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A Mathematical Model of the Production Inventory Problem for Mixing Liquid Considering Preservation Facility

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Abstract: The mixing process of liquid products is a crucial activity in the industry of essential commodities like, medicine, pesticide, detergent, and so on. So, the mathematical study of the mixing problem is very much important to formulate a production inventory model of such type of items. In this work, the concept of the mixing problem is studied in the branch of production inventory. Here, a production model of mixed liquids with price-dependent demand and a stock-dependent production rate is formulated under preservation technology. In the formulation, first of all, the mixing process is presented mathematically with the help of simultaneous differential equations. Then, the mixed liquid produced in the mixing process is taken as a raw material of a manufacturing system. Then, all the cost components and average profit of the system are calculated. Now, the objective is to maximize the corresponding profit maximization problem along with the highly nonlinear objective function. Because of this, the mentioned maximization problem is solved numerically using MATHEMATICA software. In order to justify the validity of the model, two numerical examples are worked out. Finally, to show the impact of inventory parameters on the optimal policy, sensitivity analyses are performed and the obtained results are presented graphically.

Keywords: mixing process; simultaneous differential equations; variable production rate; simulated annealing; differential evolution

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1. Introduction

The mixing problem has a great impact on different sectors of business management, viz. the medicine industry (Gautam et al. [1], Essi [2], Ploypetchara et al. [3]), cosmetics industry (Bernardo and Saraiva [4], Kim et al. [5], Zhang et al. [6]), chemical industry (Funt [7], Wu et al. [8], Jasikova et al. [9]), and so on, to produce essential commodities in our daily life. Thus, in the area of inventory control, investigation of the production inventory problem of a mixed product along with the mixing process is an intersecting research area. In this connection, Nienow et al. [10], Cheng et al. [11], Fitschen et al. [12], and many others have had a valuable influence in this area. As various inventory parameters like production rate, demand rate, deterioration rate, and preservation technology play a significant role to control a production inventory, researchers should take more care of those inventory parameters in the studying of the production inventory problem with the mixing process.

In production inventory, the production rate of the product is the key parameter that may be constant or dependent on customers' demand/stock level of the product, among others. On the other hand, owing to the failure of machines, sometimes imperfect production occurs during the production process. Thus, imperfect production is also an important factor for production firm/manufacturing firm. Several researchers developed different production models by taking various production rates and imperfect production processes. De [13] analyzed a production problem with a variable rate of production. Su and Lin [14,15] investigated two production inventory models with a demand- as well as inventory-level-dependent production rate. After that, Giri et al. [16] analyzed an unreliable production system with variable production. Roy et al. [17] studied a production inventory model for defective products with rework policy. Considering an imperfect production process, Sana [18] formulated a production inventory model. A few years later, Sharmila and Uthayakumar [19] established the optimal policy of a production problem with three different production rates. Then, Patra and Maity [20] developed a production problem for defective items with a variable production rate. Dey [21] investigated an imperfect production model under an integrated system in an imprecise environment. Succeeding them, Mishra et al. [22] studied the sustainability of a production system under controllable carbon emission. Lu et al. [23] applied the Stackelberg gaming approach to determine the optimal policy of an imperfect production inventory model with collaborative investment policy for reducing emission. Recently, Öztürk et al. [24] studied an imperfect production process with random breakdowns, rework, and inspection costs and Khara et al. [25] formulated an imperfect production model considering advanced payment and trade credit facilities. Beside these, the works of Malik et al. [26], Lin et al. [27], and Rizky et al. [28] are valuable in this area.

Demand of customers is also an important factor in inventory control. It depends on several factors, such as selling price of the product, inventory level, frequency of the advertisements, time, and so on. In reality it is seen that, if the price of a commodity increases, the demand for that commodity must decrease, i.e., the selling-price-dependent demand rate is a decreasing function. On the other side, more customers are attracted because of the large number of items in stock, i.e., the stock-dependent demand rate is an increasing function of the stock level of the items. Sometimes, the customers' demand for a new product increases drastically owing to the advertisement of the product. Thus, advertisement frequency has a great impact on the demand rate. Resh et al. [29] first introduced the variable demand rate (selling-price-dependent) in the area of inventory control and modified Harris's EOQ model. Urban [30] analyzed an inventory model with stock-linked demand. Chang [31] studied a model for optimal lot sizing with a nonlinear stock-linked demand rate. Mukhopadhyay et al. [32] and You [33] studied different types of EOQ models with price-dependent demand. After a few years, Khanra et al. [34] constructed an inventory model with a time-dependent demand rate under trade credit policy. Further, Bhunia and Shaikh [35] studied a deterministic inventory model with price-dependent demand and a three-parameter Weibull distributed deterioration rate. Prasad and Mukherjee [36] proposed an inventory model where the demand rate is connected to stock and time, along with shortages. Manna et al. [37] investigated a production inventory model with imperfect production and advertisement-dependent demand. Jain et al. [38] investigated a fuzzy inventory model where the demand for an item is dependent on time. Recently, the contributions of Alfares and Ghaithan [39], Shaikh et al. [40], Rahman et al. [41], Cardenas-Barron et al. [42], Das et al. [43], Halim et al. [44], Rahman et al. [45], and others on this topic are worth mentioning.

Deterioration is also important in the control of inventory. Most of the commodities in our daily life deteriorate with the passing of time owing to the several factors. Thus, to study an inventory problem for deteriorating items, we cannot avoid the effect of deterioration. Naturally, the deterioration rate of an item cannot be predicted accurately. However, it was taken as constant or time-dependent or probabilistic by several researchers. In their work, for the first time, Ghare and Schrader [46] proposed the concept of deterioration (constant). Then, Emmons [47] proposed the concept of stochastic deterioration with two-parameter Weibull distribution. Since then, a number of research works have been reported in the existing literature. Among those, the works of Datta and Pal [48], Wee [49], Ouyang et al. [50], Min et al. [51], Dash et al. [52], Dutta and Kumar [53], Shah [54], Tiwari et al. [55], Shaw et al. [56], Mashud et al. [57], Khakzad and Gholamian et al. [58], Mishra et al. [59], Khanna and Jaggi [60], and Naik and Shah [61] are worth mentioning.

