



Article

Innovative Investment Models with Frequent Payments of Tax on Income and of Interest on Debt

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Abstract: New modern investment models are created to be as close as possible to real investment conditions. We consider long-term as well as arbitrary duration models with payments of interest on debt and of tax on income a few times per year (semi-annually, quarterly and monthly), which could be applied in real economic practice. Their verification will lead to the creation of a comprehensive system of adequate and correct assessment of the effectiveness of the company's investment program and its investment strategy. One of the most important elements of calculating the effectiveness of investment projects is the assessment of the discount rate, the calculation methods of which are generalized for the real conditions of the implementation of investment projects. We consider the effectiveness of the investment project from two points of view: the equity owners and the owners of equity and debt. NPV for each of these cases is calculated by two different methods: with the separation of credit and investment flows (and thus discounting the flows using two different rates) and without such separation (with discounting of both flows using the same rate, and WACC can be chosen as the rate). Numerical calculations, conducted for four investment models (without flow separation) show that: (1) in the case of considering the effectiveness of an investment project for owners of equity capital, the increase in the number of payments of tax on income and of interest on debt p leads to a decrease in NPV: this means that the effectiveness of an investment project decreases with p ; (2) in the case of considering the effectiveness of an investment project for owners of equity and debt capital, the increase in the number of payments of tax on income and of interest on debt p leads to an increase in NPV: this means that the effectiveness of an investment project increases with p . In the former case, companies should pay tax on profit and interest on debt once per year, while in the latter case, more frequent payments are profitable for the effectiveness of investment. Eight innovative investment models created in this paper can assist decision-makers in the optimal design, planning and control of company investments and the development of a company's investment strategy.

Keywords: innovative investment models; effectiveness of the investment project; frequent payments of interest on debt and of tax on income; the Modigliani-Miller theory; the Brusov-Filatova-Orekhova theory



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1. Introduction

Investments play a crucial role in economy and finance. Investments in tangible and intangible assets are a necessary condition for structural adjustment and economic growth. They provide the enhancement of existing basic funds and industries and the creation of new ones. The role of investment is increased many times at the current stage. In this respect, the role of the evaluation of the efficiency of investment projects, which allows for the realization of the most effective projects in the context of scarcity and limited investment

resources, increases. Since virtually most investment projects use debt financing, the study of the influence of capital structure and debt financing on the efficiency of investment projects and determining the optimal capital structure is especially important at the present time. This is why in spite of the fact that a lot of different types of investment models have been developed—stochastic, dynamics [1], investment banking valuation models [2], etc.—the main problem, which has been discussed during the last few decades, is the impact of debt financing on the efficiency of investment projects and on the investment decisions of companies.

Below we review some approaches to these problems.

1.1. The Literature Review

Lang, L.E.; Ofek, E.; and Stulz, R. [3] used so-called “Tobin’s q ratio” and considered companies with low Tobin’s q ratios as well as high ones. Note that the q ratio, or Tobin’s q ratio, equals the market value of a company divided by its assets’ replacement cost. The equilibrium takes place when market value equals replacement cost. The q ratio expresses the relationship between market valuation and intrinsic value. It is a means of estimating the fact of whether a given business or market is undervalued or overvalued.

The authors have shown that there is a negative correlation between future growth and leverage level at the company level and, for diversified companies, at the level of business segments. This negative correlation between growth and leverage level holds for companies with low Tobin’s q ratio, but not for high-q companies or companies in high-q industries. Therefore, for companies known to have good investment opportunities, leverage does not reduce growth, but it is negatively correlated to growth for companies whose growth opportunities are either not recognized by the capital markets or are not valuable enough to overcome the effects of their debt load.

Whited [4] studied the influence of debt financing on companies’ investment decisions with pharmaceutical firms in India for 11 years, from 1998 to 2009.

To study the impact of debt financing on the firms’ investment decisions, Whited used pooling regression as well as random and fixed effect models. Leverage level, retained earnings, Tobin’s q, sales, Return on Asset, cash flow and liquidity were considered as independent variables and investment as the dependent one. Whited considered three types of companies, depending on their size: small companies, medium companies and large companies. He showed that there is a significant positive correlation between leverage level and investment for large companies, while for medium companies a negative correlation between leverage level and investment took place.

Kang [5] studied the connection between leverage level and investment decisions. “Interdependent tax models” were used to try to explain the specifics of company leverage levels by analyzing the interdependency between financing decisions and investment. These models account for the so-called “investment effect”: the influence of investment on debt tax benefit and financial risk. One of the questions is how “investment effect” influences bond financing decisions and hence the leverage level. Different “Interdependent tax models” lead to different connections between investments and leverage levels.

Some authors mentioned a positive connection via the fact that the financial risk and hence the cost of bond financing decrease with an increase in investment at a given leverage level. A negative connection has been mentioned by DeAngelo and Masulis, 1980 [6] and Dotan and Ravid, 1985 [7]. The first authors concluded this since the tax benefits of debt compete with those of capital investment. The second authors refer to the fact that financial risk and thus the cost of bond financing will increase with investment increase.

The impact of investment increase on financial risk may depend on company-specific factors, like company-specific technology (Dammon and Senbet, 1988) [8]. An analysis of the impact of corporate and personal taxes on a firm’s optimal investment and financing decisions under uncertainty is provided in this paper. By endogenizing firms’ investment decisions, it extends the DeAngelo and Masulis capital structure model. The authors’ results indicate that the existing predictions about the relationship between investment-

related and debt-related tax shields must be modified in cases where investment is allowed to adjust optimally. The authors show that increases in investment-related tax shields due to changes in the corporate tax code are not necessarily associated with reductions in leverage level at the individual company level. Companies with higher investment-related tax shields (as cross-sectional analysis shows) need not have lower debt-related tax shields (normalized by expected earnings) unless all companies utilize the same production technology. Differences in production technologies across companies may thus explain why the empirical results of recent cross-sectional studies have not conformed to the predictions of DeAngelo and Masulis [6]. A model of company leverage level choice is formulated in this paper, in which corporate and differential personal taxes exist and supply-side adjustments by companies enter into the determination of equilibrium prices of debt and equity. The presence of corporate tax shield substitutes for debt such as depletion allowances, accounting depreciation and investment tax credits are shown to imply a market equilibrium in which each company has a unique interior optimum leverage level decision. The optimal leverage level model yields a number of interesting predictions regarding cross-sectional and time-series properties of firms' capital structures. Below we discuss some portfolio investment models as well as behavioral aspects of investors, which play an important role in investments.

Among those portfolio investment models is the well-known Black–Litterman model, which was created by Fischer Black and Robert Litterman in 1992 [9]. They developed the model to address the problems that institutional investors have encountered during the application of modern portfolio theory in practice. Starting with an asset allocation based on the equilibrium assumption, the model then modifies that allocation by accounting for the investors' opinions with respect to future asset performance.

Anthony Loviscek [10] has applied the Black–Litterman model of modern portfolio theory to well-known index mutual funds: one guided by the classic 60%/40% stock/bond allocation and one based on an all-equity allocation. The period under their study is from 2000 to 2020. Although statistical evidence supports that the efficacy of a precious metal allocation is elusive, the results suggest an average allocation of about 2% for “buy-and-hold” investors who seek one. He has shown that, from 2003 to 2010 and from 2016 to 3Q2020, the allocations were in the range of 5–10%. Other periods, however, register only a little more than 0%.

Nusret Cakici and Adam Zaremba [11], using data on 65,000 stocks from 23 countries, reconsidered the performance of the Fama–French factors in global markets. As their results showed, the value, profitability and investment factors are far less reliable than what is commonly thought. Their performance depends strongly on the geographical regions and periods examined. Moreover, most factor returns are driven by the smallest companies. Virtually no value or investment effects are present among the big companies representing most of the total market capitalization worldwide. These results cast doubt on the five-factor model's applicability in international markets, citing that the smallest companies are typically not invested in by major financial institutions.

A growing number of investors want to use firm sustainability information in their investment decision processes to avoid risk, satisfy their own asset preference, or find a new alpha-generating factor. Not too many users of environment, social and governance (ESG) data understand how ESG ratings change over time. Bahar Gidwani [12] used the CSRHub data set to show that ESG ratings regress strongly toward the mean. These ratings include both data from 640 sources and from most commercial ESG ratings firms. The observed regression persists during nine years within the ratings data, for a sample set of more than 8000 firms. Newly-rated firms show even more reversion than “seasoned” firms. Firms can only rarely maintain an especially high or low ESG rating. Investors and firm managers should understand that ESG ratings are likely to change toward the mean. This however does not mean that a good firm is getting worse or a bad one is getting better.

Keith C. Brown, W. V. Harlow and Hanjiang Zhang [13] have developed statistics (holdings-based) to estimate the volatility with time of investment style characteristics of

funds. They found that funds with lower levels of style volatility significantly outperform funds with higher levels of style volatility on a risk-adjusted basis. The authors have shown that style volatility has a distinct impact on fund performance in the future compared to expenses of funds or past risk-adjusted returns, with the level of indirect style volatility being the primary determinant of the overall effect. It was concluded that deciding to maintain a less volatile investment style is an important aspect of the portfolio management process.

