risks

MDPI

Article

Analyzing the Influence of Risk Models and Investor Risk-Aversion Disparity on Portfolio Selection in Community Solar Projects: A Comparative Case Study

Mahmoud Shakouri [1,*], Chukwuma Nnaji [2], Saeed Banihashemi [3] and Khoung Le Nguyen [4]

[1] Department of Construction Management, Colorado State University, Fort Collins, CO 80523, USA
[2] Department of Construction Science, Texas A&M University, College Station, TX 77843-3337, USA; cnnaji@tamu.edu
[3] School of Built Environment, Faculty of Design, Architecture, and Building, University of Technology Sydney, 15 Broadway, Ultimo, NSW 2007, Australia; saeed.banihashemi@canberra.edu.au
[4] School of Design and Built Environment, University of Canberra, Canberra 2617, Australia; khuong.lenguyen@canberra.edu.au
* Correspondence: mahmoud.shakouri@colostate.edu

Abstract: This study examines the impact of risk models and investors' risk aversion on the selection of community solar portfolios. Various risk models to account for the volatility in the electrical power output of community solar, namely variance (Var), SemiVariance (SemiVar), mean absolute deviation (MAD), and conditional value at risk (CVaR), were considered. A statistical model based on modern portfolio theory was employed to simulate investors' risk aversion in the context of community solar portfolio selection. The results of this study showed that the choice of risk model that aligns with investors' risk-aversion level plays a key role in realizing more return and safeguarding against volatility in power generation. In particular, the findings of this research revealed that the CVaR model provides higher returns at the cost of greater volatility in power generation compared to other risk models. In contrast, the MAD model offered a better tradeoff between risk and return, which can appeal more to risk-averse investors. Based on the simulation results, a new approach was proposed for optimizing the portfolio selection process for investors with divergent risk-aversion levels by averaging the utility functions of investors and identifying the most probable outcome.

Keywords: portfolio theory; community solar; risk aversion; PV systems

Citation: Shakouri, Mahmoud, Chukwuma Nnaji, Saeed Banihashemi, and Khoung Le Nguyen. 2024. Analyzing the Influence of Risk Models and Investor Risk-Aversion Disparity on Portfolio Selection in Community Solar Projects: A Comparative Case Study. *Risks* 12: 75. https://doi.org/10.3390/risks12050075

Academic Editor: Chengguo Weng

Received: 11 February 2024
Revised: 25 April 2024
Accepted: 28 April 2024
Published: 30 April 2024

1. Introduction

There is a growing interest among individuals and companies in considering renewable energy sources as an alternative to the conventional electrical grid supply. Among various renewable energy sources, the decreasing trend in the cost of solar photovoltaics (PV) and the potential to contribute a significant share to cleaner and more sustainable electricity generation has made solar energy one of the most promising sources of electricity generation worldwide (Dusonchet and Telaretti 2010). In this regard, community solar projects (CSPs) stand as a pioneering model in the renewable energy sector, enabling multiple participants to benefit from a single, centrally located solar power installation. This model allows individuals and businesses, particularly those without suitable rooftops for solar panels, like renters or properties with shaded roofs, to invest in or subscribe to a portion of a solar energy project and receive credits on their electricity bills proportional to their share of the power produced. The recent literature explores various aspects of CSPs, ranging from operational strategies to governance models. For instance, Oh (2022) introduced an operational strategy for CSPs within smart energy communities aimed at balancing the minimization of electricity costs with equitable resource distribution. This strategy, articulated as a mixed-integer linear problem, showcased the potential for optimal resource allocation through centralized control. It also highlighted the benefits of a distributed approach under scenarios with partial information, ensuring fair distribution of electricity bill savings.

Moreover, Lee et al. (2021) presented the concept of virtual community-owned solar and storage systems as a transformative model. This approach grants individual shareholders autonomous control over their energy contributions and consumption, thus reducing grid dependency and fostering a more resilient energy ecosystem. Despite the numerous advantages of CSPs, challenges persist in community engagement, including stakeholder identification, sustained participation, and transparent decision-making processes (Prehoda et al. 2019). The call for collaborative governance strategies to overcome these challenges while emphasizing the importance of reducing risks underscores the need for inclusive and adaptive frameworks in CSP implementation.

In recent years, a new concept of investment in CSPs known as portfolio-based CSP has emerged that emphasizes optimizing the expected return and minimizing the volatility in power generation of the PV system by distributing the solar panels among several houses in a community (Shakouri et al. 2015; Shakouri and Lee 2016; Shakouri et al. 2017). In this cooperative solar investment model, community members agree to install PV panels on their properties. Considering each house's orientation and shading conditions, those with more suitable conditions—and sufficient roof space—may install additional panels to compensate for houses that are less favorable to PV installation or electricity generation. At the end of the month, the generated benefits are distributed as credits or any form agreed upon among investors. As opposed to an investment in a stand-alone PV system or a central CSP in which a single owner incurs all the costs and risks associated with the investment, the costs and risks associated with investing in a portfolio-based CSP system are diversified and distributed among the investors in the community, helping the participating investors to realize a more stable profit margin. The portfolio-based community solar proposed by Shakouri et al. (2015) proved advantageous to communities with heterogeneous building morphology (i.e., orientation, height, and roof slope) and those that do not typically meet the optimal conditions required for generating electricity.

In a portfolio-based CSP, the classic solution to modern portfolio theory (Markowitz 1952) is generally utilized to optimize the return or minimize the risks of an investment in the CSP. In short, modern portfolio theory aims to find a set of portfolios with higher returns or lower risks than any other portfolio. The portfolio choice, however, depends mainly on the risk-aversion nature of the investors. Risk-averse investors who look for safe investments and stay away from high-stakes investments typically choose the portfolios with the lowest risk. On the other hand, risk-loving investors go for the portfolios that yield higher returns at the cost of higher risks.

In portfolio-based CSPs, any fluctuations in power generation are considered a risk and are often quantified by the variance of generated power (i.e., analogous to the variance of returns in the stock market) (Shakouri et al. 2015; Shakouri and Lee 2016; Shakouri et al. 2017). One drawback to such an approach in quantifying risk is that any deviation from the average power generation is considered a risk. However, in a portfolio-based CSP, only the portion of the variability that falls below the average is suboptimal, and investors welcome higher power generation. Therefore, though modern portfolio theory helps investors in a CSP realize more return or less risk, some modifications may be required to accommodate the risks specific to community solar (Hunjra et al. 2020).

In recent years, numerous alternatives to the variance (Var) of return as a risk index have been proposed: SemiVariance (SemiVar), mean absolute deviations (MAD) of returns, and conditional value at risk (CVaR), to name a few. Although these alternative risk models have been widely accepted by practitioners in the stock market (Sarykalin et al. 2008), there is no clear information regarding which model is more suitable for project valuation and investment in a CSP.

Assuming that each house in a CSP is analogous to an asset in the market, one significant difference between a portfolio-based CSP and an asset-market portfolio is that the former undergoes less frequent changes (Speranza 1993). Implementing changes in portfolio-based CSP (e.g., redistributing the panels or modifying the set-up) can be challenging and costly, if not impossible. Another fundamental difference between a portfolio-based CSP and an asset-market portfolio is that the generated power from the individual houses in a CSP has "generally" a high spatial correlation. In contrast, in a financial market, the correlations between the returns of various assets can be more heterogeneous. Sometimes, an increase in the price of one asset may negatively influence other assets (Shakouri et al. 2015). Finally, it is crucial to acknowledge that in a community with houses featuring markedly diverse orientations, surrounding vegetation, and rooftop shading conditions, the heterogeneity in correlation among the houses increases, mirroring the variability observed in asset markets (Shakouri et al. 2017). Therefore, it is essential to consider the influence of the morphology of the community on the performance of the portfolio-based community solar project.

