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A University Teachers' Teaching Performance Evaluation Method Based on Type-II Fuzzy Sets

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Abstract: This paper proposes a university teachers' teaching performance evaluation method based on type-II fuzzy sets (T2 FSs), which solves the problems of fuzziness, complexity and uncertainty in teaching performance evaluation. Firstly, the evaluation indicator system is constructed from the aspects of teaching attitude, teaching contents, teaching professionalism, teaching methods and teaching effects. Then, T2 FSs theory and the perceptual computing method are introduced to model subjective judgments and capture uncertainties, effectively handling higher levels of uncertainty in the evaluation process. Furthermore, the linguistic weighted average operator is applied as the computing with words engine to aggregate scores and weights of indicators, which effectively integrates the uncertain information in the input data into the final evaluation conclusion and guarantees the accuracy of the evaluation results. Finally, the effectiveness of the method of this study is evaluated by simulation experiments. The computational results demonstrate that it can capture more uncertain and complex information, and is more accurate and reliable than the type-I fuzzy sets method.

Keywords: teaching performance evaluation; type-II fuzzy sets; computing with words; linguistic weighted average

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1. Introduction

The evaluation of the teaching performance of teachers in universities is an important means to assess the teachers' teaching activities, which aims at making a reasonable and scientific judgment on teaching level. This can enhance teachers' enthusiasm and improve teaching quality. At the same time, it is also the key to deepen the reform of the personnel system and strengthen the formation of the teaching body in universities [1]. Therefore, it is important to establish a reasonable and scientific teaching performance evaluation model.

Many factors and indicators (attributes) are related to university teachers' teaching performance, and an individual indicator, such as teaching content, can hardly reflect the overall teaching performance. At the same time, owing to the complexity of the teaching performance evaluation process and the uncertainty of human cognition, evaluation of university teachers involves many fuzzy and uncertain factors. For example, due to the fuzziness and uncertainty of human cognition, evaluation experts and students tend to provide scores and weights of indicators in the form of fuzzy words like "very good" and "important". Therefore, we can formulate university teachers teaching performance evaluation as a fuzzy multi-attribute decision making (MADM) problem.

Over the past years, numerous researchers have systematically studies teaching performance evaluation from the aspects of influencing factors, evaluation indicator system and evaluation methods, and have seen some achievements. For example, Jeanette et al. [2]

investigated the quality of classwork and homework to determine that homework is an important characteristic of teaching performance, and took its effectiveness as a measure of teaching performance. By using a statistical analysis method, Gupta et al. [3] studied influencing factors, such as the gender and economic status of students and teachers, on teaching performance. Graham et al. [4] investigated the associations between teachers' years of experience and teaching quality. Fuentes et al. [5] established the indicator system with four aspects and adopted the three-stage data envelopment analysis method for evaluating teaching performance. Recently, Zhang [6] constructed an improved educational evaluation indicator system and then adopted the principal component analysis method to evaluate teaching performance. Jian [7] constructed an evaluation indicator system for multimedia teaching performance evaluation. Li et al. [8] constructed a multidimensional evaluation indicator of students, peers, and leadership, and developed a backward propagation neural network based model for university teachers' teaching performance evaluation. These studies investigated the influencing factors or presented an effective design for teaching performance evaluation indicator systems. However, they adopted numerical values to represent the scores and weights of indicators, which did not give adequate consideration to the imprecise, fuzzy, and uncertain characteristics of teaching performance evaluation.

Considering the fuzzy and uncertainty characteristics of the problem, several studies adopted type-I fuzzy sets (T1 FSs) to quantify and deal with the fuzzy and imprecise factors [9–12]. For example, Basaran et al. [9] introduced fuzzy reasoning rules to represent teaching performance assessment criteria and its fuzzy factors. Zhu et al. [10] adopted triangular fuzzy numbers to represent fuzzy factors and proposed a teaching performance evaluation method using fuzzy envelopment analysis. Yang et al. [11] adopted the fuzzy comprehensive method to evaluate the quality of simulation teaching in the Fundamental Nursing Curriculum. T1 FSs theory is useful for managing uncertain information, which is widely used in various domains. However, a linguistic word involves interpersonal uncertainty and intrapersonal uncertainty simultaneously. T1 FSs theory uses precise numbers to represent the degrees of elements' membership, but this is still inadequate in describing interpersonal uncertainty. Therefore, applying T1 FSs to university teachers' teaching performance evaluation still has some defects. A theoretical tool is needed to deal with various types of uncertainty.

Unlike T1 FSs, type-II fuzzy sets (T2 FSs) use T1 FSs to describe the element membership degree, which enhances the ability to describe the uncertainty of objective factors [13,14]. The concept of T2 FSs was proposed by Zadeh, which is an extension of T1FSs [13,14]. T2FSs characterize fuzziness by primary membership function and secondary membership function, which can provide greater freedom and flexibility for expressing uncertainties. Each element of the T2 FSs has a membership degree that is a T1 FS in $[0, 1]$, while, in T1 FSs, the membership degree is a precise number in $[0, 1]$. Based on Zadeh's concept of a T2 FS, Karnik et al. [15] developed operations on T2 FSs and introduced the concept of type-reduction for T2 FSs. Since the late 1990s, many researchers have studied and discussed T2 FSs, and made great efforts to promote them in real applications. Mendel et al. [16–18] provided representations such as wavy slice representation, α-plane representation, and zSlice representation for T2 FSs. Based on these representations, many researchers have analyzed and explored T2 FSs from the aspects of aggregation operators, similarity measure, type-reduction methods and so on [19–31]. For example, Mendel et al. [19–22] conducted an in-depth study on the MADM theory and perceptual computing (Per-C) theory based on T2 FSs and their applications. Celik et al. [23] made a comprehensive review of MADA approaches based on interval T2 FSs (IT2 FSs). Hamza et al. [24] reviewed the recent advances in the application of meta-heuristic optimization algorithms to optimize type-II fuzzy logic systems in intelligent control. Yu et al. [14] presented a development overview, the dynamic evolution of the main topics and the knowledge diffusion trajectory of T2 FSs. They concluded that MADM has become the core theme of T2 FSs. Recently, T2 FSs have become an important theoretical basis for dealing with various types of uncertainty in the real world, which have been widely used in the fields of tracking control [25], data processing [26] and so on. However, the T2 FSs theory has not yet been applied to university teachers' teaching performance evaluation.

