mathematics

Article

A Cyclic Iterative Algorithm for Multiple-Sets Split Common Fixed Point Problem of Demicontractive Mappings without Prior Knowledge of Operator Norm

Nishu Gupta [1,†], Mihai Postolache [2,3,4,5,*,†], Ashish Nandal [6,†] and Renu Chugh [7,†]

1 Department of Mathematics, Pt NRS Government College, Rohtak 124001, India; nishu2052.he@hry.gov.in
2 Department of General Education, China Medical University, Taichung 40402, Taiwan
3 Department of Interior Design, Asia University, Taichung 41354, Taiwan
4 Gh. Mihoc-C. Iacob Institute of Mathematical Statistics and Applied Mathematics, Romanian Academy, 050711 Bucharest, Romania
5 Department of Mathematics and Informatics, University "Politehnica" of Bucharest, 060042 Bucharest, Romania
6 Government College, Bhainswal Kalan 131001, India; ashish2048.he@hry.gov.in
7 Department of Mathematics, Maharshi Dayanand University, Rohtak 124001, India; chughrenu1988@mdurohtak.ac.in
* Correspondence: mihai@mathem.pub.ro
† These authors contributed equally to this work.

Citation: Gupta, N.; Postolache, M.; Nandal, A.; Chugh, R. A Cyclic Iterative Algorithm for Multiple-Sets Split Common Fixed Point Problem of Demicontractive Mappings without Prior Knowledge of Operator Norm. *Mathematics* **2021**, *9*, 372. https://doi.org/10.3390/math9040372

Academic Editor: Christopher Goodrich

Received: 7 January 2021
Accepted: 10 February 2021
Published: 13 February 2021

Abstract: The aim of this paper is to formulate and analyze a cyclic iterative algorithm in real Hilbert spaces which converges strongly to a common solution of fixed point problem and multiple-sets split common fixed point problem involving demicontractive operators without prior knowledge of operator norm. Significance and range of applicability of our algorithm has been shown by solving the problem of multiple-sets split common null point, multiple-sets split feasibility, multiple-sets split variational inequality, multiple-sets split equilibrium and multiple-sets split monotone variational inclusion.

Keywords: demicontractive operators; iterative algorithm; multiple-sets split common fixed point problem; split equilibrium problem

MSC: 47H09; 47H10; 47J25

1. Introduction

In 1994, Censor et al. [1] firstly suggested the split feasibility problem (SFP) for modelling inverse problems. SFP requires one to identify a point in a non-empty closed and convex set in a space such that its image belongs to another non-empty closed and convex set in the image space under a bounded linear operator. Now, SFPs have been used in several areas such as image restoration [2,3], computer tomography [4], intensity modulated radiation therapy (IMRT) [4,5], signal processing [6] etc. Recently, various split type problems have been introduced by many authors (see [4,7–13]). For important results in this direction or in similar subjects, see [14–24].

Multiple-sets split feasibility problems (MSSFPs) were introduced by Censor et al. [4] in 2005, inspired by inverse problem of intensity modulated radiation therapy (IMRT). As a generalization of convex feasibility problem, split feasibility problem and multiple-sets split feasibility problem, split common fixed point problems (SCFPPs) were introduced by Censor and Segal [7], in 2009. The SCFPP requires one to identify a fixed point of an operator in a space such that its image is a fixed point of another operator in image space under a bounded linear operator. SCFPP has received much attention in recent years because it has many applications in different fields like image reconstruction, signal processing, intensity modulated radiation therapy (IMRT), modelling inverse problems, electron microscopy etc. (see [7,25]).

In this paper, we will find the common solution of the fixed point problem and multiple-sets split common fixed point problem for demicontractive operators which is stated as finding $z^* \in H_1$ such that

$$z^* \in \bigcap_{i=1}^{N} \mathrm{Fix}(S_i) \cap \mathrm{Fix}(U) \text{ and } Az^* \in \bigcap_{j=1}^{M} \mathrm{Fix}(T_j), \tag{1}$$

where $N, M \geq 1$ are integers, $S_i : H_1 \to H_1$ and $T_j : H_2 \to H_2$ are two families of demicontractive operators with constants $0 \leq \eta_i < 1$ and $0 \leq \mu_j < 1$ respectively for $1 \leq i \leq N$ and $1 \leq j \leq M$. Additionally, $A : H_1 \to H_2$ is a bounded linear operator and $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \leq \kappa < 1$.

To find out the solution of SCFPP, Censor et al. [7] introduced the following iteration algorithm. For any $x_1 \in H_1$, $\{x_n\}$ is defined by

$$x_{n+1} = U(x_n - \rho A^*(I - T)Ax_n), \tag{2}$$

where T and U are directed operators, $\rho \in (0, 2/||A||^2)$, then sequence $\{x_n\}$ converges weakly to a solution of SCFPP. Subsequently, this result was extended by Moudafi [26] for demicontractive operators and obtained a weak convergence result.

However, one can observe that the iterative algorithm suggested by Censor [7] and Moudafi [26] needs to find the norm of bounded linear operator. In some cases however, it is impossible or becomes very difficult to compute $\|A\|$. So, alternate ways for adopting variable step-size have been considered to overcome this difficulty. In 2012, Lopez et al. [27] firstly introduced the self adaptive method for solving SFP to select the step size which do not require prior knowledge of operator norm.

In 2015, Shehu and Cholamjiak [28] extended the result of Censor and Segal [7] for demi-contractive operators and obtain the strong convergence result but the step size still depends on operator norm. To overcome this problem, Jirakitpuwapat et al. [25], in 2019, introduced the new iterative method to solve SCFPP without prior knowledge of operator norm and obtained a strong convergence result.

In this paper, inspired and motivated by above work, we present a new cyclic iterative scheme without prior knowledge of operator norm and prove its strong convergence for approximating a common solution of fixed point problem and multiple-sets split common fixed point problem for demicontractive operators in real Hilbert spaces. Moreover, we extend and improve many results related to various split type problems.

2. Preliminaries

In this paper, H is assumed to be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$. $\mathrm{Fix}(S)$ represents the set of all fixed points of mapping S. We will use the notations $x_n \to x$ and $x_n \rightharpoonup x$ to represent strong and weak convergence of $\{x_n\}$ to x respectively. Additionally, we use $\omega_w(x_n) = \{x : \text{there exists } x_{n_k} \rightharpoonup x\}$ to represent weak ω-limit set of $\{x_n\}$.

Definition 1. *Let $S : H \to H$ be an operator. Then S is called*

1. *Nonexpansive if*
   $\|Sv - Sw\| \leq \|v - w\|$ *for all* $v, w \in H$.
2. *Contraction if there exists a constant $\gamma \in [0, 1)$ such that*
   $\|Sv - Sw\| \leq \gamma\|v - w\|$ *for all* $v, w \in H$.
3. *α- inverse strongly monotone (ism) if there exists $\alpha > 0$ such that*
   $\langle Sv - Sw, v - w\rangle \geq \alpha\|Sv - Sw\|^2$ *for all* $v, w \in H$.
4. *τ-demicontractive if $Fix(S) \neq \phi$ and there exists $\tau \in [0, 1)$ such that*

$$\|Sv - w\|^2 \leq \|v - w\|^2 + \tau\|v - Sv\|^2 \text{ for all } v \in H \text{ and } w \in Fix(S),$$

which is equivalent to

$$\langle v - Sv, v - w\rangle \geq \tfrac{1-\tau}{2}\|v - Sv\|^2$$

$$\text{or} \quad \langle v - Sv, w - Sv\rangle \leq \tfrac{1+\tau}{2}\|v - Sv\|^2.$$

Definition 2. *Let $\{x_n\}$ be a sequence in H and $S : H \to H$ be an operator, then $I - S$ is said to be demiclosed at zero if $x_n \rightharpoonup z^*$ and $(I - S)x_n \to 0$ implies $Sz^* = z^*$ i.e., $z^* \in Fix(S)$.*

Lemma 1 ([29])**.** *Let $S : Q \to H$ be a nonexpansive mapping defined on a closed and convex subset Q of H, then $I - S$ is demiclosed at any $y \in H$.*

Lemma 2. *Let $v, w \in H$ then*

$$\|v + w\|^2 \leq \|v\|^2 + 2\langle w, v + w\rangle.$$

Definition 3 ([30])**.** *Let Q be a nonempty closed and convex subset of H then the metric projection $P_Q : H \to Q$ is given as*

$$\|v - P_Q v\| = \inf\{\|v - w\|; w \in Q\} \text{ for all } v \in H. \tag{3}$$

Note that every metric projection is nonexpansive.

Lemma 3 ([30])**.** *Let Q be a nonempty closed convex subset of H then P_Q, a metric projection from H onto Q is characterized by $P_Q v \in Q$ and*

$$\langle v - P_Q v, w - P_Q v\rangle \leq 0 \text{ for all } w \in Q. \tag{4}$$

Lemma 4 ([31])**.** *Let $\{s_n\} \subset [0, \infty)$, $\{\gamma_n\} \subset [0, 1]$ and $\{b_n\}$ be three sequences of real numbers which satisfy the following inequality*

$$s_{n+1} \leq (1 - \gamma_n)s_n + \gamma_n b_n \text{ for all } n \geq 0. \tag{5}$$

Suppose $\sum_{n=0}^{+\infty} \gamma_n = +\infty$ and $\limsup\limits_{n\to+\infty} b_n \leq 0$, then $\lim\limits_{n\to+\infty} s_n = 0$.

