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A Spatial Panel Structural Vector Autoregressive Model with Interactive Effects and Its Simulation

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Abstract: The existing spatial panel structural vector auto-regressive model can effectively capture the time and spatial dynamic dependence of endogenous variables. However, the hypothesis that the common factors have the same effect for all spatial units is unreasonable. Therefore, incorporating time effects, spatial effects, and time-individual effects, this paper develops a more general spatial panel structural vector autoregressive model with interactive effects (ISpSVAR) that can reflect the different effects of common factors on different spatial units. Additionally, based on whether or not the common factors can be observed, this paper proposes procedures to estimate ISpSVAR separately and studies the finite sample properties of estimators by Monte Carlo simulation. The simulation results show the effectiveness of the proposed ISpSVAR model and its estimation procedures.

Keywords: spatial panel data; structural vector autoregressive model; interactive effects

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1. Introduction

The structural vector autoregressive (SVAR) model can reflect the contemporaneous relationship and dynamic effects of endogenous variables, and has been widely used in the dynamic analysis of economic relationships [1]. However, the drawbacks of SVAR analyses have become increasingly apparent in the application of regional science and spatial econometrics, since the spatial relationship of endogenous variables among spatial neighboring areas cannot be described in these models. Anselin and Getis [2] and Elhorst [3], for example, have noted that there is an obvious spatial dependence for spatial data. For this reason, Di Giacinto [4] and Beenstock and Felsenstein [5] constructed the spatial panel structural vector auto regressive (SpSVAR) model to overcome the shortcomings of SVAR in analyzing regional macro-economic problems. Essentially, they introduced spatial dependence into the SVAR model.

SpSVAR contains time and spatial dynamic relations of endogenous variables simultaneously. It plays an important role in many fields, including the studies of regional economic growth and environmental pollution. For instance, Monteiro [6] analyzes a hypothesis known as "pollution heaven" using the spatial panel vector autoregression (SpVAR) approach, and demonstrates that spatial spillovers seem to play a key role in FDI, openness, and environmental regulation. Marquez et al. [7] provide a SpVAR analysis of growth spillovers for Spanish regions, and suggest the existence of strong spatiotemporal regional growth spillovers.

However, SpSVAR still has flaws that cannot be ignored. Obviously, the hypothesis that exogenous variables have the same effect for all spatial units is unreasonable. Bai [8] attempts to introduce interactive effects into the panel data model to reflect unobservable common shocks and their heterogeneous impacts on different individuals. Yang et al. [9] assert that individual common factors in a fixed-effect panel data model generate the same effect for different individuals, and is not consistent with the economic theory and empirical intuition.

The existing literature on SpSVAR rarely involves these interactive effects. For instance, Di Giacinto [10] considers the spatial effects of economic variables, but the interactive effect of variables is ignored. Lee and Yu [11] discuss the interactive effects between time and space, but they use a data transformation method to eliminate these effects.

This paper suggests the incorporation of spatial effects, time effects and time-individual interactive effects, and develops a more general SpSVAR model based on interactive effects (ISpSVAR hereafter). Not only is the establishment of ISpSVAR an important extension of the spatial panel structural vector auto-regression model, it also provides a powerful tool for the study of regional economic issues.

The organization of the paper is structured as follows: Section 2 proposes ISpSVAR, Section 3 provides the estimation method of the model, Section 4 studies the finite sample properties of estimators on the basis of the Monte Carlo simulation, Section 5 serves as the conclusion.

