risks

Review

A Systematic Literature Review of Insurance Claims Risk Measurement Using the Hidden Markov Model

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Abstract: In the rapidly evolving field of insurance, accurate risk measurement is crucial for effective claims management and financial stability. Therefore, this research presented a systematic literature review (SLR) on insurance claims risk measurement using the Hidden Markov Model (HMM). Bibliometric analysis was conducted using VOSviewer 1.6.20 and ResearchRabbit software to map research trends and collaboration networks in this topic. This review explored the implementation of the HMM in predicting the frequency and severity of insurance claims, with a focus on the statistical distribution methods used. In addition, the research emphasized the influence of the number of hidden states in the HMM on claims behavior, both in terms of frequency and magnitude, and provided interpretations of these hidden dynamics. Data sources for this review comprised three databases, namely, Scopus, ScienceDirect, and Dimensions, and additional papers from a website. The article selection process followed updated PRISMA 2020 guidelines, resulting in twelve key papers relevant to the topic. The results offered insights into the application of the HMM for forecasting the frequency and severity of insurance claims and opened avenues for further investigation on distribution models and hidden state modeling.

Keywords: insurance claims; risk measurement; SLR; bibliometric analysis; Hidden Markov Model

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1. Introduction

In the contemporary insurance landscape, accurately measuring and managing claims risk is important for ensuring financial stability and operational efficiency. According to Omari et al. (2018), insurance companies face challenges in predicting claims occurrences and severities. Even though traditional statistical models have been consistently used for these purposes, their limitations in capturing the dynamic and often hidden patterns in claims data necessitate the exploration of more sophisticated methodologies. The Hidden Markov Model (HMM) has originated as a powerful tool in this regard, offering improved capabilities for modeling sequential data and uncovering latent states that influence observable outcomes (Zucchini and MacDonald 2009).

Initially developed for speech recognition and bioinformatics, the HMM is currently being applied in various fields, including finance and insurance (Oflaz et al. 2019). Awad and Khanna (2015) stated that the model was particularly adept at handling time series data, where the underlying process was assumed to follow a Markov process with unobserved (hidden) states. This unique characteristic makes the HMM suitable for analyzing insurance claims data, often exhibiting temporal dependencies and underlying factors not directly observable.

Several research have analyzed the application of the HMM (see Mor et al. 2021; Odumuyiwa and Osisiogu 2019; Ramaki et al. 2018). Although these pieces of research provided a systematic review of the HMM and specialized applications, there is no specific focus on insurance claims modeling. Therefore, a literature review of this topic presents a novel contribution. The application of the HMM in measuring insurance claims is crucial for providing practical guidance to insurance companies in implementing the HMM and improving the claims process. This may include recommendations regarding model parameters, simulation data, emerging trends, and other practical applications. The current research offered bibliometric analysis and a systematic literature review (SLR), with a focus on the HMM and insurance claims risk measurement. The first investigation on the role of the HMM in insurance claims can be traced back to Paroli et al. (2000). Subsequently, Lu and Zeng (2012) and Oflaz et al. (2019) developed the role of the HMM in insurance claims, particularly in modeling claims counts in a bivariate context.

SLR aimed to provide a comprehensive analysis of the applications of the HMM in measuring insurance claims risk. The review covered the period from the early 2000s to the present, reflecting the growing interest and advancements in this area. The following research questions were addressed: How does the implementation of the HMM impact the prediction of frequency or severity in insurance claims? How does the number of hidden states in the HMM affect the pattern of frequency or severity in claims? By synthesizing existing results, this review aimed to identify the strengths and limitations of the HMM in the context of insurance claims and identify areas for future investigations.

To achieve the stated objectives, the Scopus, Science Direct, and Dimensions databases were used, considering keywords related to the research questions. Bibliometric analysis was conducted using VOSviewer software and ResearchRabbit website. In essence, this research aimed to contribute to the broader understanding and implementation of advanced modeling in the insurance sector, fostering more accurate and effective risk management strategies. The subsequent section of this paper is structured as follows: Section 2 provides a preview of the HMM and the methods used to collect articles. In Section 3, the results of bibliometric analysis are presented and described. Section 4 provides a discussion and addresses the research questions. Lastly, Section 5 presents conclusions and recommendations for future investigations.

2. Materials and Methods

2.1. A Review of Hidden Markov Model

According to Koerniawan et al. (2020), the HMM is a discrete-time stochastic process consisting of pairs X and Y. Y is a Markov Chain representing events that cannot be observed directly, while X is an observation depending on Y. These are the main components of the HMM, as described by Orfanogiannaki and Karlis (2018).

- Hidden states $Y = \{y_1, y_2, \ldots, y_n\}$ through which the system transitions. These states are not directly visible.
- A set of possible observations $X = \{x_1, x_2, \ldots, x_m\}$ that can be seen. Each observation corresponds to a particular hidden state.
- Transition probability matrix $A = [a_{ij}]$ where a_{ij} is the probability of transitioning from state y_i to state y_j:

$$a_{ij} = P(y_{t+1} = y_j | y_t = y_i). \tag{1}$$

  The rows of this matrix sum to 1.
- Emission probabilities matrix $B = [b_j(k)]$ where $b_j(k)$ is the probability of observing x_k from state y_j:

$$b_j(k) = P(x_t = x_k | y_t = y_j). \tag{2}$$

  Each row in this matrix also sums to 1.
- Initial probabilities vector $\pi = [\pi_i]$ where π_i is the probability of starting in state y_i:

$$\pi_i = P(y_1 = y_i). \tag{3}$$

There are three main problems in the HMM. First, the evaluation problem, given the HMM and a sequence of observations, determines the probability of the sequence occurring. This can be addressed using the Forward–Backward Algorithm. Second is the decoding problem; given the HMM and a sequence of observations, it can determine the most likely sequence of hidden states that has led to the observations. This can be addressed using the Viterbi Algorithm. The third is the learning problem; given a sequence of observations and the general structure of the HMM, it can determine the model parameters (transition and emission probability) that best explain the observed sequence. This can be addressed using the Baum–Welch Algorithm, a form of the Expectation–Maximization algorithm. Preserve and Yin (2018) presented parameter estimation.

In the insurance context, the HMM can be used to model the dynamics of insurance claims by accounting for observable variables and hidden variables that influence claims frequency. For instance, in the context of auto insurance claims, observable variables are directly available from claims records and data from policyholders. Examples are claims frequency, severity, and time, as well as driver and vehicle information. Meanwhile, hidden variables influence the claims process but cannot be directly observed. An example of a hidden variable is the driver's risk state, which may change over time (e.g., careful versus reckless drivers). The HMM can be specifically applied as follows. Claims frequency is denoted X (an observable variable) and the driver's risk state is denoted Y. The driver's risk state is assumed to be unobservable but affects claims frequency. A driver's risk condition can be classified as low, medium, or high risk, corresponding to different claims frequencies, and denoted as $Y = \{y_1, y_2, y_3\}$, respectively. The initial probabilities are denoted by $\pi = \{\pi_1, \pi_2, \pi_3\}$, where π_i is the probability that the driver is in state y_i at $t = 0$. The transition probabilities $A = [a_{ij}]$, as explained previously, describe the likelihood of transitioning from one risk state to another. The emission probabilities describe the probability of observing a claim x_k from state y_j.

