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Can Multi-Peril Insurance Policies Mitigate Adverse Selection?

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Abstract: The objective of this paper is to pursue an intuitive idea: for a consumer who represents an "unfavorable" health risk but an "excellent risk" as a driver, a multi-peril policy could be associated with a reduced selection effort on the part of the insurer. If this intuition should be confirmed, it will serve to address the decade-long concern with risk selection both in the economic literature and on the part of policy makers. As an illustrative example, a two-peril model is developed in which consumers deploy effort in search of a policy offering them maximum coverage at the current market price while insurers deploy effort designed to stave off unfavorable risks. Two types of Nash equilibria are compared: one in which the insurer is confronted with high-risk and low-risk types, and another one where both types are a "better risk" with regard to a second peril. The difference in the insurer's selection effort directed at high-risk and low-risk types is indeed shown to be lower in the latter case, resulting in a mitigation of adverse selection.

Keywords: adverse selection; risk selection; consumer search effort; insurer selection effort

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1. Introduction and Motivation

For several decades, adverse selection in competitive markets has been an issue for insurance companies (insurers henceforth), economists, and policy makers. Insurers have been afraid of losing their favorable risks to a competitor, as predicted by Rothschild and Stiglitz (1976); economists continue to analyze if and how equilibria in insurance markets might dissolve [e.g., Wilson (1977); Engers and Fernandez (1987); Asheim and Nilssen (1996); and again Rothschild and Stiglitz (1997)]; and policy makers worry about unfavorable risks being discriminated against [e.g., Rosenbaum (2009); Petkantchin (2010); Avraham et al. (2014)].

Yet, there are two striking observations regarding this literature. The first is that in the age of multiline insurers, much of the analysis has revolved exclusively around one risk. However, Crocker and Snow (2011) noted that many insurance contracts bundle several perils, referred to as multi-peril policies, often with differing deductibles[1]. They developed a model in which low-risk consumers can signal their type through their deductible choices and show that this may enhance the efficiency of self-sorting to a degree that the market approaches a stable Nash equilibrium. This result holds regardless of the intuitive argument that a multiline insurer's concern about adverse selection would be mitigated because a future expected loss in one line is likely to be balanced out by an expected gain in the other. Picard (2019) asked the opposite question: why are so-called umbrella policies rare, whereas the coverage of similar risks is split? However, since no market equilibrium is derived, the analysis remains silent about the effect of this splitting on adverse selection.

The second observation is that consumers are seen as seeking out a policy without deploying costly search effort, while insurers undertake a risk selection effort that is costless. Ever since Rothschild and Stiglitz (1976), low-risk consumers have been implicitly assumed to find the contract suiting them without deploying any effort. Yet, the Internet is replete with websites offering advice on how to choose an insurance policy.[2] On the part of the insurer, the implementation of separating contracts designed to stave off adverse selection

also entails costly effort. Attracting high-risk types is relatively easy; all it takes is to offer a policy with a relatively high degree of coverage at a fair premium. However, the insurer also needs to launch a contract with limited coverage at a lower premium to attract low-risk types. In this, it faces two challenges: first, to prevent high-risk types from infiltrating this contract also through renegotiation [see e.g., Dionne and Doherty (1994)] and second, to prevent a competitor from siphoning off its low-risk types through clever contract design. In the model of Crocker and Snow (2011), the self-sorting of risks achieves this at no cost; however, many firms offer advice on developing and marketing insurance policies in the Internet.[3] Evidently, risk selection is a costly activity; in the case of health insurance, it has also been shown to be risky because high-risk types may become low-risk ones over time and vice versa (Beck et al. 2010).

Against this background, the present theoretical study introduces costly effort on both sides of the market. Consumers generally exhibit a significant search cost when evaluating insurance policies (Schlesinger and Von der Schulenburg 1991; Pauly et al. 2006; Akın and Platt 2014). Consumers need to deploy search effort to identify the policy offering them the highest coverage for a given premium, while insurers set their risk selection effort to maximize expected profit. Accordingly, in this study, Nash equilibria are derived in the effort space to model the interaction of high-risk and low-risk consumers with an insurer. These equilibria can be shown to be less far apart in the effort space if risk types are a "better risk" regarding a second peril than otherwise. In addition, the equilibrium characterizing the insurer's interaction with the high-risk type involves a reduced risk selection effort compared with a single-risk policy.

The article proceeds as follows. The next section offers a general model of a two-peril policy, with the combination of health and auto insurance serving as an example. Admittedly, such a combination hardly exists at present due to the regulatory separation of life and nonlife lines of insurance. However, as argued in the concluding section, lifting this separation may well be beneficial. Indeed, it is shown that a multi-peril policy is associated with both a reduced consumer search effort and a reduced risk selection effort on the side of the insurer (and therefore constitutes a Pareto improvement). In Section 3, these results are projected into the consumer wealth level space, where the rationing constraint inherent in separating equilibria is taken into account. The last section briefly discusses the findings and concludes.