Quantifying the risk-aversion level of an investor for an asset-based portfolio is an empirical process that typically involves filling out a standardized survey that lays out various investment scenarios (e.g., consumption and saving decisions, investment, insurance purchases, tax compliance, gambling, and other behaviors) (Eisenhauer and Ventura 2003). The quantified risk aversion, often expressed as the coefficient of risk aversion, can be fed into a utility function to obtain risk-free returns. The portfolio with the highest risk-free return is then selected as the optimum choice. One question that may arise in the context of portfolio-based CSP is how to consider the risk-aversion level of the community members when designing a community solar project.

Although ample research has been done to investigate an individual's risk aversion, there is scant information regarding how the divergence in the risk-aversion level of a group of investors can impact the portfolio selection in a CSP. This paper contributes significantly to the existing literature on community solar projects by introducing a nuanced analysis of how different risk models and the diversity in investor risk-aversion affect CSP portfolio selection. Its novelty lies in exploring alternative risk indices, such as SemiVar, MAD, and CVaR, offering a more tailored approach to quantifying risks in CSP investments than traditional variance-based models. This adjustment acknowledges that in the context of CSPs, not all deviations from average power generation are undesirable, particularly those that result in higher energy output. Furthermore, this paper addresses the challenge of integrating individual and collective risk preferences into the CSP portfolio design. This topic has received limited attention in previous research. By providing a framework that considers the static nature of solar installations and the spatial correlation of power generation, the study offers valuable insights for optimizing CSP portfolio selection in line with diverse investor risk appetites.

The following sections discuss risk characteristics in a portfolio-based CSP, provide a brief overview of modern portfolio theory, and introduce the risk models that can be applied to portfolio-based CSPs. Section four covers the mathematics of measuring risk aversion and how it can be implemented in the portfolio selection process. Section four also proposes a model incorporating the investor risk-aversion level divergence into the community solar portfolio selection process. In section five, the influence of different risk models on the generating power of a community solar project is studied. In addition, the application of the proposed risk-aversion model is shown in various scenarios.

2. Risk Characteristics in Community Solar Projects

Community solar projects present an opportunity to make renewable energy more accessible. However, potential investors need to take into account the possible risks involved. In some cases, these risks are analogous to those encountered by investors in the stock market since, in both cases, the investors will be dealing with varying levels of uncertainty in their future returns.

One of the most important risk factors in community solar projects is associated with the performance of solar panels. Factors such as weather conditions and technical issues can impact the efficiency of panels (Fouad et al. 2017; Said et al. 2018; Santhakumari and

Sagar 2019). Investors' expected returns could be affected if the panels underperform due to soiling or inadequate sunlight. There is also the potential issue of subscription risks in a CSP (Burger and Luke 2017; Chan et al. 2017). In communities where the demand is lower than expected, the financial viability of the project may be compromised. Thus, the returns to participants may be affected.

There are some notable differences when assessing the investors' risk-aversion in a CSP vis-à-vis individuals in the stock market. CSP investors often prefer a more predictable and less volatile system to help them offset their energy expenses as they participate in a community solar project (Curtin et al. 2019; Guno et al. 2021). Conversely, the dynamic and unpredictable nature of stock markets contributes to the wider range of risk tolerance among stock investors (Kuang 2021; Weixiang et al. 2022). Fluctuations in the market caused by geopolitical events, economic conditions, and company specificities make the stock market inherently more volatile than community solar projects, which is mainly influenced by weather conditions. Furthermore, the return expectations differ between the two investment domains. CSP investors focus on securing a reliable source of savings by offsetting their electricity bills (Nolden et al. 2020; Zhou et al. 2020). The emphasis is on stability and consistency rather than on pursuing substantial capital appreciation (Gjorgievski et al. 2021). On the contrary, stock market investors often prioritize capital growth. This goal comes with higher volatility and the potential for higher returns.

Another major difference between investing in a CSP and stock market is the investment duration. Investors in a CSP typically commit to a longer investment horizon, often spanning the operational life of the solar project, which can range from 20 to 30 years (Chowdhury et al. 2020; Nishanthy et al. 2023). This extended timeline aligns to offset energy costs over an extended period gradually. In contrast, stock market investors exhibit a broader spectrum of investment horizons, ranging from short-term traders seeking quick gains to long-term buy-and-hold investors aiming for sustained growth over many years. Additionally, investments in CSPs tend to be illiquid due to their tie to the project's operational lifespan. Once invested, participants generally have limited options to withdraw their funds before the project's completion (Cross and Neumark 2021; Parsonnet 2021). In contrast, stock market investments offer greater liquidity, allowing investors to buy and sell shares more readily, reflecting the broader accessibility and ease of entry and exit in the stock market.

3. Model Formulation

The process of optimizing the installation of PV panels across a CSP begins by collecting or simulating essential data for each household. This paper uses plane-of-array (POA) irradiance, which provides a direct correlation between irradiance and electrical output of PV panels, as the main input data for constructing portfolios. The choice of POA irradiance in this study streamlines the process by circumventing the intricate details involved in the specifications of diverse PV systems. POA irradiance data can be measured directly with a pyranometer, reference cell, or reference module mounted in the same orientation as the roof or indirectly by using simulation software to model the roof condition and incorporate environmental and weather data to predict the POA irradiance. POA irradiance data are typically presented numerically, measured in watts per square meter (W/m^2), and often collected over time so that they can be presented as a time series. This means a continuous sequence of values will be recorded at regular intervals, such as every minute, hour, or day.

To accurately present a generalized mathematical model for a CSP portfolio optimization scenario that can utilize various risk measures and aims to maximize the utility functions of investors with varying levels of risk aversion, the following notations are first introduced:

- $\mathbf{R}$: Matrix of returns (i.e., POA irradiance), where each column corresponds to one house, and each row corresponds to a time period;
- μ_t: The POA irradiance of house i at time t;

- **w**: Vector of portfolio weights, representing the proportion of total PV panels allocated to each house in the CSP;
- **c**: Vector of upper bounds for **w**, where c_i is defined as the ratio of possible number of PV panels installed on house i to the total available number of panels;
- λ_j: Risk-aversion coefficient for investor j;
- η_j: Contribution coefficient of investors j;
- U_j (**w**): Utility function for investor j, which depends on the portfolio weights and incorporates the investor's risk preferences;
- $\mathbf{\Sigma}$: Covariance matrix of asset returns, used in risk calculations;
- Φ (**w**, **R**): Risk model;
- **p:** Vector of portfolio returns for each time period;
- μ_p: Mean of **p**.

The optimization goal is to maximize the total weighted sum of investors' utility with varying risk aversions, subject to constraints on the asset allocations. This can be formally expressed as follows:

$$\begin{array}{ll} \text{maximize} & U(\mathbf{w}) = \sum_{j=1}^{n} \eta_j U_j(\mathbf{w}, \lambda_j, \Phi) \\ \text{subject to} & \sum\limits_{i=1}^{m} w_i = 1, \\ & 0 \leq w_i \leq c_i, \quad \forall i \end{array} \tag{1}$$

The utility function for each investor j, U_j, reflects the tradeoff between expected return and risk, considering the investor's risk-aversion coefficient and selected risk model. The general form of this utility function can be expressed as follows:

$$U_j = \mathbf{w}^T\mathbf{R} - \lambda_j.\Phi(\mathbf{w}, \mathbf{R}) \tag{2}$$

In mean-variance portfolio analysis, variance, shown in Equation (3), is commonly used as a measure of risk Φ (Markowitz 1959).

$$\text{VaR}(\mathbf{w}, \mathbf{R}) = \mathbf{w}^T\mathbf{\Sigma}\mathbf{w} \tag{3}$$

In generating renewable energies, one of the drawbacks of using variance as a risk measure is that any deviations from the expected return (i.e., POA irradiance that is either less or more than average) are considered a risk. However, in reality, investors welcome deviations above the portfolio's expected POA irradiance and are more interested in minimizing the downside risk. Therefore, the variance model may not be a helpful risk indicator when creating a portfolio-based CSP. Under these conditions, other alternative risk models such as SemiVar, MAD, and CVaR may be more appropriate.