Some behavioral aspects of investors have been considered by Marcos Escobar-Anel, Andreas Lichtenstern and Rudi Zagst [14], who introduced a strategy generalizing the CPPI (Constant Proportion Portfolio Insurance) approach. The target of this strategy is to guarantee the investment goal or floor during participation in the performance of the assets and limiting the downside risk of the portfolio at the same time. The authors show that the strategy accounts for the following behavioral aspects of investors: a risk-averse behavior for gains, distorted probabilities recognition and a risk-seeking behavior for losses. The developed strategy turns out to be optimal within the Cumulative Prospect Theory framework by Tversky and Kahneman [15].

1.2. Some Problems under the Evaluation of the Effectiveness of the Investment Projects

Some of the major problems under the evaluation of the effectiveness of investment projects are suggested as follows:

1. Which financial flows should be taken into account when calculating the parameters of efficiency of a project (NPV, IRR, etc.)?
2. How many discount rates should be used for discounting various cashflows?
3. How can these discount rates be accurately evaluated?

The first two problems are still under intensive discussion. Concerning the third issue, we need to note that, in the last decade, significant progress in the accurate determination of the cost of the equity and company weighted average cost, which just are the discount rates when evaluating the effectiveness of the project, has been achieved. The progress is mainly associated with the studies by Brusov, Filatova and Orekhova (BFO theory) [16–18], in which a general theory of capital cost of the company and its capital structure was established, and the dependence of capital cost on leverage level and on the age of a company was found for the companies of arbitrary age. The main difference between their theory and Modigliani-Miller theory is that the former one removes the assumption of perpetuity for the companies under discussion, which leads to a significantly different new theory from the theory established by the Nobel laureates Modigliani and Miller [19–21].

In modern conditions, the requirements for improving the quality of assessing the effectiveness of investments have increased. The modern investment models, which have been well-tested in real economic situations, have been developed by Brusov, Filatova and Orekhova [16,18]. They have created long-term as well as arbitrary duration models and have considered the effectiveness of the investment project from two points of view: from the equity holders and from the owners of equity and debt. NPV in each of these cases could be calculated by two different methods: with the division of credit and investment flows and using two different discounting rates, and without such a division and using a general discounting rate (for which WACC can, obviously, be chosen).

Applying their modern investment models on the evaluation of the dependence of the effectiveness of investments on debt financing of one telecommunication company in 2010–2012 from the point of view of optimal structure of investment, the authors showed that in 2012, the company lost 675 million USD on average, because the investment structure had been far from the optimal one. The ability to calculate the correct optimal capital structure is one important feature implied by the Brusov, Filatova and Orekhova modern investment models [16–18].

1.3. The Discount Rates

As we mentioned above, one of the most important elements of calculating the effectiveness of investment projects is the assessment of the discount rate. In the case of

long-term investment models without the division of credit and investment flows, the discount rate WACC has been calculated using the Modigliani-Miller formula [19–21]

$$WACC = k_0 \cdot (1 - w_d t) \quad (1)$$

while in the case of arbitrary duration models, the Brusov–Filatova–Orekhova formula for WACC [9–11]

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{1 - (1 + k_0)^{-n}}{k_0 \cdot \left(1 - w_d t \left[1 - (1 + k_d)^{-n}\right]\right)} \quad (2)$$

has been used.

Here and below, WACC is the weighted average cost of capital; k_0 is the equity cost at zero leverage ($L = 0$); k_d is the debt cost; w_d is the debt share; t is the tax on profit; n is the project duration; k_e is the equity cost; and L is the leverage level.

In the case of long-term investment models with the division of credit and investment flows, the discount rate for discounting the investment flows (equity cost k_e) has been calculated using the Modigliani-Miller formula [19–21]

$$k_e = k_0 + L \cdot (k_0 - k_d)(1 - t) \quad (3)$$

while in the case of arbitrary duration models, equity cost k_e has been calculated from the formula

$$WACC = k_e w_e + k_d w_d (1 - t) \quad (4)$$

using the Brusov–Filatova–Orekhova value for WACC [16–18].

The calculation methods of the discount rates (WACC, equity cost k_e) has been generalized in [22] for the real conditions of the implementation of investment projects: for arbitrary frequency of payment of tax on profit.

In this paper, new modern investment models, both long-term and arbitrary duration, will be created, as close as possible to real investment conditions. They will account the payments of interest on debt and of tax on income a few times per year (semi-annually, quarterly, monthly), which are applied in real economic practice. Their verification will lead to the creation of a comprehensive system of adequate and correct assessment of the effectiveness of the company's investment program and its investment strategy.

1.4. The Structure of the Paper

The structure of the paper is as follows:

1. In Section 1 above, we presented:
 - 1.1. The literature review.
 - 1.2. Some problems under the evaluation of the effectiveness of the investment projects.
 - 1.3. Discount rates.
2. In Section 2, we consider the effectiveness of the investment project from the perspective of the equity holders only.

In Section 2.1, we consider a case with flow separation.
In Section 2.2, we consider a case without flow separation.
3. In Section 3, we consider the effectiveness of the investment project from the perspective of the owners of equity and debt.

In Section 3.1, we consider a case with flow separation.
In Section 3.2, we consider a case without flow separation.
4. In Section 4, the problem of calculation of discount rates is discussed and expressions for their modified values are obtained.

5. In Section 5, we study numerically with the use of Microsoft Excel the effectiveness of the four models, created by us in this paper. We consider long-term projects as well as projects of arbitrary duration from two perspectives: the owners of equity and debt and the equity holders only without the division of credit and investment flows.
6. In Conclusions, we discuss obtained results and their impact on the correctness of valuation of efficiency of investment projects.

2. The Effectiveness of the Investment Project from the Perspective of the Equity Holders Only

We will consider the effectiveness of the investment project from two points of view: from the equity holders and from the owners of equity and debt. NPV in each of these cases could be calculated by two different methods: with the division of credit and investment flows and using two different discounting rates, and without such a division and using a general discounting rate (for which WACC can, obviously, be chosen).

The following designations are used below:

The equity value S , the investment value I , the net operating income NOI, the leverage level L , the profitability of investments β , the tax on profit t , the project duration n , the equity cost k_0 , the debt cost k_d and the number of payments of interest on debt p_1 and of tax on income p_2 ; D is the debt value.

In the first case (from the perspective of the equity holders), at the initial moment in time, $T = 0$ investments are equal to $-S$ and the flow of capital, CF, for the period is equal to:

$$CF = (NOI - k_d D)(1 - t). \quad (5)$$

In addition to the tax shields, $k_d D t$ includes a payment of interest on a loan, $-k_d D$.

We suppose below that interests on the loan are paid in equal shares of $k_d D$ during all periods and principal repayment is made at the end of the project.

In the second case (from the perspective of the equity holders), the negative flows (the interest and duty paid by owners of equity) are returned to the project and they are exactly equal to the positive flows obtained by owners of debt capital. Thus, in this case the only effect of debt financing is the effect of the tax shield, generated from the tax relief: interest on the loan is entirely included in the cost and thus reduces the tax base. After-tax flow of capital, CF, for each period is equal to

$$CF = NOI(1 - t) + k_d D t \quad (6)$$

and the value of investments at the initial moment in time $T = 0$ is equal to $-I = -S - D$.

Below two different ways of discounting will be considered:

- (1) If operating and financial flows are not separated, both flows are discounted by the general rate. In this case, the weighted average cost of capital (WACC) can be selected as a discounting rate. For long-term projects, we will use the Modigliani-Miller formula for WACC [19–21], modified by us for the case of payments of interest on debt and of tax on income a few times per year (semi-annually, quarterly, monthly) and for projects of finite (arbitrary) duration we will use the Brusov–Filatova–Orekhova formula for WACC [17,18], modified by us for the case of payments of interest on debt and of tax on income a few times per year (semi-annually, quarterly, monthly).
- (2) Operating and financial flows are separated and are discounted at different rates: the operating flow at the rate which is equal to the equity cost k_e , depending on leverage and on the number of payments of interest on debt and of tax on income, and credit flow at the rate which is equal to the debt cost k_d .

Note that loan capital is the least risky because the credit (including the interest on credit) is paid after taxes in the first place. Therefore, the cost of credit will always be less than the equity cost, whether for ordinary or for preference shares $k_e > k_d$; $k_p > k_d$. Here k_e ; k_p is the equity cost of ordinary or of preference shares consequently.

2.1. With Flow Separation

In this case, the expression for NPV (net present value) per period has a view

$$NPV = NOI(1 - t) - k_d D(1 - t) = NOI(1 - t) + k_d Dt - k_d D. \tag{7}$$

Here, the first term is the value of operating income from the investment project after tax deduction, the second term is the value of the tax shield, the third term is the value of interest on debt for one period.