On the other side, the economy of an industry is badly affected by reckless deterioration. Thus, in the case of more deterioration, the control of deterioration is highly required. Usually, to prevent more deterioration, some policies/techniques are adopted, named preservation policies/technologies. For the first time, Hsu et al. [62] investigated the concept of preservation technology in the area of inventory control. After that, Dye [63] discussed the preservation investment effect on deterioration rate. Zhang et al. [64] solved an inventory problem for perishable goods by considering stock-dependent demand and investment in preservation technology. Yang et al. [65] proposed an inventory model under preservation technology and trade-credit policy. Tayal et al. [66] studied an inventory problem for a perishable product with a permissible delay in payment along with investment in preservation technology. Dhandapanin and Uthayakumar [67] analyzed the optimal policy of a multi-item inventory model under preservation technology. Recently, Shaikh et al. [68], Das et al. [69], Saha et al. [70], Mashud et al. [71], Sepehri et al. [72], and others contributed through their works on preservation technology.

The organization of the paper is according to Figure 1. In this work, a production problem for mixed liquid and price-dependent demand is formulated. In this formulation, at first, the mixing process is presented mathematically by the simultaneous differential equations under some restrictions. Then, the corresponding optimization problem related to this model is obtained as the profit maximization problem. Because of the high nonlinearity of the objective function (average profit), the mentioned maximization problem is solved by differential evolution and simulated annealing in Mathematica software. Then, to investigate the validation of the model, two numerical examples are solved. Finally, sensitivity analyses are performed graphically and this work is concluded with some future scopes. A summary of some of the literature is presented in Table 1.

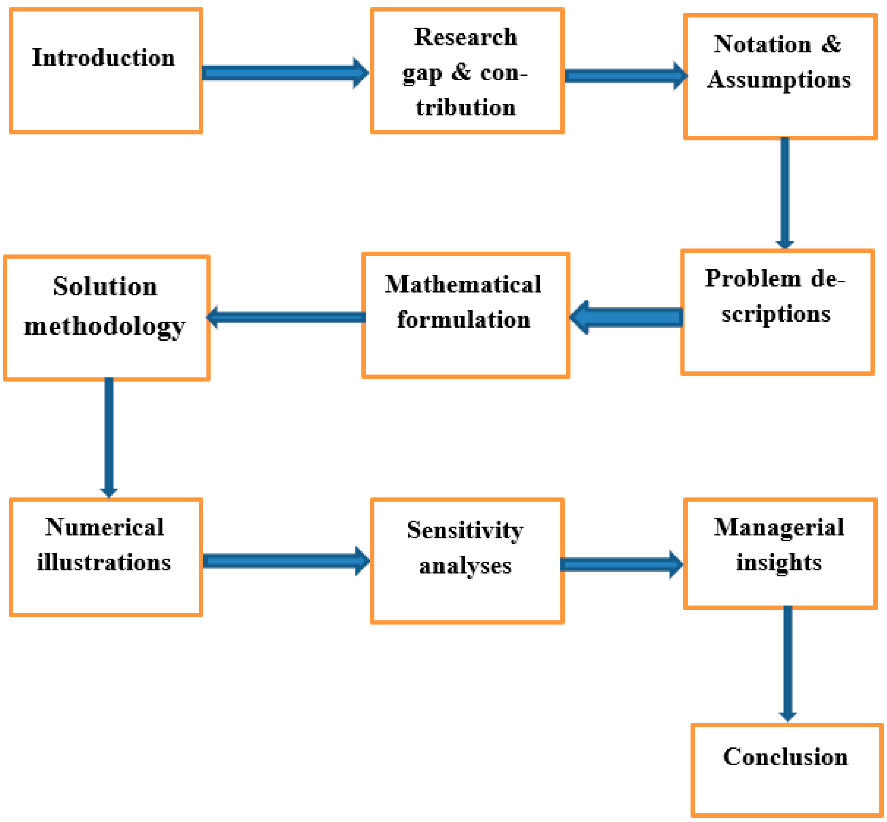

Figure 1. Organization of the paper.

Table 1. Summary of the related literature.

Literature	Simultaneous Differential Equation	Production Rate	Demand	Deterioration
Su and Lin [14]	No	Variable	Variable	_____
Kapuscinki and Tayur [73]	No	Constant	Periodic	_____
Sana et al. [74]	No	Constant	Time varying	Constant
Lo et al. [75]	No	Constant	Constant	Weibull distributed
Roy et al. [17]	No	Imprecise	Imprecise	Imprecise
Sarkar [76]	No	Constant	Constant	Probabilistic
Samanta [77]	No	Constant	Constant	Probabilistic
Bhunia et al. [78]	No	Constant	Variable	_____
Rastogi and Singh [79]	No	Demand-dependent	Selling-price-dependent	Time-dependent
Ullah et al. [80]	No	Constant	Constant	Constant
Salas-Navarro et al. [81]	No	Constant	Probabilistic	_____
Das and Islam [82]	No	Time-dependent	Time-dependent and imprecise	_____
Saren et al. [83]	No	Constant	Selling-price- and time-dependent	Constant
Khanna and Jaggi [60]	No	_____	Price- and stock-dependent	Preservation-technology-dependent
Sepehri et al. [72]	No	Constant	Selling-price-dependent	Constant
This Work	Yes	Variable	Selling-price-dependent	Preservation-technology-dependent

2. Research Gap and Contributions

After a brief survey of the literature, it is conjectured that many works have been accomplished on production inventory (Table 1) for different types of products, such as food, electrical goods, garments, medicine, and so on, with various assumptions regarding the production rate, demand rate, deterioration rate, and so forth. On the other hand, the concepts of the mixing problem are essential in the production manufacturing of liquid products (like, medicine, juice, cosmetics, and so on). To the best of our knowledge, few works on the mixing process (viz. Essi [2], Ploypetchara et al. [3], Kim et al. [5], Jasikova et al. [9], and Fitschen et al. [12], among others) are available in the literature. However, very few researchers ([84,85]) considered the combination of the mixing process as well as manufacturing process in his/her work. Though Su et al. [84] accomplished their work on production inventory for mixed products, they did not consider the mathematical formulation of the mixing process.

To fill this gap, a production inventory model for mixed liquid was formulated by defining the mixing process mathematically. Here, the mixing process of liquids is considered as a part of the production process. The mixing process is presented mathematically by simultaneous differential equations. Then, in the manufacturing part of this modelling, the variable production rate (dependent on the stock level of mixed liquid) and preservation technology are considered. The mentionable contributions of this study are as follows:

(i). Application of simultaneous linear differential equations (to the present mixing process) in the production inventory system.
(ii). Linkage between the mixing process and manufacturing process.
(iii). Consideration of the variable production rate in the manufacturing process dependent upon the stock level of mixed liquid.

All of the above represent the novelty of this work.

3. Notation and Assumptions

The following notations and assumptions are used thought the manuscript.