SemiVar only considers the variability of returns that fall below a certain threshold, typically the mean or a target return level (Boasson et al. 2011). To determine SemiVar, let **p** denote a vector of portfolio returns (i.e., $\mathbf{p} = \mathbf{w}^{\mathrm{T}}\mathbf{R}$), μ_{p} denote the mean of **p,** and μ_{t} denote the POA irradiance of house i at time t. Then, the SemiVar risk model for the total number of periods T in **p** can be expressed as shown in Equation (4):

$$\text{SemiVaR}(\mathbf{w}, \mathbf{R}) = \frac{1}{T}\sum_{t=1}^{T} \max(0, \mu_p - \mu_t)^2 \tag{4}$$

MAD measures the average absolute deviation from the mean return, offering a different perspective on risk more aligned with how individuals perceive deviations from expected outcomes (Konno and Yamazaki 1991). Mathematically, MAD can be expressed as follows:

$$\text{MAD}(\mathbf{w}, \mathbf{R}) = \frac{1}{T}\sum_{t=1}^{T} |\mu_t - \mu_p| \tag{5}$$

CVaR provides an expected loss, assuming that a loss is beyond the value at risk (VaR_α) at a certain confidence level α, typically 95% (Rockafellar and Uryasev 2000). VaR_α

calculates the quantile (or percentile) of the portfolio returns p at the level 1−α. Using this definition, CVaR at confidence level α can be expressed as follows:

$$\mathrm{CVaR}_\alpha(\mathbf{w}, \mathbf{R}) = \frac{1}{(1-\alpha)T} \sum\nolimits_{\mu_t \leq \mathrm{VaR}_\alpha} (\mu_t) \tag{6}$$

4. Illustrative Example

Figure 1 provides a hierarchy of the steps required to design a CSP. The process begins with data collection, which entails obtaining simulation data to model potential CSP performance or gathering field data to capture real-world performance metrics. This dataset, including POA irradiance measurements, often requires cleaning and preprocessing to ensure it is ready for analysis. The third step involves selecting a risk model to evaluate potential performance variabilities or uncertainties within the CSP operation. In step four, a portfolio analysis is performed to identify the optimal portfolios on the efficient frontier, characterized by either the lowest possible risk for a given level of return or the highest return for a given level of risk. Once the efficient frontier is determined, the fifth step is to calculate the utility function for each investor across all portfolios on the efficient frontier, considering individual levels of risk aversion. This calculation results in a distribution of utility values for each investor. Subsequently, each investor's contribution is determined based on their financial contributions to the CSP. This contribution is then utilized to compute the weighted sum of the utility function. The final step in this process involves identifying the portfolio index that results in the highest weighted sum of utilities and, consequently, the optimal weight for each house in the CSP. Once these optimal weights are determined, PV modules can be distributed and installed accordingly on the rooftops of the participating houses in the CSP.

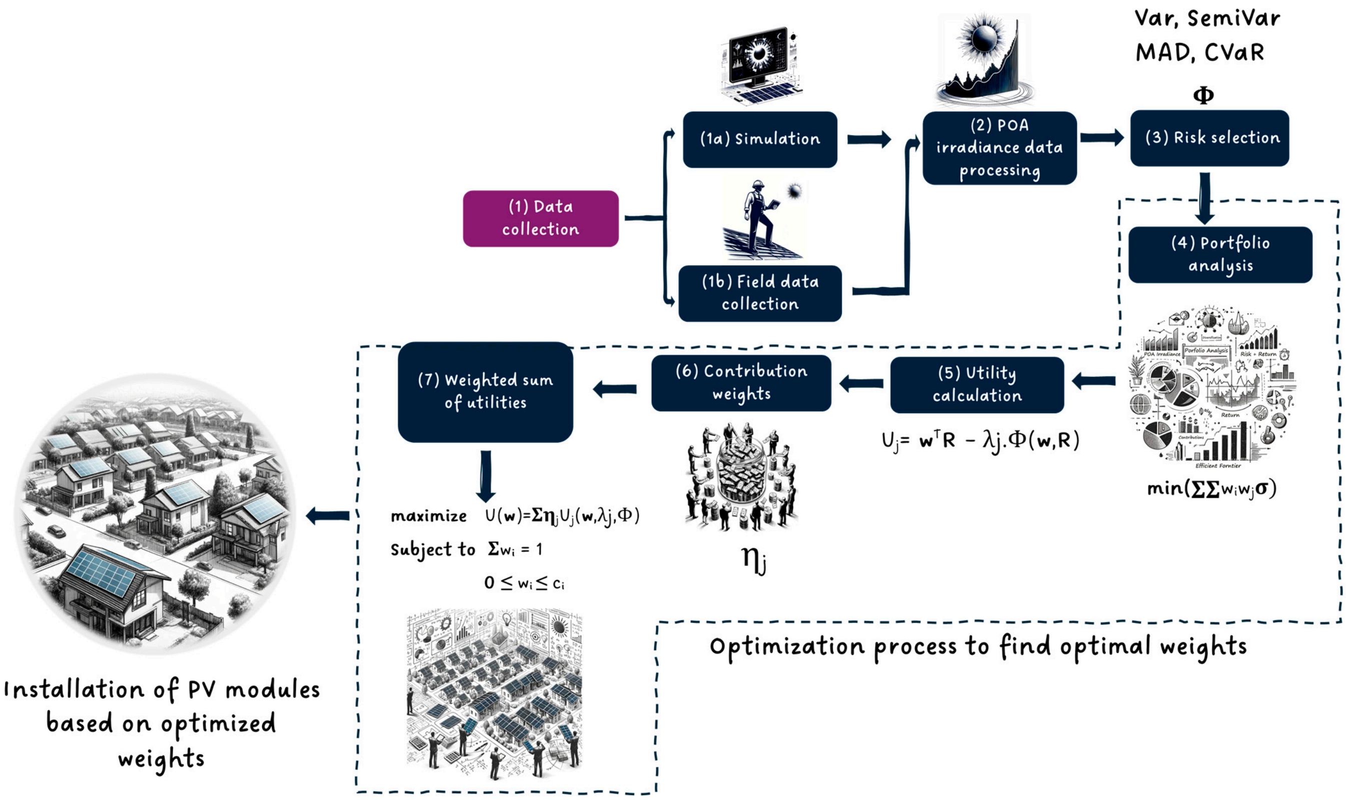

Figure 1. A step-by-step methodology for designing a portfolio-based community solar project.

4.1. Data Collection

In this study, POA irradiance data were obtained by simulating a case study comprising three houses using the System Advisor Model (SAM) software developed by the National Renewable Energy Laboratory (Blair et al. 2018). Typical-year weather data for a city in the Midwest USA (40.6993° N, 99.0817° W) were used in simulating this case study. Figure 2 shows the simulated case study, and Table 1 provides information on the physical properties of the houses, including roof slope, roof elevation, roof azimuth, available roof area, and the maximum possible number of 250 W PV modules that can be installed on each roof, assuming that the area of each module is 1.63 m^2. The maximum possible number of modules on each house serves as a constraint for portfolio analysis, as denoted by the parameter c_i in Equation (1). This study assumes that the investors are interested in installing a 12 kW package comprising 48 PV modules, each rated at 250 W.

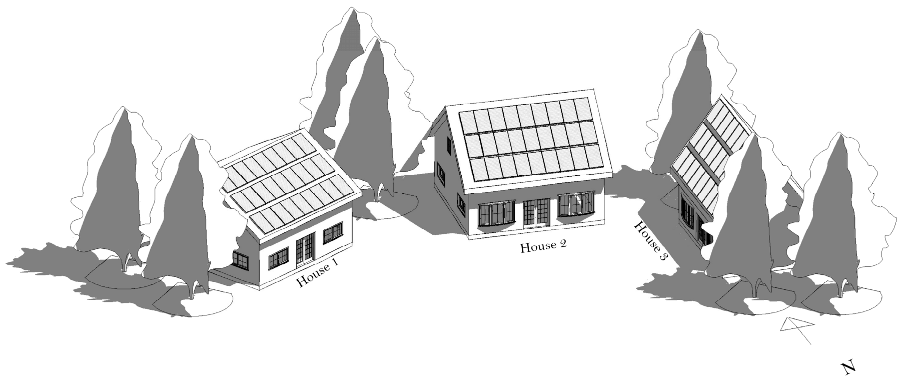

Figure 2. Aerial view of the CSP case study.