We will need the following auxiliary formulas for summing the reduced values of financial flows (7) when calculating NPV:

For annual payments of interest on debt and of tax on income:

$$\sum_{i=1}^n \frac{1}{(1 + k_d)^i} = \frac{1}{1 + k_d} \cdot \frac{1 - (1 + k_d)^{-n}}{1 - \frac{1}{1 + k_d}} = \frac{1}{k_d} \cdot (1 - (1 + k_d)^{-n}) \tag{8}$$

For more frequent payments (p times per period) of interest on debt and of tax on income (semi-annually, quarterly, monthly):

$$\sum_{i=1}^{np} \frac{R}{p(1 + k_d)^{\frac{i}{p}}} = \frac{R}{p(1 + k_d)^{\frac{1}{p}}} \cdot \frac{1 - (1 + k_d)^{-n}}{1 - \frac{1}{(1 + k_d)^{\frac{1}{p}}}} = \frac{R}{p} \cdot \frac{(1 - (1 + k_d)^{-n})}{(1 + k_d)^{\frac{1}{p}} - 1} \tag{9}$$

Similar formulas are obtained using the cost of equity k_e and WACC as the discount rates.

Summing up the given values of financial flows for each period (7), we obtain for NPV of n -years project in the case of separated flows:

$$NPV = -S + \sum_{i=1}^n \frac{NOI(1 - t)}{(1 + k_e)^i} + \sum_{i=1}^n \frac{k_d Dt}{(1 + k_d)^i} - \sum_{i=1}^n \frac{k_d D}{(1 + k_d)^i} - \frac{D}{(1 + k_d)^n} \tag{10}$$

Here, the second term is the reduced value of operating income from the investment project, the second term is the reduced value of the tax shield, the third term is the reduced value of interest paid annually (at the end of the year), and the fourth term is the reduced value of the debt paid at the end of the project.

After summing, we have the following expression for NPV:

$$NPV = -S + \frac{NOI(1 - t)}{k_e} \cdot \left(1 - \left(\frac{1}{1 + k_e}\right)^n\right) + Dt(1 - (1 + k_d)^{-n}) - D(1 - (1 + k_d)^{-n}) - \frac{D}{(1 + k_d)^n} \tag{11}$$

In the case of more frequent (p -times per year) payment of income taxes (p_1) and frequent payments of interest on debt (p_2) we have:

$$NPV = -S + \sum_{i=1}^n \frac{NOI(1 - t)}{(1 + k_e)^i} + \sum_{i=1}^{np} \frac{k_d Dt}{p_1(1 + k_d)^{\frac{i}{p_1}}} - \sum_{i=1}^{np} \frac{k_d D}{p_2(1 + k_d)^{\frac{i}{p_2}}} - \frac{D}{(1 + k_d)^n}; \tag{12}$$

After summing, we have the following expression for NPV:

$$NPV = -S + \frac{NOI(1 - t)}{k_e} \cdot \left(1 - \left(\frac{1}{1 + k_e}\right)^n\right) + \frac{k_d Dt(1 - (1 + k_d)^{-n})}{p_1 \left((1 + k_d)^{\frac{1}{p_1}} - 1\right)} - \frac{k_d D(1 - (1 + k_d)^{-n})}{p_2 \left((1 + k_d)^{\frac{1}{p_2}} - 1\right)} - \frac{D}{(1 + k_d)^n} \tag{13}$$

Long-term investment projects

To obtain the expression for NPV of long-term investment projects, one should find the limit of (13) $n \rightarrow \infty$.

$$NPV = -S + \frac{NOI(1-t)}{k_e} + \frac{k_dDt}{p_1 \left((1+k_d)^{\frac{1}{p_1}} - 1 \right)} - \frac{k_dD}{p_2 \left((1+k_d)^{\frac{1}{p_2}} - 1 \right)} \tag{14}$$

2.2. Without Flow Separation

In the case of no separation between operating and financial flows, both flows are discounted by the general rate WACC. The credit reimbursable at the end of the project can be discounted either at the same rate WACC or at the debt cost rate k_d . Below, a uniform rate for WACC has been used.

Summing up the given values of financial flows for each period (3), we obtain for NPV of n -years project in the case without separated flows:

$$NPV = -S + \sum_{i=1}^n \frac{NOI(1-t)}{(1+WACC)^i} + \sum_{i=1}^n \frac{k_dDt}{(1+WACC)^i} - \sum_{i=1}^n \frac{k_dD}{(1+WACC)^i} - \frac{D}{(1+WACC)^n} \tag{15}$$

After summing, we have the following expression for NPV:

$$NPV = -S + \frac{NOI(1-t)}{WACC} \cdot \left(1 - \left(\frac{1}{1+WACC} \right)^n \right) + \frac{k_dDt(1-(1+WACC)^{-n})}{WACC} - \frac{k_dD(1-(1+WACC)^{-n})}{WACC} - \frac{D}{(1+WACC)^n} \tag{16}$$

In the case of more frequent (p -times per year) payment of income taxes and frequent payments of interest on the debt we have:

$$NPV = -S + \sum_{i=1}^n \frac{NOI(1-t)}{(1+WACC)^i} + \sum_{i=1}^{np} \frac{k_dDt}{p_1(1+WACC)^{\frac{i}{p_1}}} - \sum_{i=1}^{np} \frac{k_dD}{p_2(1+WACC)^{\frac{i}{p_2}}} - \frac{D}{(1+WACC)^n} \tag{17}$$

After summing, we have the following expression for NPV:

$$NPV = -S + \frac{NOI(1-t)}{WACC} \cdot \left(1 - \left(\frac{1}{1+WACC} \right)^n \right) + \frac{k_dDt(1-(1+WACC)^{-n})}{p_1 \left((1+WACC)^{\frac{1}{p_1}} - 1 \right)} - \frac{k_dD(1-(1+WACC)^{-n})}{p_2 \left((1+WACC)^{\frac{1}{p_2}} - 1 \right)} - \frac{D}{(1+WACC)^n} \tag{18}$$

Long-term investment projects

To obtain the expression for NPV of long-term investment projects, one should find the limit of (18) $n \rightarrow \infty$.

$$NPV = -S + \frac{NOI(1-t)}{WACC} + \frac{k_dDt}{p_1 \left((1+WACC)^{\frac{1}{p_1}} - 1 \right)} - \frac{k_dD}{p_2 \left((1+WACC)^{\frac{1}{p_2}} - 1 \right)} \tag{19}$$

3. The Effectiveness of the Investment Project from the Perspective of the Owners of Equity and Debt

3.1. With Flow Separation

In this case, operating and financial flows are separated and are discounted using different rates: the operating flow at the rate equal to the equity cost k_e , depending on leverage, and credit flow at the rate equal to the debt cost k_d , which remain constant until fairly large values of leverage and start to grow only at high values of leverage L , when there is a danger of bankruptcy.

After-tax flow of capital for each period in this case is equal to (6):

$$CF = NOI(1 - t) + k_dDt \tag{20}$$

After summing over n-periods, one gets the following expression for NPV:

$$NPV = -I + \sum_{i=1}^n \frac{NOI(1 - t)}{(1 + k_e)^i} + \sum_{i=1}^n \frac{k_dDt}{(1 + k_d)^i} = -I + \frac{NOI(1 - t)}{k_e} (1 - (1 + k_e)^{-n}) + Dt (1 - (1 + k_d)^{-n}) \tag{21}$$

In the case of a few (p) payments of tax on profit per period, the expression for NPV will be modified:

$$NPV = -I + \sum_{i=1}^n \frac{NOI(1 - t)}{(1 + k_e)^i} + \sum_{i=1}^{np} \frac{k_dDt}{p(1 + k_d)^{\frac{i}{p}}} \tag{22}$$

After summing over n-periods, one gets the following modified expression for NPV:

$$NPV = -I + \frac{NOI(1 - t) [1 - (1 + k_e)^{-n}]}{k_e} + \frac{k_dDt [1 - (1 + k_d)^{-n}]}{p [(1 + k_d)^{\frac{1}{p}} - 1]} \tag{23}$$

To obtain the expression for NPV of long-term investment projects, one should find the limit of (23) $n \rightarrow \infty$.

$$NPV = -I + \frac{NOI(1 - t)}{k_e} + \frac{k_dDt}{p [(1 + k_d)^{\frac{1}{p}} - 1]} \tag{24}$$

3.2. Without Flow Separation

In the case of no separation between operating and financial flows, both flows are discounted by the general rate WACC.

