

Appendix 2. Original task and rubric by group 1

Read the following paragraph and answer the questions.

A bus company was concerned about calculating the profit earned at the time of work (from 6 am to 9 am) for a bus passing on route A three years ago. If the time interval between buses is too long, the number of passengers may decrease, resulting in a decrease in revenue. If the time interval is too short, the operating cost increases as the number of buses is dispatched, and thus revenue may decrease. Based on this, the profit function of the time interval was approximated by $f(t) = -(t - 10)^2 + 300$ where t stands for the time interval in minutes. The company can maximize its profit to 300 (ten-thousand won) when $t = 10$, and so they have scheduled vehicles every 10 minutes for the last three years. (Let's assume that only the number of passengers and operating costs affect revenue.)

However, some staff have suggested using the cubic function approximation method for a more accurate approximation than the quadratic function approximation. For example, if the time interval decreases from 10 minutes to 1 minute and becomes 9 minutes, we are trying to find a cubic function approximation.

This bus company accepted this employee's opinion and after investigating relevant factors, it is expected that when the time interval decreases by t_1 ($0 \leq t_1 < 10$) from 10 minutes during rush hours, the profit will change by $-\frac{9}{8}t_1^3 - \frac{9}{8}t_1^2 + 63t_1$. When the time interval increases by t_2 ($0 \leq t_2 \leq 170$) from 10 minutes, the profit will change by $g(t_2)$. For example, if the time interval between buses is reduced by 2 minutes, $(-9 - 4.5 + 126 = 112.5)$ ten-thousand won will change, so a profit of $333 + 112.5 = 421.5$ ten-thousand won can be expected.

(1) The following table shows a part of a chart that an employee of this company used to find $h(t)$ on a closed interval $[0, 170]$. Find $h(t)$.

t	1	4
$h(t)$	80	140
$h'(t)$	80	

(2) In the table above, what is the meaning of the derivative $h'(1)$? Explain its meaning in relation to the profit.

(3) Draw, by using the table of the increase and decrease in y-values, a sketch of the graphs, $y = g(x)$ defined on the interval $(-10, 0)$ and $y = h(x)$ defined on the interval $[0, 170]$.

(4) Considering this particular bus line, find the range of time intervals between vehicles during rush hours that would generate more profit than the last three years, and find the time interval between vehicles during rush hours that will maximize profits.

	score	criteria
(1)	0	In case of a blank or meaningless statement
	1	In case that $h(x) = at^3 + bt^2 + ct + d$ is denoted
	2	In the case of establishing a system of four equations by substituting $h(0)$
	3	In the case of where $h(t)$ was not found correctly because a, b, c, d was obtained incorrectly due to a calculation mistake in the process of performing the system of equations.
	4	When the system of equation is solved correctly without calculation errors and $h(t)$ is obtained correctly.

	score	criteria
(2)	0	In case of a blank or meaningless statement
	2	When only mathematical meanings such as instantaneous rate of change are described
	4	In the case of correlating revenue with mathematical meaning, such as the instantaneous rate of change of revenue when the time interval is 11 minutes

	score	criteria
(3)	0	In case of a blank or meaningless statement
	1	When the derivative of $g(x)$ is found
	2	When a table showing the increase and decrease of $g(x)$ is drawn but the table is drawn incorrectly
	3	When a table showing the increase and decrease of $g(x)$ is drawn correctly
	4	When the graph of $g(x)$ is drawn correctly using the above table.

* $h(x)$ is scored in the same way and given 4 points each, i.e. a total of 8 points

	score	criteria
(4)	0	In case of a blank or meaningless statement
	1	When $g(t)$ and $h(t)$ are factored separately
	2	If only one of the case where the profit increases and the case of where the profit is maximum is found correctly
	3	If one solved the problem logically but didn't write the exact meaning like $3 < x < 15$ ($x \neq 10$), $x=13$
	4	When the time interval is between 3 and 10 minutes or between 10 and 15 minutes, the profit increases, and when the time interval is 13 minutes, the profit is maximum, etc.