2.2. Methods

According to Ortega-Rodríguez et al. (2020), SLR allows for the clear identification, analysis, and synthesis of the existing body of literature. The analysis was carried out in four phases. The eligibility criteria followed a PRISMA diagram (Figure 1), and each phase is described as follows. First, based on the research question, the related keywords were ("Hidden Markov Model" OR "HMM") AND (("insurance" OR "frequency" OR "severity") AND "claim" OR "claims"). These were used to search for publications in selected databases, namely, Scopus, ScienceDirect, and Dimensions. The databases were selected because they contain significant publications on the research topic. The journals included in Scopus and Science Direct adhere to high standards of impact and quality. Dimensions, a newly free scholarly database, includes journal articles and citation counts and is considered a plausible alternative for supporting some types of research evaluations (Thelwall 2018). Previous literature reviews also used these databases (see Ortega-Rodríguez et al. 2020; Sukono et al. 2022; Firdaniza et al. 2022).

A total of 15 papers were obtained from Scopus, 301 from Science Direct, and 623 from Dimensions. Furthermore, in this second phase, Jabref Software was used to identify and remove duplicate articles, leaving a total of 922 unique papers. The papers retrieved from the database must be in the .bib format if they are to be imported into the JabRef software.

In the third phase, an initial screening of titles and abstracts was conducted using Jabref Software, resulting in the selection of 28 papers. This software facilitates researchers in viewing the titles and abstracts of papers without the need to download the full text. These papers met the criteria by being published after 1999, the year when the first article on the HMM application in insurance was published by Paroli et al. (2000). This paper became a key reference for subsequent research related to the application of the HMM in insurance. Papers discussing the HMM but in the field of speech recognition were excluded in this phase.

Finally, after an in-depth reading of each article, seven papers were identified as being directly related to the research topic, specifically focusing on the HMM in the context of claims in insurance, which discussed the frequency or severity of claims. However, these seven papers are still very limited and were deemed insufficient to explore research on the application of the HMM in insurance claims, particularly concerning the frequency and severity of claims. Therefore, additional related studies were included through a website search, which is permitted under the updated PRISMA 2020 guidelines. The type of publications allowed in the search may include journal articles, preprints, conference abstracts, study register entries, clinical study reports, dissertations, unpublished manuscripts, government reports, or any other documents providing relevant information (see Page et al. (2021) for further details).

The website search began with the keyword "Hidden Markov Model for Insurance Claims", resulting in numerous publications related to the topic, but only 25 papers were downloadable for screening. Similar to the papers sourced from databases, the second phase involved screening the titles and abstracts, leading to 10 papers selected for in-depth reading. After a thorough examination in the third phase, five papers remained relevant to the topic. These five papers were combined with the screened results from databases, yielding a total of 12 papers related to the topic. Subsequently, these 12 papers were used for bibliometric analysis and to address the research questions.

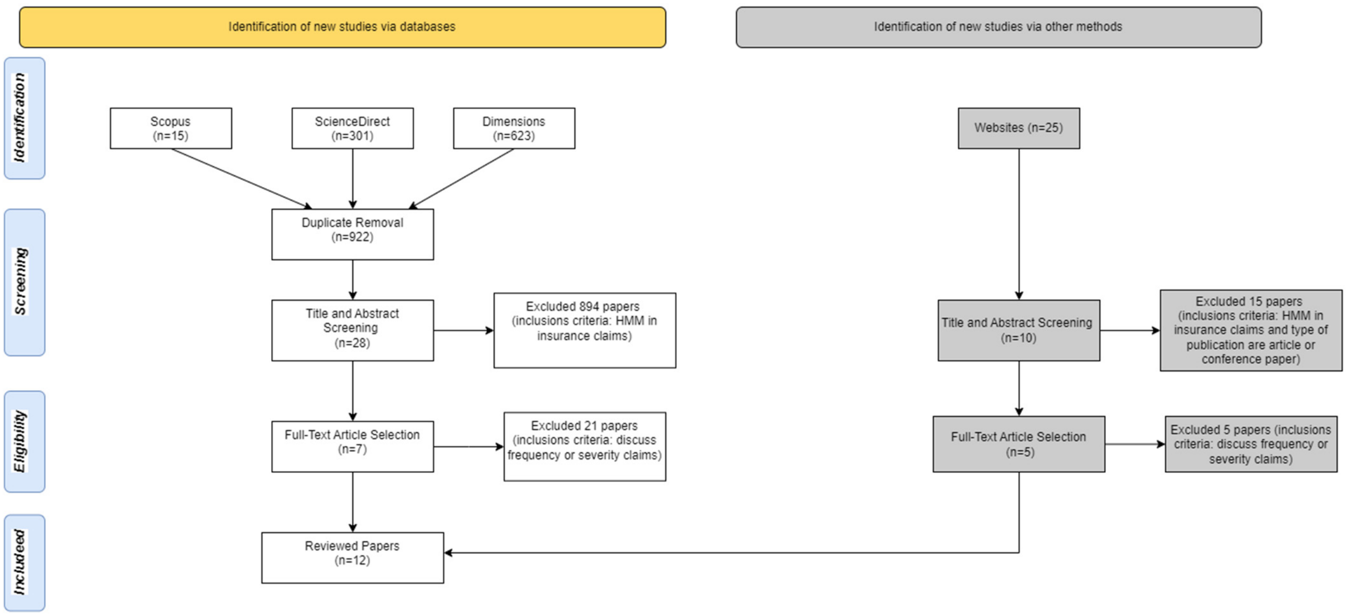


Figure 1. Phases of SLR presented on PRISMA diagram.

From the 12 selected articles, bibliometric analysis was conducted to provide a comprehensive overview of the research being conducted, helping other research experts to identify trends, collaboration opportunities, and future research directions. This current research used VOSviewer software to analyze and visualize bibliometric data, while the ResearchRabbit website was used to recommend relevant articles and provide network visualizations showing relationships between articles and authors.

3. Results

3.1. Results from Bibliometric Analysis

The first analysis addressed the trend of publications related to the application of the HMM in insurance. Based on the established criteria and the analysis in Figure 2, the publication of articles in this research area followed a linear trend. This could be attributed to the relatively limited amount of research conducted on this specific topic. However, a minimum of one article has been published every two years over the last 10 years that focuses on the application of the HMM in insurance. This steady stream of publications showed literature on this topic remained relevant and important. Despite the relatively low volume of publications, the consistent interest by research experts showed the potential and ongoing importance of the HMM in advancing insurance risk measurement and modeling. The steady output shows that, while the field may be niche, it holds significant value and continues to garner attention in the academic community.

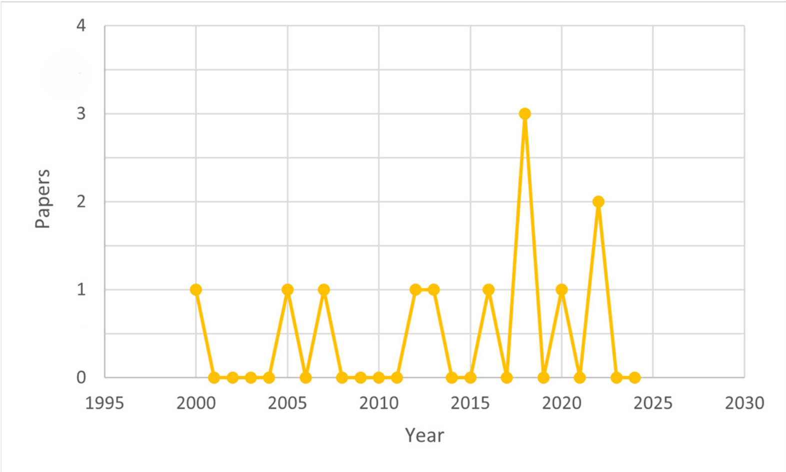


Figure 2. The annual number of papers on the application of the HMM in insurance claims.