After summing the expression (6) over n-periods, one gets the following expression for NPV:

$$NPV = -I + \sum_{i=1}^n \frac{NOI(1 - t)}{(1 + WACC)^i} + \sum_{i=1}^n \frac{k_dDt}{(1 + WACC)^i} \tag{25}$$

$$NPV = -I + \frac{NOI(1 - t)}{WACC} (1 - (1 + WACC)^{-n}) + \frac{k_dDt}{WACC} (1 - (1 + WACC)^{-n}) \tag{26}$$

In the case of a few (p) payments of tax on profit per period, the expression for NPV will be modified:

$$NPV = -I + \sum_{i=1}^n \frac{NOI(1 - t)}{(1 + WACC)^i} + \sum_{i=1}^{np} \frac{k_dDt}{p(1 + WACC)^{\frac{i}{p}}} \tag{27}$$

After summing over n-periods, one gets the following modified expression for NPV:

$$NPV = -I + \frac{NOI(1 - t)}{WACC} \cdot \left(1 - \left(\frac{1}{1 + WACC}\right)^n\right) + \frac{k_dDt (1 - (1 + WACC)^{-n})}{p \left((1 + WACC)^{\frac{1}{p}} - 1\right)} \tag{28}$$

To obtain the expression for NPV of long-term investment projects, one should find the limit of (28) $n \rightarrow \infty$.

$$NPV = -I + \frac{NOI(1 - t)}{WACC} + \frac{k_dDt}{p \left((1 + WACC)^{\frac{1}{p}} - 1\right)} \tag{29}$$

4. Discount Rates

In the case without the division of credit and investment flows (both flows are discounted using the same rate, for which WACC can obviously be chosen), WACC is calculated by the following formulas:

For arbitrary project duration [15]:

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{1 - (1 + k_0)^{-n}}{k_0 \cdot \left(1 - \frac{k_d w_d t}{p} \frac{[1 - (1 + k_d)^{-n}]}{(1 + k_d)^{\frac{1}{p}} - 1}\right)} \tag{30}$$

For long-term projects [15]:

$$WACC = k_0 \cdot \left(1 - \frac{k_d w_d t}{p} \frac{1}{(1 + k_d)^{\frac{1}{p}} - 1}\right) \tag{31}$$

In the case of the division of credit and investment flows (and thus discounting of the payments using two different rates, equity cost k_e and debt cost k_d), k_e should be found from the equation

$$WACC = k_e w_e + k_d w_d (1 - t) \tag{32}$$

where we substitute WACC from the formula (30) for a project with arbitrary duration and from the formula (31) for a long-term project.

Note that formulas (30) and (31) are quite different from the original formulas by Brusov–Filatova–Orekhova [17,18] and Modigliani–Miller [19–21], where payments of interest on debt and of tax on income are made once per year and are turned into them if we put $p = 1$:

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{1 - (1 + k_0)^{-n}}{k_0 \cdot \left(1 - w_d t [1 - (1 + k_d)^{-n}]\right)} \tag{33}$$

$$WACC = k_0 \cdot (1 - w_d t) \tag{34}$$

5. Results and Discussions

In this section, we study numerically with the use of Microsoft Excel the effectiveness of the four models created above by us. We consider long-term projects as well as projects of arbitrary duration from two perspectives: the owners of equity and debt and the equity holders only. We will study the case without flow separation. In this case, operating and financial flows are not separated and both are discounted, using the general rate, for which we select the weighted average cost of capital, WACC. Let us start from the study numerically on the dependence of the discount rates (weighted average cost of capital, WACC) on leverage level L at different frequencies of payment of tax on profit p .

5.1. Numerical Calculation of the Discount Rates

5.1.1. The Long-Term Investment Projects

For long-term investment projects (the Modigliani–Miller limit), the discount rate (WACC) in the case of arbitrary frequency of payment of tax on profit p is described by formula (31) [22]:

$$WACC = k_0 \cdot \left(1 - \frac{k_d w_d t}{p} \frac{1}{(1 + k_d)^{\frac{1}{p}} - 1}\right)$$

For WACC calculation, we will use the following parameters: $k_0 = 0.22$; $k_d = 0.14$; $t = 20\%$; $L = 0; 1; 2 \dots ; 10$.

Dependence of WACC on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ is shown in Table 1 and Figure 1.

Table 1. Dependence of WACC on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$.

L	WACC		
	$p = 1$	$p = 6$	$p = 12$
0	0.2200	0.2200	0.2200
1	0.1980	0.1967	0.1966
2	0.1907	0.1890	0.1888
3	0.1870	0.1851	0.1849
4	0.1848	0.1828	0.1826
5	0.1833	0.1812	0.1810
6	0.1823	0.1801	0.1799
7	0.1815	0.1793	0.1791
8	0.1809	0.1787	0.1784
9	0.1804	0.1781	0.1779
10	0.1800	0.1777	0.1775

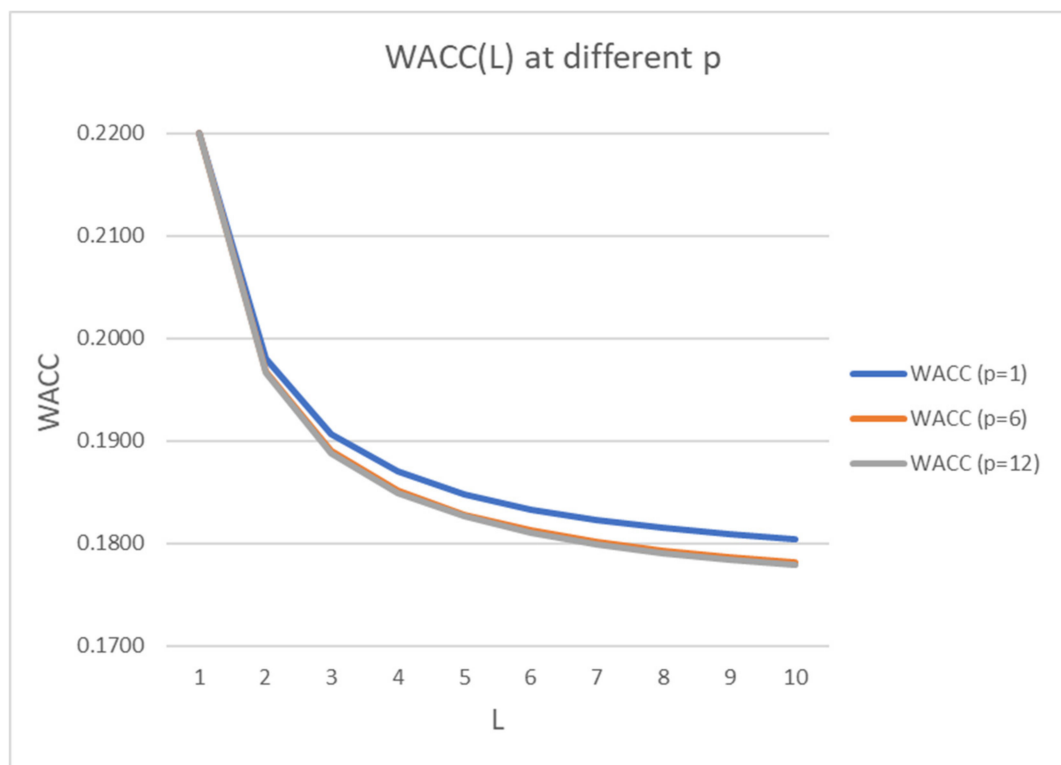


Figure 1. Dependence of the weighted average cost of capital, WACC, on leverage level L at different frequencies of payments of tax on profit, $p = 1; 6; 12$.

5.1.2. The Arbitrary Duration Investment Projects

For arbitrary duration investment projects, the discount rate (WACC) in the case of arbitrary frequency of payment of tax on profit p is described by formula (30) (modified BFO formula [22]).

For WACC calculation, we will use the following parameters: $k_0 = 0.22$; $k_d = 0.14$; $t = 20\%$; $L = 0; 1; 2 \dots ; 10$; $p = 1; 6; 12$; $n = 3$

$$\frac{1 - (1 + WACC)^{-n}}{WACC} = \frac{1 - (1 + k_0)^{-n}}{k_0 \cdot \left(1 - \frac{k_d w_d t [1 - (1 + k_d)^{-n}]}{p (1 + k_d)^{\frac{1}{p}} - 1} \right)}$$

Dependence of WACC on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year company is shown in Table 2 and Figures 2 and 3.

Table 2. Dependence of WACC on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year company.

L	WACC		
	$p = 1$	$p = 6$	$p = 12$
0	0.2200	0.2200	0.2200
1	0.1987	0.1974	0.1973
2	0.1915	0.1899	0.1897
3	0.1879	0.1861	0.1859
4	0.1858	0.1838	0.1836
5	0.1843	0.1823	0.1821
6	0.1833	0.1812	0.1810
7	0.1825	0.1804	0.1802
8	0.1819	0.1798	0.1795
9	0.1815	0.1793	0.1790
10	0.1811	0.1788	0.1786