3.1. Notation

The notations used in this paper are as follows:

$x(t)$	Concentration of liquid at time t in container-I (%)
$y(t)$	Concentration of liquid at time t in container-II (%)
$q(t)$	Stock level of mixed liquid (L)
A	Capacity of container-I (L)
B	Capacity of container-II (L)
α	Incoming rate of liquid with concentration k in container-I (L/unit time)
β	Outgoing rate of liquid from container-I to container-II (L/unit time)
γ	Incoming rate of liquid from container-II to container-I (L/unit time)
δ	Outgoing rate of liquid 2 from container-II (L/unit time)
η	Initial concentration of liquid in container-II (%)
k	Concentration of liquid supplied from outside (%)
a, b	Demand parameters
$D(p)$	Demand of the customers
P	Production rate (L/time unit)
θ_1	Wastage rate during production without preservation
m	Preservation controlling parameter
ξ	Preservation investment ($)
c_p	Processing cost ($/L)
p	Selling price per unit ($/L)
C_o	Set up cost ($/order)
h	Carrying cost per unit per unit time ($/L/time unit)
t_1	Duration of production (time unit)
T	Cycle length (time unit)
$TP(t_1, p, \xi)$	Average profit ($/time unit)

3.2. Assumptions

(i). This work deals with a mixture of three different concentrations of a liquid.

(ii). The capacity of container-I filled with liquid with an initial concentration of zero (container) is less than the capacity of container-II filled with liquid with an initial concentration of k.

(iii). At first, the liquid with concentration η is sent to container-I at the rate α. Then, the mixture of liquid is sent to container-II at the rate β, and the liquid is sent back to container-I from container-II at the rate γ; this process continues to obtain the best desirable mixture. After reaching the desired mixture, the mixed liquid from container-II at the rate δ is used in the production process.

(iv). The production rate of the mixed product $P(t)$ is proportional to the level of mixed liquids ($y(t)$). The mathematical form of $P(t)$ is $P(t) = \frac{\delta}{B} y(t)$.

(v). The wastage/deterioration rate θ during production is dependent on preservation technology. The mathematical form of the deterioration rate is $\theta = \theta_1 e^{-m\xi}$, where ξ is the preservation investment, m is the preservation controlling parameter, and θ_1 is the original deterioration rate.

(vi). The demand of an item is dependent on selling price and its mathematical form is $D(p) = a - bp,\ a, b, p > 0$, such that $p < \frac{a}{b}$.

(vii). Shortages are not allowed.

(viii). Time horizon is infinite and lead time is constant.

4. Problem Description

The problem of the proposed model has two parts: (i) the mixing problem and (ii) the production inventory problem. In the mixing problem, the process of mixing takes place on an instrument made by two containers (Figure 2). In this instrument, container-I is connected to container-II by a pipe line so that the liquid can pass from container-I to container-II, and vice versa. Initially, liquids of two different concentrations (η and k) are taken to make the initial mixture. During mixing, the liquid with a concentration η is passed through container-I at the rate α and then from container-I to container-II with the rate β. Again, the mixed liquid is returned back from container-II to container-I with the rate γ, and this process is continued to obtain the desired mixed liquid. Finally, the desired mixed liquid exits from container-II at the rate δ. The entire process of mixing is presented in Figure 2. Then, in the part of production process, the desired mixture is taken as a raw material and a single product is produced at the production rate $P(t)\left(P(t) = \frac{\delta}{B}y(t)\right)$. During the production period, owing to the customers' demand, the produced product is stored with the rate $(P - D)$ per unit time and the level of inventory reaches its pick level at time $t = t_1$. After that, the level of stock gradually decreases because of fulfilling the demand of the customer and the stock level reaches zero at time $t = T$. The variation in the level of inventory at any time t is shown in Figure 3.

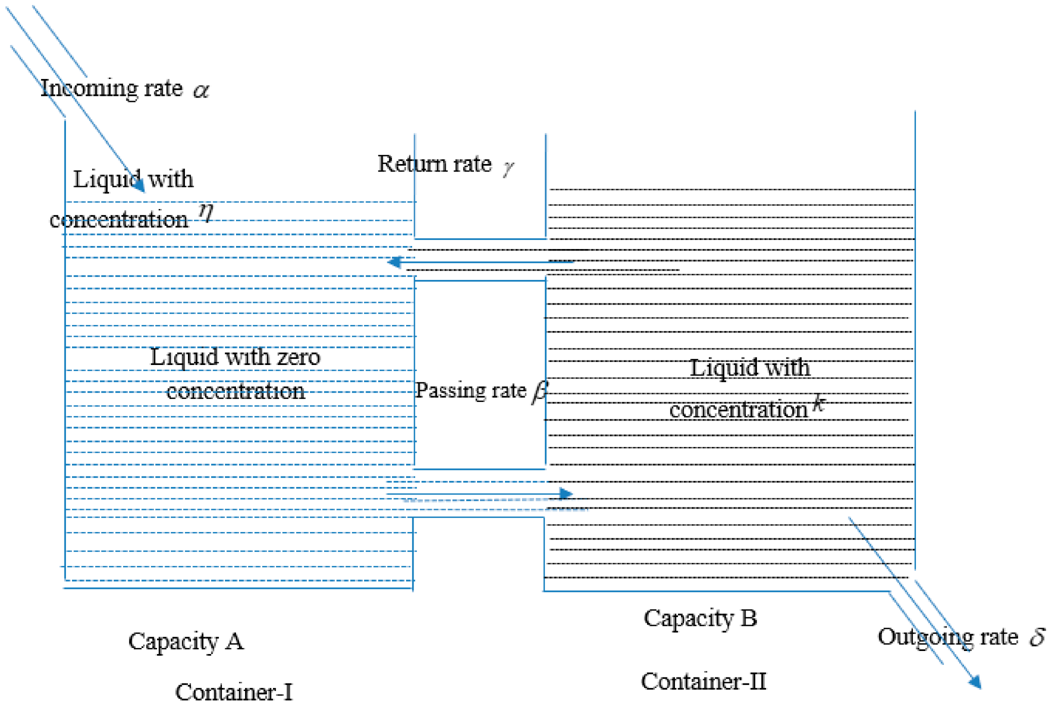

Figure 2. Representation of the mixing procedure in the production process.