3.1.1. Type of Publication

The publication types related to research on the application of the Hidden Markov Model (HMM) in insurance claims are shown in Figure 3. The most common type of publication is a journal article, of which there are seven papers, accounting for the majority of the total publications. This indicates that research on the application of the HMM in insurance claims is predominantly published in scientific journals, reflecting a strong focus on quality and higher scientific contribution. Conference proceedings were the second most common, with two papers included, indicating that many early research findings or recent developments in HMM application are presented at academic forums before being formally published in journals. Meanwhile, there are two theses, indicating a smaller proportion of the total, suggesting that this topic is often used as a research subject in graduate programs, particularly in mathematics, statistics, and actuarial science. On the other hand, a report is the least common type of publication, indicating that research in the form of formal reports is less prevalent compared to other types of publications. Overall, it can be concluded that journal articles are the primary medium for publishing research related to the HMM and insurance claims.

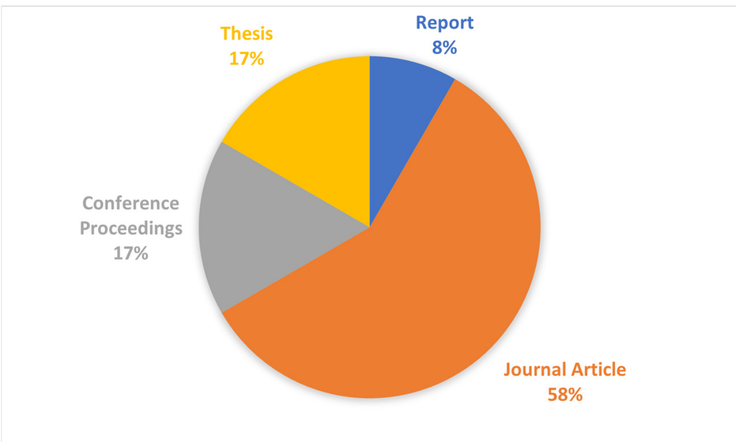


Figure 3. Type of publication of papers on the applications of the HMM in insurance claims.

3.1.2. Visualization and Analysis of Journals

Figure 4 shows that the ASTIN Bulletin is a prominent journal with a history of having published two articles specifically related to the application of the HMM in the insurance sector. In contrast, other journals have only published one article on this topic. Therefore, ASTIN Bulletin has a stronger inclination to accept and publish articles on HMM applications within the insurance context, reflecting a particular interest and focus in this niche area. This track record shows a significant opportunity for experts to publish work on the HMM in insurance. The journal's established expertise in this subject area increases the likelihood of acceptance for relevant research submissions. Conversely, journals with only a single publication on the topic also present untapped potential. These journals may be open to expanding the coverage of HMM applications in insurance, suggesting additional avenues for publication. Due to these observations, articles on the HMM in insurance should consider a dual method, namely, targeting the ASTIN Bulletin for its demonstrated interest while also submitting to other journals with preliminary engagement with the topic. By applying a strategic publication plan that includes both established and emerging journals, research experts can increase their chances of article acceptance and contribute to the broader discourse on the HMM in insurance.

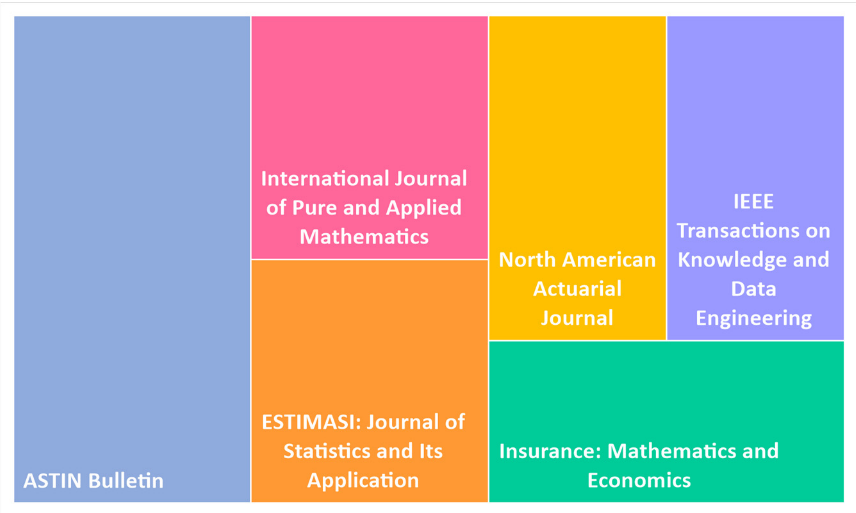


Figure 4. List of journals that publish HMM applications in insurance claims.

An analysis was conducted on the journals that published seven articles, focusing on their quartile ranking in the Scimago Journal Rank (SJR) and inclusion in reputable indexers. The SJR is a metric that evaluates the scientific impact of academic journals by considering the number of citations received by articles published in those journals. Journals with high SJR scores were regarded as more prestigious and of higher quality, typically attracting more citations compared to others with comparable quality (Kumar et al. 2023).

The reputable indexers considered in this research were Scopus and Web of Science (WoS). Both indexers were highly esteemed in the academic community for rigorous publication standards and selective indexing processes. This ensured that the articles included were of high quality and relevant within respective scientific fields, functioning similarly to a curation process with high academic standards.

The results are summarized in Table 1, providing an overview of the journals' impact and adherence to high-quality publication standards, as reflected by SJR rankings and indexing status. This detailed evaluation helped to understand the credibility and academic significance of the journals where the selected articles were published.

Based on Table 1, the journals in question are predominantly ranked as SJR Q1. The designation showed that these journals were in the top 25% of their field, reflecting their significant influence and high standing in their corresponding scientific disciplines

(Kumar et al. 2023). Furthermore, this high ranking showed the prestige and impact of these journals in the academic community.

Table 1. Analysis results of journals that publish HMM applications in insurance claims.

Journal	SJR	Indexed by	
		Scopus	**WoS**
ASTIN Bulletin	Q1	Yes	Yes
International Journal of Pure and Applied Mathematics	-	No (discontinued in 2016)	No
ESTIMASI: Journal of Statistics and Its Application	-	No	No
Insurance: Mathematics and Economics	Q1	Yes	Yes
IEEE Transactions on Knowledge and Data Engineering	Q1	Yes	Yes
North American Actuarial Journal	Q2	Yes	Yes

The journals covered a wide range of fields, with the most prominent areas of publication being Applied Mathematics and Mathematics and Statistics, followed by Economics and Econometrics, as well as Computer Science. This distribution showed the interdisciplinary nature of the articles published in these journals.

For a more detailed view, Figure 5 provides a visualization of the fields represented in these journals. The visualization showed that the applications of the HMM transcended multiple disciplines. In addition, the broad applications explained the versatility and utility of the HMM across various scientific domains, reinforcing its significance as a robust analytical instrument in diverse research areas.