Lemma 5 ([32])**.** *Assume that $\{b_n\}$ is a sequence of real numbers and there is a subsequence $\{n_j\}$ of $\{n\}$ satisfying $b_{n_j} < b_{n_j+1}$ for all $j \in \mathbb{N}$. Then there exists a nondecreasing sequence $\{m_q\} \subset \mathbb{N}$ such that $m_q \to +\infty$ and satisfies*

$$b_{m_q} \leq b_{m_q+1} \text{ and } b_q \leq b_{m_q+1}, \tag{6}$$

for all (suitably large) numbers $q \in \mathbb{N}$ where $m_q = \max\{k \leq q : b_k < b_{k+1}\}$.

Lemma 6 ([33])**.** *Let $U : H \to H$ be an η-demicontractive operator where $\eta < 1$. Consider $U_\lambda := (1 - \lambda)I + \lambda U$ for any $\lambda \in (0, 1 - \eta)$, then for any $z \in H$ and $z^* \in Fix(U)$*

$$\|U_\lambda(z) - z^*\|^2 \leq \|z - z^*\|^2 - \lambda(1 - \eta - \lambda)\|z - Uz\|^2. \tag{7}$$

Lemma 7 ([33])**.** *Let $A : H_1 \to H_2$ be a bounded linear operator and $T : H_2 \to H_2$ be a μ-demicontractive operator where $\mu < 1$. If $A^{-1}(FixT) \neq \phi$, then*

1. *$(I - T)Az = 0$ iff $A^*(I - T)Az = 0$ for all $z \in H_1$,*
2. *$\|z - \rho A^*(I - T)Az - z^*\|^2 \leq \|z - z^*\|^2 - \frac{(1-\mu)^2\|(I-T)Az\|^4}{4\|A^*(I-T)Az\|^2}$,*

where $z \in H_1$, $Az \neq T(Az)$ and $\rho = \frac{(1-\mu)\|(I-T)Az\|^2}{2\|A^*(I-T)Az\|^2}$.

Lemma 8 ([34])**.** *Let $\{x_n\}$ be a bounded sequence in a Hilbert space H and $J = \{1, 2, \dots, N\}$ where N is a positive integer. If $\lim_{n\to+\infty} \|x_{n+1} - x_n\| = 0$ and $z^* \in \omega_w(x_n)$, then for any $i \in J$, there exists a subsequence $\{x_{m_k}\}$ of $\{x_n\}$ such that $m_k(modN) + 1 = i$ and $x_{m_k} \rightharpoonup z^*$.*

3. Main Result

In this section, we present our main result and prove the strong convergence of our iterative algorithm for approximating a common solution of fixed point problem and multiple-sets split common fixed point problem involving demicontractive operators without prior knowledge of operator norm.

Theorem 1. *Assume that H_1 and H_2 are two real Hilbert spaces and $A : H_1 \to H_2$ is a bounded linear operator. Consider $S_i : H_1 \to H_1$ and $T_j : H_2 \to H_2$ are two families of demicontractive operators with constants $0 \le \eta_i < 1$ and $0 \le \mu_j < 1$ respectively such that $I - S_i$ and $I - T_j$ are demiclosed at zero for $1 \le i \le N, 1 \le j \le M$ with $\eta = max\ \eta_i$ and $\mu = max\ \mu_j$. Assume that $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \le \kappa < 1$ such that $I - U$ is demiclosed at zero and $U_{\lambda_n} := (1 - \lambda_n)I + \lambda_n U$ for $\lambda_n \in (\epsilon, 1 - \kappa - \epsilon)$, where $\epsilon > 0$ is arbitrary. For any $x_1 \in H_1$, $\{x_n\}$ is defined by*

$$\begin{cases} y_n = \alpha_n g(x_n) + (1 - \alpha_n)U_{\lambda_n}(x_n - \rho_n A^*(I - T_n)Ax_n), \\ x_{n+1} = (1 - \beta_n)y_n + \beta_n S_n y_n, & \text{for all } n \ge 1 \end{cases} \tag{8}$$

where $\rho_n = \begin{cases} \frac{(1-\mu_n)\|(I-T_n)Ax_n\|^2}{2\|A^*(I-T_n)Ax_n\|^2}, & Ax_n \neq T_n(Ax_n) \\ 0, & \text{otherwise} \end{cases}$
and g is a γ contraction operator defined on H_1, $\gamma \in [0, 1)$. Define $S_n = S_{n(modN)+1}$ and $T_n = T_{n(modM)+1}$ for all $n \ge 1$. Suppose Γ is a set of all solutions to the problem (1) and $\Gamma \neq \phi$. Additionally, let $\{\alpha_n\}, \{\beta_n\}$ be sequences of positive real numbers satisfying the following conditions:

1. $\lim_{n\to+\infty} \alpha_n = 0$ *and* $\sum_{n=0}^{+\infty} \alpha_n = +\infty$ *where* $\alpha_n \in (0, 1)$,
2. $\beta_n \in (\delta, 1 - \eta - \delta)$, $\delta > 0$.

Then sequence $\{x_n\}$ converges strongly to $z^* \in \Gamma$ where z^* is a unique fixed point of contraction $P_\Gamma g$.

Proof. Take an arbitrary element $z^* \in \Gamma$ and put $a_n = x_n - \rho_n A^*(I - T_n)Ax_n$ for all $n \in \mathbb{N}$. First we claim that $\{x_n\}$ is bounded.

If $\rho_n = 0$, then we have $a_n = x_n$ and using Lemma 6, we get

$$\begin{aligned} \|U_{\lambda_n} a_n - z^*\|^2 &= \|U_{\lambda_n} x_n - z^*\|^2 \\ &\le \|x_n - z^*\|^2 - \lambda_n(1 - \kappa - \lambda_n)\|x_n - Ux_n\|^2 \\ &\le \|x_n - z^*\|^2. \end{aligned} \tag{9}$$

If $\rho_n \neq 0$, then from Lemmas 6 and 7, we have

$$\begin{aligned} \|U_{\lambda_n} a_n - z^*\|^2 &\le \|a_n - z^*\|^2 - \lambda_n(1 - \kappa - \lambda_n)\|a_n - Ua_n\|^2 \\ &= \|x_n - \rho_n A^*(I - T_n)Ax_n - z^*\|^2 - \lambda_n(1 - \kappa - \lambda_n)\|a_n - Ua_n\|^2 \\ &\le \|x_n - z^*\|^2 - \frac{(1 - \mu_n)^2\|(I - T_n)Ax_n\|^4}{4\|A^*(I - T_n)Ax_n\|^2} - \lambda_n(1 - \kappa - \lambda_n)\|a_n - Ua_n\|^2 \\ &\le \|x_n - z^*\|^2. \end{aligned} \tag{10}$$

Thus, we obtain

$$\|U_{\lambda_n} a_n - z^*\| \leq \|x_n - z^*\|. \tag{11}$$

Now define $(S_n)_{\beta_n} = (1-\beta_n)I + \beta_n S_n$ for all $n \in \mathbb{N}$. Again using Lemma 6, we have

$$\begin{aligned} \|x_{n+1} - z^*\|^2 &= \|(S_n)_{\beta_n}(y_n) - z^*\|^2 \\ &\leq \|y_n - z^*\|^2 - \beta_n(1-\eta_n-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &\leq \|y_n - z^*\|^2 - \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &\leq \|y_n - z^*\|^2. \end{aligned} \tag{12}$$

Using Equation (11), we get

$$\begin{aligned} \|x_{n+1} - z^*\| &\leq \|y_n - z^*\| \\ &= \|\alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n} a_n - z^*\| \\ &= \|\alpha_n (g(x_n) - z^*) + (1-\alpha_n)(U_{\lambda_n} a_n - z^*)\| \\ &\leq \alpha_n \|g(x_n) - z^*\| + (1-\alpha_n)\|x_n - z^*\| \\ &\leq \alpha_n \|g(x_n) - g(z^*)\| + \alpha_n\|g(z^*) - z^*\| + (1-\alpha_n)\|x_n - z^*\| \\ &\leq \alpha_n \gamma \|x_n - z^*\| + \alpha_n\|g(z^*) - z^*\| + (1-\alpha_n)\|x_n - z^*\| \\ &= (1-(1-\gamma)\alpha_n)\|x_n - z^*\| + \alpha_n\|g(z^*) - z^*\| \\ &\leq \max\{\|x_n - z^*\|, \frac{1}{1-\gamma}\|g(z^*) - z^*\|\} \\ &\leq \max\{\|x_0 - z^*\|, \frac{1}{1-\gamma}\|g(z^*) - z^*\|\}, \end{aligned} \tag{13}$$

which shows that $\{x_n\}$ is bounded. Further, we can prove that $\{g(x_n)\}$, $\{y_n\}$, and $\{a_n\}$ are also bounded.

Since the set of fixed points of demicontractive mapping is closed and convex, the solution set Γ is nonempty, closed and convex subset of H_1. Therefore, the nearest point projection P_Γ is well defined. Additionally, we have that $P_\Gamma g : H_1 \to H_1$ is a contraction mapping, hence there exists $z^* \in \Gamma$ such that $z^* = P_\Gamma g(z^*)$.