Table 1. Physical properties of the houses in the selected community.

House	Roof Slope (°)	Roof Elevation (m)	Roof Azimuth (North 0°)	Available Roof Area (m^2)	Maximum Possible Number of PV Panels, c_i
1	31	3.65	150	61	27
2	40	3.65	183	63	27
3	45	3.65	270	60	s27

4.2. POA Irradiance Data Processing

SAM's output data include annual hourly POA irradiance. Since the roofs receive no irradiance from the sun during nighttime, the software outputs a value of 0 for nighttime POA irradiance. Consequently, the raw data for each house contain many zeros, causing the POA irradiance for the houses to be highly correlated and overshadowing the influence of differences in roof orientation and slopes. Therefore, before using these data for portfolio analysis, all zero POA irradiance values associated with nighttime were removed from the dataset, resulting in 4709 data points for each house. Figure 3 shows the resulting distribution of POA irradiance values for the simulated case study, and Table 2 shows the covariance (correlation) matrix developed based on the simulated POA irradiance of these three houses.

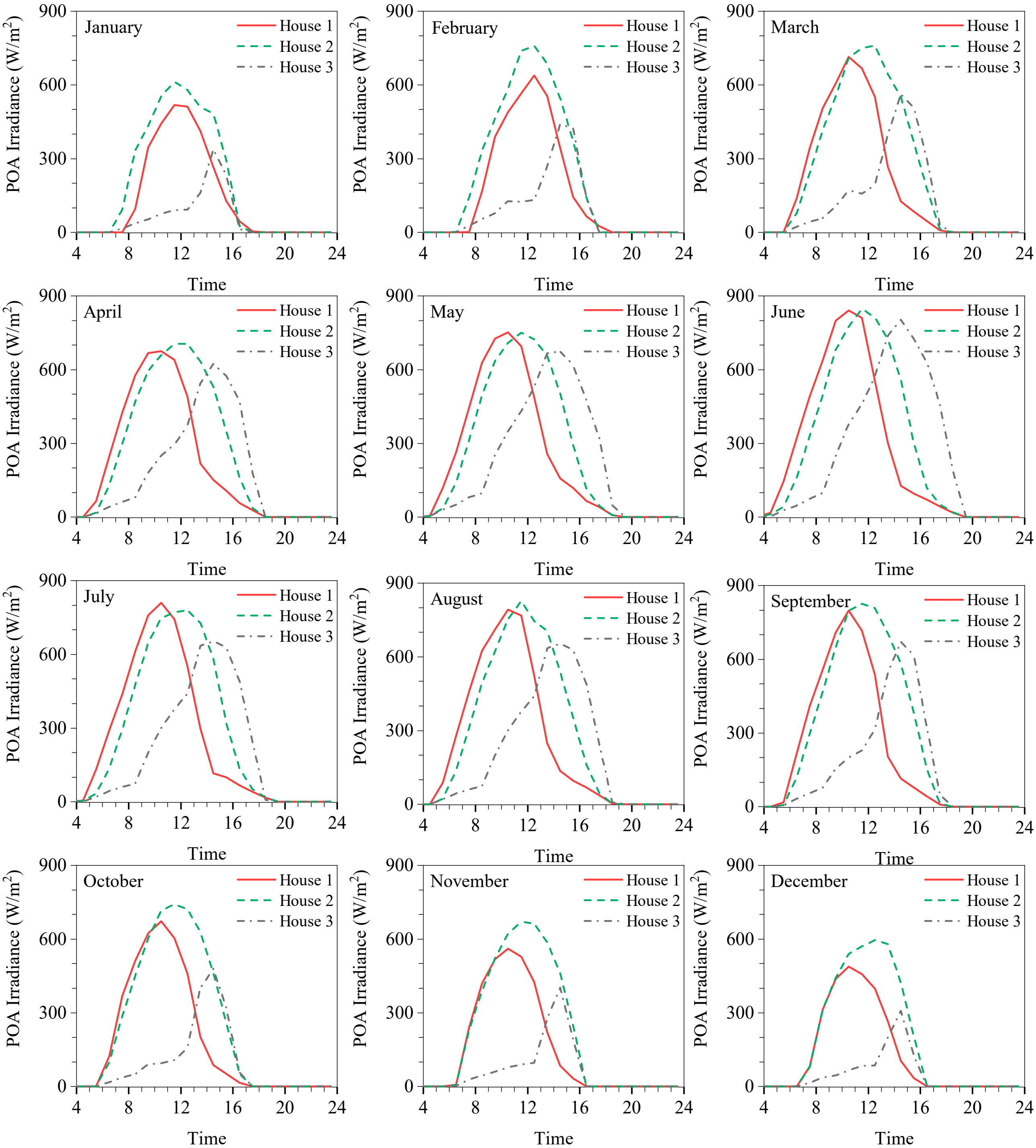

Figure 3. Simulated POA irradiance data for the CSP case study.

Table 2. Covariance (correlation) matrix of POA irradiance (W/m^2) for houses in the CSP.

	House 1	House 2	House 3
House 1	97,718 (1.0)	84,014 (0.79)	6884 (−0.08)
House 2	84,014 (0.79)	115,161 (1.0)	36,757 (0.38)
House 3	−6884 (−0.08)	36,757 (0.38)	78,487 (1.0)

According to the simulation results, House #1, with a 31-degree southeast-facing roof, shows a significant overlap in irradiance patterns with House 2, which has a steeper

40-degree south-facing roof, as reflected in a correlation coefficient 0.79. This indicates a strong linear relationship and a substantial similarity in the magnitude of variability in irradiance between these two houses across different months. House #3, which has the steepest 45-degree slope and is west-facing, exhibits a lower correlation with House 1 of −0.08, suggesting almost no linear relationship. The negative covariance value of −6884 between these two houses further supports this lack of alignment, likely due to differences in peak irradiance timings throughout the day and year. House 2 and House #3 have a correlation coefficient of 0.38, indicating that despite their different orientations, the variability in their irradiance does show some degree of synchronicity, although it is moderate and not as pronounced as between Houses 1 and 2. The irradiance curves in Figure 3 provide visual confirmation of these statistical relationships. House #1 and House 2 have similarly shaped curves, peaking earlier due to their southerly orientations, hence the relatively higher correlation. House 3's curve peaks later due to its westerly orientation, leading to a different irradiance pattern, especially when compared with House 1, which is reflected in the low correlation value.

4.3. Risk Model Selection and Portfolio Analysis

In this study, the Financial Toolbox in MATLAB was used to construct portfolios based on POA irradiance data. The toolbox comes with built-in functions for Var, MAD, and CVaR risk models. We developed a customized code to calculate SemiVar and an algorithm for calculating the utility function, weighted sum utilities, and retrieving the optimal portfolio weights associated with the maximum value of the weighted sum of utilities.

Figure 4 illustrates the efficient frontiers obtained using simulated POA data for four risk models. Notably, due to the significant differences in the risk scale of the CVaR model compared to the other three models, it is represented separately in Figure 4b. A comparison between the Var, SemiVar, and MAD models reveals distinct differences in returns at equivalent levels of risk. Specifically, the SemiVar model generates higher returns than the other two risk models. The SemiVar model incorporates downside risk by considering tail-end returns, while the MAD model focuses on minimizing return dispersion. Consequently, the model offers the potential for higher returns at equivalent risk levels, albeit with increased variability.