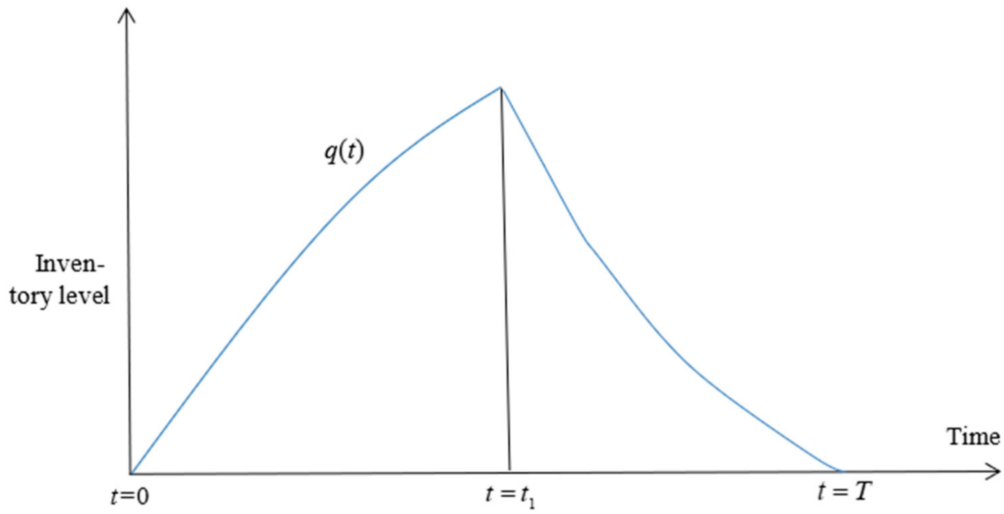

Figure 3. Changes in inventory level with respect to time.

5. Mathematical Formulation

Here, we have discussed the mathematical formulation of mixing and the production inventory system.

5.1. Mathematical Formulation of Mixing Problem

The mixing process described in the previous section is presented mathematically by the following simultaneous differential equations:

$$\begin{aligned} \dot{x} &= \eta\alpha - \tfrac{\beta}{A}x + \tfrac{\gamma}{B}y \\ \dot{y} &= \tfrac{\beta}{A}x - \tfrac{\gamma+\delta}{B}y \end{aligned} \tag{1}$$

Subject to the initial conditions $x(0) = 0$, $y(0) = kB$, where $\delta < \gamma < \beta$ and $0 < k < 1$. Moreover, from the principle of flow, we get

$$\beta = \alpha + \gamma = \gamma + \delta \tag{2}$$

Solving the system (1), one can obtain the concentrations of the liquids in container-I and container-II as follows:

$$x(t) = \exp\left(-\frac{k_1}{2}t\right)\{c_1\exp(k_4 t) + c_2\exp(-k_4 t)\} + \frac{k_3}{k_2} \tag{3}$$

$$y(t) = \frac{B}{\gamma}\left[c_1\left(k_4 - \frac{k_1}{2} + \frac{\beta}{A}\right)\exp\left\{\left(k_4 - \frac{k_1}{2}\right)t\right\} + c_2\left(-k_4 - \frac{k_1}{2} + \frac{\beta}{A}\right)\exp\left\{\left(-k_4 - \frac{k_1}{2}\right)t\right\}\right] - \frac{\eta}{\gamma}\alpha B + \frac{\beta B k_3}{\gamma A k_2} \tag{4}$$

where

$$\begin{aligned} k_1 &= \tfrac{\beta}{A} + \tfrac{\gamma+\delta}{B}, \\ k_2 &= \tfrac{\beta\delta}{AB} \\ k_3 &= \eta\tfrac{\alpha(\gamma+\delta)}{B} \\ k_4 &= \tfrac{\sqrt{k_1{}^2 - 4k_2}}{2} \\ c_1 &= -\tfrac{k_3}{2k_2} + \tfrac{1}{2k_4}\left[k\gamma + \eta\alpha + \tfrac{k_3}{k_2}\left(\tfrac{\beta}{A} - \tfrac{k_1}{2}\right) - \tfrac{\beta k_3}{Ak_2}\right] \end{aligned}$$

and

$$c_2 = -\frac{k_3}{2k_2} - \frac{1}{2k_4}\left[k\gamma + \eta\alpha + \frac{k_3}{k_2}\left(\frac{\beta}{A} - \frac{k_1}{2}\right) - \frac{\beta k_3}{Ak_2}\right].$$

5.2. Mathematical Formulation of the Production Problem

The inventory level of the problem at any time t satisfies the governing differential equations

$$\frac{dq(t)}{dt} + \theta q(t) = P(t) - D \text{ for } 0 \le t \le t_1 \tag{5}$$

$$\frac{dq(t)}{dt} + \theta q(t) = -D \text{ for } t_1 < t \le T \tag{6}$$

with the conditions $q(0) = 0$, $q(t_1) = Q$ and $q(T) = 0$.

The solutions of Equations (5) and (6) are given by

$$\begin{aligned} q(t) &= \left(\frac{\beta\delta k_3}{\gamma\theta k_2 A} - \frac{D}{\theta} - \frac{\eta\alpha\delta}{\gamma\theta}\right)\{1 - \exp(-\theta t)\} \\ &+ \frac{c_1}{\left(k_4 - \frac{k_1}{2} + \theta\right)}\left\{\left(k_4 - \frac{k_1}{2}\right)\frac{\delta}{\gamma} + \frac{\beta\delta}{\gamma A}\right\}\left[\exp\left\{\left(k_4 - \frac{k_1}{2}\right)t\right\} - \exp(-\theta t)\right] \\ &+ \frac{c_2}{\left(k_4 + \frac{k_1}{2} - \theta\right)}\left\{\left(k_4 + \frac{k_1}{2}\right)\frac{\delta}{\gamma} - \frac{\beta\delta}{\gamma A}\right\}\left[\exp\left\{-\left(k_4 + \frac{k_1}{2}\right)t\right\} - \exp(-\theta t)\right] \text{ for } 0 < t \le t_1 \end{aligned} \tag{7}$$

and

$$q(t) = \frac{D}{\theta}\{\exp(\theta(T - t)) - 1\} \text{ for } t_1 < t \le T \tag{8}$$