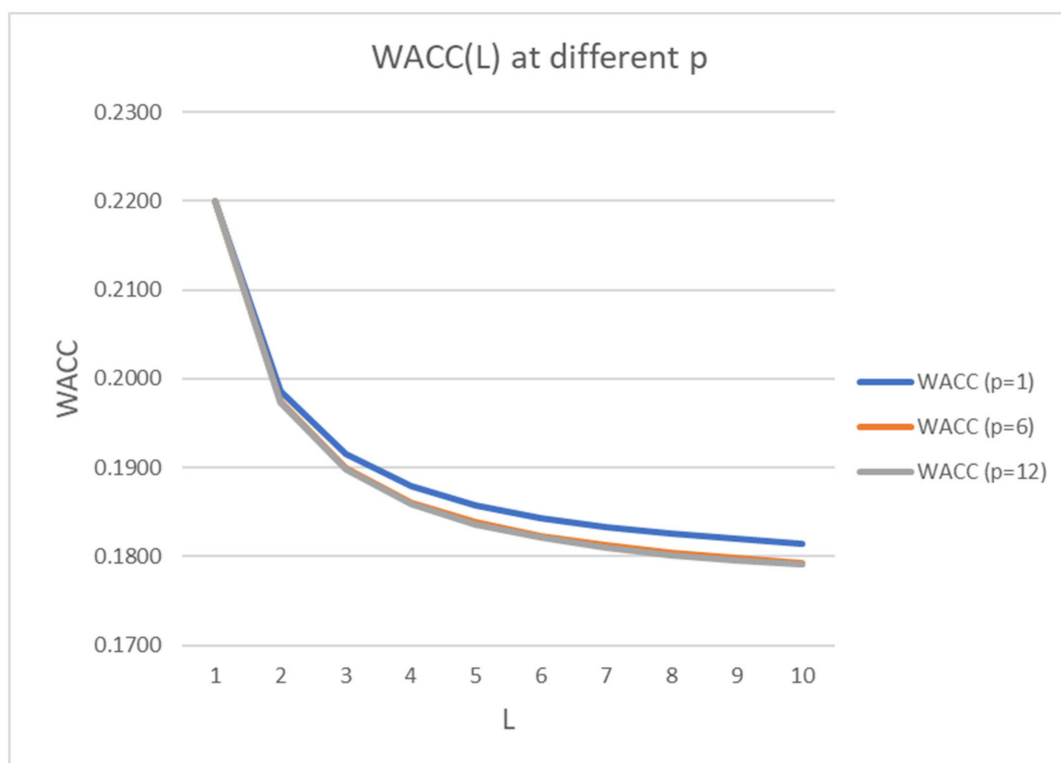


Figure 2. Dependence of WACC on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project.

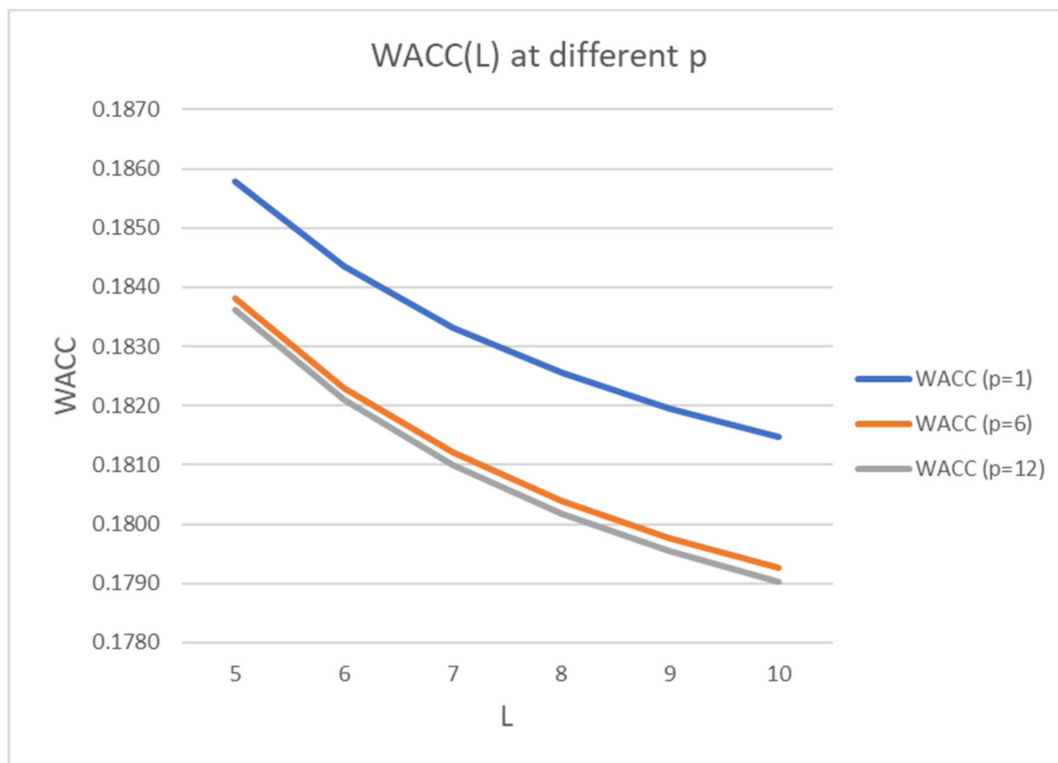


Figure 3. Dependence of WACC on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project (larger scale).

Note that the dependences of WACC on leverage level L obtained above will be used below under a study of the effectiveness of long-term as well arbitrary duration investment projects, from the perspective of the owners of equity capital and of the owners of equity and debt.

5.2. The Effectiveness of the Long-Term Investment Project from the Perspective of the Owners of Equity Capital

For long-term investment projects, the discount rate (WACC) in the case of arbitrary frequency of payment of tax on profit p is described by formula (19) and for NPV one has:

$$NPV = -S + \frac{NOI(1-t)}{WACC} + \frac{k_d D t}{p_1 \left((1+WACC)^{\frac{1}{p_1}} - 1 \right)} - \frac{k_d D}{p_2 \left((1+WACC)^{\frac{1}{p_2}} - 1 \right)}$$

For NPV calculation, we will use the following parameters: $k_0 = 0.22$; $k_d = 0.14$; $t = 20\%$; $L = 0; 1; 2 \dots ; 10$; $p_1 = p_2 = p = 1; 6; 12$; $S = 1000$; $D = LS$; $NOI = 800$.

It is seen from Figures 4 and 5 that in the case of considering the effectiveness of long-term investment projects for owners of equity capital, NPV will vary with the change of p , but with not much variation (it is seen from Table 3 and from Figure 5, but not from Figure 4).

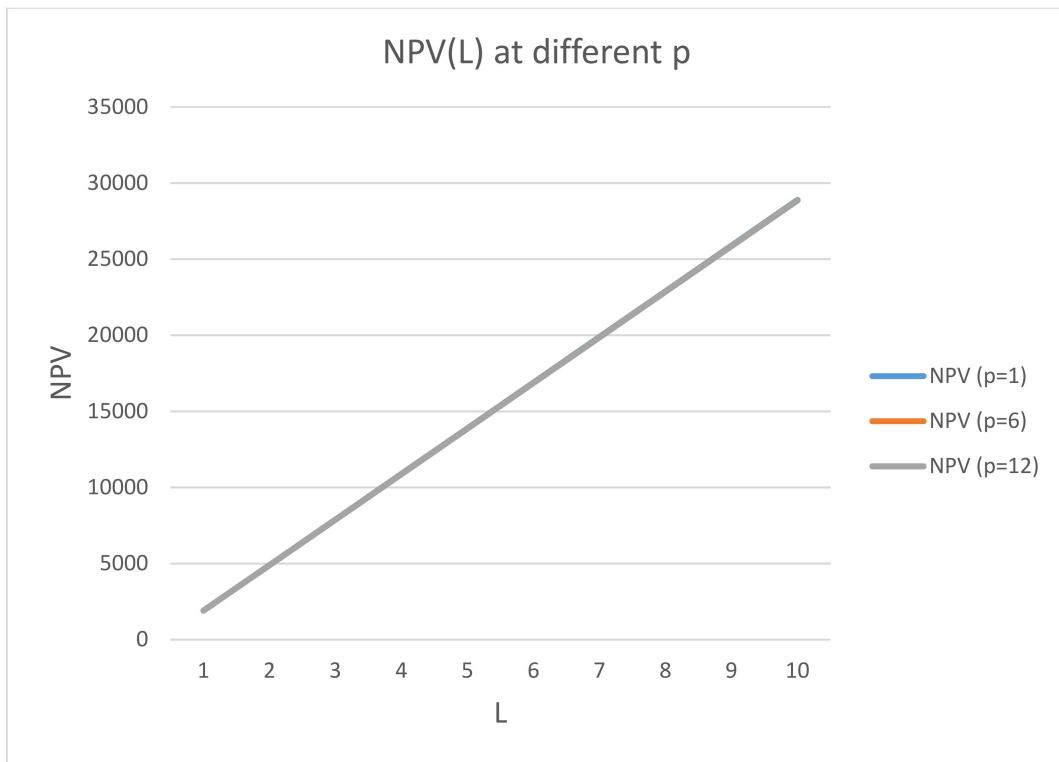


Figure 4. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a long-term project.