If $\rho_n = 0$, then from Lemma 2 and using Equations (9) and (12), we have

$$\begin{aligned} \|x_{n+1} - z^*\|^2 &\leq \|y_n - z^*\|^2 - \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &= \|\alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n} x_n - z^*\|^2 - \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &\leq (1-\alpha_n)\|U_{\lambda_n} x_n - z^*\|^2 + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle \\ &\quad - \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &\leq (1-\alpha_n)[\|x_n - z^*\|^2 - \lambda_n(1-\kappa-\lambda_n)\|x_n - Ux_n\|^2] \\ &\quad + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle - \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &\leq \|x_n - z^*\|^2 - \lambda_n(1-\kappa-\lambda_n)\|x_n - Ux_n\|^2 + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle \\ &\quad - \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2. \end{aligned} \tag{14}$$

If $\rho_n \neq 0$, then again using Lemma 2 and Equations (10) and (12), we obtain

$$\begin{aligned}
\|x_{n+1}-z^*\|^2 &\le \|y_n-z^*\|^2-\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2\\
&= \|\alpha_n g(x_n)+(1-\alpha_n)U_{\lambda_n}a_n-z^*\|^2-\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2\\
&\le (1-\alpha_n)\|U_{\lambda_n}a_n-z^*\|^2+2\alpha_n\langle g(x_n)-z^*,y_n-z^*\rangle\\
&\quad -\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2\\
&\le (1-\alpha_n)[\|x_n-z^*\|^2-\frac{(1-\mu_n)^2\|(I-T_n)Ax_n\|^4}{4\|A^*(I-T_n)Ax_n\|^2}\\
&\quad -\lambda_n(1-\kappa-\lambda_n)\|a_n-Ua_n\|^2]\\
&\quad +2\alpha_n\langle g(x_n)-z^*,y_n-z^*\rangle-\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2\\
&\le \|x_n-z^*\|^2-\frac{(1-\mu)^2\|(I-T_n)Ax_n\|^4}{4\|A^*(I-T_n)Ax_n\|^2}-\lambda_n(1-\kappa-\lambda_n)\|a_n-Ua_n\|^2\\
&\quad +2\alpha_n\langle g(x_n)-z^*,y_n-z^*\rangle-\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2.
\end{aligned} \tag{15}$$

Let $s_n=\|x_n-z^*\|^2$. Now, we prove $x_n\to z^*$. For this, we study two cases:

Case 1. Suppose that there is a $n_0\in\mathbb{N}$ such that $\{s_n\}$ is decreasing for all $n\ge n_0$. Since $\{s_n\}$ is monotonic and bounded and hence convergent.

If $\rho_n=0$, then from Equation (14) we have

$$\begin{aligned}
0 &\le \lambda_n(1-\kappa-\lambda_n)\|x_n-Ux_n\|^2+\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2\\
&\le \|x_n-z^*\|^2-\|x_{n+1}-z^*\|^2+2\alpha_n\langle g(x_n)-z^*,y_n-z^*\rangle.
\end{aligned}$$

Thus, we obtain

$$0\le\lambda_n(1-\kappa-\lambda_n)\|x_n-Ux_n\|^2\le s_n-s_{n+1}+\alpha_nK$$

$$\text{and}\ \ 0\le\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2\le s_n-s_{n+1}+\alpha_nK,$$

where $K=\sup_{n\in\mathbb{N}}\{2\langle g(x_n)-z^*,y_n-z^*\rangle\}$.

Since $\{s_n\}$ is convergent and $\lim_{n\to+\infty}\alpha_n=0$, hence we get

$$\begin{aligned}
&\lim_{n\to+\infty}\lambda_n(1-\kappa-\lambda_n)\|x_n-Ux_n\|^2=0\\
\text{and}\ \ &\lim_{n\to+\infty}\beta_n(1-\eta-\beta_n)\|S_n(y_n)-y_n\|^2=0,
\end{aligned}$$

which implies

$$\lim_{n\to+\infty}\|x_n-Ux_n\|=0 \tag{16}$$

$$\text{and}\ \lim_{n\to+\infty}\|S_n(y_n)-y_n\|=0. \tag{17}$$

Additionally, from $\rho_n=0$, we get

$$\lim_{n\to+\infty}\|(I-T_n)Ax_n\| = 0. \tag{18}$$

Now,

$$\begin{aligned}\|y_n - x_n\| &= \|\alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n}x_n - x_n\| \\ &\le \alpha_n\|g(x_n) - x_n\| + (1-\alpha_n)\|U_{\lambda_n}x_n - x_n\| \\ &= \alpha_n\|g(x_n) - x_n\| + (1-\alpha_n)\lambda_n\|x_n - Ux_n\|.\end{aligned}$$

Using Equation (16) and $\lim_{n\to+\infty}\alpha_n = 0$, we get

$$\lim_{n\to+\infty}\|y_n - x_n\| = 0. \tag{19}$$

Consider

$$\begin{aligned}\|x_{n+1} - x_n\| &= \|(1-\beta_n)y_n + \beta_n S_n(y_n) - x_n\| \\ &\le \|y_n - x_n\| + \beta_n\|S_n(y_n) - y_n\|.\end{aligned}$$

From Equation (17) and (19), we get $\lim_{n\to+\infty}\|x_{n+1} - x_n\| = 0$ and hence $\{x_n\}$ is asymptotically regular.

As $\{x_n\}$ is bounded, take a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \rightharpoonup p \in H_1$ and hence $p \in \omega_w(x_n)$. Additionally, $\lim_{n\to+\infty}\|x_n - Ux_n\| = 0$ so by demiclosedness of $(I-U)$ at zero, we get $p \in \mathrm{Fix}(U)$. We fix an index $i \in \{1,2,\ldots,N\}$. Since the pool of indexes is finite and hence using $\lim_{n\to+\infty}\|x_{n+1} - x_n\| = 0$ and Lemma 8, we can find a subsequence $\{x_{m_k}\}$ of $\{x_n\}$ such that $m_k(modN) + 1 = i$ and $x_{m_k} \rightharpoonup p$. From Equation (19) we obtain $y_{m_k} \rightharpoonup p$. Since

$$\lim_{k\to+\infty}\|S_i(y_{m_k}) - y_{m_k}\| = \lim_{k\to+\infty}\|S_{m_k}(y_{m_k}) - y_{m_k}\| = 0$$

and $I - S_i$ is demiclosed at zero for each $i \in \{1,2,\ldots,N\}$, it follows that $p \in \bigcap_{i=1}^N Fix(S_i)$. Again using Lemma 8, we can find a subsequence $\{x_{m_t}\}$ of $\{x_n\}$ such that $m_t(modM) + 1 = j$ for each $j \in \{1,2,\ldots,M\}$ and $x_{m_t} \rightharpoonup p$. Additionally, A is a bounded linear operator and hence continuous, so $x_{m_t} \rightharpoonup p$ implies $Ax_{m_t} \rightharpoonup Ap$. Since,

$$\lim_{t\to+\infty}\|(I-T_j)Ax_{m_t}\| = \lim_{t\to+\infty}\|(I-T_{m_t})Ax_{m_t}\| = 0$$

and $I - T_j$ is demiclosed at zero, it follows that $Ap \in Fix(T_j)$ for each $j \in \{1,2,\ldots,M\}$. Hence, we obtain $p \in \bigcap_{i=1}^N Fix(S_i) \cap Fix(U)$ and $Ap \in \bigcap_{j=1}^M Fix(T_j)$ which implies $p \in \Gamma$.

If $\rho_n \neq 0$, then from Equation (15) and using $s_n = \|x_n - z^*\|^2$, we have

$$\begin{aligned}0 &\le \frac{(1-\mu)^2\|(I-T_n)Ax_n\|^4}{4\|A^*(I-T_n)Ax_n\|^2} + \lambda_n(1-\kappa-\lambda_n)\|a_n - Ua_n\|^2 \\ &\quad + \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \\ &\le s_n - s_{n+1} + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle.\end{aligned}$$

Hence,

$$0 \leq \frac{(1-\mu)^2\|(I-T_n)Ax_n\|^4}{4\|A^*(I-T_n)Ax_n\|^2} \leq s_n - s_{n+1} + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle,$$

$$0 \leq \lambda_n(1-\kappa-\lambda_n)\|a_n - Ua_n\|^2 \leq s_n - s_{n+1} + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle$$

and

$$0 \leq \beta_n(1-\eta-\beta_n)\|S_n(y_n) - y_n\|^2 \leq s_n - s_{n+1} + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle.$$

Consider $K = \sup_{n\in\mathbb{N}}\{2\langle g(x_n) - z^*, y_n - z^*\rangle\}$.