Again, using the continuity of $q(t)$ at $t = t_1$, we have

$$\begin{aligned} T = \frac{1}{\theta}\log\Bigg[\frac{\theta}{D}\Bigg[&\left(\frac{\beta\delta k_3}{\gamma\theta k_2 A} - \frac{D}{\theta} - \frac{\eta\alpha\delta}{\gamma\theta}\right)\{1 - \exp(-\theta t_1)\} \\ &+ \frac{c_1}{\left(k_4 - \frac{k_1}{2} + \theta\right)}\left\{\left(k_4 - \frac{k_1}{2}\right)\frac{\delta}{\gamma} + \frac{\beta\delta}{\gamma A}\right\}\left[\exp\left\{\left(k_4 - \frac{k_1}{2}\right)t_1\right\} - \exp(-\theta t_1)\right] \\ &+ \frac{c_2}{\left(k_4 + \frac{k_1}{2} - \theta\right)}\left\{\left(k_4 + \frac{k_1}{2}\right)\frac{\delta}{\gamma} - \frac{\beta\delta}{\gamma A}\right\}\left[\exp\left\{-\left(k_4 + \frac{k_1}{2}\right)t_1\right\} - \exp(-\theta t_1)\right]\Bigg] + 1\Bigg] + t_1 \end{aligned} \tag{9}$$

5.3. Various Components of the System

The various components of the system are calculated as follows:

(i). Sales revenue (SR):

$$SR = p\int_0^T D\,dt = pDT$$

(ii). Ordering cost (C_o):

(iii). Holding cost (HC):

$$\begin{aligned} HC &= h\int_0^{t_1} q(t)\,dt + h\int_{t_1}^{T} q(t)\,dt \\ &= h\left(\frac{\beta\delta k_3}{\gamma\theta k_2 A} - \frac{D}{\theta} - \frac{\eta\alpha\delta}{\gamma\theta}\right)\left\{t_1 - \frac{1}{\theta}(1 - \exp(-\theta t_1))\right\} \\ &\quad + \frac{hc_1}{\left(k_4 - \frac{k_1}{2} + \theta\right)}\left\{\left(k_4 - \frac{k_1}{2}\right)\frac{\delta}{\gamma} + \frac{\beta\delta}{\gamma A}\right\}\left[\frac{\exp\left\{\left(k_4 - \frac{k_1}{2}\right)t_1\right\} - 1}{\left(k_4 - \frac{k_1}{2}\right)} - \frac{1}{\theta}(1 - \exp(-\theta t_1))\right] \\ &\quad + \frac{hc_2}{\left(k_4 + \frac{k_1}{2} - \theta\right)}\left\{\left(k_4 + \frac{k_1}{2}\right)\frac{\delta}{\gamma} - \frac{\beta\delta}{\gamma A}\right\}\left[\frac{1 - \exp\left\{-\left(k_4 + \frac{k_1}{2}\right)t_1\right\}}{\left(k_4 + \frac{k_1}{2}\right)} - \frac{1}{\theta}(1 - \exp(-\theta t_1))\right] \\ &\quad + \frac{hD}{\theta^2}[\exp\{\theta(T - t_1)\} - 1] - \frac{hD}{\theta}(T - t_1) \end{aligned}$$

(iv). Production cost (PC):

$$\begin{aligned}
\text{PC} &= c_p \int_0^{t_1} P(t)\,dt \\
&= c_p \frac{\delta}{B} \int_0^{t_1} y(t)\,dt \\
&= c_p \frac{\delta}{\gamma}\left[\frac{c_1\left(k_4-\frac{k_1}{2}+\frac{\beta}{A}\right)}{\left(k_4-\frac{k_1}{2}\right)}\left(\exp\left\{\left(k_4-\frac{k_1}{2}\right)t_1\right\}-1\right)+\frac{c_2\left(-k_4-\frac{k_1}{2}+\frac{\beta}{A}\right)}{\left(k_4+\frac{k_1}{2}\right)}\left(1-\exp\left\{\left(-k_4-\frac{k_1}{2}\right)t_1\right\}\right)\right] \\
&\quad + c_p \frac{\delta}{\gamma}\left(\frac{\beta k_3}{A k_2}-\eta\alpha\right)t_1
\end{aligned}$$

(v). Preservation cost: $CP = \xi T$.

Therefore, the profit per unit time of the system is given by

$$TP(t_1, p, \xi) = \frac{1}{T}[SR - PC - HC - C_o - CP]$$

Now, the corresponding maximization problem of the system is given by

$$\begin{gathered}
\text{Maximize } TP(t_1, p, \xi) \\
\text{subject to } t_1 > 0,\ 0 < p < \tfrac{a}{b}
\end{gathered} \tag{10}$$

6. Solution Methodology

The corresponding optimization problem (10) of the proposed production system is clearly highly non-linear in nature with respect to the decision variables t_1, p, ξ. It is difficult to solve (10) by any analytical method, such as the gradient-based technique, Lagrange's multiplier method, Newton's method, saddle point optimization techniques, and so on. Thus, in order to solve the mentioned optimization problem (10), the following algorithms built in MATHEMATICA software are used:

(i) Differential evolution (Price, 1996);
(ii) Simulated annealing (Marchesi, 1988).

The discussions of the above-mentioned algorithms are done based on the following generalized optimization problem:

$$\begin{gathered}
\text{Maximize } f(u) \\
\text{subject to } t \in S \subseteq \mathbb{R}^n \\
\text{where } f : S \to \mathbb{R}
\end{gathered}$$

(i) Differential Evolution (DE)

Differential evolution is one of the popular search techniques in the area of optimization. The algorithm of this optimizer has the following attributes:

- The initial positions of the population of size m are $\{u_1, u_2, \ldots, u_m\}$, $m \gg n$
- In the evaluation process for each iteration, the algorithm generates a new population with m points. Using the three points u_u, u_v and u_w, the algorithm generated the jth new point randomly from the previous population.
- The mathematical form is $u_s = u_w + s(u_u - u_v)$, where s is a scaling parameter.
- The new point u_{new} is created from u_j and u_s with the help of the ith coordinate from u_s along with probability ρ, otherwise it will take the coordinate from u_j.
- If $f(u_{new}) > f(u_j)$, then u_{new} replaces u_j in the new population.
- The probability ρ is controlled by the "cross probability" option.

Generally, this process is converged if deviation in between the best functional values in the new position and old population as well as the deviation between the new best point and the old best point are less than the tolerances.

The values of parameters used in the Differential Evolution are given in Table 2.

Table 2. The values of parameters used in the Differential Evolution.

Operator Name	Default Value	Descriptions
"Cross Probability"	0.5	Probability of a gene taken from t_i
"Random Seed"	0	It is a starting value of random number generator
"Scaling Factor"	0.6	Scale applied to the deviation vector in creating a mate
"Tolerance"	0.001	It is accepting constraint violations

(ii) Simulated Annealing (SA)

Simulated annealing is another random search-based meta-heuristic maximizer. The algorithm of this maximizer is inspired by physical activity of annealing, in which a metallic object is warmed up to an extreme temperature and allowed to cool gently. In this process, the atomic structure of metal reaches the lower energy level from the upper, and thus becomes a tougher metal. Exploring this concept in optimization, the algorithm of simulated annealing allows to move away from a local minimizer, and to traverse and settle on a better position and, ultimately, on the global maximizer.