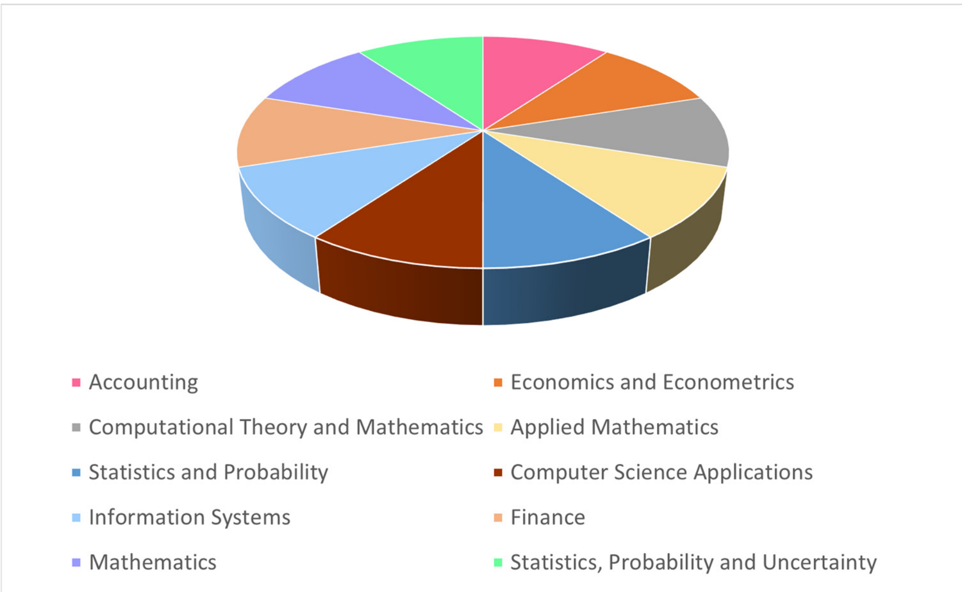


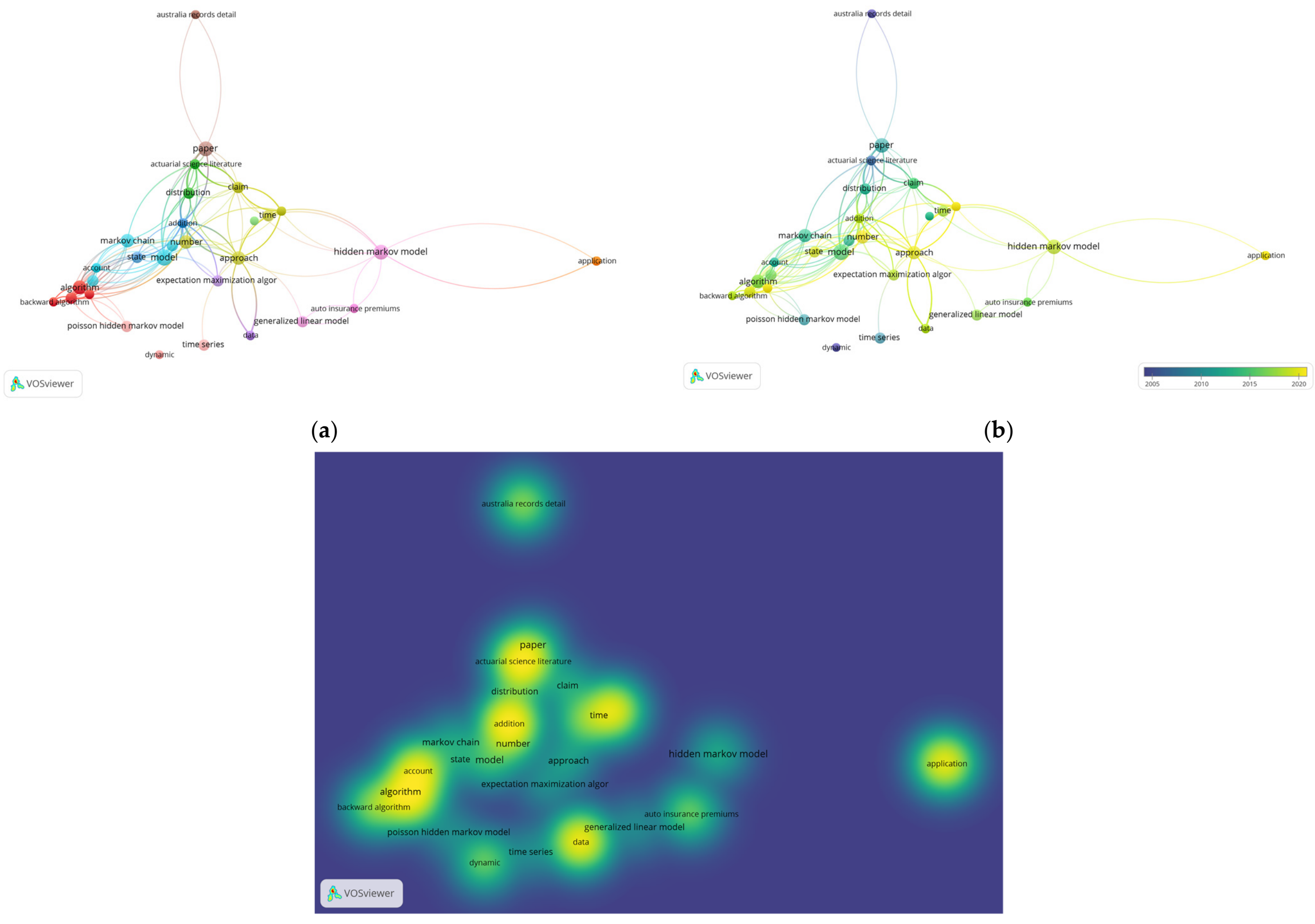
Figure 5. Field area research of journals that publish HMM applications in insurance claims.