Since $\{s_n\}$ is convergent and $\lim_{n\to+\infty} \alpha_n = 0$, it follows that

$$\lim_{n\to+\infty} \frac{\|(I-T_n)Ax_n\|^2}{\|A^*(I-T_n)Ax_n\|} = \lim_{n\to+\infty} \|S_n(y_n) - y_n\| = \lim_{n\to+\infty} \|a_n - Ua_n\| = 0. \tag{20}$$

Since,

$$\begin{aligned}\|A^*(I-T_n)Ax_n\| &\leq \|A^*\|\|(I-T_n)Ax_n\| \\ &= \|A\|\|(I-T_n)Ax_n\|.\end{aligned}$$

Hence,

$$\|(I-T_n)Ax_n\| = \frac{\|A\|\|(I-T_n)Ax_n\|^2}{\|A\|\|(I-T_n)Ax_n\|} \leq \frac{\|A\|\|(I-T_n)Ax_n\|^2}{\|A^*(I-T_n)Ax_n\|}.$$

Now from Equation (20) we get

$$\lim_{n\to+\infty} \|(I-T_n)Ax_n\| = 0. \tag{21}$$

Moreover,

$$\begin{aligned}\lim_{n\to+\infty} \|x_n - a_n\| &= \lim_{n\to+\infty} \|\rho_n A^*(I-T_n)Ax_n\| \\ &= \lim_{n\to+\infty} \frac{(1-\mu_n)\|(I-T_n)Ax_n\|^2}{2\|A^*(I-T_n)Ax_n\|} \\ &= 0.\end{aligned} \tag{22}$$

Now,

$$\begin{aligned}\|y_n - x_n\| &= \|\alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n}a_n - x_n\| \\ &\leq \alpha_n\|g(x_n) - x_n\| + (1-\alpha_n)\|x_n - U_{\lambda_n}a_n\| \\ &\leq \alpha_n\|g(x_n) - x_n\| + (1-\alpha_n)\|x_n - a_n\| + (1-\alpha_n)\|a_n - U_{\lambda_n}a_n\| \\ &\leq \alpha_n\|g(x_n) - x_n\| + \|x_n - a_n\| + \lambda_n\|a_n - Ua_n\|.\end{aligned}$$

Using $\lim_{n\to+\infty} \alpha_n = 0$ and Equations (20) and (22), we get

$$\lim_{n\to+\infty}\|y_n - x_n\| = 0. \tag{23}$$

Now,

$$\begin{aligned}\|x_{n+1} - x_n\| &= \|(1-\beta_n)y_n + \beta_n S_n(y_n) - x_n\| \\ &\le \|y_n - x_n\| + \beta_n\|S_n(y_n) - y_n\|. \end{aligned} \tag{24}$$

Using Equations (20) and (23), we get $\lim_{n\to+\infty}\|x_{n+1} - x_n\| = 0$. Hence $\{x_n\}$ is asymptotically regular.

As $\{x_n\}$ is bounded, take a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \rightharpoonup p \in H_1$. Now, $\lim_{n\to+\infty}\|x_n - a_n\| = 0$ and boundedness of $\{a_n\}$ implies there is a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ such that $a_{n_k} \rightharpoonup p$. Since $\lim_{n\to+\infty}\|a_n - Ua_n\| = 0$, so by demiclosedness of $I - U$ at zero, we have $p \in Fix(U)$. Proceeding similarly as for case $\rho_n = 0$ and using Equations (20)–(24) and Lemma 8, we conclude that $p \in \bigcap_{i=1}^{N} \mathrm{Fix}(S_i) \cap \mathrm{Fix}(U)$ and $Ap \in \bigcap_{j=1}^{M} \mathrm{Fix}(T_j)$. Hence $p \in \Gamma$.

Now, using Lemma 3, we get

$$\begin{aligned}\limsup_{n\to+\infty}\langle g(z^*) - z^*, y_n - z^*\rangle &= \lim_{k\to+\infty}\langle g(z^*) - z^*, y_{n_k} - z^*\rangle \\ &= \langle g(z^*) - z^*, p - z^*\rangle \\ &= \langle g(z^*) - P_\Gamma g(z^*), p - P_\Gamma g(z^*)\rangle \\ &\le 0. \end{aligned} \tag{25}$$

Now, we prove that $x_n \to z^*$.

$$\begin{aligned}\|x_{n+1} - z^*\|^2 &= \|(S_n)_{\beta_n}(y_n) - z^*\|^2 \\ &\le \|y_n - z^*\|^2 - \beta_n(1 - \eta_n - \beta_n)\|S_n(y_n) - y_n\|^2 \\ &\le \|y_n - z^*\|^2 - \beta_n(1 - \eta - \beta_n)\|S_n(y_n) - y_n\|^2 \\ &\le \|y_n - z^*\|^2. \end{aligned}$$

Moreover, using Lemma 2 and Equation (11), we obtain

$$\begin{aligned}\|y_n - z^*\|^2 &= \|\alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n}a_n - z^*\|^2 \\ &\le (1-\alpha_n)^2\|U_{\lambda_n}a_n - z^*\|^2 + 2\alpha_n\langle g(x_n) - z^*, y_n - z^*\rangle \\ &\le (1-\alpha_n)^2\|x_n - z^*\|^2 + 2\alpha_n[\langle g(x_n) - g(z^*), y_n - z^*\rangle + \langle g(z^*) - z^*, y_n - z^*\rangle] \\ &\le (1-\alpha_n)^2\|x_n - z^*\|^2 + 2\alpha_n\gamma\|x_n - z^*\|.\|y_n - z^*\| + 2\alpha_n\langle g(z^*) - z^*, y_n - z^*\rangle \\ &\le (1-\alpha_n)^2\|x_n - z^*\|^2 + \alpha_n\gamma[\|x_n - z^*\|^2 + \|y_n - z^*\|^2] \\ &\quad + 2\alpha_n\langle g(z^*) - z^*, y_n - z^*\rangle, \end{aligned}$$

which implies

$$(1-\alpha_n\gamma)\|y_n - z^*\|^2 \le [(1-\alpha_n)^2 + \alpha_n\gamma]\|x_n - z^*\|^2 + 2\alpha_n\langle g(z^*) - z^*, y_n - z^*\rangle$$

and hence,

$$\begin{aligned}
\|y_n - z^*\|^2 &\le \left(\frac{(1-\alpha_n)^2 + \alpha_n\gamma}{1-\alpha_n\gamma}\right)\|x_n - z^*\|^2 + \frac{2\alpha_n}{1-\alpha_n\gamma}\langle g(z^*) - z^*, y_n - z^*\rangle \\
&= \left(1 - \frac{2\alpha_n(1-\gamma)}{1-\alpha_n\gamma}\right)\|x_n - z^*\|^2 + \frac{\alpha_n^2}{1-\alpha_n\gamma}\|x_n - z^*\|^2 \\
&\quad + \frac{2\alpha_n}{1-\alpha_n\gamma}\langle g(z^*) - z^*, y_n - z^*\rangle \\
&= \left(1 - \frac{2\alpha_n(1-\gamma)}{1-\alpha_n\gamma}\right)\|x_n - z^*\|^2 + \frac{2\alpha_n(1-\gamma)}{1-\alpha_n\gamma}\left(\frac{1}{1-\gamma}\langle g(z^*) - z^*, y_n - z^*\rangle\right. \\
&\quad \left. + \frac{\alpha_n}{2(1-\gamma)}\|x_n - z^*\|^2\right).
\end{aligned}$$

So, we get

$$s_{n+1} \le (1-a_n)s_n + a_n b_n, \tag{26}$$

where $s_n = \|x_n - z^*\|^2, a_n = \frac{2\alpha_n(1-\gamma)}{1-\alpha_n\gamma}$ and

$$b_n = \frac{1}{1-\gamma}\langle g(z^*) - z^*, y_n - z^*\rangle + \frac{\alpha_n}{2(1-\gamma)}\|x_n - z^*\|^2.$$

From Equation (25), we have $\limsup_{n\to+\infty} b_n \le 0$. Additionally, $\sum_{n=0}^{+\infty} a_n = +\infty$. Hence, it follows from Lemma 4 that

$$\lim_{n\to+\infty} s_n = \lim_{n\to+\infty}\|x_n - z^*\|^2 = 0.$$

Therefore $x_n \to z^*$.

Case 2. Suppose there is no such $n_0 \in \mathbb{N}$ such that $\{s_n\}$ is decreasing for all $n \ge n_0$. Thus there is a subsequence $\{s_{n_l}\}$ of $\{s_n\}$ such that

$$s_{n_l} < s_{n_l+1} \text{ for all } l \in \mathbb{N}. \tag{27}$$

Then from Lemma 5 there exists a nondecreasing sequence of natural numbers $\{n_q\} \subset \mathbb{N}$ such that $n_q \to +\infty$ as $q \to +\infty$ and

$$\begin{aligned}
&\|x_{n_q} - z^*\| \le \|x_{n_q+1} - z^*\| \\
\text{and } &\|x_q - z^*\| \le \|x_{n_q+1} - z^*\|.
\end{aligned} \tag{28}$$

If $\rho_{n_q} = 0$, then from Equation (14), we obtain

$$\begin{aligned}
0 &\le \lambda_{n_q}(1-\kappa-\lambda_{n_q})\|x_{n_q} - Ux_{n_q}\|^2 + \beta_{n_q}(1-\eta-\beta_{n_q})\|S_{n_q}(y_{n_q}) - y_{n_q}\|^2 \\
&\le s_{n_q} - s_{n_q+1} + \alpha_{n_q}K \\
&\le \alpha_{n_q}K,
\end{aligned}$$

where $K = \sup_{n_q\in\mathbb{N}}\{2\left\langle g(x_{n_q}) - z^*, y_{n_q} - z^*\right\rangle\}$.