During the iterative process, a new point u_{new} is created in the neighboring point u. Thus, the radius of the neighborhood is decreased from iteration to iteration. The best-found point u_{best} obtained so far is tracked as follows:

If $f(u_{new}) > f(u_{best})$,u_{new} replaces u_{best} and u.

Otherwise, u_{new} replaces u with a probability $e^{b(i,\Delta f,f_0)}$, where b is the Boltzmann exponent, I is the current iteration, Δf is the change in the objective value, and f_0 is the last iteration objective function value.

The default function for b is taken as $\frac{-\Delta f \log(i+1)}{10}$.

Simulated annealing is used for multi-initial points and obtains an optimizer among them. In general, the default number of initial points is taken as $\min\{2n, 50\}$.

The starting points is repeated until achieving of the maximum number of iterations and this method converges to a point.

The values of the parameters of the Simulated annealing are given in Table 3.

Table 3. The values of parameters used in the Simulated annealing.

Option Name	Default Value	Descriptions
"Level Iterations"	50	Maximum number of iterations to stay at a given point
"Perturbation Scale"	1.0	Scale for the random jump
"Random Seed"	0	It is a starting value of random number generator
"Tolerance"	0.001	Tolerance for accepting constraint violations

Solution Procedure

To solve the optimization Problem (10), the following steps are followed:

Step 1: Set the initial values of all input inventory parameters.

Step 2: Define the objective Function (10) in MATHEMATICA.

Step 3: Use the following comments:

"NMaximize [objective, decision variables, Method → "SimulatedAnnealing"]

"NMaximize [objective, decision variables, Method → "DifferentialEvaluation"]

Step 4: Compile and execute.

Step 5: Check the result.

Step 6: If the program is convergent and the results are feasible, go to **Step 8**, otherwise go to **Step 7**.

Step 7: Repeat **Steps 1** to **6**.

Step 8: Print the optimal results.

Step 9: Stop.

7. Numerical Illustrations

Here, we have discussed validation of the proposed work. During the validation process, two numerical examples of a hypothetical system are considered as follows:

Example 1: In this numerical example, the hypothetical data of input parameters are taken in the following way:

$A = 690$, $B = 700$, $\alpha = 325$, $\beta = 690$, $\gamma = 365$, $\delta = 325$, $\theta_1 = 0.22$, $\eta = 0.9$, $k = 0.2$, $a = 150$, $b = 0.5$, $C_o = 450$, $c_p = 50, h = 0.5$.

Example 2: Here, the values of preservation parameters are taken as $m = 0.7$ and the other input parameters are taken to be the same as Example 1.

Example 1 and Example 2 are solved by DE and SA, which are codded in Mathematica software, and the obtained results are displayed in Tables 4 and 5, respectively. Moreover, to show the concavity of the objective function, the pictorial representations of the average profit function versus independent variables taken two at a time w.r.t. Example 2 are depicted in Figures 4–6.

Table 4. Best-found solution of Example 1.

Unknown Parameters	Best-Found Result Obtained by DE	Best-Found Result Obtained by SA
Production time (t_1) (month)	1.9851	1.9851
Cycle length (T) (month)	2.2615	2.2615
Selling price (p) ($/L)	170.746	170.746
Average profit (TP) ($/month)	7591.65	7591.65

Table 5. Best-found solution of Example 2.

Unknown Parameters	Best-Found Result Obtained by DE	Best-Found Result Obtained by SA
Production time (t_1) (month)	3.92474	3.92474
Cycle length (T) (month)	7.005	7.005
Selling price (p) ($/L)	174.89	174.89
Preservation investment (ξ) ($)	8.97837	8.97836
Average profit (TP) ($/month)	7703.15	7703.15

Discussion

From the solution of Examples 1 and 2 (cf. Tables 2 and 3), the following findings are observed.

(i) The average profit of Example 2 (model with preservation technology) is higher than that of the Example 1 (model without preservation technology). From this finding, it may be concluded that the model with preservation technology is more economical than the model without preservation technology.

(ii) The best-found results of both Examples 1 and 2 obtained by DE and SA are same up to a certain degree of accuracy. Thus, from here, it can also be concluded that both of the algorithms are equally efficient to solve the corresponding optimization problem of the proposed model.

(iii) Figures 4–6 indicate pictorial evidence for the near optimality of the obtained results for Examples 1 and 2.

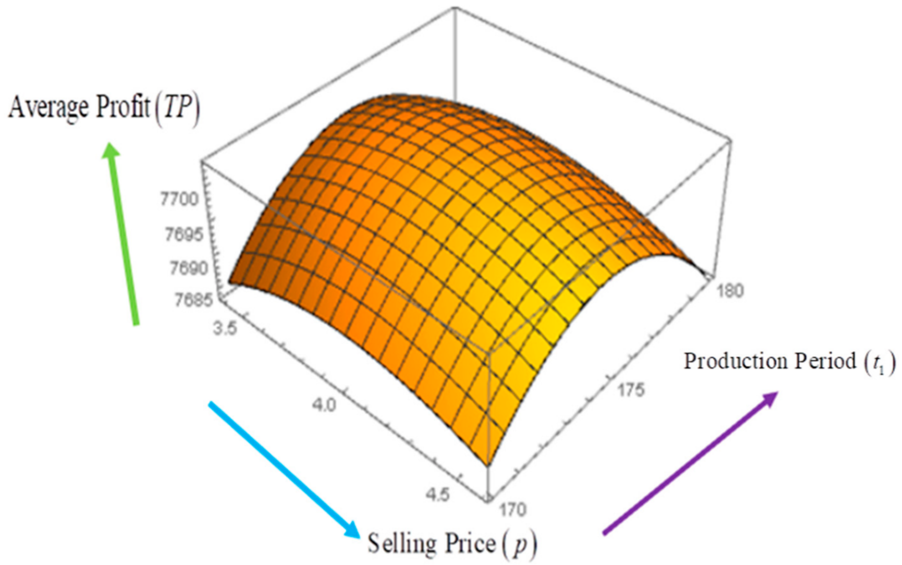

Figure 4. Profit function with respect to t_1 and p.