According to the analysis above, this study formulates university teachers' teaching performance evaluation as a fuzzy MADM problem and proposes a teaching performance evaluation method by using T2 FSs. Firstly, we analyze the influencing factors and establish the indicator system of teaching performance evaluation. Then, we introduce T2 FSs to represent human decisions, effectively quantifying and dealing with the uncertainties of the evaluation process. Furthermore, we use the linguistic weighted average (LWA) operator as a computing with words (CWW) engine to aggregate the indicator scores, ensuring that the fuzziness and uncertainty in the evaluation process are sufficiently taken into account and reflected in the final results. Finally, we test the feasibility of the proposed method by practical teaching performance evaluation examples.

The rest of this study is structured as follows. Section 2 outlines the framework of the university teachers' teaching performance assessment approach and establishes the indicator system. Section 3 introduces T2 FSs for managing the uncertainties inherent in human decisions. Section 4 analyzes the computational results, and Section 5 summarizes the study.

2. Teaching Performance Evaluation Modeling

This section presents the general scheme of the proposed university teachers' teaching performance assessment approach and establishes the hierarchical indicator system.

2.1. General Framework

As mentioned above, teaching performance evaluation of university teachers can be regarded as a fuzzy MADM problem, which involves lots of fuzzy information. The solution process for MADM problems mainly includes the acquisition of decision information, the aggregation of decision information, and the sorting and classification of schemes (or alternatives). The decision information mainly includes scores and preferences of indicators. In the problem of teaching performance evaluation of university teachers studied in this study, the indicator scores of the teachers are provided by the students participating in the teaching evaluation process, and the indicator weights, representing the preferences of evaluation experts to the indicators, are provided by evaluation experts. They are all linguistic words involving a lot of fuzziness.

The fuzziness and uncertainty of the input evaluation data sources make the final evaluation results uncertain. The Per-C method using T2 FSs is a novel method for aggregating decision information and solving MADM problems [19–22], which is a computing with words technique that models linguistic words using interval T2 FSs (IT2 FSs). As shown in Figure 1, the architecture of Per-C is composed of three components: an encoder, a CWW engine, and a decoder. The encoder can map the linguistic words provided by humans into IT2FSs. The CWW engine aggregates the outputs of the encoder. The decoder transforms the outputs of the CWW engine into recommendations in the form of a word, rank, or class. The Per-C method is effective in handling inherent uncertainties in words and is used extensively in solving MADM problems. However, its application to university teachers' teaching performance evaluation is new. To integrate the uncertainty information within these data sources into the final evaluation results, series of sub-indicators and indicators are integrated by using the Per-C method in this study.

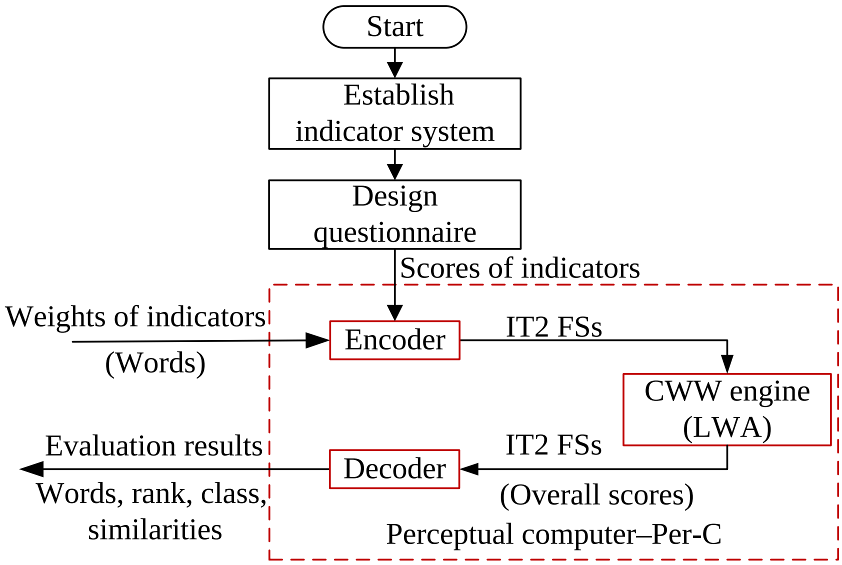

Figure 1. Flow chart of the proposed university teachers teaching performance evaluation approach based on T2 FSs and the Per-C method.

Figure 1 presents the structure diagram of the proposed teaching performance evaluation method, which mainly includes the following steps:

1. Establish the hierarchical indicator framework of university teachers' teaching performance evaluation.
2. Design a questionnaire according to the established evaluation indicator system, and ask students to give scores of the teachers to be evaluated on the indicators by using linguistic words.
3. Consult evaluation experts, who provide linguistic weight of each indicator.
4. Considering the ambiguity and uncertainty of natural language, adopt IT2 FSs to model words based on the interval endpoint method (Encoder).
5. Adopt the CWW engine in the manner of the LWA method to aggregate decision information and obtain the IT2 FSs representing the overall scores of the teachers' teaching performance (CWW).
6. Analyze the overall scores of the teachers, provide rankings of the teachers' teaching performance based on the average centroid ranking method, the similarities among the teachers' teaching performance based on the Jaccard similarity measure, etc. (Decoder).