2. The Model

The contemporaneous relations between endogenous variables in VAR have never been modeled explicitly, and they can only be reflected by the instantaneous covariance of the error term. A limitation of the model is that it is almost useless for structural analysis, such as impulse response and decomposition of prediction error. Thus, the following form of SVAR is proposed by Amisano and Giannini [12]

$$C_0 y_t = C_1 y_{t-1} + \ldots + C_p y_{t-p} + \Lambda F_t + \alpha_i + \varepsilon_t \quad (1)$$

where $\mathrm{y}_t = [y_{11t}, \ldots, y_{N1t}, \ldots, y_{1Kt}, \ldots, y_{NKt}]\prime$ is $NK \times 1$ dimensional endogenous variables, N is the number of spatial units, K is the number of endogenous variables, F_t denotes exogenous variables, C_0 is $NK \times NK$ dimensional instantaneous structural parameter, reflecting instantaneous structure relations of variables, C_i is the $NK \times NK$ dimensional coefficients matrix that reflects long-term variable relationships, Λ reflects the effects of common factors on individuals, suggesting that they affect different individuals identically, α_i represents the spatial fixed effects that reflect a fixed space and do not vary with time. ε_t ($\varepsilon_t = [\varepsilon_{11t}, \ldots, \varepsilon_{N1t}, \ldots, \varepsilon_{1Kt}, \ldots, \varepsilon_{NKt}]\prime$) is assumed to follow a normal distribution with zero mean and variance matrix Ω, and Ω is a diagonal matrix, that is

$$E(\varepsilon_t) = 0$$
$$E(\varepsilon_t \varepsilon_t') = \Omega = diag\{[\omega_{11}, \ldots, \omega_{N1}, \ldots, \omega_{1K}, \ldots, \omega_{NK}]\}$$
$$E(\varepsilon_t \varepsilon_{t-h}') = 0, \; h = 1, 2, \ldots$$

Equation (1) is unidentified since its parameter matrix is unconditional. To solve this problem, the basic idea is to impose reasonable limitations on parameters. Following Di Giacinto [4] and Beenstock and Felsenstein [5], we can restrict the parameter matrix on the basis of spatial structure directly by setting

$$C_h = \begin{bmatrix} A_{11}^{(h)} & A_{12}^{(h)} & \ldots & A_{1k}^{(h)} \\ A_{21}^{(h)} & A_{22}^{(h)} & \ldots & A_{2k}^{(h)} \\ \ldots & \ldots & \ldots & \ldots \\ A_{k1}^{(h)} & A_{k2}^{(h)} & \ldots & A_{kk}^{(h)} \end{bmatrix} \quad (2)$$

with

$$A_{kr}^{(h)} = \sum_{l=0}^{s} \Phi_{kr}^{(hl)} W_{kr}^{(l)}, \;\; \Phi_{kr}^{(hl)} = \varphi_{kr} I_N \quad (3)$$

where k, r = 1, 2 ... K, h = 1, 2 ... p. K is the number of endogenous variables, p is time lag orders, I_N denotes a $N \times N$ unit matrix, $W_{kr}^{(l)}$ is the $N \times N$ spatial weight matrix of order

l, whose elements $w_{kr}^{(l)}(i,j)$ are known and positive if locations i and j are neighbors of order l.

Obviously, the influence of exogenous variable F_t on different individuals is not distinguished in Equation (1). To reflect the different influence of common factors on individuals in processing panel data with unobservable common factors [8], we can establish an ISpSVAR model, which can be written as

$$C_0 y_t = \delta + C_1 y_{t-1} + \ldots + C_p y_{t-p} + \Lambda_i F_t + \alpha_i + \varepsilon_t \quad (4)$$

where δ is a constant term, and Λ_i reflects that common factors F_t affect individuals in different ways. Hence, Equation (4) contains time effects, spatial effects, and time-individual interactive effects.

However, observation of common factors differs according to different economic problems or areas of concern. For example, in the study of regional economic growth, regional units are not only affected by their own consumption, investment, and government expenditure, but also by some common factors like monetary and fiscal policy. In these cases, common factors like monetary and fiscal policy can be observed or quantized. However, in the study of environment pollution, regional units are not only affected by their own energy consumption, industrial structure, economic growth and other factors, but also by national environmental management policy, which is difficult to measure.

Therefore, according to whether or not common factors F_t can be observed, the estimation of ISpSVAR should be considered separately.