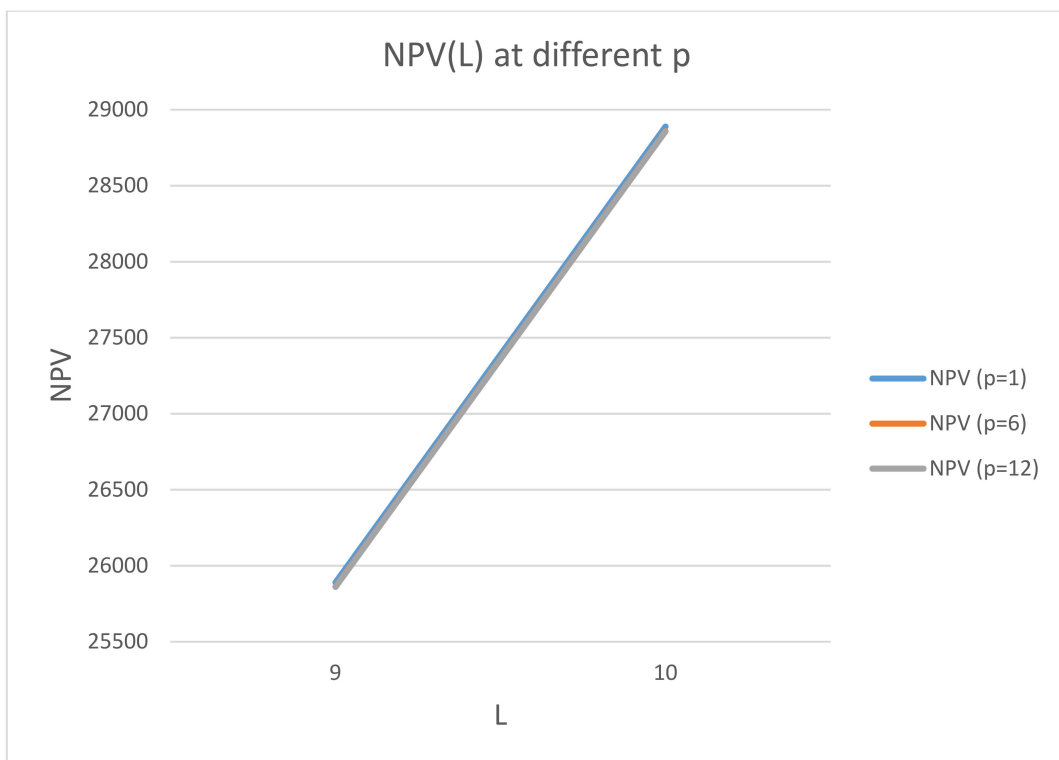


Figure 5. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a long-term project (larger scale).

Table 3. Dependence of WACC and NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$.

L	NPV		
	$p = 1$	$p = 6$	$p = 12$
0	1909	1909	1909
1	4899	4891	4891
2	7895	7883	7882
3	10,893	10,878	10,877
4	13,892	13,874	13,873
5	16,891	16,871	16,869
6	19,890	19,868	19,866
7	22,890	22,865	22,863
8	25,889	25,863	25,860
9	28,889	28,860	28,858
10	31,889	31,858	31,855

5.3. The Effectiveness of the Long-Term Investment Project from the Perspective of the Owners of Equity and Debt

For long-term investment projects, NPV in the case of arbitrary frequency of payment of tax on profit p is described by formula (29):

$$NPV = -I + \frac{NOI(1 - t)}{WACC} + \frac{k_d D t}{p \left((1 + WACC)^{\frac{1}{p}} - 1 \right)}$$

For NPV calculation, we will use the following parameters: $k_0 = 0.22$; $k_d = 0.14$; $t = 20\%$; $L = 0; 1; 2 \dots ; 10$; $p_1 = p_2 = p = 1; 6; 12$; $S = 1000$; $D = LS$; $NOI = 800$.

Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a long-term project is shown in Table 4 and Figures 6 and 7.

Table 4. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$.

L	NPV		
	$p = 1$	$p = 6$	$p = 12$
0	1909	1909	1909
1	4606	4659	4665
2	7364	7478	7489
3	10,139	10,316	10,334
4	12,922	13,163	13,188
5	15,709	16,015	16,047
6	18,498	18,870	18,908
7	21,289	21,726	21,771
8	24,081	24,583	24,635
9	26,874	27,441	27,500
10	29,667	30,300	30,365

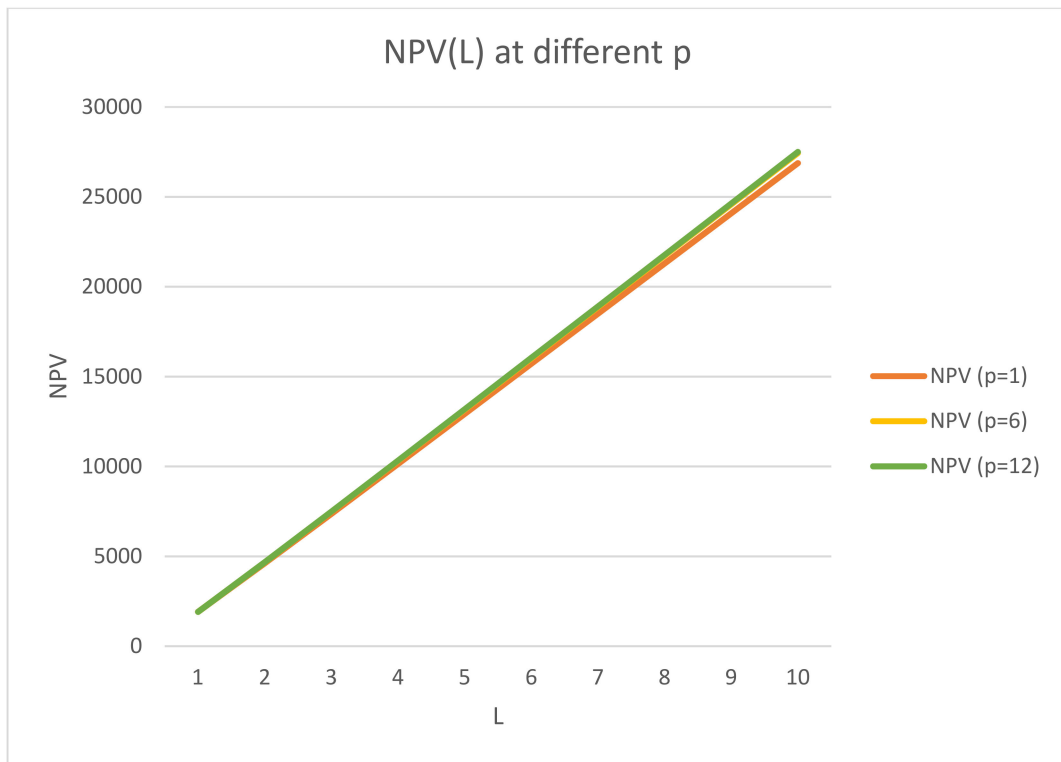


Figure 6. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a long-term project.

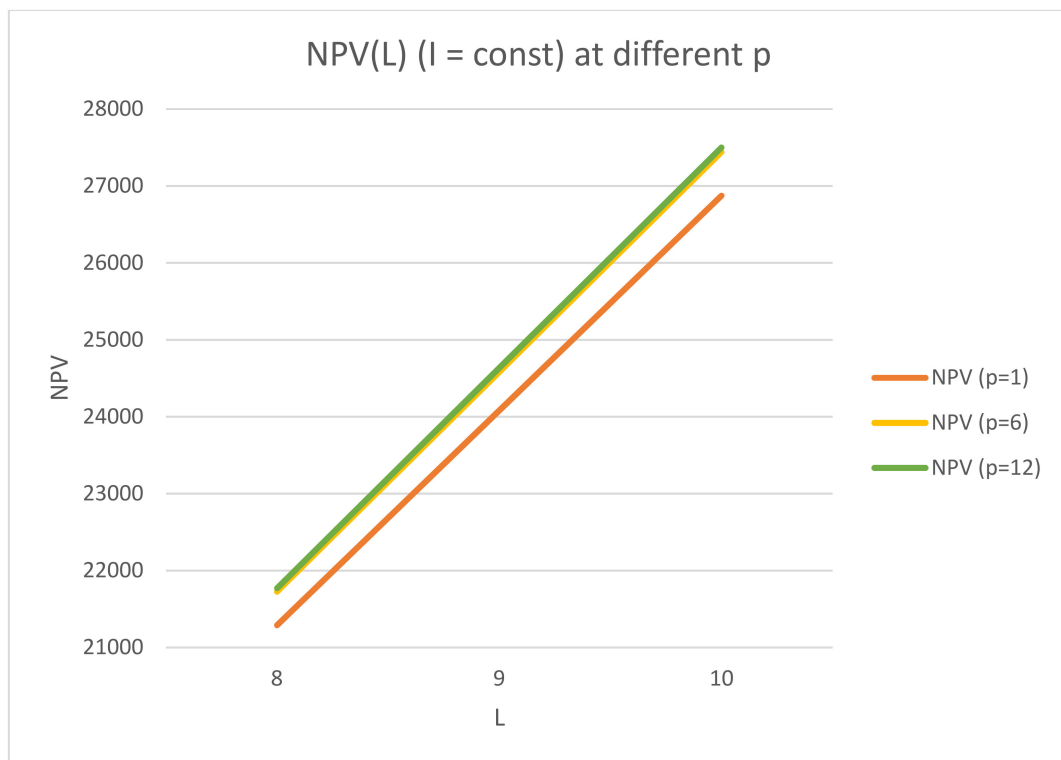


Figure 7. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a long-term project (larger scale).

5.4. The Effectiveness of the Arbitrary Duration Investment Projects from the Perspective of the Owners of Equity Capital

For the arbitrary duration investment projects, NPV in the case of arbitrary frequency of payment of tax on profit p is described by formula (18):

$$NPV = -S + \frac{NOI(1-t)}{WACC} \cdot \left(1 - \left(\frac{1}{1+WACC}\right)^n\right) + \frac{k_d D t (1-(1+WACC)^{-n})}{p_1 \left((1+WACC)^{\frac{1}{p_1}} - 1\right)} - \frac{k_d D (1-(1+WACC)^{-n})}{p_2 \left((1+WACC)^{\frac{1}{p_2}} - 1\right)} - \frac{D}{(1+WACC)^n}$$

For NPV calculation, we will use the following parameters: $k_0 = 0.22$; $k_d = 0.14$; $t = 20\%$; $L = 0; 1; 2 \dots ; 10$; $p_1 = p_2 = p = 1; 6; 12$; $S = 1000$; $D = LS$; $NOI = 800$; $n = 3$.

Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project is shown in Table 5 and Figures 8 and 9.

Table 5. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project.

L	NPV		
	$p = 1$	$p = 6$	$p = 12$
0	307	307	307
1	885	869	867
2	1438	1406	1403
3	1985	1937	1932
4	2530	2464	2458
5	3072	2991	2983
6	3615	3516	3506
7	4156	4042	4030
8	4698	4566	4553
9	5239	5091	5076
10	5780	5615	5599

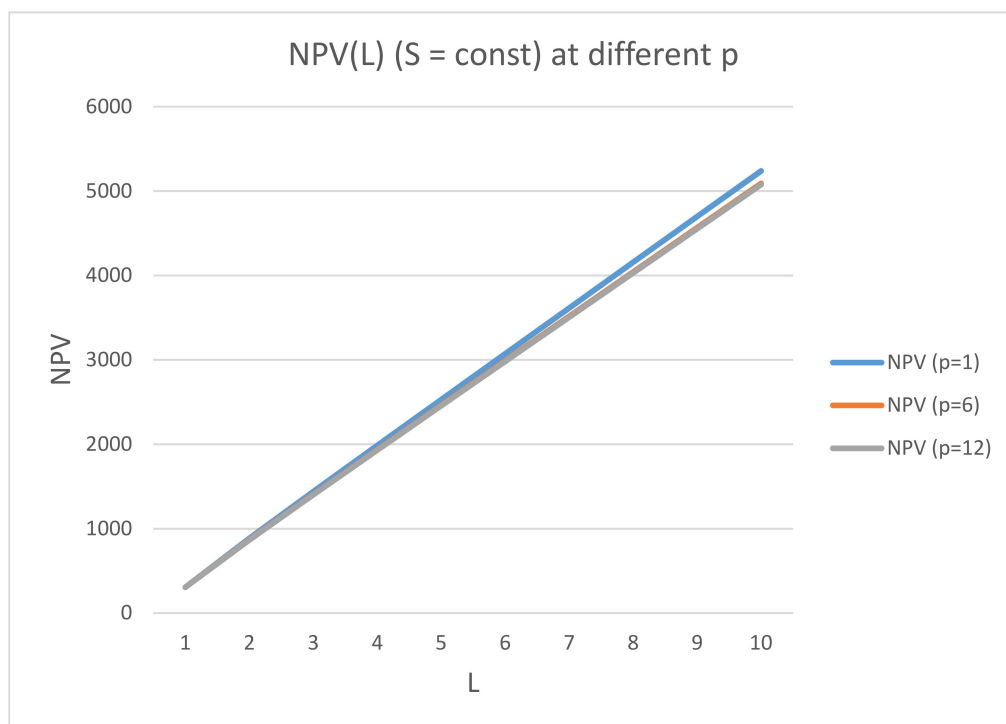


Figure 8. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project.

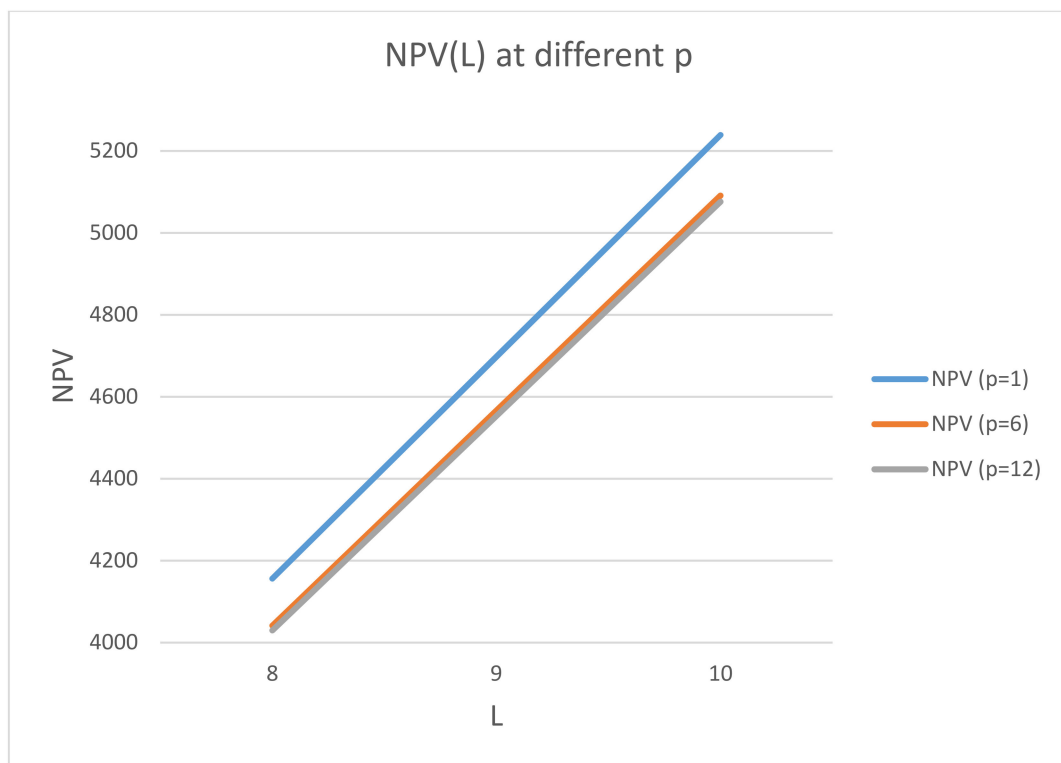


Figure 9. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project (larger scale).

5.5. The Effectiveness of the Arbitrary Duration Investment Project from the Perspective of the Owners of Equity and Debt

For the arbitrary duration investment projects, NPV in the case of arbitrary frequency of payment of tax on profit p is described by formula (28):

$$NPV = -I + \frac{NOI(1-t)}{WACC} \cdot \left(1 - \left(\frac{1}{1+WACC}\right)^n\right) + \frac{k_dDt(1 - (1+WACC)^{-n})}{p \left((1+WACC)^{\frac{1}{p}} - 1\right)}$$

For NPV calculation, we will use the following parameters: $k_0 = 0.22$; $k_d = 0.14$; $t = 20\%$; $L = 0; 1; 2 \dots ; 10$; $p_1 = p_2 = p = 1; 6; 12$; $S = 1000$; $D = LS$; $NOI = 800$; $n = 3$.

Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project is shown in Table 6 and Figures 10 and 11.

Table 6. Dependence of NPV on leverage level L at $k_0 = 0.22$, $k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project.

L	NPV		
	p = 1	p = 6	p = 12
0	307	307	307
1	761	771	772
2	1218	1238	1240
3	1676	1707	1710
4	2135	2175	2179
5	2594	2644	2649
6	3053	3113	3119
7	3511	3582	3589
8	3970	4051	4059
9	4429	4520	4529
10	4888	4989	4999

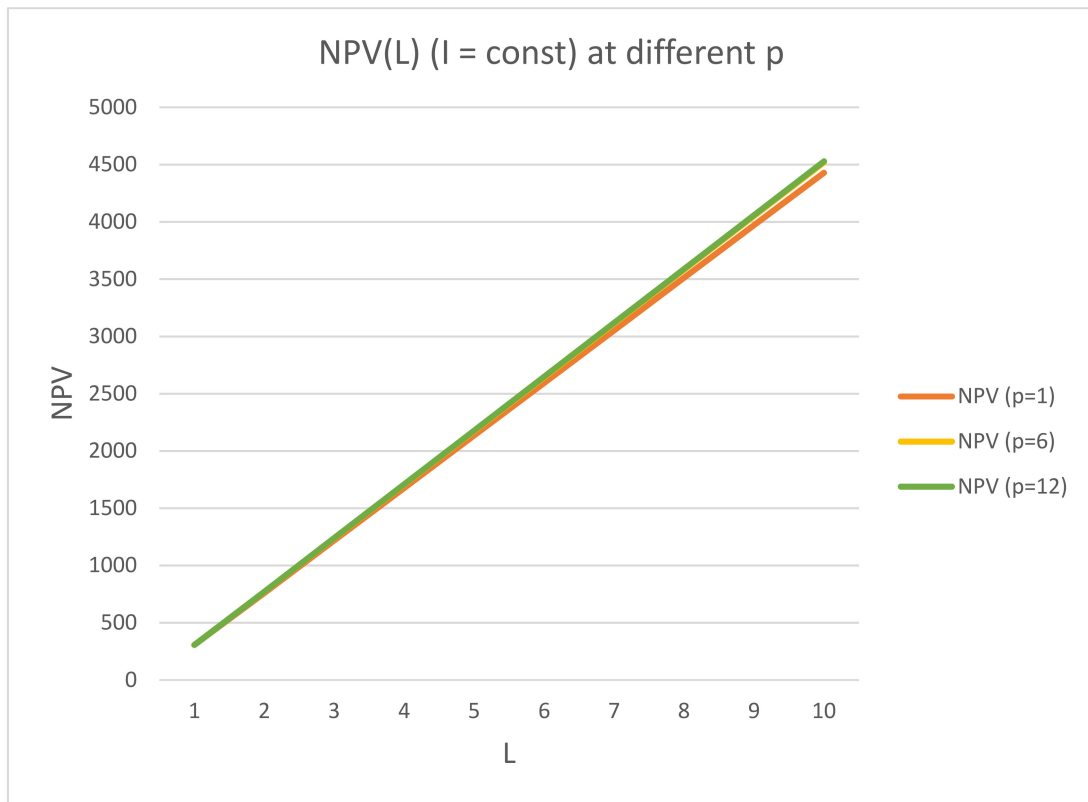


Figure 10. Dependence of NPV on leverage level L at $k_0 = 0.22, k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project.