2.2. Construct the Indicator System

Teaching performance evaluation of university teachers is a complicated decision making problem, which involves many evaluation indicators, such as teaching attitude, teaching content, and so on. In order to construct a relatively objective and accurate indicator system, the principles of scientificity, consistency, integrity, independence and development should be followed. According to the above principles, a comprehensive indicator system for university teacher' teaching performance evaluation is established in this study. As shown in Table 1, the indicator system includes five first level indicators, which are teaching attitude, teaching contents, teaching methods, teaching professionalism and teaching effect, respectively. These five indicators can be divided into 23 sub-indicators, such as teaching plan, blackboard writing, and so on.

Table 1. University teachers' teaching performance evaluation indicator system.

Quality of classroom teaching T	Teachingattitude T_1	The teaching plan is clear and the lessons are well prepared T_{11} Follows teaching disciplines, maintains scientific and effective classroom management techniques T_{12} Assigns proper amount of homework and does marking carefully, answers questions of students in a timely manner T_{13} Teaching and cultivating, rigorous scholarship, teaches by precept and examples T_{14} Well-groomed, enthusiastic, energetic and well-mannered T_{15}
	Teaching contents T_2	The concepts and theory are correct T_{22} Use appropriate emphasis, reasonably, detailed and the degree of difficulty is appropriate T_{22} Conform to the teaching plan T_{23} Integrate with the development of the discipline and absorb new achievements T_{24} Rich in practical knowledge, link theory with practice, pay attention to ability training T_{25}
	Teaching methods T_3	Good at inspiring and guiding, the choices of typical examples are appropriate, the scene designs are clever T_{31} Treats students as individuals, and the teaching methods are simple, easy to understand T_{32} The teaching methods are novel, good at stimulating students' interests in learning T_{33} Pays attention to teaching research, makes innovations in teaching reform and experience T_{34} Has strong classroom organization ability, manages classroom discipline effectively T_{35} Improves classroom atmosphere, strengthens the interactions between the teacher and students T_{36}
	Teaching professionalism T_4	Blackboard writing is neat and standard T_{41} Keeps voice volume moderate, has no serious accent, being clear and precise in expression T_{42} Has a solid academic background T_{43}
	Teachingeffect T_5	Through the teaching, students master the knowledge required by the course T_{51} Through the teaching, the students' learning methods have been expanded, and theiranalytical ability and comprehensive application ability are improved T_{52} Through the teaching, the students' enthusiasm and initiative are promoted T_{53} Explains knowledge points and examples with clear thinking, and the students'thinking ability is improved T_{54}

3. Evaluate Teaching Performance Using Type-II Fuzzy Sets

This section proposes using T2 FSs and the Per-C theory for handling the various types of uncertainty in the evaluation process and evaluating the teaching performance of university teachers.

3.1. Type-II Fuzzy Sets

Let $\widetilde{A}$ represent a T2 FS in the universe of discourse X. $\widetilde{A}$ can be described by a type-II membership function $u_{\widetilde{A}}$, expressed as [16]:

$$\widetilde{A} = \left\{(x,u), u_{\widetilde{A}}(x,u) \middle| \forall x \in X, u \in J_x \subseteq [0,1]\right\} \tag{1}$$

where x and u represent the main variable and the secondary variable, respectively. J_x represents a subinterval of $[0, 1]$.

For a T2 FS set $\widetilde{A}$, if all its type-II membership degrees are 1, then $\widetilde{A}$ is called an IT2 FS. In addition, membership functions of a T2 FS can be expressed in many forms, such as triangle, Gaussian and trapezoid. The trapezoidal membership function is widely used because of its simple design and few parameters. As shown in Figure 2, the lower and upper membership functions of a trapezoidal IT2 FS $\widetilde{A}$ are both trapezoidal T1 FSs, expressed as [16,19]:

$$\widetilde{A} = (A^U, A^L) \tag{2}$$

where $A^U = (a_1^U, a_2^U, a_3^U, a_4^U; H_1(A^U), H_2(A^U))$ and $A^L = (a_1^L, a_2^L, a_3^L, a_4^L; H_1(A^L), H_2(A^L))$ are T1 FSs, which represent the upper and lower membership functions of $\widetilde{A}$, respectively. $H_1(A^T)$ and $H_2(A^T)$ represents the membership grade of the membership function at a_2^T and a_3^T, $T \in \{U, L\}$, respectively.

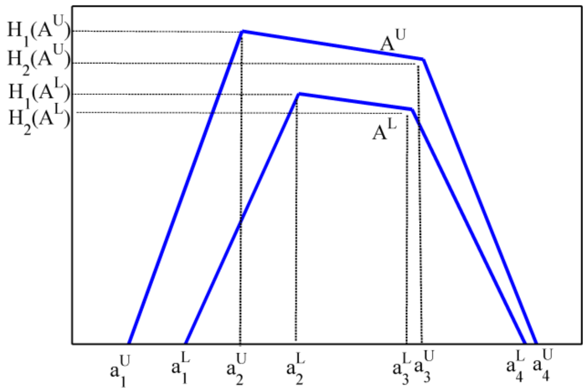

Figure 2. Primary membership function of trapezoidal IT2 FS.

3.2. University Teaching Performance Evaluation: A Per-C Method

According to the established teaching performance evaluation indicator system, a teaching questionnaire is designed to allow university students to rate the teaching performance of the teachers to be evaluated, and then the indicator scores can be collected. Due to the complexity and fuzziness of the teaching process, students often find it difficult to give accurate ratings, but tend to use fuzzy words such as "very good" to give their ratings. Considering the cognition and expression habits of the students, students can give the teachers' ratings on each indicator by using five linguistic variables, which are "Excellent", "Good", Adequate", "Marginal" and "Poor". Furthermore, to guarantee the accuracy and credibility of the conclusions, the experts with rich teaching experience in the teaching supervision group of the university are consulted, and they provide the weight of each indicator and sub-indicator. The indicator weights can also be divided into five grades, which are "Very Important" (VI), "Medium Important" (MI), "Important" (I), "Medium Unimportant" (MU) and "Very Unimportant" (VU), respectively.