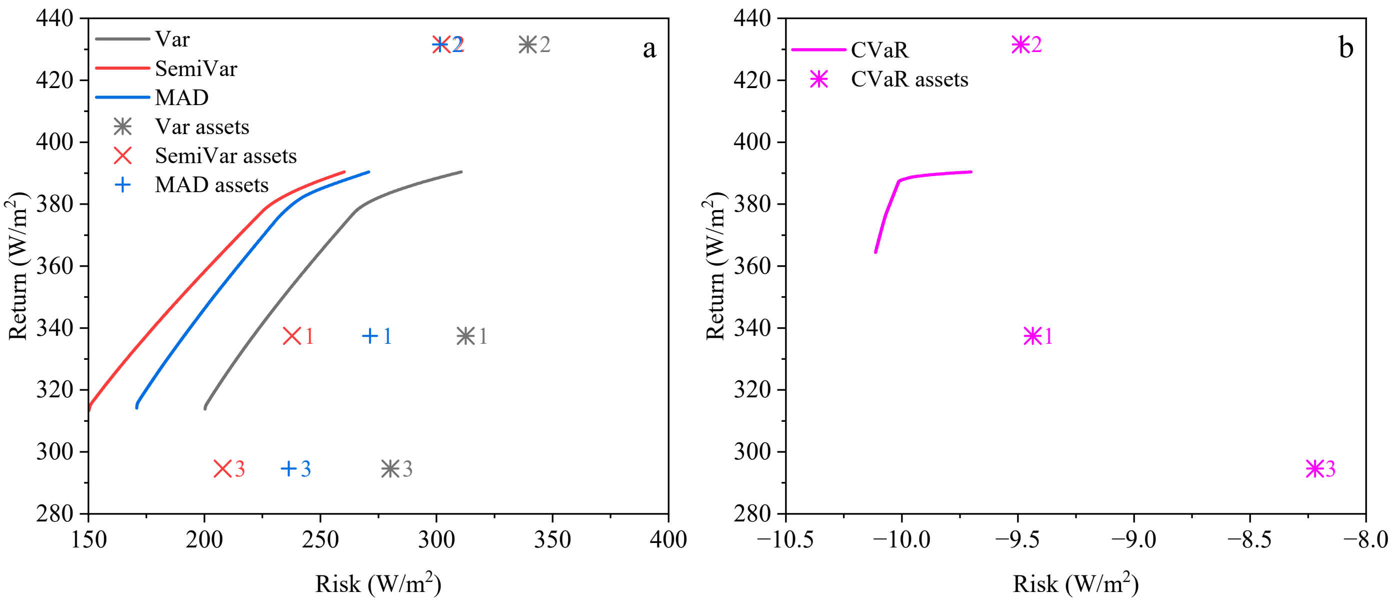

Figure 4. Efficient frontiers of portfolios constructed based on different risk models. (**a**) Var, SemiVar, and MAD; (**b**) CVAR model.

The difference between the scale of risk in CVaR and other risk models is a consequence of the unique nature of the CVaR model, which specifically focuses on capturing and quantifying the risk associated with extreme events or tail-end returns (Pflug 2000; Rockafellar and Uryasev 2000). The negative risk values indicate that all portfolios have potential losses below their expected returns. However, the portfolio with a larger negative risk has a larger buffer against potential losses (Sarykalin et al. 2008). In contrast to the CVaR model, the other models exhibit longer and less concave efficient frontiers. This characteristic implies a broader range of potential risk–return tradeoffs. These models allow for the possibility of achieving higher returns, albeit with increased levels of risk, as they do not place as much emphasis on extreme downside events.

It is important to emphasize that the choice between the CVaR and the other models depends on the investment objectives. The shorter and more concave efficient frontier in the CVaR model suggests a focus on conservative risk management, particularly in mitigating extreme downside risks (Guo et al. 2019). However, it may not be the preferred model for investors seeking higher returns or those with a greater willingness to tolerate higher levels of risk. The location of individual assets relative to the efficient frontiers in Figure 4 indicates that a 100% investment in Houses #1 and #3 would be inefficient, as they are situated below the efficient frontiers. However, House #2 is above the efficient frontier in all models, attributed to the spatial constraint c_i defined in Equation (1) and given in Table 1. According to Table 1, House #2 can accommodate a maximum of 27 out of the 48 available PV modules rated at 250 W. This imposes an upper-bound constraint of $c_i = 0.5625$ on the weights of this house, explaining why a 100% investment in House #2 is not feasible.

Figure 5 displays the density distributions of risk and return from all four models analyzed in this study. In Figure 5a, the density distributions of the Var, SemiVar, and MAD models, each with a mean value of 372 W/m^2, completely overlap. However, the CVaR distribution, with a mean of 385 W/m^2, yields a 3% higher return. Despite this difference, the overlap between the distribution of return in CVaR and the other three distributions suggests no statistical significance in returns among the four models. Figure 5b,c illustrate the density distributions of risk from all models, with the CVaR model presented separately due to its distinct risk scale. Comparing the distribution of risks in Figure 5b suggests that, on average, the SemiVar model with the mean risk and standard deviation of 220.38 and 44.8 W/m^2 has the lowest mean and highest standard deviation of risk among all models. The Var model, with the mean risk and standard deviation of 262.2 and 40.53 W/m^2, has the highest risk compared to other models. The MAD model exhibits the lowest variability in risk compared to Var and SemiVar models, with mean and standard deviation values of 230.6 W/m^2 and 38.3 W/m^2, respectively. Since the CVaR model has a different risk scale, meaningful comparisons with the other models would not be appropriate. However, it is worth noting that the density distribution of risk in the CVaR model is right-skewed, confirming its emphasis on selecting portfolios with lower risks. As the return distributions in Var, SemiVar, and MAD models are not statistically different, the choice among these distributions primarily hinges on their differences in their corresponding risk distributions. Considering that investors generally prefer lower volatility for a given level of return, the most reasonable choice appears to be the CVaR distribution.

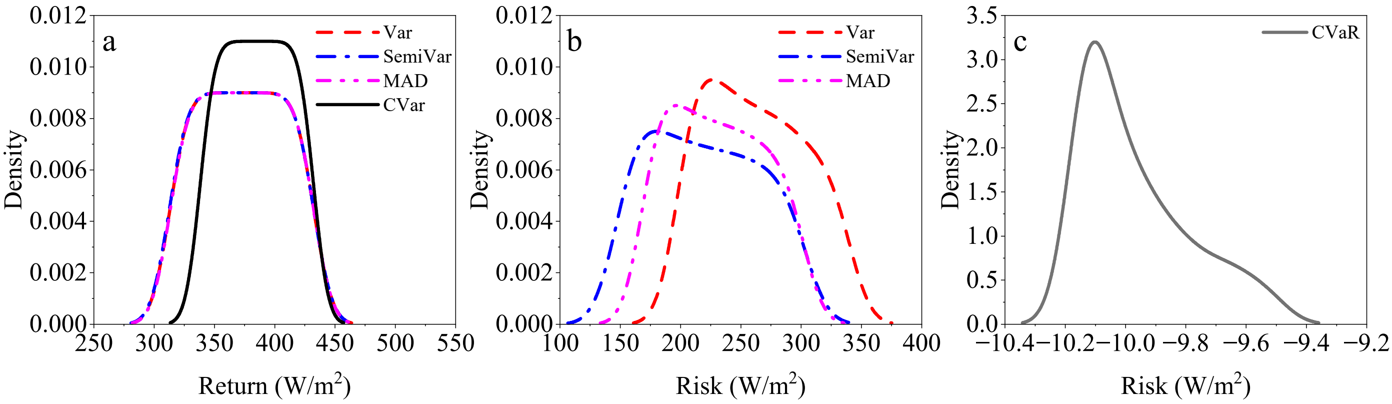

Figure 5. Distribution of risk and return in different models. (**a**) Return distributions; (**b**) risk distribution of Var, SemiVar, and MAD models; (**c**) risk distribution of CVaR model.