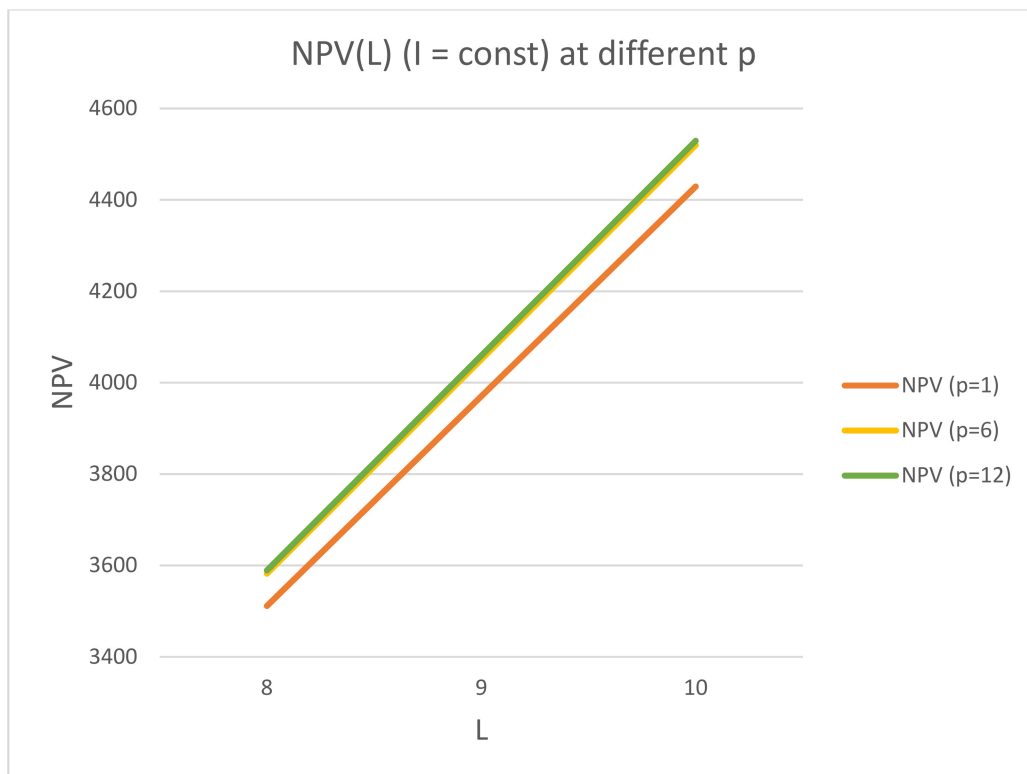


Figure 11. Dependence of NPV on leverage level L at $k_0 = 0.22, k_d = 0.14$ and different $p = 1, 6, 12$ for a three-year project (larger scale).

5.6. Discussions

The analysis of results from Tables 1–6 and Figures 1–11 shows that:

- (1) The weighted average cost of capital, WACC, decreases with leverage level L at all values of frequency of payments of tax on profit p (see Tables 1 and 2 and Figures 1–3);
- (2) With an increase in p , WACC decreases: WACC(L) curves lie lower with increase of p ;
- (3) The difference between curves corresponding to $p = 1$ and $p = 6$ is much more than the difference between the curves corresponding to $p = 6$ and $p = 12$. This difference decreases with p . It turns out that an increase in the number of payments of tax on profit per year p leads to a decrease in the cost of attracting capital (WACC). Will this decrease in the discount rate increase the effectiveness of investment projects? As we see from the Tables 3–6 and Figures 4–11, the situation is different for owners of equity capital and for owners of equity and debt capital.
- (4) NPV practically linearly increases with leverage level at all values of frequency of payments of tax on profit p and all frequency of payments of interest on debt;
- (5) In the case of considering the effectiveness of long-term investment projects for owners of equity capital, NPV is changed with a change of p (both p_1 and p_2 are equal everywhere below) but by a very small value (it is seen in the Tables, but not in the Figures);
- (6) In the case of considering the effectiveness of long-term investment projects for owners of equity and debt, NPV is changed with a change of p more significantly (it is seen in the Tables and in the Figures);
- (7) For arbitrary duration projects, this difference in NPV with a change of p is more significant and should be accounted under valuation of the effectiveness of an investment project;
- (8) In the case of considering the effectiveness of an investment project for owners of equity capital, we need to note that an increase in p leads to a decrease in NPV: this means that the effectiveness of an investment project decreases with p ;
- (9) In the case of considering the effectiveness of an investment project for owners of equity and debt capital, we need to note that the situation is opposite and an increase in p leads to increase in NPV: this means that the effectiveness of an investment project increases with p ;
- (10) The above results show that in the former case, companies should pay tax on profit and interest on debt once per year, while in the latter case, more frequent payments are profitable for the effectiveness of an investment;
- (11) Thus, while for long-term projects' NPV, the impact of more frequent payments of both values p_1 and p_2 is insignificant, for arbitrary duration projects the account of the frequency of both types of payments could be important and could lead to more significant influence on the effectiveness of an investment project, decreasing it (in the former case), or increasing it (in the latter case). Note that the specific value of the effect depends on the values of the parameters in the project (k_0, k_d, n, t, S etc).

6. Conclusions

There are too few investment models which can numerically value the effectiveness of investment projects, among them the investment models developed by the authors of this paper. Moreover, investment models are practically absent which account for the conditions of the real functioning of investment projects. This paper covers this gap in the literature and science in the field of investments and develops innovative investment models which are much closer to economic practice.

We developed here for the first time eight innovative investment models: long-term (described by Equations (14), (19), (24) and (29)) as well arbitrary duration (described by Equations (13), (18), (23) and (28)), which account payments of interest on debt and of tax on income a few times per year (semi-annually, quarterly, monthly) as it happened in practice. Note that no one before had investigated the impact of the frequency of payments of taxes and of debt interest on the effectiveness of investment projects. These investment models

allow for investigation of the impact of all main parameters of investment projects (equity value S , investment value I , net operating income NOI, leverage level L , profitability of investments β , tax on profit t , project duration n , equity cost k_0 , debt cost k_d and number of payments of interest on debt p_1 and of tax on income p_2) on the main indicator of effectiveness of investment projects NPV (net present value). They could be used for investigation of different problems of investments, such as the influence of debt financing, leverage level, taxing, project duration, method of financing, number of payments of interest on debt and of tax on income and some other parameters on efficiency of investments and other problems. In particular, they will improve the issue of project ratings [23]. These new models allow for making more correct evaluations of effectiveness of investment projects long-term as well as of arbitrary duration.

Numerical calculations, conducted for four investment models (without flow separation) show that:

- In the case of considering the effectiveness of an investment project for owners of equity capital, the increase in the number of payments of tax on income and of interest on debt p leads to a decrease in NPV: this means that the effectiveness of an investment project decreases with p ;
- In the case of considering the effectiveness of an investment project for owners of equity and debt capital, the increase in the number of payments of tax on income and of interest on debt p leads to an increase in NPV: this means that the effectiveness of an investment project increases with p .

In the former case, companies should pay tax on profit and interest on debt once per year, while in the latter case, more frequent payments are profitable for the effectiveness of an investment.

Thus, while for long-term projects' NPV, the impact of more frequent payments of both values p_1 and p_2 is insignificant, for arbitrary duration projects the account of the frequency of both types of payments could be important and could lead to more significant influence on the effectiveness of an investment project, decreasing it (in the former case), or increasing it (in the latter case). Note that the specific value of the effect depends on the values of the parameters in the project (k_0, k_d, n, t, S , etc.).

There are some limitations of the applicability of the proposed models:

- (1) For long-term projects, they are connected with the limitations of the Modigliani-Miller theory;
- (2) For consideration of without flow separation, they are connected with the well-known limitations of the WACC approach;
- (3) For arbitrary duration projects (using BFO theory), they are connected with the fact that not all the conditions of real investments are accounted yet.

The contribution of this study to finance and economics is mainly related to the goal that new modern investment models have been created to be as close as possible to real investment conditions. Our models, namely, models with payments of interest on debt and of tax on income which occur a few times per year (semi-annually, quarterly or monthly), could be more successfully applied in real economic practice.

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