Based on the above analysis, the indicator weights and scores are represented by linguistic words in the teaching performance evaluation problem, which contains a lot of fuzziness and uncertainties. The fuzziness and uncertainties of the evaluation data sources make the final evaluation conclusions uncertain. As a novel method for solving MADM problems, the Per-C method based on IT2 FSs is effective in handling inherent uncertainties in linguistic words and is widely applied in the field of MADM [19–22]. In order to integrate the uncertainties in the data sources into the final evaluation conclusions, this study applies the Per-C method to solve the university teachers' teaching performance evaluation problem.

As shown in Figure 3, the method proposed in this study mainly includes the following steps.

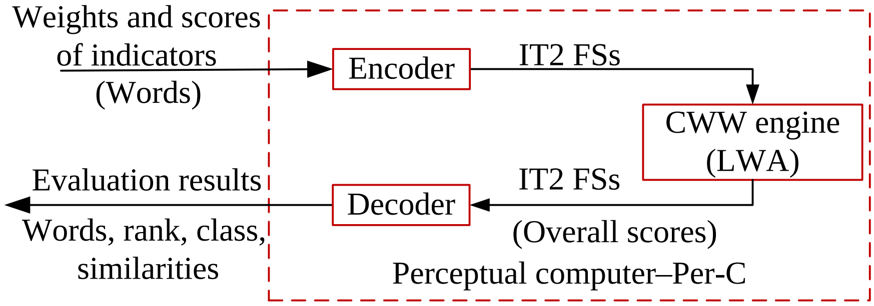

Figure 3. Conceptual structure of the proposed Per-C method based university teachers' teaching performance evaluation scheme.

1. Map the input of Per-C, which are linguistic words representing the weights and scores of the indicators to their corresponding IT2 FSs, by using the interval endpoint method (Encoder).
2. The CWW engine integrates the output of the encoder into the overall scores of teaching performance. The type of CWW engine can be selected according to the characteristics of the problem. A widely used form of CWW engine is the LWA operator, which has been proven to be effective in handling MADM problems with fuzzy weights [19–22]. In this study, the LWA operator is adopted as the CWW engine to integrate outputs of the encoder, and thus obtain the IT2 FSs that represent the overall teaching performances of the teachers (CWW).
3. Decode the output of the CWW engine based on the average centroid ranking method and the Jaccard similarity measure, and the final evaluation conclusions can be obtained, such as the linguistic words describing the overall teaching performance of teachers, the rankings and ranking values of teachers' overall teaching performances, the similarities among teachers' overall teaching performances and the uncertain interval of the ranking values (Decoder).

3.3. Encoder: Modeling Words Based on Type-II Fuzzy Sets

As Figures 3 and 4 show, the encoder adopts the interval endpoint method to transform linguistic words into their corresponding trapezoidal IT2 FSs. As shown in Table 2, the linguistic variables in the indicator weight sets can be transformed into trapezoidal interval type-II fuzzy numbers $\widetilde{1}$, $\widetilde{3}$, $\widetilde{5}$, $\widetilde{7}$, and $\widetilde{9}$.

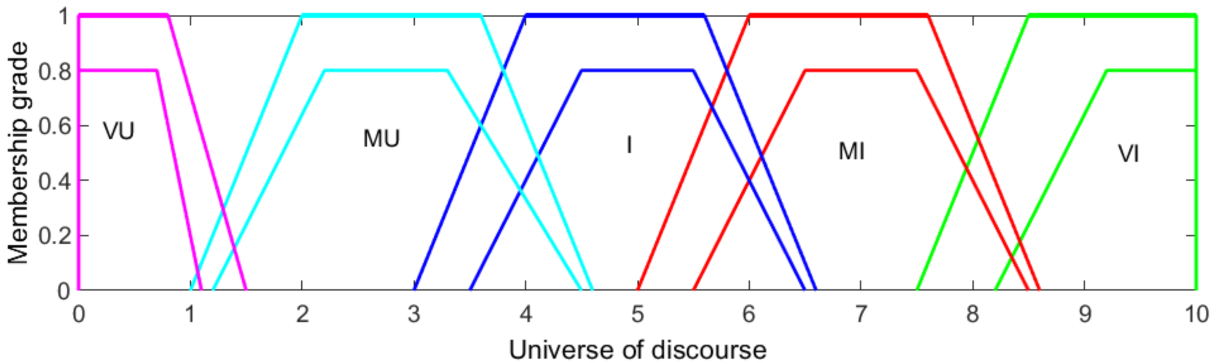

Figure 4. IT2 FSs of the linguistic words describing the indicator weights.

Table 2. Words describing the indicator weights and their IT2 FSs.

Linguistic Term	Fuzzy Number	Membership Function
Very Unimportant (VU)	$\widetilde{1}$	$[(0,0,0.8,1.5;1,1),\ (0,0,0.7,1.1;0.8,0.8)]$
Medium Unimportant (MU)	$\widetilde{3}$	$[(1,2,3.6,4.6;1,1),\ (1.2,2.2,3.3,4.5;0.8,0.8)]$
Important (I)	$\widetilde{5}$	$[(3,4,5.6,6.6;1,1),\ (3.5,4.5,5.5,6.5;0.8,0.8)]$
Medium Important (MI)	$\widetilde{7}$	$[(5,6,7.6,8.6;1,1),\ (5.5,6.5,7.5,8.5;0.8,0.8)]$
Very Important (VI)	$\widetilde{9}$	$[(7.5,8.5,10,10;1,1),\ (8.2,9.2,10,10;0.8,0.8)]$

Words adopted for scoring each teacher regarding each indicator or the overall teaching performance can be represented by their corresponding synonyms in Table 2. For example, the linguistic word "very good" is represented by the trapezoidal IT2 FS $[(7.5,8.5,10,10;1,1),\ (8.2,9.2,10,10;0.8,0.8)]$.