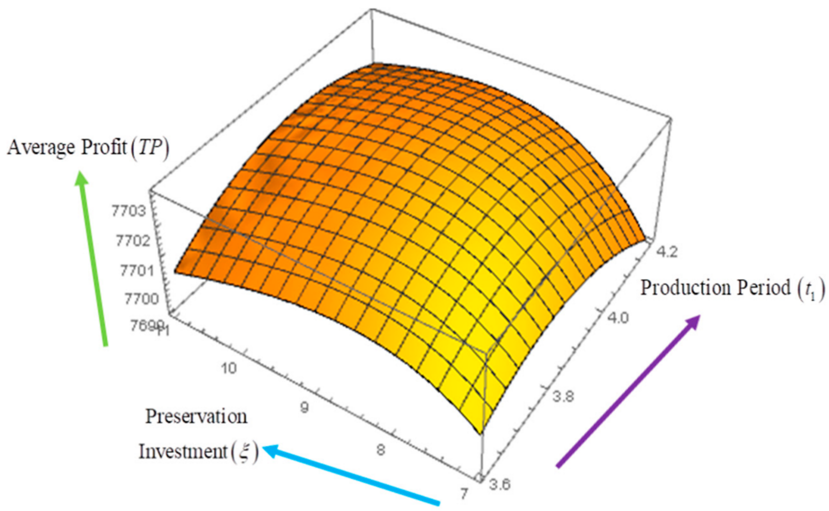

Figure 5. Profit function with respect to t_1 and ξ.

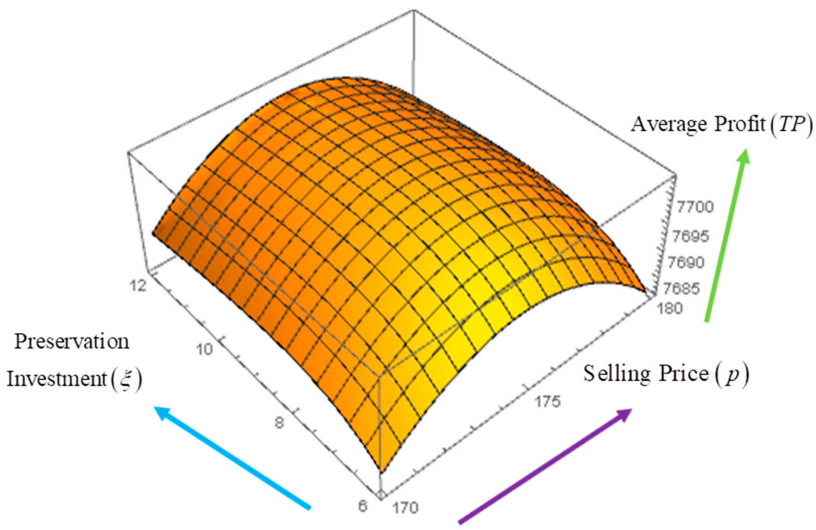

Figure 6. Profit function with respect to ξ and p.

8. Sensitivity Analyses

To show the impact of various known inventory parameters on the average profit (TP), production time (t_1), selling price (p), and cycle length (T), sensitivity analyses are performed with respect to Example 2 by changing the parameters from −20% to 20%. Then, the obtained results of these analyses are depicted graphically in Figures 7–12.

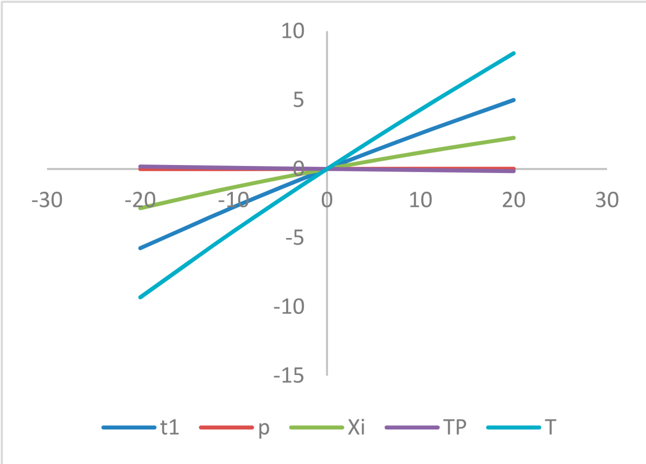

Figure 7. Impact of C_o on the optimal policy.

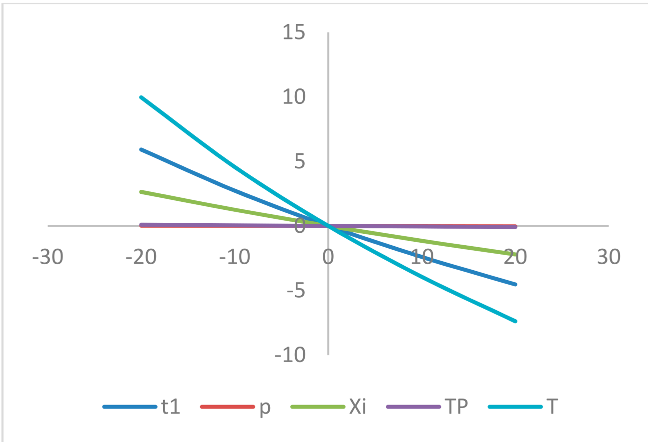

Figure 8. Impact of h on the optimal policy.

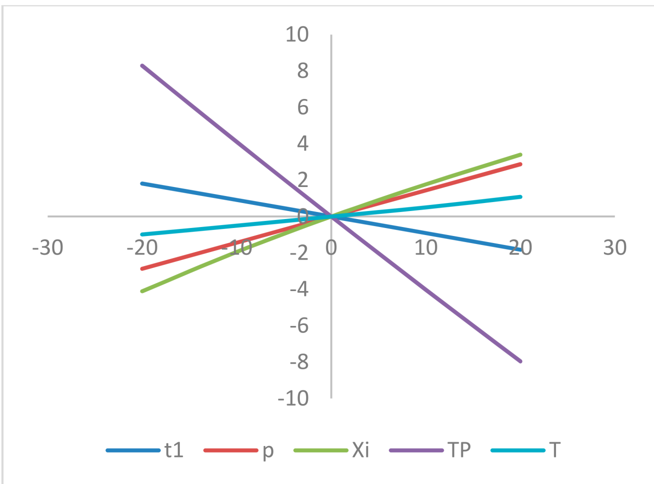

Figure 9. Impact of c_p on the optimal policy.