4.4. Estimating Individual and Weighted Sum of Utilities

According to Equations (2), determining the individual utility values of the investors for portfolios located on the efficient frontier requires three input parameters: the expected risk and return of portfolios, which were estimated in the previous section, and the risk-aversion coefficient of investors. Historically, the risk-aversion coefficient of investors λ_j has been quantified via empirical methods such as surveys and questionnaires (Kimball 1993; Bodnar et al. 2018). In this study, λ_j was assumed to range from 0 to 1, with values close to 0 indicating risk-loving, values close to 0.5 indicating risk-neutral, and λ_j values close to 1 indicating risk-averse attitudes. A stochastic approach was implemented to account for uncertainty in λj by simulating λ_j from a lognormal distribution (Aıt-Sahalia and Lo 2000). The lognormal distribution is suitable for simulating the risk-aversion parameter λ_j because it ensures non-negativity, realistically models a wide and skewed range of economic behaviors, and offers flexible parameterization. To account for disparities in risk-aversion levels among investors, the mean κ and standard deviation τ of the lognormal distributions for each investor were selected carefully to ensure minimal overlap between their λ_j distributions. Additionally, it was assumed that each investor contributed equally to the CSP, meaning the parameter η_j given in Equation (1) had a value of 1/3. Table 3 provides a summary of the parameters used for estimating the individual utilities and the weighted sum of utilities, and Figure 6 shows the distribution of individual utilities and weighted sum of utilities for different risk models.

Table 3. Input parameters used for calculating individual and weighted sum of utilities.

	$\lambda_j \sim$ **Lognormal (κ,τ²)**		η_j
	κ	**τ**	
Investor 1	−5.21	1.33	1/3
Investor 2	−0.64	0.09	1/3
Investor 3	−0.14	0.05	1/3

Figure 6 demonstrates that investors' risk attitudes significantly impact their utility distributions within CSP investments. Risk-averse investors typically favor investments with a narrower spread of outcomes, which results in utility distributions with a steeper peak and minimal skewness, highlighting their preference for certainty. On the other hand, risk-loving investors are more comfortable with a broader spread of outcomes and potentially more skewed distributions, as they are willing to accept the possibility of higher returns at the cost of greater risks. Risk-neutral investors generally contribute to a more even or symmetrical distribution, indicating a proportional valuation of returns to risk.

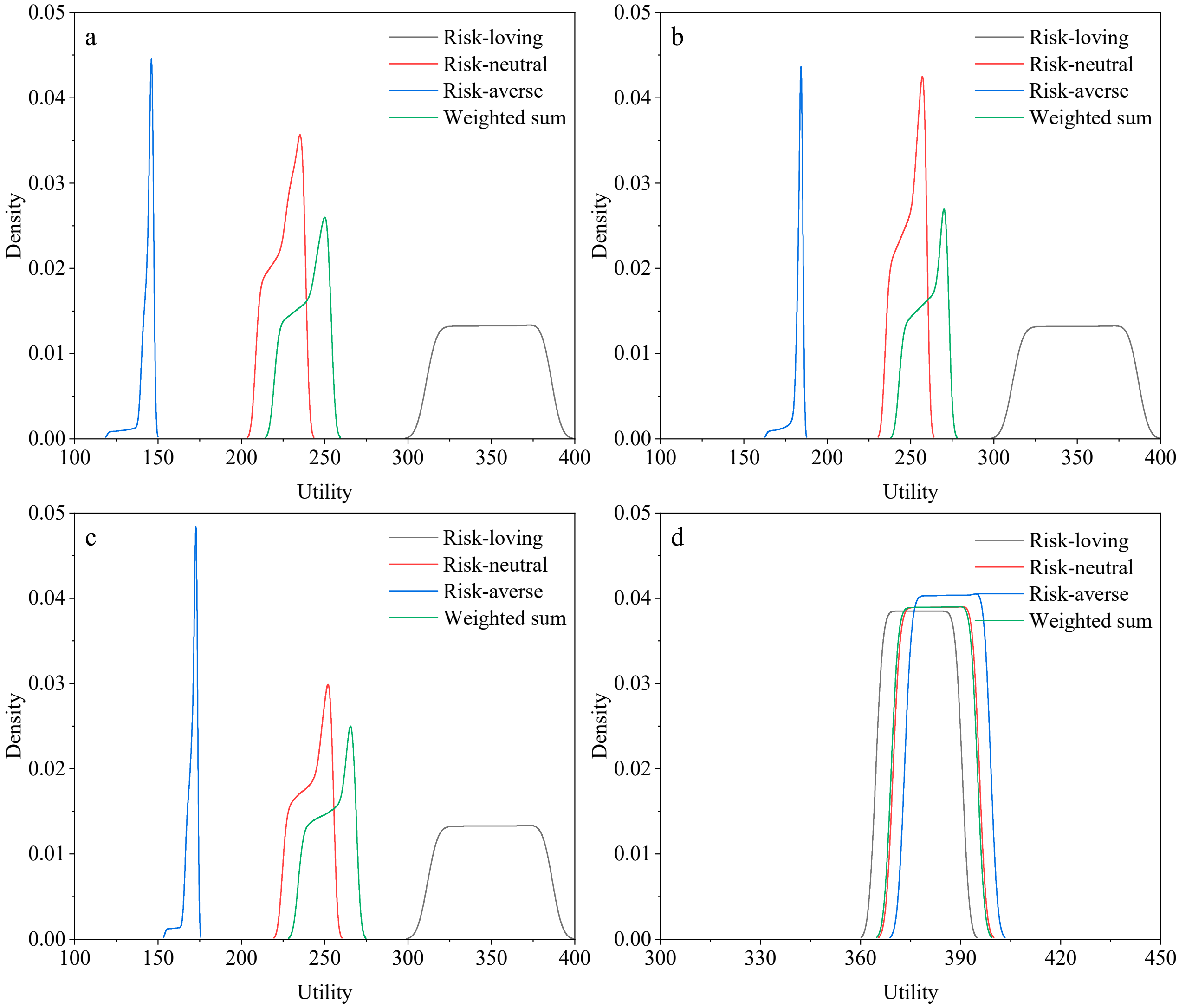

Figure 6. Distribution of individual utilities of investors and their weighted sum of utilities for different risk models. (**a**) Var, (**b**) SemiVar, (**c**) MAD, and (**d**) CVaR.

There appears to be no significant difference when comparing the utility distributions across Var, SemiVar, and MAD risk models. This indicates that the characteristics of CSP investments yield comparable risk–reward profiles across these diverse risk measurement frameworks. Such consistency may arise from the nature of CSP investments, where returns (e.g., POA irradiance values) exhibit similar variability based on these risk models, as depicted in Figure 3. Consequently, the distinctions between risk measures like Var and SemiVar, which concentrate on worst-case scenarios, and MAD, which accounts for average deviations, become subdued.

In contrast, the utility distributions from the CVaR model stand out distinctly from those of the Var, SemiVar, and MAD models, owing to CVaR's unique risk assessment approach. CVaR evaluates not only the potential losses identified by Var but also the severity of losses beyond that point, emphasizing the tail end of the potential loss distribution and calculating the average of the worst-case losses (Kisiala 2015). CVaR is likely to align with the perspectives of risk-averse investors with heightened concerns over severe losses (Holt and Laury 2002). Given that the return distribution illustrated in Figure 5a exhibits fat tails, which signals a higher likelihood of extreme outcomes than a normal distribution would suggest, CVaR would ascribe lower utility values to such investments, mirroring a more prudent stance. This implies that while Var, SemiVar, and MAD models might present an acceptable level of risk to investors, CVaR may indicate that worst-case scenarios are intolerable, thereby distinguishing the risk–reward profiles more clearly. The overlapping distribution of utility values in the CVaR model suggests that irrespective of whether an investor is risk-averse or risk-loving, the CVaR model projects a comparatively uniform expectation of loss in the tail end, leading to similar utility values for all investors.