3.4. The Compute with Words Engine

After encoding, all weights and scores are represented by trapezoidal IT2 FSs. Then, we can adopt the LWA operator to aggregate these data to compute the overall scores of teaching performances. For the indicator set $T = \{T_1, T_2 \ldots, T_n\}$, LWA is expressed as [19–22]:

$$\widetilde{Y} = \sum_{i=1}^{n} \widetilde{W}_i \widetilde{X}_i / \sum_{i=1}^{n} \widetilde{W}_i \tag{3}$$

where $\widetilde{W}_i$ and $\widetilde{X}_i$ represent the trapezoidal IT2 FSs of the weight and score of indicator T_i, respectively. IT2 FS $\widetilde{Y}$ represents the aggregated score.

Let $\widetilde{X}_{11m} \sim \widetilde{X}_{15m}$ represent the scores of teacher A_i on indicator $T_{11} \sim T_{15}$ given by the mth ($m = 1, 2 \ldots, N$, where N represents the number of students participating in the teaching evaluation activity) student, and $\widetilde{W}_{11} \sim \widetilde{W}_{15}$ represent the weights of indicators $T_{11} \sim T_{15}$, respectively. The score of teacher A_i ($i = 1, 2, \ldots, M$, where M represents the number of teachers to be evaluated) on teaching attitude T_1, $\widetilde{X}_1(i)$, is calculated as follows:

$$\widetilde{X}_1(i) = \sum_{k=1}^{5} \widetilde{W}_{1k} \widetilde{X}_{1k} / \sum_{k=1}^{5} \widetilde{W}_{1k}, \ \widetilde{X}_{1k} = \frac{1}{N} \sum_{m=1}^{N} \widetilde{X}_{1km} \tag{4}$$

Similarly, the LWA operator can be adopted as the CWW engine to compute the scores of teacher A_i on the indicators of $T_2 \sim T_5$. Then, using the LWA operator to aggregate the scores of the five first level indicators, the overall score of teaching performance of teacher A_i can be obtained, which is expressed as:

$$\widetilde{A}_i = \sum_{k=1}^{5} \widetilde{W}_k \widetilde{X}_k(i) / \sum_{k=1}^{5} \widetilde{W}_k \tag{5}$$

where $\widetilde{A}_i$ is an IT2 FSs representing the overall score of teaching performance of teacher A_i. $\widetilde{W}_k$ and $\widetilde{X}_k(i)$ ($k = 1, 2, \ldots, 5$) represent the trapezoidal IT2 FSs of the weights and scores of teacher A_i regarding the indicators of $T_1 \sim T_5$, respectively.

3.5. Decoder: Analysis of the Overall Scores

The CWW engine can output the IT2 FSs describing the overall scores of the teachers' teaching performances. These IT2 FSs can be ranked and classified in the decoding process to obtain the final evaluation conclusions. The overall score $\widetilde{A}_i$ ($i = 1, 2, \ldots, M$, where M represents the number of teachers to be evaluated) of the teachers' teaching performances is ranked based on the average centroid ranking method. The center of the centroid of the IT2 FS $\widetilde{A}$, $c(\widetilde{A})$, is defined as [19]:

$$c(\widetilde{A}) = (c_l(\widetilde{A}) + c_r(\widetilde{A}))/2 \tag{6}$$

where $c_l(\widetilde{A})$ and $c_r(\widetilde{A})$ are the minimum and maximum centroids of all embedded fuzzy sets of $\widetilde{A}$. The larger the $c(\widetilde{A}_i)$, the better teacher A_i teaches.

The Jaccard similarity measure can be adopted to calculate the similarities between the overall score $\widetilde{A}_i$ and the IT2 FS of each word in the evaluating criteria term sets, which can be expressed as [19,27]:

$$S(\widetilde{A}, \widetilde{B}) = \frac{\int_X \min(\underline{u}_{\widetilde{A}}(x), \underline{u}_{\widetilde{B}}(x))dx + \int_X \min(\overline{u}_{\widetilde{A}}(x), \overline{u}_{\widetilde{B}}(x))dx}{\int_X \max(\underline{u}_{\widetilde{A}}(x), \underline{u}_{\widetilde{B}}(x))dx + \int_X \max(\overline{u}_{\widetilde{A}}(x), \overline{u}_{\widetilde{B}}(x))dx} \tag{7}$$

where $\widetilde{A}$ and $\widetilde{B}$ are IT2 FSs, $\underline{u}_{\widetilde{A}}(x)$ ($\underline{u}_{\widetilde{B}}(x)$) and $\overline{u}_{\widetilde{A}}(x)$ ($\underline{u}_{\widetilde{B}}(x)$) are the lower and upper member functions of $\widetilde{A}(\widetilde{B})$, respectively. The linguistic term in the evaluating indicator term sets that has the maximum similarity to $\widetilde{A}_i$ can be selected to describe the overall teaching performance of teacher A_i.

4. Experiment Results

In this section, we assess the method of this study to show its effectiveness and performance. We also compare it with the evaluation method based on T1 FSs.

The proposed method and the evaluation method based on T1 FSs were coded in MATLAB, and run on a computer with Intel Core i9-9900 CPU 3.60 GHz and 32 GB RAM running Windows 10.

4.1. University Teachers' Teaching Performance Evaluation Data

To evaluate the feasibility of the proposed method, a typical university teachers' teaching performance evaluation example is selected. In this example, a total of three teachers who teach the Computer Network course in Nanjing University of Posts and Telecommunications will be evaluated and ranked. According to the evaluation indicator system shown in Table 1, produce the teaching performance evaluation questionnaire, and then randomly deliver 100 questionnaires among the students so that each teacher teaches by sampling method. The sampled students use linguistic words such as "very good" and "moderate" to evaluate their teachers' teaching performance on the sub-indicators. Table 3 shows the scores of a teacher given by four of his students on the sub-indicators.