3.1.3. Visualization of Keywords and Authors

The VOSviewer software was used to analyze keywords from the 12 selected papers. This software generated a visual map of related keywords, showing their distribution and interconnections. VOSviewer was specifically used to analyze keywords in the titles and abstracts of the articles. Figure 6a presents the analysis results, showing ten different colors representing ten clusters of HMM applications research in insurance claims. The pink cluster shows the Hidden Markov Model, Generalized Linear Model, Auto Insurance premiums, and Application. This cluster illustrates the primary application of the HMM in various contexts, particularly in the field of automobile insurance and premium determination. The Generalized Linear Model (GLM) is often used in conjunction with the HMM to model premiums based on past claims, especially within the context of auto insurance (see Berry 2016). The yellow cluster includes Claim, Time, and Approach. Focusing on the core variables related to insurance claim prediction, time, and approach methods are key elements in understanding the frequency and severity of claims, particularly in how the HMM is used to model changes in claim patterns over time. The green cluster includes Actuarial Science Literature and Distribution. This cluster is related to the scientific literature in the field of actuarial science, with a particular focus on claim distribution and claim numbers analyzed using the HMM. It highlights how the HMM is combined with probability distributions in claim analysis. The red cluster (Algorithm, Poisson Hidden Markov Model, Backward Algorithm) focuses on the algorithms used in the parameter estimation process of the HMM, such as the Poisson Hidden Markov Model using the Backward Algorithm (see Paroli et al. 2000; Koerniawan et al. 2020). This indicates the use of complex computational methods in HMM-based insurance claim analysis, which are crucial for obtaining accurate estimates. The blue cluster (Markov Chain, Model, Dynamic) highlights the fundamental technical aspects of HMM, namely, the Markov chain and hidden state models. The main focus is on how these models are used to represent dynamic hidden states within claim data, which indirectly affect the frequency and magnitude of claims. The chocolate cluster (Paper and Australia Records Detail) contains keywords such as Australian records detail, indicating that some research or empirical data on the application of the HMM may be based in Australia or related to specific case studies in the region (see Tsoi et al. 2005) utilizing detailed insurance claim data. The dark blue cluster (Addition and State) focuses on the state of the HMM. The orange cluster (Application) highlights the direct application of the HMM in various insurance claim scenarios. It reinforces the focus on how this model is not only studied theoretically but also implemented in real-world scenarios, particularly in insurance applications. The peach cluster (Time Series) emphasizes the role of data, particularly time series data, in the application of the HMM. Time series data are highly relevant in the context of insurance claims, as claim patterns often occur in temporal sequences, and the HMM is a well-suited model to capture these dynamics. The violet cluster (Expectation Maximization Algorithm and Data) focuses on the Expectation Maximization (EM) algorithm and the dynamic nature of claim data; this cluster illustrates the parameter estimation methods in the HMM, particularly in dealing with dynamic data and uncertainty at each step of the claims process.

Furthermore, Figure 6b shows the topic overlay of the 12 articles. The brightness of the color confirmed the recency of the topic, with brighter colors representing more recent articles. The latest topic on the application of the HMM is shown in the brightest color. Figure 6c shows the density of research topics, where the brighter the color (yellow), the more articles on those keywords. The algorithm in the HMM was a particularly well-studied topic, as evidenced by the increasingly thick yellow color in the circle shown in Figure 6c. The HMM algorithm was comprehensively analyzed by Alwansyah and Rachmawati (2024).

Figure 7 shows the mapping of the authors of the 12 selected papers. There was no overlapping network as the authors were not related as they had distinct interests regarding the application of the HMM in insurance. This lack of interconnection confirmed that each research expert or group worked independently, focusing on different aspects of HMM applications in the insurance domain. This situation presented a valuable opportunity for collaboration. By fostering connections between the authors, research experts could provide avenues for shared discussions and the exchange of ideas. Furthermore, collaboration could improve the quality of research by incorporating diverse perspectives and expertise and even lead to more robust methodologies, improved data analysis techniques, and better interpretations of results. Collaboration could also advance the field of HMM applications in insurance and lead to more comprehensive and innovative outcomes.



(**a**) (**b**)

(**c**)

Figure 6. Bibliographic mapping by the co-occurrence of keywords: (**a**) network visualization; (**b**) overlay visualization; (**c**) density visualization.

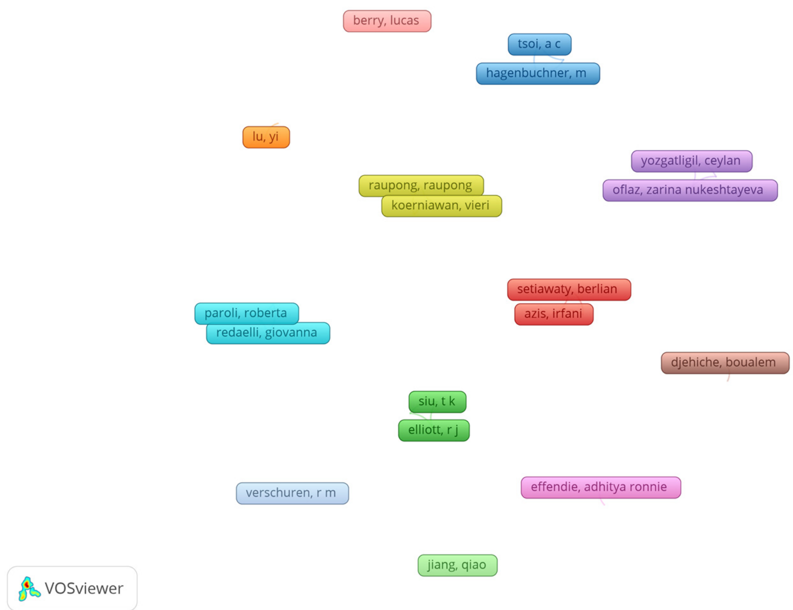
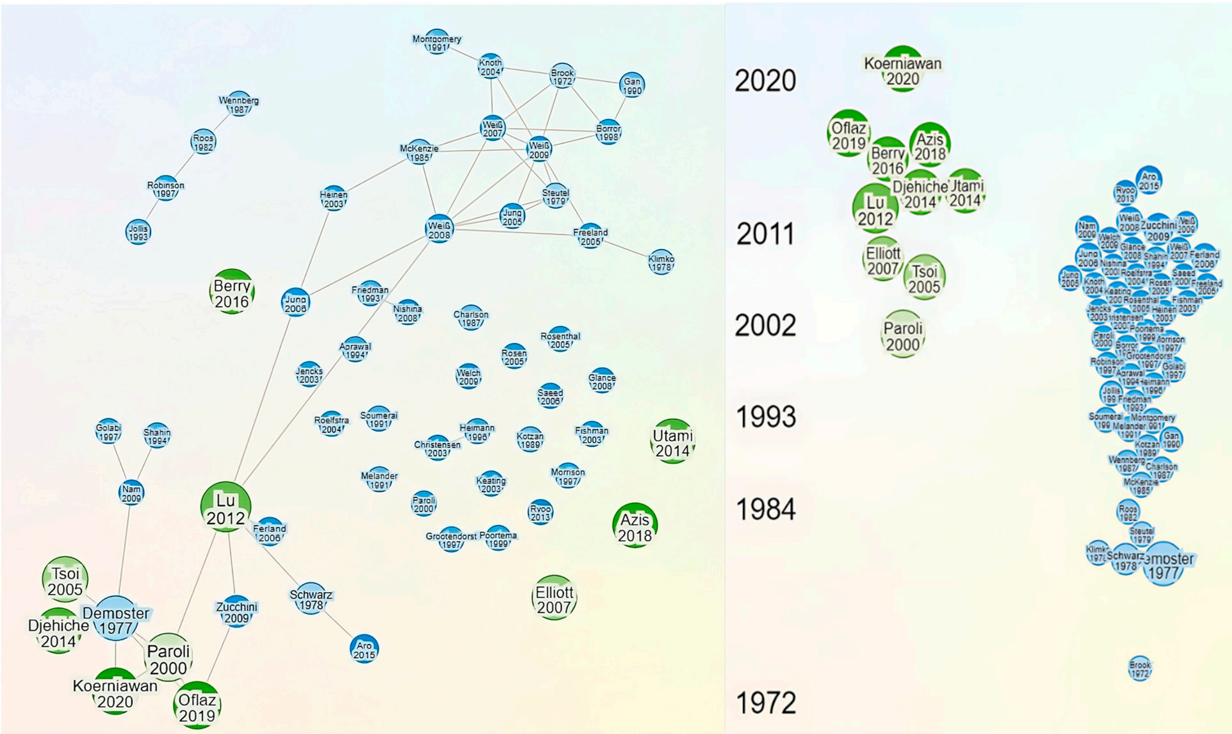


Figure 7. Co-authorship of 12 papers selected.