Figure 7 presents the weighted sum of utilities for 1000 simulated portfolios on the efficient frontier. In this figure, lower portfolio indices signify portfolios with comparatively lower risk and return, while higher indices correspond to portfolios that potentially yield higher returns at the expense of increased risk. The utilities are arranged in descending order from the CVaR model to the Var model: CVaR > SemiVar > MAD > Var. This ordering aligns with the positioning of the efficient frontiers, as illustrated in Figure 4. The SemiVar model, being the second highest, indicates that investors potentially find some value in this model, which takes into account only the downside risk rather than both the upside and downside as Var and MAD do.

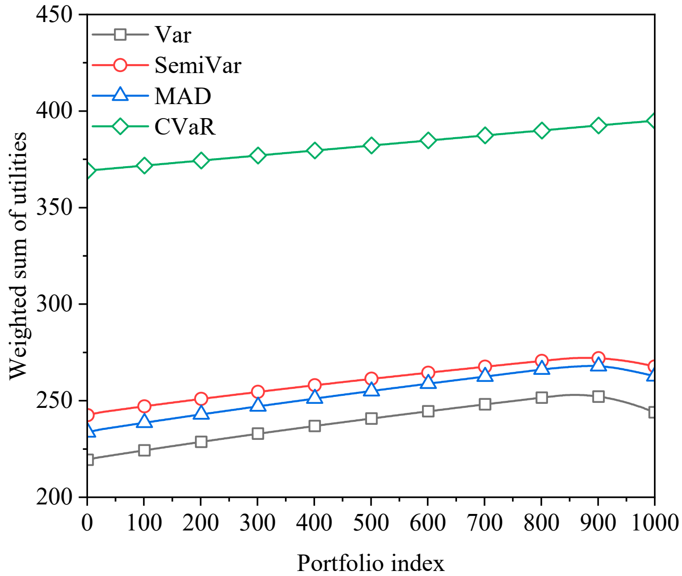

Figure 7. Weighted sum utilities of investors for various risk models.

4.5. Optimal Investment Weights and PV Distribution

Table 4 provides the information associated with the maximum weighted sum of utilities (U_{max}) for different risk models shown in Figure 7, along with the optimal asset weights and the portfolio's risk and return that would yield U_{max}. The results show that the CVaR model had the highest U_{max} at 394.975, which correlates with a portfolio strategy that heavily weights House 1 (0.4375) and House #2 (0.5625) and excludes House #3 entirely. In contrast, House 2 maintains the maximum weight of 0.5625 across all models, reflecting its optimal characteristics for solar energy production. The Var model shows the lowest U_{max} at 253.004, with more evenly distributed weights between Houses 1 and 3. The SemiVar and MAD models exhibit intermediate U_{max} values of 272.185 and 267.998, respectively, with portfolio weights that suggest a balance between managing downside risk and achieving returns. Return values are highest for the CVaR model at 390.4006, suggesting that the portfolio composition aligned with this model's methodology may yield the greatest returns according to the simulation.

Assuming that 48 PV modules are available for distribution among these three houses, the allocation would be guided by each house's optimal weights (w_1, w_2, w_3) as determined by the respective risk models. For the Var model, where w_1 is 0.18541, w_2 is 0.5625, and w_3 is 0.25209, House #1 would receive approximately 9 modules, House #2 would receive the full 27 modules as per its capacity, and House 3 would get 12 modules. Similarly, following the SemiVar model's weights, House 1 would get 10 modules, House 2 again would receive 27 modules, and House 3 would be allocated roughly 11 modules. For the MAD model, the distribution would be about 11 modules to House #1, 27 to House 2, and the remaining 10 to House 3. According to the CVaR model, House #1 would be allocated approximately 21 modules, and House 2 would receive 27 modules, with House 3 receiving no modules. This distribution strategy would adhere to the weightings derived from the maximum weighted sum of utilities that the respective risk model portfolios would yield.

Table 4. Estimated optimal portfolio information.

Risk Model	U_{max}	w_1	w_2	w_3	Risk	Return
Var	253.004	0.18541	0.5625	0.25209	268.4965	379.5937
SemiVar	272.185	0.2162	0.5625	0.2213	230.6128	380.9133
MAD	267.998	0.23988	0.5625	0.19762	241.6483	381.9287
CVaR	394.975	0.4375	0.5625	0	−9.70197	390.4006

5. Conclusions

This study introduced a mathematical model to optimize the distribution of PV panels in a community solar project, using the plane of array irradiance as the main input data. The model employed a range of risk measures to cater to investors with different levels of risk aversion. The results show that the allocation strategy under the CVaR significantly diverged from traditional risk models, reflected in negative risk values. Key findings from the risk distributions showed no statistically significant differences in returns among the Var, SemiVar, and MAD models, while the CVaR model displayed a marginally higher mean return, making it an attractive option for those seeking higher returns. In terms of risk, the SemiVar model had the lowest mean risk but the highest variability; the MAD model showed the least variability, indicating a more consistent risk profile; and the CVaR model's risk distribution was right-skewed, favoring the selection of lower-risk portfolios. Investors' risk aversion impacted utility functions across risk models, with increased risk aversion correlating with decreased utility values. The utility values across the Var, SemiVar, and MAD models were notably similar. In contrast, the utility behavior in the CVaR model was distinct due to its negative risk sign, suggesting a preference for lower utility (higher risk and return) within this model.

The contributions of this study to CSP optimization and design are manifold. Leveraging diverse risk models provides a blueprint for investors to enhance returns while minimizing risks, thereby increasing the attractiveness and viability of solar investments. It also offers practical guidance for CSP design by analyzing the spatial correlation of power generation potential within communities, assisting urban planners and architects in creating efficient and equitable solar installations. Furthermore, the study's implications extend to risk management within renewable energy portfolios and could influence policy-making to foster CSP development and regulation. It also emphasizes the importance of a participatory approach in CSPs, promoting project alignment with community risk preferences and fostering broader community engagement.

In terms of limitations, the potential constraint of a limited sample size of investors may limit the generalizability of the results of this study. Additionally, in this study, the role of external and behavioral factors in portfolio selection was ignored. Future research should investigate various utility functions, including a broader range of influential factors, and closely examine investor behaviors. Empirical validation of the portfolio selection methodology through sensitivity analyses, comparative studies, and assessments of real-world implementations is essential for refining investment strategies and optimizing risk-adjusted outcomes in CSPs.

Author Contributions: Conceptualization, M.S.; methodology, M.S.; software, M.S.; validation, M.S.; formal analysis, M.S.; investigation, M.S.; resources, M.S.; data curation, M.S.; writing—original draft preparation, M.S.; writing—review and editing, M.S., C.N., S.B. and K.L.N.; visualization, M.S.;

supervision, M.S.; project administration, M.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to concerns regarding privacy and the potential for misinterpretation or misuse of the specialized data outside of its intended context.

Conflicts of Interest: The authors declare no conflicts of interest.

References

Aıt-Sahalia, Yacine, and Andrew W. Lo. 2000. Nonparametric Risk Management and Implied Risk Aversion. *Journal of Econometrics* 94: 9–51. [CrossRef]

Blair, Nate, Aron P. Dobos, Janine Freeman, Ty Neises, Michael Wagner, Tom Ferguson, Paul Gilman, and Steven Janzou. 2018. *System Advisor Model, Sam 2014.1. 14: General Description*. Golden: National Renewable Energy Laboratory.

Boasson, Vigdis, Emil Boasson, and Zhao Zhou. 2011. Portfolio Optimization in a Mean-SemiVariance Framework. *Investment Management and Financial Innovations* 8: 58–68.