Table 3. A teacher's scores on the sub-indicators given by four of his students.

Indicators	Evaluation Scores
$(T_{11}, T_{12}, \ldots, T_{15})$	$(\widetilde{9},\widetilde{7},\widetilde{5},\widetilde{7},\widetilde{7})$,$(\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{5})$,$(\widetilde{9},\widetilde{5},\widetilde{7},\widetilde{7},\widetilde{7})$,$(\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{7})$
$(T_{21}, T_{22}, \ldots, T_{25})$	$(\widetilde{7},\widetilde{5},\widetilde{7},\widetilde{7},\widetilde{9})$,$(\widetilde{7},\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{9})$,$(\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{9})$,$(\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{9})$
$(T_{31}, T_{32}, \ldots, T_{36})$	$(\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{9},\widetilde{5})$,$(\widetilde{7},\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{9},\widetilde{5})$,$(\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{5},\widetilde{9})$,$(\widetilde{7},\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{9},\widetilde{7})$
(T_{41}, T_{42}, T_{43})	$(\widetilde{7},\widetilde{5},\widetilde{9})$, $(\widetilde{7},\widetilde{7},\widetilde{9})$, $(\widetilde{9},\widetilde{7},\widetilde{7})$, $(\widetilde{7},\widetilde{7},\widetilde{7})$
$(T_{51}, T_{52}, T_{53}, T_{54})$	$(\widetilde{7},\widetilde{7},\widetilde{7},\widetilde{9})$,$(\widetilde{7},\widetilde{7},\widetilde{5},\widetilde{9})$, $(\widetilde{7},\widetilde{5},\widetilde{7},\widetilde{9})$, $(\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{9})$

As shown in Table 4, evaluation experts in the teaching supervision group of the university consult, and give the indicator weights by using linguistic words extracted from the weight sets.

Table 4. The indicator weights given by evaluation experts.

Indicators	Type 2 Fuzzy Weights
$(T_{11}, T_{12}, \ldots, T_{15})$	$(\widetilde{1},\widetilde{3},\widetilde{9},\widetilde{9},\widetilde{7})$
$(T_{21}, T_{22}, \ldots, T_{25})$	$(\widetilde{3},\widetilde{9},\widetilde{7},\widetilde{9},\widetilde{1})$
$(T_{31}, T_{32}, \ldots, T_{36})$	$(\widetilde{9},\widetilde{1},\widetilde{7},\widetilde{3},\widetilde{5})$
(T_{41}, T_{42}, T_{43})	$(\widetilde{1},\widetilde{7},\widetilde{9},\widetilde{7},\widetilde{7},\widetilde{3})$
$(T_{51}, T_{52}, T_{53}, T_{54})$	$(\widetilde{7},\widetilde{9},\widetilde{1})$

4.2. Teaching Performance Evaluation of University Teachers Using T2 FSs

As mentioned above, most of the existing literature has adopted numerical values to represent the scores and weights of indicators, ignoring the imprecise, fuzzy, and uncertain characteristics of teaching performance evaluation [1–8]. Some studies adopted the T1 FSs to manage fuzzy uncertain information in teaching performance evaluation process [9–12]. To assess the feasibility of the proposed method, we compare it with the evaluation method based on T1 FSs in the following experiments.

The first experiment is to evaluate the feasibility of the proposed method in dealing with uncertainties. As Figure 4 shows, all linguistic terms describing the indicator scores and weights are represented by T1 FSs in the evaluation method based on T1 FSs. By comparing Figures 4 and 5, it can be observed that T1 FSs use a determined value to describe the membership degree, which cannot deal with interpersonal uncertainty. The IT2 FS adopted in the proposed method uses a T1 FS to describe the membership degree, which has greater freedom and flexibility for capturing uncertainties, along with the capability of modelling second-order uncertainties.

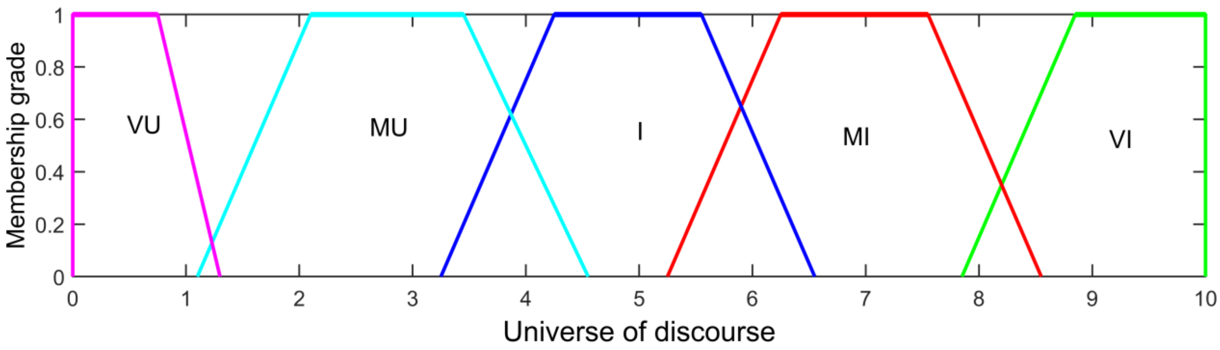

Figure 5. Linguistic words describing the weights of indicators and their T1 FSs.

According to the indicator scores and indicator weights shown in Table 4, the LWA operator is used to integrate the input data sources. As Figure 6 shows, the overall teaching performance ratings $\widetilde{A}_i$ $(i = 1, 2, 3)$ of the three teachers are also T1 FSs and IT2 FSs obtained by these two methods, respectively. By comparing Figure 6a,b, it can be observed that interpersonal uncertainty is not incorporated in the final ratings obtained by the evaluation method based on T1 FSs. The method of this study can deal with intrapersonal uncertainty and interpersonal uncertainty simultaneously, and can reflect them in the final results.

