,\mathcal{T}\xi)]\,d_{ml}(\zeta,\mathcal{T}\zeta)}{1+d_{ml}(\xi,\zeta)}\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta), \frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), \frac{d_{ml}(\xi,\mathcal{T}\xi)+d_{ml}(\zeta,\mathcal{T}\zeta)}{2}, \frac{d_{ml}(\xi,\mathcal{T}\zeta)+d_{ml}(\zeta,\mathcal{T}\xi)}{2}\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta)\right\}$$

$$\mathcal{M}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta)\right\} = d_{ml}(\xi,\zeta).$$

In every one of these cases, $\mathcal{T}$ has a unique fixed point in $\mathcal{X}$. The result can simply be obtained by putting $\alpha_i(\xi,\zeta) = 1, i = \overline{1,6}$ and $\mathcal{F}(\iota) = \iota, \mathcal{F}(\iota) = \exp(\iota), \mathcal{F}(\iota) = -\frac{1}{\iota}, \mathcal{F}(\iota) = -\frac{1}{\iota}+\iota$, $\mathcal{F}(\iota) = \frac{1}{1-\exp(\iota)}, \mathcal{F}(\iota) = \frac{1}{\exp(-\iota)-\exp(\iota)}$ in Theorem 2.

In [22], Ćirić introduced one of the most generalized contractive conditions (so-called quasicontraction) in the context of metric spaces as follows:

Definition 12. *The self-mapping* $\mathcal{T}:\mathcal{X}\to\mathcal{X}$ *on metric space* $(\mathcal{X},d)$ *is called quasicontraction (in the sense of Ćirić) if there exists* $\lambda\in[0,1)$ *such that, for all* $\xi,\zeta\in\mathcal{X}$ *holds true*

$$d(\mathcal{T}\xi,\mathcal{T}\zeta) \le \lambda\max\left\{d(\xi,\zeta), d(\xi,\mathcal{T}\xi), d(\zeta,\mathcal{T}\zeta), d(\xi,\mathcal{T}\zeta), d(\zeta,\mathcal{T}\xi)\right\}. \tag{19}$$

In [22], Ćirić proved the following result:

Theorem 3. *Each quasicontraction* $\mathcal{T}$ *on a complete metric space* $(\mathcal{X},d)$ *has a unique fixed point (say)* η*. Moreover, for all* $\xi\in\mathcal{X}$*, the sequence* $\{\mathcal{T}^n\xi\}_{n=0}^{+\infty}, \mathcal{T}^0\xi=\xi$ *converges to the fixed point* η *as* $n\to+\infty$*.*

Finally, we formulate the following notion and an open question:

Definition 13. *Let* $(\mathcal{X},d_{ml})$ *be a metric-like space and* $\alpha:\mathcal{X}^2\to[0,+\infty)$*. A mapping* $\mathcal{T}:\mathcal{X}\to\mathcal{X}$ *is said to be a triangular* $(\alpha,\mathcal{F})$*-contraction mapping of Ćirić type, if there exists* $\tau>0$ *such that, for all* $\xi,\zeta\in\mathcal{X}$ *with* $d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)>0$ *and* $\alpha(\xi,\zeta)\ge 1$ *holds true:*

$$\alpha(\xi,\zeta)\left(\tau+\mathcal{F}\left(d_{ml}(\mathcal{T}\xi,\mathcal{T}\zeta)\right)\right) \le \mathcal{F}\left(\mathcal{N}(\xi,\zeta)\right), \tag{20}$$

where

$$\mathcal{N}(\xi,\zeta) = \max\left\{d_{ml}(\xi,\zeta), d_{ml}(\xi,\mathcal{T}\xi), d_{ml}(\zeta,\mathcal{T}\zeta), d_{ml}(\xi,\mathcal{T}\zeta), d_{ml}(\zeta,\mathcal{T}\xi)\right\},$$

$\mathcal{F}:(0,+\infty)\to(-\infty,+\infty)$ *is strictly increasing function satisfying only* $(\mathcal{F}1)$*.*

An open question: *Prove or disprove the following claim: each triangular* $(\alpha,\mathcal{F})$*-contraction mapping* $\mathcal{T}:\mathcal{X}\to\mathcal{X}$ *of Ćirić type defined on* $0-d_{ml}-$ *complete metric-like space* $(\mathcal{X},d)$ *has a unique fixed point.*

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