From $\lim_{q\to+\infty} \alpha_{n_q} = 0$, we obtain

$$\lim_{q\to+\infty} \|x_{n_q} - Ux_{n_q}\| = \lim_{q\to+\infty} \|S_{n_q}(y_{n_q}) - y_{n_q}\| = 0.$$

Additionally, $\rho_{n_q} = 0$ implies $\lim_{q\to+\infty} \|(I - T_{n_q})Ax_{n_q}\| = 0$.
If $\rho_{n_q} \neq 0$, then from Equation (15), we have

$$\begin{aligned} 0 \leq& \frac{(1-\mu)^2\|(I - T_{n_q})Ax_{n_q}\|^4}{4\|A^*(I - T_{n_q})Ax_{n_q}\|^2} + \lambda_{n_q}(1-\kappa-\lambda_{n_q})\|a_{n_q} - Ua_{n_q}\|^2 \\ &+ \beta_{n_q}(1-\eta-\beta_{n_q})\|S_{n_q}(y_{n_q}) - y_{n_q}\|^2 \\ &\leq s_{n_q} - s_{n_q+1} + \alpha_{n_q}K \\ &\leq \alpha_{n_q}K, \end{aligned}$$

where $K = \sup_{n_q\in\mathbb{N}} \{2\left\langle g(x_{n_q}) - z^*, y_{n_q} - z^* \right\rangle\}$.

From $\lim_{q\to+\infty} \alpha_{n_q} = 0$, we obtain

$$\begin{aligned} \lim_{q\to+\infty} \|a_{n_q} - Ua_{n_q}\| &= 0, \\ \lim_{q\to+\infty} \|S_{n_q}(y_{n_q}) - y_{n_q}\| &= 0 \\ \text{and} \lim_{q\to+\infty} \|(I - T_{n_q})Ax_{n_q}\| &= 0. \end{aligned}$$

Now, proceeding similarly as in Case 1, we obtain

$$\begin{aligned} \lim_{q\to+\infty} \|x_{n_q+1} - x_{n_q}\| &= 0 \\ \text{and } \limsup_{q\to+\infty} \left\langle g(z^*) - z^*, y_{n_q} - z^* \right\rangle &\leq 0. \end{aligned} \tag{29}$$

Further, from Equation (26), we get

$$s_{n_q+1} \leq (1 - a_{n_q})s_{n_q} + a_{n_q}b_{n_q}, \tag{30}$$

$$\begin{aligned} a_{n_q}s_{n_q} &\leq s_{n_q} - s_{n_q+1} + a_{n_q}b_{n_q} \\ &\leq a_{n_q}b_{n_q}. \end{aligned}$$

Using $a_{n_q} > 0$, we get $s_{n_q} \leq b_{n_q}$. Hence,

$$\|x_{n_q} - z^*\|^2 \leq \frac{1}{1-\gamma}\left\langle g(z^*) - z^*, y_{n_q} - z^* \right\rangle + \frac{\alpha_{n_q}}{2(1-\gamma)}\|x_{n_q} - z^*\|^2.$$

Since $\{x_{n_q}\}$ is bounded and $\alpha_{n_q} \to 0$ as $q \to +\infty$. Hence, by using Equation (29), we get $\|x_{n_q} - z^*\| \to 0$ as $q \to +\infty$. Further, from Equation (30), we obtain $\|x_{n_q+1} - z^*\| \to 0$ as $q \to +\infty$. Additionally, Equation (28) gives $\|x_q - z^*\| \leq \|x_{n_q+1} - z^*\|$ for all $q \in \mathbb{N}$, which implies $x_q \to z^*$ as $q \to +\infty$. This completes the proof. □

We end this section with the following remarks.

Remark 1. *Theorem 1 extends and improves the result of Wang and Qin ([35], Theorem 1) from weak convergence to strong convergence and by choosing step size independent of operator norm. Further Theorem 1 improves the result in ([35], Theorem 2) without assuming the extra condition that one of the mappings* $\{S_i;\ i = 1, 2, \ldots, N\}$ *is semi-compact as assumed in ([35], Theorem 2).*

Remark 2. *Theorem 1 extends and improves the result of Tang et al. [36], where it was assumed that* $\{S_i;\ i = 1, 2, \ldots, N\}$ *and* $\{T_j;\ j = 1, 2, \ldots, M\}$ *are continuous. We leave this assumption in our result. Moreover, our result extends the result in [36] from weak convergence to strong convergence.*

Remark 3. *Theorem 1 generalizes and improves Cui and Wang's result [33] from weak convergence to strong convergence for multiple sets.*

Remark 4. *If we take* $S_1 = S_2 = \cdots = S_N = I$, $T_1 = T_2 = \cdots = T_M = T$, g *is a constant mapping and* $\lambda_n = \lambda$ *for all* n, *in Theorem 1, then we obtain Boikanyo's result ([37], Theorem 4.1) for approximating the solution of split common fixed point problem. Thus Boikanyo's result is a particular case of our result.*

Remark 5. *Our result improves the result of Tang et al. ([38], Theorem 3.2) from weak convergence to strong convergence and without assuming the extra condition that* $\{S_i;\ i = 1, 2, \ldots, N\}$ *are continuous.*

Remark 6. *By taking* $S_1 = S_2 = \cdots = S_N = g$, $T_1 = T_2 = \cdots = T_M = T$ *in Theorem 1, where* g *is a contraction mapping, we obtain Jirakitpuwapat et al.'s result [25] for solving split common fixed point problem. Thus, Theorem 1 improves and extends Jirakitpuwapat et al.'s result [25].*

Remark 7. *If we take* $S_1 = S_2 = \cdots = S_N = I$, $T_1 = T_2 = \cdots = T_M = I$ *and* $\lambda_n = \lambda$ *for all* n, *then Theorem 1 reduces to fixed point problem of Mainge ([39], Theorem 3.1). Furthermore, our result generalize the Mainge's result [39] from quasi nonexpansive mapping to more generalized demicontractive mapping.*

4. Applications

In this section, we will study different split type problems in Hilbert spaces by utilizing main result provided in this paper.

4.1. Multiple-Sets Split Common Null Point Problem

Consider $B : H_1 \to 2^{H_1}$ and $F : H_2 \to 2^{H_2}$ two set valued operators defined on Hilbert spaces H_1 and H_2 respectively and $A : H_1 \to H_2$ is a bounded linear operator, then split common null point problem (SCNPP) is to identify $z^* \in H_1$ such that $0 \in B(z^*)$ and $y^* = Az^*$ solves $0 \in F(y^*)$.

We now solve the SCNPP for multiple sets i.e., if $B_i : H_1 \to 2^{H_1}$ and $F_j : H_2 \to 2^{H_2}$ be two families of set valued operators where $1 \le i \le N$ and $1 \le j \le M$, then multiple-sets split common null point problem is to identify $z^* \in H_1$ such that

$$0 \in \bigcap_{i=1}^{N} B_i(z^*) \text{ and } y^* = Az^* \text{ solves } 0 \in \bigcap_{j=1}^{M} F_j(y^*). \tag{31}$$

Before proving the theorem, we give some basic definitions.

Definition 4 ([40])**.** *A multivalued mapping* $F : H \to 2^H$ *is a monotone mapping if* $f \in F(u)$ *and* $g \in F(v)$ *implies* $\langle u - v, f - g\rangle \ge 0$ *for all* $u, v \in H$.

Definition 5 ([40]). *A monotone mapping F is called maximal if graph of any other monotone mapping cannot contain graph of F, where graph of F is given by $G(F) = \{(u,v) \in H \times H; v \in F(u)\}$ for a multivalued mapping F.*

Definition 6 ([40]). *The resolvent operator $J_r^F : H \to H$ associated with multivalued mapping F and r is the mapping defined as $J_r^F(v) = (I + rF)^{-1}(v)$ for all $v \in H$, where $r > 0$.*

We know that for any $r > 0$, the mapping J_r^F is firmly nonexpansive and single valued and $z^* \in \text{Fix}(J_r^F)$ iff $0 \in F(z^*)$ where F is maximal monotone mapping, see [41]. Therefore, problem (31) is equivalent to the problem of finding $z^* \in H_1$ such that

$$z^* \in \bigcap_{i=1}^{N} \text{Fix}(J_r^{B_i}) \text{ and } Az^* \in \bigcap_{j=1}^{M} \text{Fix}(J_r^{F_j}).$$

Now, we solve the split common null point problem as split common fixed point problem.

Theorem 2. *Assume that $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \le \kappa < 1$ such that $I - U$ is demiclosed at zero. For any $x_1 \in H_1, \{x_n\}$ is defined by*

$$\begin{cases} y_n = \alpha_n g(x_n) + (1-\alpha_n) U_{\lambda_n}(x_n - \rho_n A^*(I - J_r^{F_n})Ax_n), \\ x_{n+1} = (1-\beta_n) y_n + \beta_n J_r^{B_n}(y_n), \qquad \textit{for all } n \ge 1 \end{cases} \tag{32}$$

where $\rho_n = \begin{cases} \frac{\|(I-J_r^{F_n})Ax_n\|^2}{2\|A^*(I-J_r^{F_n})Ax_n\|^2}, & Ax_n \neq J_r^{F_n}(Ax_n) \\ 0, & \textit{otherwise} \end{cases}$

and g is a γ contraction operator defined on H_1, $\gamma \in [0,1)$. Consider $U_{\lambda_n} := (1-\lambda_n)I + \lambda_n U$ for $\lambda_n \in (\epsilon, 1-\kappa-\epsilon), \epsilon > 0$. Define $J_r^{B_n} = J_r^{B_{n(modN)+1}}$ and $J_r^{F_n} = J_r^{F_{n(modM)+1}}$ for all $n \ge 1$. Suppose Ω is a set of all solutions to the problem (31) and $\Omega \cap Fix(U) \neq \phi$. Additionally, let $\{\alpha_n\}, \{\beta_n\}$ be two sequences of positive real numbers satisfying the following conditions:

1. $\lim_{n \to +\infty} \alpha_n = 0$ *and* $\sum_{n=0}^{+\infty} \alpha_n = +\infty$ *where* $\alpha_n \in (0,1)$,
2. $\beta_n \in (\delta, 1-\delta), \delta > 0$.