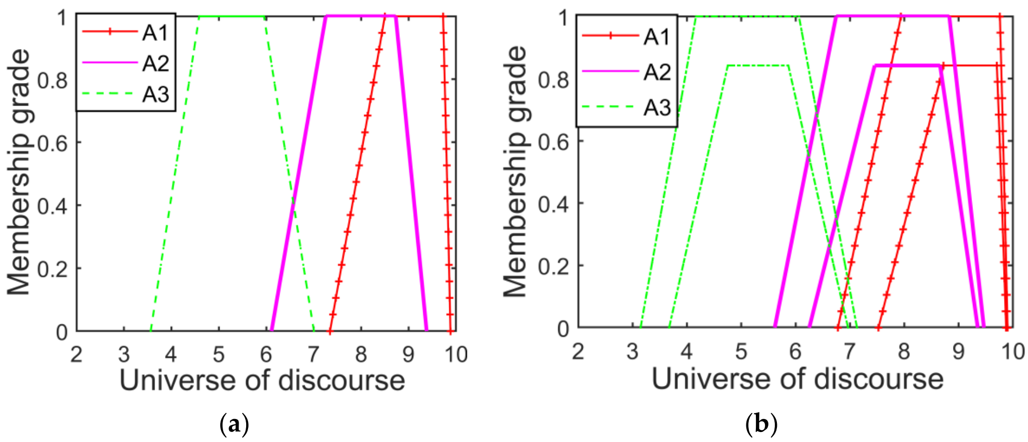

Figure 6. Comparison results of the evaluation method based on T1 FSs and the method of this study: (**a**) Overall ratings obtained by the evaluation method based on T1 FSs; (**b**) Overall ratings obtained by the method of this study.

Table 5 shows the ranking values of teaching performance obtained by the evaluation method based on T1 FSs and the method of this study. Comparisons show few differences between the ranking values these two methods give. In the proposed method, T2 FSs are introduced to represent linguistic variables, which can provide more freedom and directly manage multiple types of uncertain information. The CWW engine adopted in the proposed method can integrate the fuzzy and uncertain information existing in all data sources into the final evaluation results, which can ensure the accuracy and credibility of the evaluation results. Compared to the proposed method, the T1 FSs method is simple, but it may lose or distort the original decision information.

Table 5. Teaching performance rankings given by the evaluation method based on T1 FSs and the method of this study.

Method	$\widetilde{A}_1$	$\widetilde{A}_2$	$\widetilde{A}_3$
T1 FSs	8.21	7.03	4.84
T2 FSs	8.72 ± 1.15	7.53 ± 1.42	5.27 ± 1.44

The second experiment is to assess the adaptability of the proposed method in providing accurate and reliable results. As Table 5 shows, the method of this study can give the uncertainty interval of the final ranking values (similar to the confidence interval in probability statistics) according to the uncertainty information existing in the evaluation data sources, and this uncertainty interval can be an important decision-making basis. It can be observed from the above experiments that the overall scores of teaching performance are related to the indicator weights. When the ranking values of several teachers' teaching performance are almost the same, an incorrect conclusion may be drawn when rank teachers simply rely on ranking values. For example, indicator weights are established as in Table 6 in this experiment:

Table 6. Indicator weights given by evaluation experts.

Indicators	Type 2 Fuzzy Weights
$(T_{11}, T_{12}, \ldots, T_{15})$	$(\widetilde{1}, \widetilde{3}, \widetilde{9}, \widetilde{9}, \widetilde{7})$
$(T_{21}, T_{22}, \ldots, T_{25})$	$(\widetilde{3}, \widetilde{7}, \widetilde{9}, \widetilde{5}, \widetilde{1})$
$(T_{31}, T_{32}, \ldots, T_{36})$	$(\widetilde{9}, \widetilde{1}, \widetilde{7}, \widetilde{3}, \widetilde{5})$
(T_{41}, T_{42}, T_{43})	$(\widetilde{1}, \widetilde{7}, \widetilde{9}, \widetilde{7}, \widetilde{7}, \widetilde{3})$
$(T_{51}, T_{52}, T_{53}, T_{54})$	$(\widetilde{7}, \widetilde{9}, \widetilde{1})$

Figure 7a,b show the overall ratings of teaching performance obtained by these two methods. It can be observed from Figure 7b that the membership functions of $\widetilde{A}_1$ and $\widetilde{A}_2$ overlap a lot, resulting difficulty in distinguishing them from each other. Table 7 shows the teaching performance ranking values provided by these two methods. Depending on the ranking values alone, both methods can reach the same conclusion $\widetilde{A}_1 \succ \widetilde{A}_2 \succ \widetilde{A}_3$. However, the ranking values of $\widetilde{A}_1$ and $\widetilde{A}_2$ are almost equal, making them difficult to distinguish from each other.