3. Estimation of ISpSVAR

Equation (4) cannot be estimated directly, so the most appropriate method is to first estimate its corresponding reduced form, and then calculate the parameters of structural form. The reduced form can be written as

$$y_t = B_1 y_{t-1} + \ldots + B_p y_{t-p} + \Psi_i F_t + \delta_i + \eta_t \quad (5)$$

where $B_h = C_0^{-1} C_h,\ (h = 1, 2, \ldots, p), \Psi_i = C_0^{-1}\Lambda_i, \delta_i = C_0^{-1}\alpha_i, \eta_t = C_0^{-1}\varepsilon_t$. Factor loadings Ψ_i reflect the contemporaneous impact of common factors on the endogenous variable y. The shocks η_t in reduced form is the linear combination of shocks ε_t in structural form, representing composite impact. As shown below, we can obtain the relation between the variance–covariance matrix of structural residuals (Σ_ε) and variance–covariance matrix of the reduced form's residuals through Choleski decomposition. Formally:

$$\Sigma = {C_0}^{-1} \Sigma_\varepsilon ({C_0}^{-1})\prime \quad (6)$$

In this way, the estimation of ISpSVAR is converted into the estimation of ISpVAR. However, there is still some difference in the estimation of Equation (5) according to whether or not common factors can be observed, and different estimation procedures are necessary. Therefore, we can estimate ISpSVAR using two steps: first, the estimation of ISpVAR, second, the estimation of ISpSVAR based on ISpVAR.

3.1. Estimation of ISpVAR with Common Factors Known

Assuming that common factors can be observed, the estimation procedures of Equation (5) can be established as follows.

(1) To facilitate the estimation, individual effects can be eliminated by the within-group transformation [13]. Therefore, model (5) can be written as

$$\widetilde{y}_t = B_1 \widetilde{y}_{t-1} + \ldots + B_p \widetilde{y}_{t-p} + \Psi_i F_t + \eta_t \quad (7)$$

where $\widetilde{y}_{it} = y_{it} - \bar{y}_{.t}$.

(2) Eliminate the interactive term in model (7) through orthogonal transformation. Firstly, standardize common factor F_t, that is,$\overline{F} = 0$, $F\prime F/(T-1) = I$. Then, define projection matrix $Q = I_{T-1} - F\prime F/(T-1)$. Lastly, premultiply Q for all variables in Equation (7) so as to eliminate common factors F_t [14]. Then, Equation (7) can be expressed as

$$Z_t = B_1 Z_{t-1} + \ldots + B_p Z_{t-p} + \xi_t \tag{8}$$

where $Z_t = Q\widetilde{y}_t$. Thus, the estimation of interactive effects SpVAR with known common factors is converted into the estimation of SpVAR.

(3) Estimate SpVAR and factor loadings, Ψ_i. According to Beenstock and Felsenstein [5], we estimate SpVAR by estimating the dynamic spatial panel model, and transform Equation (8) as follows

$$Z_t = B_1 Z_{t-1} + \ldots + B_p Z_{t-p} + \hat{B}_1 Z^*_{t-1} + \ldots + \hat{B}_p Z^*_{t-p} + \xi_t \tag{9}$$

where $Z^*_{kt-1} = Q(WZ_{kt-1})$. W, as above, is the spatial weight matrix. Quasi-maximum likelihood can be applied to the estimate dynamic spatial panel model [15] to obtain the parameter estimation of $B_1, \ldots, B_p; \hat{B}_1, \ldots, \hat{B}_p$.

Finally, the value of factor loadings can be expressed as

$$\Psi_i = (T-1)^{-1} F\prime \left(y_t - B_1 y_{t-1} - \ldots - B_p y_{t-p}\right) \tag{10}$$

3.2. Estimation of ISpVAR with Common Factors Unknown

We can only attain the empirical distribution of residuals for the estimation of SpVAR when common factors are unknown, and the parameter method cannot be used for statistical interference. Therefore, in this paper, we construct a bootstrap method based on the orthogonal nonlinear tool variable method to estimate the ISpVAR model with common factors unknown. The procedures are as follows:

(1) Estimate SpVAR that doesn't include interactive effects. Model (7) can be written as

$$\widetilde{y}_t = B_1 \widetilde{y}_{t-1} + \ldots + B_p \widetilde{y}_{t-p} + \xi_t \tag{11}$$

where $\xi_t = \Psi_i F_t + \eta_t$. Using QML, we can obtain the value of $\widetilde{B}_1, \ldots \widetilde{B}_p, \widetilde{\xi}_t$ in Equation (11).