Then sequence $\{x_n\}$ converges strongly to $z^* \in \Omega \cap \text{Fix}(U)$.

Proof. Let us take $T_n = J_r^{F_n}$ and $S_n = J_r^{B_n}$ for all $n \in N$. Then S_n and T_n are firmly nonexpansive and hence 0-demicontractive. From Lemma 1, we get $I - S_n$ and $I - T_n$ are demiclosed at zero. Hence proof follows from the main Theorem 1. □

We remark that, in [11], Byrne et al. solve the SCNPP where the step size depends on operator norm which is not easy to calculate. Our result extends and generalizes the results in [11] for multiple sets without prior knowledge of operator norm. Additionally, in ([11], Theorem 4.3), maximal monotone mappings B_1 and F_1 are considered to be odd but, in Theorem 2, we have not imposed such a condition on maximal monotone mappings.

4.2. Multiple-Sets Split Feasibility Problem

Assume that $C_i (1 \le i \le N)$ and $Q_j (1 \le j \le M)$ are two families of nonempty closed and convex subsets of Hilbert spaces H_1 and H_2 respectively and $A : H_1 \to H_2$ is a bounded linear operator. Then the multiple-sets split feasibility problem is to identify a point $z^* \in H_1$ such that

$$z^* \in \bigcap_{i=1}^{N} C_i \text{ and } Az^* \in \bigcap_{j=1}^{M} Q_j. \tag{33}$$

We denote the solution set of multiple-sets split feasibility problem by Ω. It is well known that $z^* \in \text{Fix}(P_C)$ iff $z^* \in C$ where C is closed, convex subset of Hilbert space H. Now, we prove the strong convergence theorem for the multiple-sets split feasibility problem.

Theorem 3. *Assume that $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \leq \kappa < 1$ such that $I - U$ is demiclosed at zero. Suppose $\Omega \cap \text{Fix}(U) \neq \phi$ and for any $x_1 \in H_1, \{x_n\}$ is defined by*

$$\begin{cases} y_n = \alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n}(x_n - \rho_n A^*(I - P_{Q_n})Ax_n), \\ x_{n+1} = (1-\beta_n)y_n + \beta_n P_{C_n}(y_n), & \text{for all } n \geq 1 \end{cases} \tag{34}$$

where $\rho_n = \begin{cases} \frac{\|(I-P_{Q_n})Ax_n\|^2}{2\|A^*(I-P_{Q_n})Ax_n\|^2}, & Ax_n \neq P_{Q_n}(Ax_n) \\ 0, & \text{otherwise} \end{cases}$
and g is a γ contraction operator defined on H_1, $\gamma \in [0,1)$. Consider $U_{\lambda_n} := (1-\lambda_n)I + \lambda_n U$ for $\lambda_n \in (\epsilon, 1-\kappa-\epsilon), \epsilon > 0$. Define $P_{C_n} = P_{C_{n(modN)+1}}$ and $P_{Q_n} = P_{Q_{n(modM)+1}}$ for all $n \geq 1$. Additionally, $\{\alpha_n\}, \{\beta_n\}$ be two sequences of positive real numbers satisfying the following conditions:

1. $\lim_{n\to+\infty} \alpha_n = 0$ *and* $\sum_{n=0}^{+\infty} \alpha_n = +\infty$ *where* $\alpha_n \in (0,1)$,
2. $\beta_n \in (\delta, 1-\delta), \delta > 0$.

Then sequence $\{x_n\}$ converges strongly to $z^* \in \Omega \cap Fix(U)$.

Proof. Consider $S_n = P_{C_n}$ and $T_n = P_{Q_n}$ for all $n \in N$. Since metric projection is firmly non-expansive and hence 0-demicontractive. Hence proof follows from the main Theorem 1. □

Note that if we take $N = M = 1$ in Theorem 3, then multiple-sets split feasibility problem reduces to a split feasibility problem (see [1,3,4,6,27]). Further, if we take $A = I$, where I is identity mapping and $H_1 = H_2 = H$ in Theorem 3, then we get a convex feasibility problem which is formulated as finding a point in the intersection of a family of closed and convex subsets in Hilbert space H.

4.3. Multiple-Sets Split Variational Inequality Problem

Assume that C is nonempty closed and convex subset of Hilbert space H and $f : H \to H$ is an operator, then variational inequality problem is to identify $z^* \in C$ such that

$$\langle f(z^*), z - z^* \rangle \geq 0 \text{ for all } z \in C. \tag{35}$$

The solution set of variational inequality problem is denoted by VI(C, f). Note that if f is an α-ism operator on H, then $P_C(I - \lambda f)$ is nonexpansive for each $\lambda \in (0, 2\alpha)$ and $z^* \in \text{Fix}(P_C(I - \lambda f))$ iff $z^* \in \text{VI}(C, f)$ (see [41]).

Consider $C_i(1 \leq i \leq N)$ and $Q_j(1 \leq j \leq M)$ two families of nonempty closed and convex subsets of Hilbert spaces H_1 and H_2 respectively and $A : H_1 \to H_2$ is a bounded linear operator. Assume that $h_i : H_1 \to H_1$ and $f_j : H_2 \to H_2$ are two families of ν_i and μ_j-ism operators, respectively. Set $\mu = min\{\nu_i, \mu_j\}$ and take $\tau \in (0, 2\mu)$. Suppose

$$\Omega = \{z^* \in H_1 \text{ such that } z^* \in \bigcap_{i=1}^{N} \mathrm{VI}(h_i, C_i) \text{ and } Az^* \in \bigcap_{j=1}^{M} \mathrm{VI}(g_j, Q_j)\}.$$

Then multiple-sets split variational inequality problem is to identify $z^* \in \Omega$.

Now, we will present a strong convergence theorem for solving multiple-sets split variational inequality problem.

Theorem 4. *Assume that $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \leq \kappa < 1$ such that $I - U$ is demiclosed at zero. Suppose $\Omega \cap Fix(U) \neq \phi$ and for any $x_1 \in H_1, \{x_n\}$ is defined by*

$$\begin{cases} y_n = \alpha_n g(x_n) + (1-\alpha_n)U_{\lambda_n}(x_n - \rho_n A^*(I - P_{Q_n}(I - \tau f_n))Ax_n), \\ x_{n+1} = (1-\beta_n)y_n + \beta_n P_{C_n}(I - \tau h_n)(y_n), \qquad \textit{for all } n \geq 1 \end{cases} \tag{36}$$

where $\rho_n = \begin{cases} \frac{\|(I-P_{Q_n}(I-\tau f_n))Ax_n\|^2}{2\|A^*(I-P_{Q_n}(I-\tau f_n))Ax_n\|^2}, & Ax_n \neq P_{Q_n}(I - \tau f_n)Ax_n \\ 0, & \textit{otherwise} \end{cases}$

and g is a γ contraction operator defined on H_1, $\gamma \in [0,1)$. Consider $U_{\lambda_n} := (1-\lambda_n)I + \lambda_n U$ for $\lambda_n \in (\epsilon, 1-\kappa-\epsilon), \epsilon > 0$. Define $P_{C_n}(I - \tau h_n) = P_{C_{n(modN)+1}}(I - \tau h_{n(modN)+1})$ and $P_{Q_n}(I - \tau f_n) = P_{Q_{n(modM)+1}}(I - \tau f_{n(modM)+1})$. Additionally, $\{\alpha_n\}, \{\beta_n\}$ be two sequences of positive real numbers satisfying the following conditions:

1. $\lim\limits_{n\to+\infty} \alpha_n = 0$ *and* $\sum_{n=0}^{+\infty} \alpha_n = +\infty$ *where* $\alpha_n \in (0,1)$,
2. $\beta_n \in (\delta, 1-\delta), \delta > 0$.

Then sequence $\{x_n\}$ converges strongly to $z^* \in \Omega \cap \mathrm{Fix}(U)$.

Proof. Since h_i and f_j are two families of ν_i and μ_j- ism operators respectively and hence μ-ism operators. Then for each $\tau \in (0, 2\mu)$, $P_{C_i}(I - \tau h_i)$ and $P_{Q_j}(I - \tau f_j)$ are nonexpansive and hence 0-demicontractive. Hence by using Theorem 1, $\{x_n\}$ converges strongly to $z^* \in H_1$ such that

$$z^* \in \bigcap_{i=1}^{N} Fix(P_{C_i}(I - \tau h_i)) \cap Fix(U) \text{ and } Az^* \in \bigcap_{j=1}^{M} Fix(P_{Q_j}(I - \tau f_j))$$

which implies $z^* \in \Omega \cap Fix(U)$. □

We underly that, in [8], Censor et al. solved the multiple-sets split variational inequality problem through weak convergence of parallel algorithm and in addition, they assume extra conditions

$$\left\langle h_i(z), P_{C_i}(I - \lambda h_i)(z) - z^* \right\rangle \geq 0 \text{ for all } z \in H_1$$
$$\text{and } \left\langle f_j(z), P_{Q_j}(I - \lambda f_j)(z) - z^* \right\rangle \geq 0 \text{ for all } z \in H_2$$

in ([8], Theorem 6.5). However, in Theorem 4, we do not assume such conditions and improve the result from weak convergence to strong convergence.