3.1.4. Visualization of Articles and Similar Works

Visualization capabilities of the Researchrabbit website provided recommendations for relevant papers based on those already discovered. This functionality helped identify additional literature that was probably neglected. Moreover, the website was adept at tracking research trends and the latest publications in specific fields of interest, offering a dynamic and up-to-date view of ongoing research activities. Figure 8a shows that several research articles shared similarities with the 12 selected papers, identifying extensive investigations in related areas. However, it is important to note that the mapping results from the website might not always correlate perfectly with the criteria set for this particular piece of research. Despite this, some references identified through Researchrabbit could be valuable additions to the literature review. A significant example is Zucchini and MacDonald (2009); this book is related to half of the selected papers because most of them cite this book in their research.

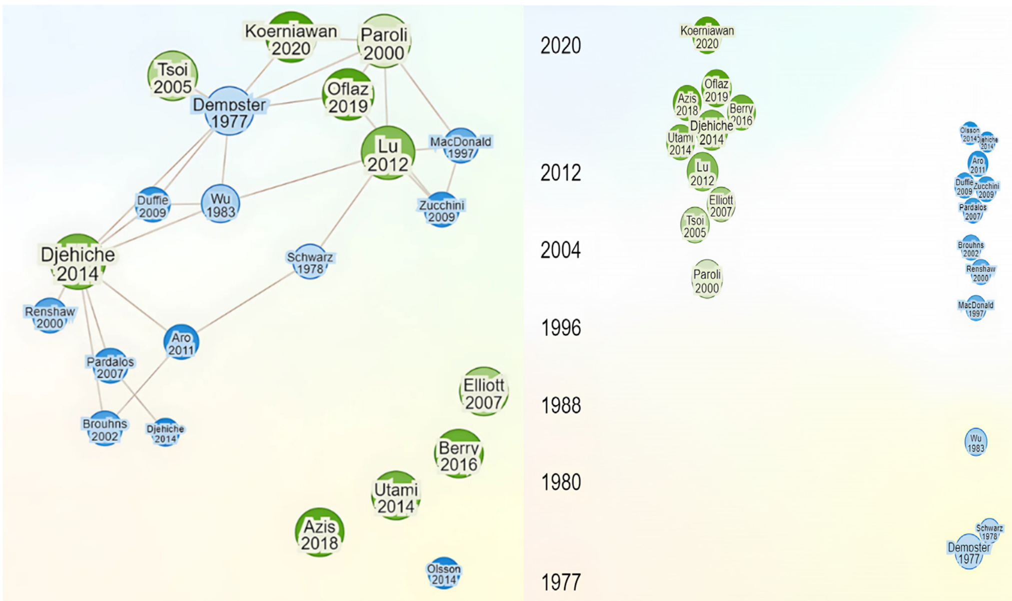


(**a**) (**b**)

Figure 8. Similar works in relation to the research of 12 selected papers: (**a**) graph type of network; (**b**) graph type of timeline.

Figure 8b shows that similar articles were marked in blue, while the primary articles from the selection were in green. Similar articles on the HMM were published between 1972 and 2012, with research trends peaking in the early 2000s. On the other hand, the selected articles were published between 2000 and 2022. This showed that the topic of the HMM has been a focus for several decades, and interest in the topic still persists. The research trends observed in the early 2000s reflected a period when HMM methods started gaining significant attention in various applications, including insurance. The applications of the HMM in the insurance field have consistently evolved with technological advancements and increasing data complexity. The more recent articles, published between 2003 and 2021, confirmed how articles in this area have developed and adapted to new challenges and opportunities. Generally, this visualization showed the importance of staying abreast of the latest developments in HMM research, as this topic remained relevant with the potential for further innovation. By considering the wide range of publications and ongoing trends, research experts could better understand the evolution of the topic and identify opportunities for new contributions in the field of the HMM and its applications in the insurance industry.

Figure 9a,b shows earlier works that were related to the research of the selected articles. Therefore, the previous research should be examined more thoroughly to ensure the suitability requirements for the article selection were met. A significant example that met the selection criteria is MacDonald and Zucchini (1997). However, this research was not published in an accredited journal, raising concerns about the credibility and accuracy of the results. While this research served as a reference, it should be considered with caution. Despite the limitations, MacDonald and Zucchini (1997) remained a reference point for subsequent research, including Paroli et al. (2000), Tsoi et al. (2005), Lu and Zeng (2012), Oflaz et al. (2019), and Koerniawan et al. (2020), due to the relevance of the topic. This model was particularly useful for modeling all kinds of HMMs. Based on Figure 8b, there was a relatively narrow gap between previous and current relevant research, as experts intensively investigated the topic. The focused nature of these pieces of research allowed for a more comprehensive and detailed understanding of the research subject. By building on the foundational work of research like MacDonald and Zucchini (1997), subsequent investigations could be carried out on specific aspects, contributing to a more nuanced understanding of the HMM. This depth of exploration not only validates and refines existing models but also opens avenues for innovations and solutions to complex problems in the field.



(**a**) (**b**)

Figure 9. Earlier works in relation to the research of 12 selected papers: (**a**) graph type of network; (**b**) graph type of timeline.

3.2. Results from Systematic Literature Review

This section addresses the research questions of the 12 selected papers.

3.2.1. The Implementation of the HMM in Predicting the Frequency or Severity of Insurance Claims

The HMM has been significantly used across various fields due to its capacity to handle time-series data and its robust framework for dealing with sequences of observable events that were dependent on underlying hidden states. In the context of insurance, the HMM can be particularly effective for predicting the frequency and severity of claims, which are crucial for risk assessment and premium calculation (Elliott and Siu 2012).

The key components of claims prediction using the HMM include hidden states (which cannot be directly observed but influence claims), transition dynamics, or probabilities between hidden states and the output (observation) probabilities of each hidden state. Based on these components, the HMM can predict the frequency or severity of future claims by estimating the probability of transitions between various hidden states. Out of the 12 selected papers, five papers, by Elliott et al. (2007), Berry (2016), Oflaz et al. (2019), Verschuren (2022), and Jiang (2022), predicted the frequency and severity of insurance claims. Meanwhile, Utami and Effendie (2013) and Azis et al. (2018) predicted the severity of insurance claims, and the rest predicted the frequency of insurance claims. These articles are presented in Figure 10. The HMM is often easier to apply when predicting claims frequency in the form of discrete sequential data, while claims severity typically requires a more complex method due to its continuous nature.