(2) Using the method of random sampling, we can generate the residuals $\xi^*_1, \ldots, \xi^*_T$ used in the bootstrap from estimated residuals $\left\{\xi_1 - \overline{\xi}, \ldots, \xi_T - \overline{\xi}\right\}$, where $\xi^*_i = \xi_i - \overline{\xi}$ (i=1, 2 … T), $\xi_t = y_t - \widetilde{B}_1 y_{t-1} - \ldots - \widetilde{B}_p y_{t-p}$, $\overline{\xi} = T^{-1} \sum \xi_t$.

(3) Factor loadings Ψ_i and the initial estimated value of common factors F_t can be obtained through principal component analysis of residuals $\xi^*_1, \ldots, \xi^*_T$. Standardize F_t in accordance with Section 3.1, and then construct projection matrix Q using the standardized common factor.

(4) Eliminate common factor F_t by transforming Equation (5) as

$$\overline{y}_t = B_1 \overline{y}_{t-1} + \ldots + B_p \overline{y}_{t-p} + \varepsilon_t \tag{12}$$

where $\overline{y}_t = Q\widetilde{y}_t$. Repeating the estimation method of SpVAR in step (1) we can obtain the estimation of $B_1 \ldots B_p, \varepsilon_t$.

(5) The parameters, common factors and factor loadings in model (5) can be obtained by iterating (2) to (4) until the estimated parameters are converged.

3.3. Estimation of ISpSVAR

There is no difference in the estimation of ISpSVAR regarding whether the common factors are known or unknown when the parameter and variance–covariance matrix of the corresponding ISpVAR has been estimated. Therefore, the most critical thing is to estimate the corresponding structural form of Equation (7) as

$$C_0\widetilde{y}_t = C_1\widetilde{y}_{t-1} + \ldots + C_p\widetilde{y}_{t-p} + \Lambda_i F_t + \varepsilon_t \tag{13}$$

where $B_h = C_0{}^{-1}C_h,\ h = 1,2,\ldots p;\ \Psi_i = C_0{}^{-1}\Lambda_i;\ \ \eta_t = C_0{}^{-1}\varepsilon_t$. Using the variance–covariance relationship between reduced form and structural form, as shown in Equation (6), we can estimate $\widetilde{C}_0$ and Σ_ε, and other paraeters can be expressed as

$$\widetilde{C}_h = \widetilde{C}_0\widetilde{B}_h, h = 1,\ldots p;\ \widetilde{\Lambda}_i = \widetilde{C}_0\widetilde{\Psi}_i \tag{14}$$

Following Di Giacinto [10], we restrict C_0 to the following structure of block triangular matrix to avoid the identification problem

$$C_0 = \begin{bmatrix} A_{11}^{(0)} & 0 & \ldots & 0 \\ A_{21}^{(0)} & A_{22}^{(0)} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ A_{K1}^{(0)} & A_{K2}^{(0)} & \ldots & A_{KK}^{(0)} \end{bmatrix} \tag{15}$$

where

$$A_{kr}^{(0)} = \begin{cases} I_N - \sum\limits_{l=1}^{s} \Phi_{kr}^{(0l)} W_{kr}^{(l)} & r = k \\ -\sum\limits_{l=0}^{s} \Phi_{kr}^{(0l)} W_{kr}^{(l)} & r < k \end{cases},\ \Phi_{kr}^{(0l)} = \phi_{kr}^{(0l)} I_N \tag{16}$$

Based on the above constraints, we can estimate the parameters of Equation (13). However, the contemporaneous relations and spatial dependence between endogenous variables are included in Equation (13). Considering the feasibility and simplicity of the estimation program, we provide a practical way to deal with Equation (13). That is, bringing the contemporaneous relationship and spatial dependence between endogenous variables into the model using the following two steps.