4.4. Multiple-Sets Split Equilibrium Problem

Let Q be a nonempty closed convex subset of real Hilbert space H and F be a bifunction from $Q \times Q$ to $\mathbb{R}$. Then the equilibrium problem for F is to identify a point $u \in Q$ such that $F(u,v) \geq 0$ for all $v \in Q$. The solution set of the equilibrium problem is denoted by EP(F).

To find the solution of the equilibrium problem, we assume that the following conditions are satisfied by bifunction F:

1. $F(u,u) \geq 0$.
2. F is monotone i.e., $F(u,v) + F(v,u) \leq 0$ for any $u, v \in Q$.
3. For each $u, v, w \in Q$, $\limsup_{t \to 0^+} F(tw + (1-t)u, v) \leq F(u,v)$.
4. For each $u \in Q, v \to F(u,v)$ is convex and lower semi-continuous.

Now, we give an important lemma to solve the equilibrium problem.

Lemma 9 ([42]). *Let Q be a nonempty closed convex subset of Hilbert space H and let $F: Q \times Q \to \mathbb{R}$ be a bifunction satisfying the four conditions given above. Consider $u \in H$ and $r > 0$, then there exists $w \in Q$ such that*

$$F(w,v) + \frac{1}{r}\langle v - w, w - u \rangle \geq 0 \text{ for all } v \in Q.$$

Further if $S_r^F(u) = \{w \in Q : F(w,v) + \frac{1}{r}\langle v - w, w - u\rangle \geq 0 \text{ for all } v \in Q\}$ then the following statements hold:

1. *S_r^F is single valued.*
2. *S_r^F is firmly nonexpansive i.e., for any $u, v \in H$*

$$\|S_r^F(u) - S_r^F(v)\|^2 \leq \left\langle S_r^F(u) - S_r^F(v), u - v \right\rangle.$$

3. Fix*(S_r^F)=EP(F).*
4. *Solution set EP(F) is closed and convex.*

From the above lemma, it is observed that the equilibrium problem can be solved as a fixed point problem.

Assume that C_i and Q_j are two families of nonempty closed, convex subsets of Hilbert spaces H_1 and H_2 respectively where $1 \leq i \leq N, 1 \leq j \leq M$. Consider $F_i : C_i \times C_i \to \mathbb{R}$ and $G_j : Q_j \times Q_j \to \mathbb{R}$ bifunctions satisfying the above four conditions and A is a bounded linear operator from H_1 to H_2, then the multiple-sets split equilibrium problem is to identify a point $z^* \in \Omega$ where

$$\Omega = \{z^* \in H_1 : z^* \in \bigcap_{i=1}^{N} EP(F_i) \text{ and } Az^* \in \bigcap_{j=1}^{M} EP(G_j)\}.$$

Now, we present a strong convergence theorem for solving the multiple-sets split equilibrium problem.

Theorem 5. *Assume that $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \leq \kappa < 1$ such that $I - U$ is demiclosed at zero. Choose $r_1, r_2 > 0$ and suppose $\Omega \cap Fix(U) \neq \phi$. For any $x_1 \in H_1$, $\{x_n\}$ is defined by*

$$\begin{cases} y_n = \alpha_n g(x_n) + (1-\alpha_n) U_{\lambda_n}(x_n - \rho_n A^*(I - S_{r_1}^{G_n}) A x_n), \\ x_{n+1} = (1-\beta_n) y_n + \beta_n S_{r_2}^{F_n}(y_n), \end{cases} \quad \text{for all } n \geq 1 \tag{37}$$

where $\rho_n = \begin{cases} \frac{\|(I-S_{r_1}^{G_n})Ax_n\|^2}{2\|A^*(I-S_{r_1}^{G_n})Ax_n\|^2}, & Ax_n \neq S_{r_1}^{G_n}(Ax_n) \\ 0, & otherwise \end{cases}$

and g is a γ contraction operator defined on H_1, $\gamma \in [0,1)$. Consider $U_{\lambda_n} := (1-\lambda_n)I + \lambda_n U$ for $\lambda_n \in (\epsilon, 1-\kappa-\epsilon), \epsilon > 0$. Define $S_{r_2}^{F_n} = S_{r_2}^{F_n(modN)+1}$ and $S_{r_1}^{G_n} = S_{r_1}^{G_n(modM)+1}$ for all $n \geq 1$. Additionally, let $\{\alpha_n\}, \{\beta_n\}$ be two sequences of positive real numbers satisfying the following conditions:

1. $\lim_{n\to+\infty} \alpha_n = 0$ *and* $\sum_{n=0}^{+\infty} \alpha_n = +\infty$ *where* $\alpha_n \in (0,1)$,
2. $\beta_n \in (\delta, 1-\delta), \delta > 0$.

Then sequence $\{x_n\}$ converges strongly to $z^* \in \Omega \cap \mathrm{Fix}(U)$.

Proof. From Lemma 9, it is clear that the mappings $S_{r_1}^{G_n}$ and $S_{r_2}^{F_n}$ are firmly nonexpansive and hence 0-demicontractive. Hence result follows from Lemma 9 and Theorem 1. □

4.5. Multiple-Sets Split Monotone Variational Inclusion Problem

Assume that H_1 and H_2 are two real Hilbert spaces and $A : H_1 \to H_2$ is a bounded linear operator. Consider $P_i : H_1 \to 2^{H_1} (1 \leq i \leq N)$ and $Q_j : H_2 \to 2^{H_2} (1 \leq j \leq M)$ as two families of set valued maximal monotone operators. Let h_i be ν_i-ism operator in H_1 for each $i = 1, 2, \ldots, N$ and f_j be μ_j-ism operator in H_2 for each $j = 1, 2, \ldots, M$. Set $\mu = \min\{\nu_i, \mu_j\}$ and take $\tau \in (0, 2\mu)$. Then the multiple-sets split monotone variational inclusion problem is to identify $z^* \in H_1$ such that

$$0 \in \bigcap_{i=1}^{N} (h_i(z^*) + P_i(z^*)) \text{ and } y^* = Az^* \text{ solves } 0 \in \bigcap_{j=1}^{M} (f_j(Az^*) + Q_j(Az^*)).$$

We denote the solution set of the multiple-sets split monotone variational inclusion problem by Ω. We know that $0 \in h(z^*) + P(z^*)$ iff $z^* \in Fix(J_\lambda^P(I - \lambda h))$, where J_λ^P is a resolvent operator associated with multivalued mapping P and $\lambda > 0$. Additionally, if $\lambda \in (0, 2\alpha)$, then $J_\lambda^P(I - \lambda h)$ is averaged mapping, where h is an α-ism operator and P is a maximal monotone operator, see [9,43].

We now present a strong convergence theorem for solving multiple-sets split monotone variational inclusion problem.

Theorem 6. *Assume that $U : H_1 \to H_1$ is a demicontractive operator with constant $0 \leq \kappa < 1$ such that $I - U$ is demiclosed at zero. Suppose $\Omega \cap \mathrm{Fix}(U) \neq \phi$ and for any $x_1 \in H_1, \{x_n\}$ is defined by*

$$\begin{cases} y_n = \alpha_n g(x_n) + (1-\alpha_n) U_{\lambda_n}(x_n - \rho_n A^*(I - J_\tau^{Q_n}(I - \tau f_n))Ax_n), \\ x_{n+1} = (1-\beta_n) y_n + \beta_n J_\tau^{P_n}(I - \tau h_n)(y_n), \qquad \text{for all } n \geq 1 \end{cases} \tag{38}$$

where $\rho_n = \begin{cases} \frac{\|(I-J_\tau^{Q_n}(I-\tau f_n))Ax_n\|^2}{2\|A^*(I-J_\tau^{Q_n}(I-\tau f_n))Ax_n\|^2}, & Ax_n \neq J_\tau^{Q_n}(I - \tau f_n)Ax_n \\ 0, & otherwise \end{cases}$

and g is a γ contraction operator defined on H_1, $\gamma \in [0,1)$. Consider $U_{\lambda_n} := (1-\lambda_n)I + \lambda_n U$ for $\lambda_n \in (\epsilon, 1-\kappa-\epsilon), \epsilon > 0$. Define $J_\tau^{P_n}(I - \tau h_n) = J_\tau^{P_n(modN)+1}(I - \tau h_{n(modN)+1})$ and $J_\tau^{Q_n}(I - \tau f_n) = J_\tau^{Q_n(modM)+1}(I - \tau f_{n(modM)+1})$ for all $n \geq 1$. Additionally, let $\{\alpha_n\}, \{\beta_n\}$ be two sequences of positive real numbers satisfying the following conditions:

1. $\lim_{n\to+\infty} \alpha_n = 0$ *and* $\sum_{n=0}^{+\infty} \alpha_n = +\infty$ *where* $\alpha_n \in (0,1)$,
2. $\beta_n \in (\delta, 1-\delta), \delta > 0$.

Then sequence $\{x_n\}$ *converges strongly to* $z^* \in \Omega \cap \mathrm{Fix}(U)$.