Bodnar, Taras, Yarema Okhrin, Valdemar Vitlinskyy, and Taras Zabolotskyy Zabolotskyy. 2018. Determination and Estimation of Risk Aversion Coefficients. *Computational Management Science* 15: 297–317. [CrossRef]

Burger, Scott P., and Max Luke. 2017. Business Models for Distributed Energy Resources: A Review and Empirical Analysis. *Energy Policy* 109: 230–48. [CrossRef]

Chan, Gabriel, Isaac Evans, Matthew Grimley, Ben Ihde, and Poulomi Mazumder Mazumder. 2017. Design Choices and Equity Implications of Community Shared Solar. *The Electricity Journal* 30: 37–41. [CrossRef]

Chowdhury, Md Shahariar, Kazi Sajedur Rahman, Tanjia Chowdhury, Narissara Nuthammachot, Kuaanan Techato, Md Akhtaruzzaman, Sieh Kiong Tiong, Kamaruzzaman Sopian, and Nowshad Amin. 2020. An Overview of Solar Photovoltaic Panels' End-of-Life Material Recycling. *Energy Strategy Reviews* 27: 100431. [CrossRef]

Cross, Jamie, and Tom Neumark. 2021. Solar Power and Its Discontents: Critiquing Off-Grid Infrastructures of Inclusion in East Africa. *Development and Change* 52: 902–26. [CrossRef]

Curtin, Joseph, Celine McInerney, Brian O. Gallachoir, and Sarah Salm. 2019. Energizing Local Communities—What Motivates Irish Citizens to Invest in Distributed Renewables? *Energy Research & Social Science* 48: 177–88.

Dusonchet, Luigi, and Enrico Telaretti. 2010. Economic Analysis of Different Supporting Policies for the Production of Electrical Energy by Solar Photovoltaics in Western European Union Countries. *Energy Policy* 38: 3297–308. [CrossRef]

Eisenhauer, Joseph G., and Luigi Ventura. 2003. Survey Measures of Risk Aversion and Prudence. *Applied Economics* 35: 1477–84. [CrossRef]

Fouad, M. M., Lamia A. Shihata, and ElSayed I. Morgan. 2017. An Integrated Review of Factors Influencing the Perfomance of Photovoltaic Panels. *Renewable and Sustainable Energy Reviews* 80: 1499–511. [CrossRef]

Gjorgievski, Vladimir Z., Snezana Cundeva, and George E. Georghiou. 2021. Social Arrangements, Technical Designs and Impacts of Energy Communities: A Review. *Renewable Energy* 169: 1138–56. [CrossRef]

Guno, Charmaine Samala, Casper Boongaling Agaton, Resy Ordona Villanueva, and Riza Ordona Villanueva. 2021. Optimal Investment Strategy for Solar Pv Integration in Residential Buildings: A Case Study in the Philippines. *International Journal of Renewable Energy Development* 10: 79. [CrossRef]

Guo, Xu, Raymond H. Chan, Wing-Keung Wong, and Lixing Zhu. 2019. Mean–Variance, Mean–Var, and Mean–CVaR Models for Portfolio Selection with Background Risk. *Risk Management* 21: 73–98. [CrossRef]

Holt, Charles A., and Susan K. Laury. 2002. Risk Aversion and Incentive Effects. *American Economic Review* 92: 1644–55. [CrossRef]

Hunjra, Ahmed Imran, Suha Mahmoud Alawi, Sisira Colombage, Uroosa Sahito, and Mahnoor Hanif. 2020. Portfolio Construction by Using Different Risk Models: A Comparison among Diverse Economic Scenarios. *Risks* 8: 126. [CrossRef]

Kimball, Miles S. 1993. Standard Risk Aversion. *Econometrica: Journal of the Econometric Society* 61: 589–611. [CrossRef]

Kisiala, Jakob. 2015. Conditional Value-at-Risk: Theory and Applications. *arXiv* arXiv:1511.00140.

Konno, Hiroshi, and Hiroaki Yamazaki. 1991. Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market. *Management Science* 37: 519–31. [CrossRef]

Kuang, Wei. 2021. Are Clean Energy Assets a Safe Haven for International Equity Markets? *Journal of Cleaner Production* 302: 127006. [CrossRef]

Lee, Stephen, Prashant Shenoy, Krithi Ramamritham, and David Irwin. 2021. Autoshare: Virtual Community Solar and Storage for Energy Sharing. *Energy Informatics* 4: 1–24. [CrossRef]

Markowitz, Harry. 1952. Portfolio Selection. *The Journal of Finance* 7: 77–91.

Markowitz, Harry. 1959. *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley Sons.

Nishanthy, J., S. Charles Raja, and J. Jeslin Drusila Nesamalar. 2023. Feasibility Analysis of Solar Pv System in Presence of Ev Charging with Transactive Energy Management for a Community-Based Residential System. *Energy Conversion and Management* 288: 117125. [CrossRef]

Nolden, Colin, Jacob Barnes, and J. Nicholls. 2020. Community Energy Business Model Evolution: A Review of Solar Photovoltaic Developments in England. *Renewable and Sustainable Energy Reviews* 122: 109722. [CrossRef]

Oh, Eunsung. 2022. Community Solar Operation Strategy for Smart Energy Communities Considering Resource Fairness. *Applied Sciences* 12: 12867. [CrossRef]

Parsonnet, Myra. 2021. What Would Expand Utility-Led Community Solar in the Southeastern Us? Master's thesis, Duke University, Durham, NC, USA.

Pflug, Georg Ch. 2000. Some Remarks on the Value-at-Risk and the Conditional Value-at-Risk. In *Probabilistic Constrained Optimization: Methodology and Applications*. Boston: Springer, pp. 272–81.

Prehoda, Emily, Richelle Winkler, and Chelsea Schelly. 2019. Putting Research to Action: Integrating Collaborative Governance and Community-Engaged Research for Community Solar. *Social Sciences* 8: 11. [CrossRef]

Rockafellar, R. Tyrrell, and Stanislav Uryasev. 2000. Optimization of Conditional Value-at-Risk. *Journal of Risk* 2: 21–42. [CrossRef]

Said, Syed A. M., Ghassan Hassan, Husam M. Walwil, and N. Al-Aqeeli. 2018. The Effect of Environmental Factors and Dust Accumulation on Photovoltaic Modules and Dust-Accumulation Mitigation Strategies. *Renewable and Sustainable Energy Reviews* 82: 743–60. [CrossRef]

Santhakumari, Manju, and Netramani Sagar. 2019. A Review of the Environmental Factors Degrading the Performance of Silicon Wafer-Based Photovoltaic Modules: Failure Detection Methods and Essential Mitigation Techniques. *Renewable and Sustainable Energy Reviews* 110: 83–100. [CrossRef]

Sarykalin, Sergey, Gaia Serraino, and Stan Uryasev. 2008. Value-at-Risk Vs. Conditional Value-at-Risk in Risk Management and Optimization. In *State-of-the-Art Decision-Making Tools in the Information-Intensive Age*. Moscow: Informs, pp. 270–94.

Shakouri, Mahmoud, and Hyun Woo Lee. 2016. Mean-Variance Portfolio Analysis Data for Optimizing Community-Based Photovoltaic Investment. *Data in Brief* 6: 840–42. [CrossRef]

Shakouri, Mahmoud, Hyun Woo Lee, and Kunhee Choi. 2015. Pacpim: New Decision-Support Model of Optimized Portfolio Analysis for Community-Based Photovoltaic Investment. *Applied Energy* 156: 607–17. [CrossRef]

Shakouri, Mahmoud, Hyun Woo Lee, and Yong-Woo Kim. 2017. A Probabilistic Portfolio-Based Model for Financial Valuation of Community Solar. *Applied Energy* 191: 709–26. [CrossRef]

Speranza, Maria Grazia. 1993. Linear Programming Models for Portfolio Optimization. *Finance* 14: 107–23.

Weixiang, Sun, Md Qamruzzaman, Wang Rui, and Rajnish Kler. 2022. An Empirical Assessment of Financial Literacy and Behavioral Biases on Investment Decision: Fresh Evidence from Small Investor Perception. *Frontiers in Psychology* 13: 977444. [CrossRef]

Zhou, Suyang, Fenghua Zou, Zhi Wu, Wei Gu, Qiteng Hong, and Campbell Booth. 2020. A Smart Community Energy Management Scheme Considering User Dominated Demand Side Response and P2p Trading. *International Journal of Electrical Power & Energy Systems* 114: 105378.