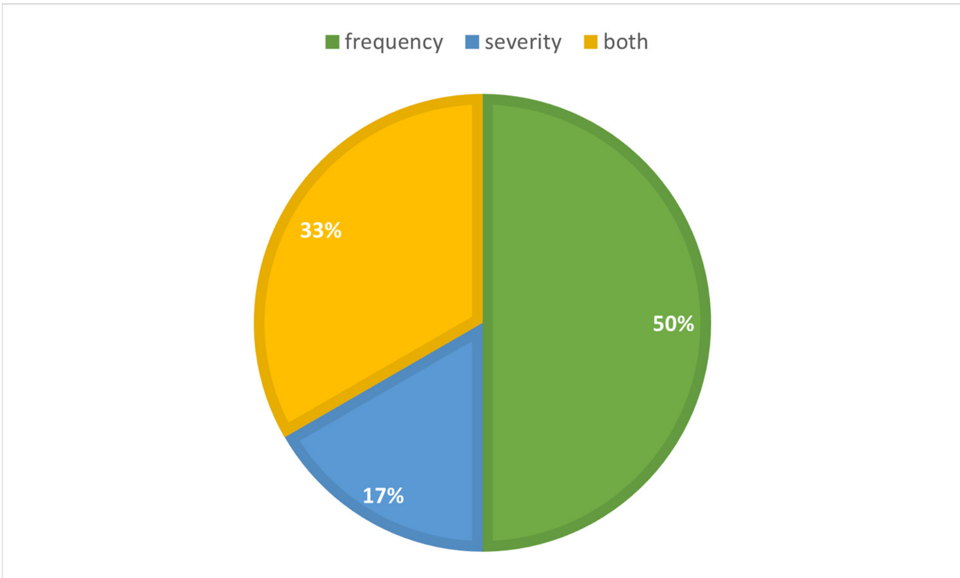


Figure 10. Frequency and severity of claims insurance in HMM applications in selected articles.

From the 12 papers in Figure 10, the distributions used in predicting claims with the HMM were analyzed. These distributions, along with corresponding probability density functions (pdf) or probability mass functions (pmf), are presented in Table 2.

Table 2. Analysis results of type distributions in selected articles.

Frequency of Claims Distribution	pmf	Severity of Claims Distribution	pdf
Poisson	$P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}$	Exponential	$f(x;\lambda)=\begin{cases}\lambda e^{-\lambda x}, x\geq 0\\ 0,\ x<0\end{cases}$
Negative Binomial	$P(X=x)=\binom{x+r-1}{x}p^r(1-p)^x$	Gamma	$f(x;\alpha;\beta)=\begin{cases}\beta^\alpha x^{\alpha-1}e^{-\beta x}, x\geq 0\\ 0,\ x<0\end{cases}$
Bernoulli	$P(X=x)=\begin{cases}p\\ 1-p\end{cases}$	Normal	$f(x\mid\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$
Binomial	$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$		

In claims risk prediction using the HMM, a Poisson distribution was used to calculate the probability of a certain number of claims occurring in the future based on the current and predicted hidden states. For instance, hidden states can represent conditions affecting claims frequency, such as economic factors, policy changes, or policyholder behavior. Each state can have a different parameter λ for Poisson distribution, reflecting the expected frequency of claims when the observation is in that state. This concept also applies to the Negative Binomial, Bernoulli, and Binomial distributions, with differences in distribution parameters. The Negative Binomial distribution can be used to model the number of claims in each hidden state, specifically, when a Poisson distribution is not sufficient to capture the variability involved. Meanwhile, the Bernoulli distribution has a simpler concept, making it suitable for modeling whether a claim occurs or does not occur within a given period. This distribution can be used when we are only concerned with whether a claim happens rather than the number of claims. The Binomial distribution is appropriate when there is a limit on the number of claims by modeling the number of claims from a fixed number of n trials, each with a constant probability of success p within a given period.

The Exponential distribution can be used to model claim severity when the system is in a particular hidden state in the HMM. Each state may have a distinct Exponential distribution. For example, a high-risk hidden state might have an Exponential distribution with a smaller parameter λ (confirming a larger average claim severity). The Exponential distribution is a special case of the Gamma distribution with $\alpha = 1$. The Gamma distribution offers additional transmission with parameter α, allowing it to capture more variability in complex severity claims. This makes the Gamma distribution more suitable for cases where claims severity exhibits non-uniform patterns or when modeling large claims with varying levels of severity. The hidden states in an HMM will have different mean μ and variance σ^2 parameters when using the Normal distribution. The mean μ represents the average claims value within that state, while the standard deviation σ indicates how much the severity of claims varies around the mean. A hidden state with a smaller standard deviation suggests that the severity of claims is more concentrated around the value of the average claim, whereas a larger standard deviation indicates greater variability in the severity of claims.

3.2.2. The Number and the Interpretation of Hidden States in the HMM

The number of hidden states in the HMM plays a crucial role in determining the model's ability to accurately represent the underlying structure of the observed data, particularly in the context of insurance claims. The number of hidden states is not typically known in advance and needs to be selected based on domain knowledge or experimentation. Typically, the number of hidden states is defined as $m = 2, 3, 4, \ldots$. Each hidden state can be interpreted as representing a different risk regime or latent condition that influences the observed frequency or severity of claims. A model with an insufficient number of hidden states might oversimplify the complexity of the data, failing to capture important variations and leading to less accurate predictions of claims risks. Conversely, having numerous hidden states can lead to overfitting, where the model becomes excessively tailored to the training data, reducing its predictive accuracy on new or hidden data. In HMM applications, more than two hidden states are often necessary because real-world systems tend to exhibit complex patterns or phases that cannot be fully captured by only two states (Awad and Khanna 2015). Table 3 shows that the pieces of research using HMMs have a minimum of two hidden states.

The number of hidden states had a significant impact on risk assessment, as it directly influenced how the model identified and differentiated between various risk levels. For example, in the insurance context, a model with multiple hidden states could distinguish between periods of low, medium, and high claims risk, allowing for more nuanced risk prediction and management strategies. This level of granularity enabled the model to capture subtle shifts in claims behavior or frequency, providing more accurate estimates of potential risk exposure. Therefore, determining the appropriate number of hidden states was critical to ensuring the HMM could adequately balance model complexity with predictive accuracy, leading to better-informed decisions in managing and pricing insurance risk. To determine the appropriate number of hidden states in the HMM, model selection criteria like Bayesian Information Criterion (BIC) or Akaike Information Criterion (AIC) were commonly used. The HMM with the lowest BIC value was considered to have the optimal number of hidden states.

Table 3. Analysis results of the number of hidden states in selected articles.

Authors	Number of Hidden States (m)	Selection Criteria
Paroli et al. (2000)	2	AIC and BIC
Tsoi et al. (2005)	6	-
Elliott et al. (2007)	-	-
Lu and Zeng (2012)	3	AIC
Utami and Effendie (2013)	4	-
Berry (2016)	2	AIC and BIC
Azis et al. (2018)	2	BIC
Djehiche and Löfdahl (2018)	4	AIC and BIC
Oflaz et al. (2019)	3	AIC and BIC
Koerniawan et al. (2020)	2	BIC
Verschuren (2022)	3	BIC
Jiang (2022)	3	AIC and BIC

Most of the selected articles used BIC to determine the appropriate number of hidden states. For example, Lu and Zeng (2012) interpreted hidden states 1, 2, and 3 as corresponding to low, moderate, and high rates of occurrence of hurricanes and tropical storms annually. Oflaz et al. (2019) defined three hidden states, with state 3 representing the lowest mean claims severity (low state), state 2 representing the highest mean claims severity (high state), and state 1 representing the medium level of claims severity (medium state). In contrast, Azis et al. (2018), Koerniawan et al. (2020), and Jiang (2022) did not define the meaning of hidden states. This was not unusual, as further investigations could show the interpretation of the hidden states. Meanwhile, Elliott et al. (2007) only developed the theoretical framework of the model without conducting simulations, so there is no information on the number of hidden states.