Proof. Let us take $T_n = J_\tau^{Q_n}(I - \tau f_n)$ and $S_n = J_\tau^{P_n}(I - \tau h_n)$ for all $n \in N$. Then S_n and T_n are averaged mapping and hence nonexpansive. Additionally, $I - S_n$ and $I - T_n$ are demiclosed at zero. Hence, the result follows from the Main Theorem 1. □

Note that Theorem 6 generalizes and extends Moudafi's result [9] from weak convergence to strong convergence without knowing the operator norm.

Author Contributions: N.G., M.P., A.N., and R.C. participated equally in the conceptualization, validation, formal analysis, investigation, writing of the original draft preparation, review and editing. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Censor, Y.; Elfving, T. A multiprojection algorithm using Bregman projections in a product space. *Numer. Algorithms* **1994**, *8*, 221–239. [CrossRef]
2. He, H.; Ling, C.; Xu, H.K. An Implementable Splitting Algorithm for the ℓ_1-norm Regularized Split Feasibility Problem. *J. Sci. Comput.* **2016**, *67*, 281–298. [CrossRef]
3. Lorenz, D.A.; Shöpfer, F.; Wenger, S. The linearized Bregman method via split feasibility problems: Analysis and generalizations. *SIAM J. Imag. Sci.* **2014**, *7*, 1237–1262. [CrossRef]
4. Censor, Y.; Elfving, T.; Kopf, N.; Bortfeld, T. The multiple-sets split feasibility problem and its applications for inverse problems. *Inverse Probl.* **2005**, *21*, 2071–2084. [CrossRef]
5. Censor, Y.; Bortfeld, T.; Martin, B.; Trofimov, A. A unified approach for inversion problems in intensity-modulated radiation therapy. *Phys. Med. Biol.* **2006**, *51*, 2353–2365. [CrossRef]
6. Byrne, C. An unified treatment of some iterative algorithms in signal processing and image reconstruction. *Inverse Probl.* **2004**, *20*, 103–120. [CrossRef]
7. Censor, Y.; Segal, A. The split common fixed point problem for directed operators. *J. Convex Anal.* 2009, *16*, 587–600.
8. Censor, Y.; Gibali, A.; Reich, S. Algorithms for the split variational inequality problem. *Numer. Algorithms* **2012**, *59*, 301–323. [CrossRef]
9. Moudafi, A. Split monotone variational inclusions. *J. Optim. Theory Appl.* **2011**, *150*, 275–283. [CrossRef]
10. He, Z. The split equilibrium problem and its convergence algorithms. *J. Inequalities Appl.* **2012**, *2012*, 162. [CrossRef]
11. Byrne, C.; Censor, Y.; Gibali, A.; Reich, S. Weak and strong convergence of algorithms for the split common null point problem. *J. Nonlinear Convex Anal.* **2012**, *13*, 759–775.
12. Sahu, D.R.; Pitea, A.; Verma, M. A new iteration technique for nonlinear ope-rators as concerns convex programming and feasibility problems. *Numer. Algor.* **2020**, *83*, 421–449. [CrossRef]
13. Usurelu, G.I. Split feasibility handled by a single-projection three-step iteration with comparative analysis. *J. Nonlinear Convex Anal.* **2021**, *22*, in printing.
14. Nandal, A.; Chugh, R.; Postolache, M. Iteration process for fixed point problems and zeros of maximal monotone operators. *Symmetry* **2019**, *11*, 655. [CrossRef]
15. Ivković, S. Semi-Fredholm theory on Hilbert C^*-modules. *Banach J. Math. Anal.* **2019**, *13*, 989–1016. [CrossRef]
16. Postolache, M.; Nandal, A.; Chugh, R. Strong Convergence of a New Generalized Viscosity Implicit Rule and Some Applications in Hilbert Space. *Mathematics* **2019**, *7*, 773. [CrossRef]
17. Nandal, A.; Chugh, R. On Zeros of Accretive Operators with Application to the Convex Feasibility Problem. *UPB Sci. Bull. Ser. A* **2019**, *81*, 95–106.
18. Hussain, N.; Nandal, A.; Kumar, V.; Chugh, R. Multistep Generalized Viscosity Iterative Algorithm for solving Convex Feasibility Problems in Banach Spaces. *J. Nonlinear Convex Anal.* **2020**, *21*, 587–603.
19. Đukić, D.; Paunović, L.; Radenović, S. Convergence of iterates with errors of uniformly quasi-Lipschitzian mappings in cone metric spaces. *Kragujev. J. Math.* **2011**, *35*, 399–410.
20. Nandal, A.; Chugh, R; Kumari, S. Convergence Analysis of Algorithms for Variational Inequalities involving Strictly Pseudocontractive operators. *Poincare J. Anal. Appl.* **2019**, *2019*, 123–136.
21. Koskela, P.; Manojlović, V. Quasi-nearly subharmonic functions and quasiconformal mappings. *Potential Anal.* **2012**, *37*, 187–196. [CrossRef]
22. Yao, Y; Li, H; Postolache, M. Iterative algorithms for split equilibrium problems of monotone operators and fixed point problems of pseudo-contractions. *Optimization*. **2020**. [CrossRef]
23. Dadashi, V; Postolache, M. Forward-backward splitting algorithm for fixed point problems and zeros of the sum of monotone operators. *Arab. J. Math.* **2020**, *9*, 89–99. [CrossRef]

24. Yao, Y; Postolache, M; Liou, YC. Strong convergence of a self-adaptive method for the split feasibility problem. *Fixed Point Theory Appl.* **2013**, *2013*, 201. [CrossRef]
25. Jirakitpuwapat, W.; Kumam, P.; Cho, Y. J.; Sitthithakerngkiet, K. A general algorithm for the split common fixed point problem with its applications to signal processing. *Mathematics* **2019**, *7*, 226. [CrossRef]
26. Moudafi, A. The split common fixed-point problem for demicontractive mappings. *Inverse Probl.* **2010**, *26*, 055007. [CrossRef] [PubMed]
27. López, G.; Martín-Márquez, V.; Wang, F.; Xu, H.K. Solving the split feasibility problem without prior knowledge of matrix norms. *Inverse Probl.* **2012**, *28*, 085004. [CrossRef]
28. Shehu, Y.; Cholamjiak, P. Another look at the split common fixed point problem for demicontractive operators. *RACSAM* **2016**, *110*, 201–218. [CrossRef]
29. Opial, Z. Weak convergence of the sequence of successive approximations for nonexpansive mappings. *Bull. Am. Math. Soc.* **1967**, *73*, 591–597. [CrossRef]
30. Takahashi, W. Nonlinear functional analysis. In *Fixed Point Theory and Its Applications*; Yokohama Publishers: Yokohama, Japan, 2000.
31. Xu, H. K. Iterative algorithms for nonlinear operators. *J. Lond. Math. Soc.* **2002**, *66*, 240–256. [CrossRef]
32. Maingé, P. E. Strong convergence of projected subgradient methods for nonsmooth and nonstrictly convex minimization. *Set-Valued Anal.* **2008**, *16*, 899–912. [CrossRef]
33. Cui, H.; Wang, F. Iterative methods for the split common fixed point problem in Hilbert spaces. *Fixed Point Theory Appl.* **2014**, *2014*, 78. [CrossRef]
34. Wang, J.; Wang, Y. Strong convergence of a cyclic iterative algorithm for split common fixed-point problems of demicontractive mappings. *J. Nonlinear Var. Anal.* **2018**, *2*, 295–303.
35. Qin, L.J.; Wang, G. Multiple-set split feasibility problems for a finite family of demicontractive mappings in Hilbert spaces. *Math. Inequal. Appl.* **2013**, *16*, 1151–1157. [CrossRef]
36. Tang, Y.C.; Peng, J.G.; Liu, L.W. A cyclic algorithm for the split common fixed point problem of demicontractive mappings in Hilbert spaces. *Math. Model. Anal.* **2012**, *17*, 457–466. [CrossRef]
37. Boikanyo, O.A. A strongly convergent algorithm for the split common fixed point problem. *Appl. Math. Comput.* **2015**, *265*, 844–853. [CrossRef]
38. Tang, Y.C.; Peng, J.G.; Liu, L.W. A cyclic and simultaneous iterative algorithm for the multiple split common fixed point problem of demicontractive mappings. *Bull. Korean Math. Soc.* **2014**, *51*, 1527–1538. [CrossRef]
39. Maingé, P.E. The viscosity approximation process for quasi-nonexpansive mappings in Hilbert spaces. *Comput. Math. Appl.* **2010**, *59*, 74–79. [CrossRef]
40. Kazmi, K.R.; Rizvi, S.H.; Ali, R. A Hybrid-extragradient iterative method for split monotone variational inclusion, mixed equilibrium problem and fixed point problem for a nonexpansive mapping. *J. Nigerian Math. Soc.* **2016**, *35*, 312–338.
41. Kitkuan, D.; Kumam, P.; Padcharoen, A.; Kumam, W.; Thounthong, P. Algorithms for zeros of two accretive operators for solving convex minimization problems and its application to image restoration problems. *J. Comput. Appl. Math.* **2019**, *354*, 471–495. [CrossRef]
42. Combettes, P.L.; Hirstoaga, S.A. Equilibrium programming in Hilbert spaces. *J. Nonlinear Convex Anal.* **2005**, *6*, 117–136.
43. Shehu, Y.; Ogbuisi, F.U. An iterative method for solving split monotone variational inclusion and fixed point problems. *RACSAM* **2016**, *110*, 503–518. [CrossRef]