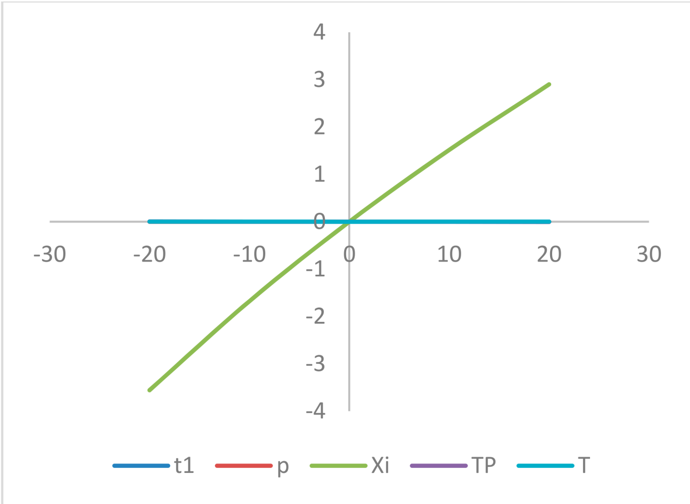

Figure 10. Impact of θ_1 on the optimal policy.

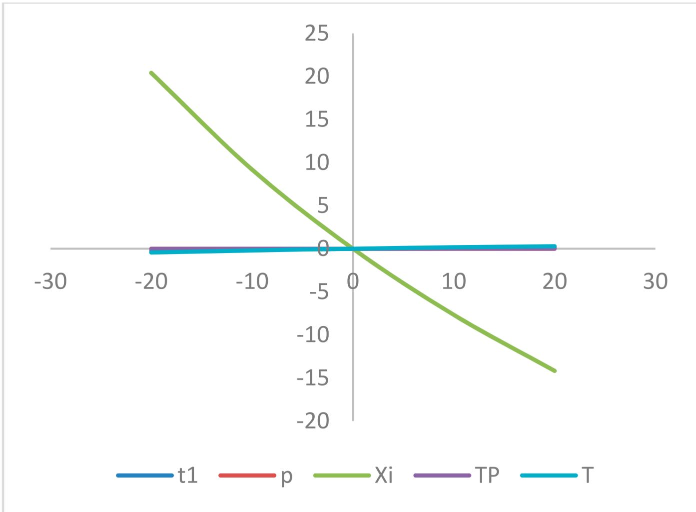

Figure 11. Impact of m on the optimal policy.

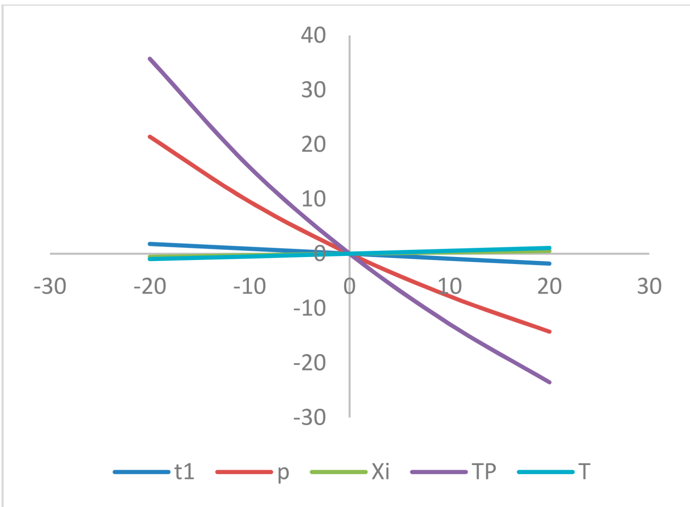

Figure 12. Impact of b on the optimal policy.

From Figures 7–12, the following observations can be made:

- The profit per unit (TP) is moderately sensitive with a reverse effect with respect to c_p, whereas it is insensitive with the change of θ_1, m, h and C_o, and highly sensitive with a reverse effect with respect to b.
- The production time (t_1) is less sensitive directly with respect to C_o, c_p, and slightly sensitive with a reverse effect with respect to h. On the other hand, it is insensitive with the changes in θ_1, m & b.
- The selling price (p) is slightly sensitive with respect to c_p, whereas it is insensitive with the changes in C_o, h, θ_1 & m, and fairly sensitive with a reverse effect with respect to b.
- The preservation investment (ξ) is slighter sensitive with respect to C_o, c_p, h & θ_1, whereas it is insensitive with the changes in b, and highly sensitive with a reverse effect with respect to m.
- The cycle length (T) is slightly sensitive with respect to c_p, and moderately sensitive with a reverse effect with respect to h. On the other hand, it is insensitive with the changes in b, θ_1 and m and fairly sensitive with respect to C_o.

9. Managerial Implications

From the numerical and sensitivity analyses, a few advisories or awareness may be given to the manager of the manufacturing system of mixed products, which are presented below:

(i). As the model with preservation technology is more economical than the model without preservation technology, it will be a good choice for the manager to consider the preservation facility during the manufacturing process of perishable products.

(ii). On the other hand, the manager should be careful about the preservation controlling factor (m), which has a high reverse effect on the preservation investment, the ignorance of which may be the cause of higher compensation on preservation technology.

(iii). The average profit is highly sensitive with respect to the demand controlling parameter b and inventory cost components in the reverse sense, thus a manager/model analyst should take more care about these parameters when making the optimal decision.

10. Conclusions

In this work, the concept of the mixing problem is implemented in the production inventory model for a liquid product with selling-price-dependent demand and a variable production rate under preservation technology. The mixing process is formulated mathematically by the system of differential equations. The non-linear average profit is maximized numerically by the meta-heuristic optimizers: differential evaluation and simulated annealing.

It may be concluded from the numerical result that, if the enterprise/organization applies the preservation facility, it will be more beneficial for them. From the sensitivity analyses, it can also be concluded that the demand parameters and different inventory costs have a significant negative impact on average profit.

As a practical implication, the concept of this proposed model can be applied in various industries, such as medicine, cosmetics, detergent, food industries, and so on. Although the concept of this model can be implemented in the various fields mentioned above, this work has some limitations. Firstly, there is no theoretical proof of the optimal policy of the proposed model. Secondly, under uncertainty, this model cannot be directly implemented in the such industrial sectors and, finally, the shortages case is not considered in this model.

Keeping the above limitations of the proposed model in mind, in the future, the concept of the mixing problem can be extended in other production inventory models, such as models with shortages, a production model with an imperfect production process, and a model with trade credit policy, among others. Finally, the concept of this work may be extended in an uncertain environment-fuzzy, stochastic, fuzzy-stochastic, and interval environment, among others.

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