(1) Considering contemporaneous relations of endogenous variables y, the structural form of Equation (7) can be written as

$$C_{01}\widetilde{y}_t = C_{10}\widetilde{y}_{t-1} + \ldots + C_{p0}\widetilde{y}_{t-p} + \Lambda_{i0} F_t + \varepsilon_{t0} \tag{17}$$

where $C_{01} = \begin{bmatrix} \Phi_{11} & 0 & \ldots & 0 \\ \Phi_{21} & \Phi_{22} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \Phi_{K1} & \Phi_{K2} & \ldots & \Phi_{KN} \end{bmatrix}$; $\Phi_{kr} = \phi_{kr} I_N$. The parameters in model (7) have been estimated; thus, we can adopt full information maximum likelihood (FIML) to estimate $\widetilde{C}_{01}, \widetilde{C}_{10}, \ldots \widetilde{C}_{p0},\ \widetilde{\Lambda}_{i0}, \sum \varepsilon_{t0}$ [12].

(2) Furthermore, we consider the spatial dependence of endogenous variables. Equation (17) can be written in tis reduced form

$$\widetilde{y}_t = C_{10}\widetilde{y}_{t-1} + \ldots + C_{p0}\widetilde{y}_{t-p} + \Lambda_{i0} F_t - \widetilde{C}_{01} y_t + \varepsilon_{t0} \tag{18}$$

where $\widetilde{C}_{01} = \begin{bmatrix} \Phi_{11} - I_N & 0 & \ldots & 0 \\ \Phi_{21} & \Phi_{22} - I_N & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \Phi_{K1} & \Phi_{K2} & \ldots & \Phi_{KN} - I_N. \end{bmatrix}$ Bringing contemporaneous spatial correlation item into Equation (18), we have

$$C_{02}\widetilde{y}_t = C_{11}\widetilde{y}_{t-1} + \ldots + C_{p1}\widetilde{y}_{t-p} + \Lambda_{i1} F_t - \widetilde{C}_{011}\widetilde{y}_t + \varepsilon_{t1} \tag{19}$$

where $C_{02} = \begin{bmatrix} \widetilde{A}_{11}^{(0)} & 0 & \dots & 0 \\ \widetilde{A}_{21}^{(0)} & \widetilde{A}_{22}^{(0)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \widetilde{A}_{K1}^{(0)} & \widetilde{A}_{K2}^{(0)} & \dots & \widetilde{A}_{KK}^{(0)} \end{bmatrix}$, $\widetilde{A}_{kr}^{(0)} = -\sum_{l=1}^{s} \Phi_{kr}^{(0l)} W_{kr}^{(l)}$, $\Phi_{kr}^{(0l)} = \phi_{kr}^{(0l)} I_N$. I_N denotes a $N \times N$ unit matrix, W denotes the spatial weight matrix.

Then, parameters $C_{02}, C_{11}, \dots C_{p1}, \Lambda_{i1}, \widetilde{C}_{011}$ in Equation (19) can be estimated by FIML. So far, we have completed the estimation of the structural form of Equation (13).

4. Monte Carlo Simulation

This part uses the interactive effects spatial panel structural vector auto regression model of one-period-lagged time and space (assuming there is only one common factor) as an example to study the finite sample properties of estimators.