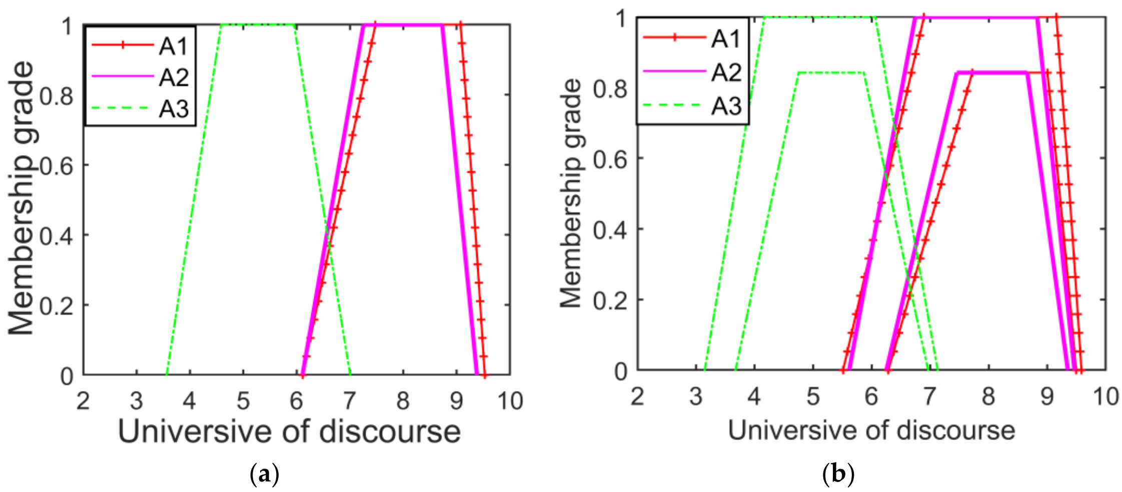

Figure 7. Comparison results of the evaluation method based on T1 FSs and the method of this study: (**a**) Overall ratings obtained by the evaluation method based on T1 FSs; (**b**) Overall ratings obtained by the method of this study.

Table 7. Teaching performance rankings given by the evaluation method based on T1 FSs and the method of this study.

Method	$\widetilde{A}_1$	$\widetilde{A}_2$	$\widetilde{A}_3$
T1 FSs	7.29	7.26	4.85
T2 FSs	7.92 ± 1.48	7.78 ± 1.42	5.28 ± 1.45

Table 8 shows the similarities among the overall scores $\widetilde{A}_i$ $(i = 1, 2, 3)$ in Figure 7b. It can be observed that the similarity between $\widetilde{A}_1$ and $\widetilde{A}_2$ is up to 91.49%. Therefore, it is hard to judge who teaches better based on the T1 FSs method. However, the coverage of the uncertainty band $r(\widetilde{A}_1)$ obtained by the method of this study is wider than that of the uncertainty band $r(\widetilde{A}_2)$. Students may in some cases prefer a teacher with a more stable teaching performance. In that situation, teacher A_2 teaches better than teacher A_1. However, the evaluation method based on T1 FSs cannot provide the uncertainty band information, and it may draw incorrect conclusions because it is simply relying on ranking numbers to rank the teachers. In the proposed method, the IT2 FS result of the performance evaluation contains uncertainty information, like the uncertain band, similarity and entropy information, which is much more information than just a single ranking number. This uncertain information can provide an important decision-making basis for the decision makers to make correct conclusions. In short, the proposed method is more flexible and accurate in preserving and processing uncertainties in human decisions, and can reflect the uncertainties in the final conclusions. It better reflects the fuzziness and uncertainty of the decision data and the decision process, which is more in accordance with human cognitive habits and ways of thinking, and can provide more accurate evaluation conclusions.

Table 8. Similarities among the overall scores in Figure 7b.

IT2 FS	$\widetilde{A}_1$	$\widetilde{A}_2$	$\widetilde{A}_3$
$\widetilde{A}_1$	1.000	0.9149	0.2857
$\widetilde{A}_2$	0.9149	1.000	0.2951
$\widetilde{A}_3$	0.2857	0.2951	1.000

5. Discussion

In the proposed method, all the input data sources are words, represented by IT2 FSs, overcoming the limitations of previous models. By using the Per-C architecture and the LWA method for modeling and aggregating the input data sources, all the uncertainties associated with the words, indicator weights and indicator scores can be integrated in the aggregation process and be reflected in the final results, which can guarantee accurate and reliable evaluation results. As the experimental results demonstrate, the uncertainties incorporated in the final results can provide an important decision-making basis for the decisions makers. In summary, the proposed method can provide accurate and reliable evaluation conclusions, and can provide a useful tool for evaluating the teaching performance of university teachers in a more flexible and intelligent manner.

Nonetheless, this study holds several limitations. The evaluation of teaching performance of university teachers involves many indicators and factors. This study mainly considers five first level indicators and 23 sub-indicators. In practical applications, the indicator system should be extended or improved according to different requirements, in order to improve the adaptability of the model. At the same time, there are often some relationships between the indicators. However, the relationship between indicators is difficult to define due to the complexity of the teaching and evaluation process. How to consider the relevance and relationship between indicators in the evaluation process still deserves further investigation. Moreover, T2 FSs have higher computational complexity and costs compared with T1FSs. How to reduce the computational costs of T2 FSs should also be further studied. Research could possibly be carried out on computer technology to solve such problems. Besides, although operators like the LWA operator adopted in the Per-C method are proven to be effective and are widely applied in the field of fuzzy MADM, they still have some disadvantages and there is still a wide domain for improving these operators. The design of more effective and reliable operators should be investigated in the future.

6. Conclusions

This study proposes a comprehensive university teachers' teaching performance evaluation method based on type-II fuzzy sets. By investigating and analyzing the teaching characteristics of university teachers, the indicator system of university teachers teaching performance evaluation is established. Then, considering the large amount of fuzzy and uncertainty information inherent in human decisions, the trapezoidal interval type-II fuzzy sets are introduced to represent the input data sources, managing the uncertainty of human decisions effectively. Then, the linguistic weighted average operator is used as a computing with words engine to integrate the indicators and sub-indicators, enabling the fuzzy uncertainty existing widely in the data sources to be effectively integrated into the final conclusions and guaranteeing the accuracy of the evaluation results. Finally, practical examples are adopted to verify the validity and feasibility of the method. Compared with the type-I fuzzy sets method, the proposed method can provide more accurate and reliable evaluation results, and has better practicability.

Multi-attribute decision making problems exist extensively in the areas of science and engineering. Although this study adopts the proposed evaluation method to solve the teaching performance evaluation problem, the ideas of the method are universal. Therefore, the proposed evaluation method can also be used to solve multi-attribute decision making problems in various areas, such as dynamical systems, cloud computing, etc.

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