4. Discussion

4.1. The State of the Art of the HMM Applications in Insurance Claims

The applications of the HMM in the insurance industry have gained significant traction in recent years due to the ability to model stochastic processes with unobservable states. The HMM is particularly well-suited for handling the uncertainty and dynamic nature of insurance claims, where underlying risk factors and claims patterns evolve over time. By allowing for transitions between hidden states, the HMM provides a robust framework for capturing the complex temporal dependencies inherent in insurance data. This section reviews the state-of-the-art applications of the HMM in insurance, emphasizing common methodologies, the evolving use of distribution models across various types of insurance claims, and the outputs of the HMM, which can be in the form of probabilities or forecasts.

Based on Table 4, previous research on insurance claims using the HMM showed that the Poisson distribution was commonly used to describe the frequency of claims. The Poisson distribution was frequently chosen due to its simplicity and suitability for modeling discrete events, such as insurance claims, occurring over a specified time interval. It allowed the model to predict the number of claims events within a given period, assuming the events were independent and occurred randomly at a constant average rate. This fundamental property made it effective in scenarios where the claims frequency was low to moderate, justifying its usage in the HMM to map claims patterns from policyholders.

The Poisson distribution, however, has limitations that make it unsuitable for all types of claims data. One of its main assumptions is that the mean and variance of the distribution are equal. In the context of insurance claims, particularly for more complex types like catastrophe or fire claims, the data often exhibit overdispersion, where the variance exceeds the mean. Therefore, the Poisson distribution often fails to capture the greater variability in insurance claims data with high frequencies or significant variations between claims.

The inability to handle this condition results in inaccuracies when modeling more complex claims data.

Table 4. The state of the art of five selected articles.

Authors	Models	Dataset	Results: Probabilities or Forecasts
Paroli et al. (2000)	Poisson HMM	Corporate Accident Insurance	Probabilities
Tsoi et al. (2005)	Normal HMM	Health Insurance	Probabilities
Elliott et al. (2007)	Compound Poisson HMM	-	-
Lu and Zeng (2012)	Non-Homogeneous Poisson HMM	Catastrophe Insurance	Probabilities
Utami and Effendie (2013)	Normal HMM	Health Insurance	Probabilities
Berry (2016)	Poisson-Gamma HMM	Automobile Insurance	Probabilities
Azis et al. (2018)	Exponential HMM	Vehicle Insurance	Probabilities and Forecasts
Djehiche and Löfdahl (2018)	Binomial HMM	Health Insurance (Disability)	Probabilities
Oflaz et al. (2019)	Bivariate HMM	Automobile Insurance	Probabilities and Forecasts
Koerniawan et al. (2020)	Poisson HMM	Life Insurance	Probabilities
Verschuren (2022)	Poisson–Gamma HMM	Automobile Insurance	Probabilities
Jiang (2022)	Bernoulli–Gamma HMM	Automobile Insurance	Probabilities

The Gamma distribution is widely used for modeling the severity of claims due to its flexibility and its ability to handle claim data that are strictly positive and often skewed. However, the Gamma distribution has relatively light tails, making it unsuitable for claim data with heavy tails, where there is a significantly higher probability of very large claims than what the Gamma distribution can accommodate. Additionally, the Gamma distribution only applies to positive values, which is advantageous for modeling claim amounts. However, in some cases, claims severity may approach zero or be very small, requiring a different distribution. While claims severity cannot be negative, a distribution with more flexibility around small or near-zero claims is assumed to better capture the characteristics of claims severity.

Based on previous research, like that of Azis et al. (2018) and Oflaz et al. (2019), the parameter estimation results from the HMM, particularly the transition probabilities, have been further developed into exact event forecasts. This allows the model to not only estimate the likelihood of transitions between hidden states but also predict specific future events with a degree of precision. Although this extension of the HMM parameter estimation into forecasting represents a significant advancement, the models used for prediction have not been able to achieve high levels of accuracy. In many cases, the predicted outcomes deviate from actual observations, particularly in complex or highly variable datasets. Therefore, improvements in model structure, parameter tuning, or the inclusion of additional factors were necessary to improve forecasting performance.

4.2. Research Gaps

In addressing the limitations of the Poisson distribution in the HMM, some researchers have attempted to combine it with other distributions or use modifications, such as Poisson–XLindley. These combinations aim to improve the model's ability to handle data with overdispersion or underdispersion (see Ahsan-Ul-Haq et al. 2022), as well as to account for situations where claims frequency varies greatly, or events were rare but with significant impacts. Such hybrid distributions provide better flexibility in capturing the complexity of insurance claims data, ensuring that the HMM can deliver more accurate and realistic predictions in line with real-world claims patterns.

The Lognormal or Generalized Pareto distributions (GPDs) are alternative choices that are assumed to be more flexible in modeling claims near the lower bound (which cannot be effectively captured by the Gamma distribution). The combination of the Lognormal and GPD in modeling claim amounts within the HMM is a highly effective approach, especially when claim data exhibit different characteristics across various parts of the distribution. This combination allows for the modeling of small to medium claims (using the Lognormal distribution) as well as large and extreme claims (using the GPD), making the model more flexible in capturing the complexity of claim data variability.

After selecting the appropriate distribution model, it is crucial to support the HMM with more accurate forecasting models to improve predictive performance. While the HMM excels at capturing hidden state transitions and estimating associated probabilities, integrating it with robust forecasting techniques can improve the precision of future event predictions. Several forecasting models can be used in conjunction with HMM parameters, such as Autoregressive Integrated Moving Average (ARIMA), which can handle time series data by incorporating past observations and residual errors. Another option is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which is useful for predicting volatility, specifically in financial or insurance data with time-varying variances. In addition, Bayesian forecasting models are combined with the HMM to provide probabilistic forecasts, capturing uncertainty more effectively, or the State–Space Model can be used to represent changes in hidden states over time and generate predictions of claim severity or claim frequency based on the available observations. By combining the HMM with one of these models, the resulting framework can leverage the strength of the HMM's hidden state transitions while improving the accuracy of the prediction process.

5. Conclusions

In conclusion, this research presented the SLR of the applications of the HMM in insurance claims. A total of five articles were selected through a screening process from four digital libraries, namely, Scopus, Science Direct, and Dimensions. The results of the SLR, particularly in terms of the bibliometric analysis, showed that research on the applications of the HMM in insurance claims had been actively conducted in the last 10 years, as detailed in the provided journal list. Furthermore, the journals that published articles on this topic were accredited, ensuring reliable results, and the authors of this research had not been able to collaborate extensively. This presented opportunities for further collaboration to produce more articles and contribute beneficial and novel insights.

Analysis of the ResearchRabbit website showed that similar investigations in the 12 selected papers occurred between the early 2000s and 2012, offering insights into previous and related articles. From a literature review standpoint, more than 50% of the 12 papers focused on insurance claims from the frequency perspective. Even though the Poisson distribution was often used to assess the risk of claims frequency, it needed adjustment according to the characteristics of claims within the specific insurance scheme; this also applies to the distribution of claim severity. Moreover, the optimal number of hidden states in the HMM was determined using AIC and BIC methods, with results showing a minimum of two states, which could be interpreted or assumed according to research requirements.

Future research was recommended to expand the SLR of the HMM applications in assessing insurance claims risk with a broader scope. In addition to addressing claims frequency and severity, it is crucial to involve the risk profiles of policyholders and the detection of fraudulent claims.

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