4.1. Data Generation

Set the model of generating data as follows

$$C_0 y_t = C_1 y_{t-1} + \Lambda_i F_t + \varepsilon_t \tag{20}$$

Without loss of generality, we set the parameters of Equation (20) as follows

$$C_0 = \begin{bmatrix} I_N + \theta_{11} W & 0 \\ \theta_{21}^{(00)} I_N + \theta_{21}^{(01)} W & I_N + \theta_{22} W \end{bmatrix},\ C_1 = \begin{bmatrix} \varphi_{11}^{(00)} I_N + \varphi_{11}^{(01)} W & \varphi_{12}^{(00)} I_N + \varphi_{12}^{(01)} W \\ \varphi_{21}^{(00)} I_N + \varphi_{21}^{(01)} W & \varphi_{22}^{(00)} I_N + \varphi_{22}^{(01)} W \end{bmatrix},$$

where $\begin{matrix} \theta_{11} = -0.23,\ \theta_{21}^{(00)} = -0.45,\ \theta_{21}^{(01)} = 0.16,\ \theta_{22} = -0.32 \\ \varphi_{11}^{(00)} = 0.41, \varphi_{11}^{(01)} = -0.37, \varphi_{12}^{(00)} = 0.53, \varphi_{12}^{(01)} = 0.28 \\ \varphi_{21}^{(00)} = -0.52, \varphi_{12}^{(01)} = 0.19, \varphi_{22}^{(00)} = 0.21, \varphi_{22}^{(01)} = 0.36 \end{matrix}$, This paper uses a first-order Rook matrix, assuming factor loadings, Λ_i is a uniform distribution of intervals $[-1, 1]$, setting $y_1 \sim 2 + N(0,1)$, supposing common factor F_t obeys standard normal distribution, setting random error term ε_t follows a normal distribution, $\varepsilon_t \sim i.i.d\ N(0, \Omega)$, $\Omega = \begin{bmatrix} 0.2 & 0 \\ 0 & 1.1 \end{bmatrix}$.

We set a different cross-section (N = 10, 20, 30) and time (T = 5, 10, 30) in order to study the influence of sample sizes on estimators and simulate 500 times.

4.2. Finite Sample Properties of Estimators

We can measure the estimators from the perspective of unbiasedness and stability by using bias and Root Mean Square Error (RMSE) [16]. The formulas are listed as follows

$$Bias = \frac{1}{n} \sum_{i=1}^{n} (\hat{\beta}_i - \beta) \tag{21}$$

$$RMSE = \frac{1}{n} \sqrt{\sum_{i=1}^{n} (\hat{\beta}_i - \beta)^2} \tag{22}$$

where $\hat{\beta}_i$ denotes the estimated values of every simulation, β is initial values. The result of simulation is shown in Table 1.

The performance of estimation procedures depends on the RMSE value, and the general value of RMSE is 0.05 [17]. As shown in Table 1, the value of Bias and RMSE for most estimators is relatively small and reasonable. This demonstrates the rationality of the procedures for estimating interactive effects SpSVAR. Additionally, when cross-section N (or time T) is the same, the absolute value of Bias and RMSE will decrease with the increase in time T (or cross-section N) for most parameter estimations. This shows that the accuracy of the estimation will significantly improve with an increase in sample sizes.

Table 1. Simulation results of parameter estimation.

		N = 10 T = 5	N = 10 T =10	N = 10 T =30	N = 20 T =5	N = 20 T =10	N = 20 T =30	N = 30 T =5	N = 30 T =10	N = 30 T =30
θ_{11}	*Bias*	0.0911	0.0201	0.0126	0.0325	0.0213	0.0074	0.0574	0.0271	0.0234
	RMSE	0.0186	0.0109	0.0050	0.0143	0.0084	0.0046	0.0133	0.0069	0.0045
$\theta_{21}^{(00)}$	*Bias*	0.2207	0.1658	0.2125	0.0996	0.1792	0.1645	−0.2216	−0.1839	−0.2716
	RMSE	0.0528	0.0307	0.0245	0.0352	0.0266	0.0200	0.0301	0.0223	0.0221
$\theta_{21}^{(01)}$	*Bias*	−0.1419	−0.1185	−0.1293	−0.1137	−0.1266	−0.1191	−0.0601	−0.0521	−0.0427
	RMSE	0.0162	0.0135	0.0132	0.0136	0.0134	0.0122	0.0237	0.0068	0.005
θ_{22}	*Bias*	0.0658	0.0274	0.0208	0.048	0.0443	0.0313	0.0655	0.0588	0.034
	RMSE	0.0175	0.0094	0.0057	0.0144	0.0109	0.0052	0.0622	0.0097	0.0054
$\varphi_{11}^{(00)}$	*Bias*	−0.0518	−0.0169	−0.0063	−0.0282	−0.0188	−0.0046	−0.0237	−0.0088	−0.0043
	RMSE	0.0092	0.004	0.0021	0.0067	0.0034	0.0019	0.0051	0.0024	0.0013
$\varphi_{11}^{(01)}$	*Bias*	0.0437	0.0188	0.0096	0.0148	0.019	0.019	0.0216	0.0166	0.0062
	RMSE	0.0107	0.0047	0.0027	0.0083	0.0045	0.0061	0.0065	0.0037	0.0019
$\varphi_{12}^{(00)}$	*Bias*	0.0223	0.0047	0.0017	0.0036	0.0014	0.0042	0.0127	0.0032	0.0037
	RMSE	0.0078	0.0042	0.0022	0.0048	0.0026	0.0015	0.0033	0.0022	0.0011
$\varphi_{12}^{(01)}$	*Bias*	0.0811	0.0134	0.0094	0.0386	0.013	0.0048	0.0505	0.0175	0.0186
	RMSE	0.0162	0.0096	0.0046	0.0135	0.0075	0.004	0.0113	0.0058	0.0039
$\varphi_{21}^{(00)}$	*Bias*	0.0376	0.051	0.0891	−0.0126	0.0499	0.062	0.1009	−0.0945	−0.0917
	RMSE	0.0287	0.0181	0.0117	0.0227	0.0142	0.0093	0.0123	0.0125	0.0103
$\varphi_{21}^{(01)}$	*Bias*	−0.0252	−0.0499	−0.1060	0.0135	−0.0593	−0.0786	0.113	0.0494	0.0514
	RMSE	0.0246	0.017	0.013	0.024	0.0146	0.0105	0.022	0.0199	0.0075
$\varphi_{22}^{(00)}$	*Bias*	−0.0187	0.0282	0.0983	−0.0544	0.0438	0.0668	−0.2105	−0.1329	−0.1272
	RMSE	0.0294	0.0173	0.0127	0.0238	0.0132	0.0094	0.0268	0.0162	0.0135
$\varphi_{22}^{(01)}$	*Bias*	0.0962	0.0205	0.0214	0.0387	0.0336	0.0115	−0.0805	−0.0642	−0.1054
	RMSE	0.0311	0.0158	0.0096	0.0199	0.0124	0.0067	0.0183	0.013	0.012
$\overline{\Lambda}$	*Bias*	0.0067	0.0065	0.0116	−0.0093	−0.0075	−0.0102	−0.0076	−0.0004	−0.0001
	RMSE	0.0419	0.0308	0.0196	0.0342	0.0264	0.0161	0.0478	0.0341	0.022

Notes. To save space, we list the average results of factor loadings Λ_i.

5. Conclusions

This paper constructs an interactive effects SpSVAR (ISpSVAR) which includes time effects, spatial effects, and time-individual interactive effects. Based on whether or not the common factors can be observed, we propose procedures to estimate ISpSVAR separately. ISpSVAR brings time and spatial dynamic relations of endogenous variables into the model, which plays an important role in many areas, such as regional macro-economic growth, environmental pollution, etc. Finally, this paper studies the finite sample properties of estimators by Monte Carlo simulation—the results demonstrate the effectiveness of the estimation procedures.

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Abbreviations

VAR	Vector AutoRegression Model
SVAR	Structural Vector AutoRegression Model
SpVAR	Spatial Panel Vector AutoRegression Model
SpSVAR	Spatial Panel Structural Vector AutoRegression Model
ISpSVAR	Interactive Effects Spatial Panel Structural Vector AutoRegression Model

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