2. A Simple Model of a Two-Peril Policy

This section develops a simple game-theoretic model to determine Nash equilibria of the conventional Rothschild and Stiglitz (1976) type for unfavorable ("high") and favorable ("low") risk types in the effort space (for the projection into the conventional wealth level space, see Section 3). Starting in the effort space takes account of costly consumer search, which is implicit in the literature on adverse selection. It also permits the integration of risk selection effort on the part of the insurer (developing contract variants is a costly activity, as argued in Section 1). In the present model, costly search effort and costly risk selection effort are the decision variables controlled by the respective players.

2.1. Consumers

Following the general adverse selection literature, consumers are seen as expected utility maximizers. For simplicity, assume there are only two risk types of consumers in the market: low-risk types and high-risk types. It is also assumed that a consumer may represent, e.g., an "unfavorable" health risk but an "excellent risk" as a driver. Consumers thus represent two risks simultaneously: a good one and a bad one at the same time. High- (low-) risk consumers undertake an observable search effort for securing maximum amounts of coverage or indemnity payments I and J for their two risks, $I^H\left(I^L\right)$ and $J^H\left(J^L\right)$,

respectively, at the current insurance market price, $P^H(P^L)$, which they view as exogenous for now.[4] Assuming they are expected utility maximizers, their decision problem reads

$$\begin{aligned} \max_{c^H} EU^H = {} & \rho^H \cdot (\rho^H\Delta) v^H[W_0 + I^H(c^H,e) + J^H(c^H,e) - K - L - P^H] \\ & + (1-\rho^H) \cdot (\rho^H\Delta) v^H[W_0 + J^H(c^H,e) - K - P^H] \\ & + \rho^H \cdot (1-\rho^H\Delta) v^H[W_0 + I^H(c^H,e) - L - P^H] \\ & + (1-\rho^H) \cdot (1-\rho^H\Delta) v^H[W_0 - P^H] - c^H, \text{ with} \\ & P^H = 2\overline{\rho}(1+\lambda^H(e)); \end{aligned} \tag{1a}$$

$$\begin{aligned} \max_{c^L} EU^L = {} & \rho^L(\rho^L\Delta) v^L[W_0 + I^L(c^L,e) + J^L(c^L,e) - K - L - P^L] \\ & + (1-\rho^L)(\rho^L\Delta) v^L[W_0 + J^L(c^L,e) - K - P^L] \\ & + \rho^L(1-\rho^L\Delta) v^L[W_0 + I^L(c^L,e) - L - P^L] \\ & + (1-\rho^L)(1-\rho^L\Delta) v^L[W_0 - P^L] - c^L, \text{with} \\ & P^L = 2\overline{\rho}(1+\lambda^L(e)). \end{aligned} \tag{1b}$$

Here, $EU^H(EU^L)$ denotes the expected utility of the high- (low-) risk type, with the superscripts H and L not explained separately below unless necessary. v^H is the VNM utility function with $v^{H\prime} > 0$ and $v^{H\prime\prime} < 0$ $\left(v^{L\prime} > 0 \text{ and } v^{L\prime\prime} < 0\right)$. Both risk types are exposed to two perils, the first with probability ρ^H ($\rho^L < \rho^H$ for the low-risk type) and the second peril with lower probabilities $\rho^H\Delta$ and $\rho^L\Delta$, respectively, where $0 < \Delta < 1$ indicates that both types are "better risks" concerning the second peril.[5] The common value of Δ serves a purpose: if only the high-risk consumer in terms of health benefited from being a low risk as a driver, the two types would become more similar, with less selection effort on the part of the insurer as the natural consequence. However, as shown in Section 2, this will also be the outcome if both types are "better" risks as drivers to the same extent. W_0 denotes exogenous initial wealth, and $I^H(c^H,e)$ the indemnity payment in case of a loss depending on both a consumer's search cost c^H or c^L (with $\frac{\partial I^H}{\partial c^H} > 0$, $\frac{\partial I^L}{\partial c^L} > 0$ and $\frac{\partial^2 I^H}{\partial c^{H2}} < 0$, $\frac{\partial^2 I^L}{\partial c^{L2}} < 0$), as well as on the insurer's selection effort e (with $\frac{\partial I^H}{\partial e} < 0$, $\frac{\partial J^H}{\partial e} < 0$ and $\frac{\partial I^L}{\partial e} < 0$, $\frac{\partial J^L}{\partial e} < 0$). This is because regardless of risk type, consumers are burdened by having to provide additional information to the insurer.[6] Note that the indemnity payments are fully determined by c^H, c^L, and e; only the occurrence of losses is governed by chance. Finally, K and L denote the exogenous losses pertaining to the first and second peril, respectively, with inequalities $K \le I^H$, $K \le I^L$, $L \le J^H$, and $L \le J^L$ not explicitly imposed in order to avoid the complication of Kuhn–Tucker conditions.[7]

In view of the information asymmetry, the premium covers the two risks based on a population average value, $\overline{\rho}$. As should be expected, loadings increase with selection effort $\lambda^H(e)$ and $\lambda^L(e) < \lambda^H(e)$, which reflects the separating contracts that will characterize the market equilibrium.

The interior optimum is determined by the first-order condition (FOC)[8]

$$\begin{aligned} \frac{dEU^H}{dc^H} = {} & \rho^H \cdot (\rho^H\Delta) v^{H\prime}[W_0 + I^H(c^H,e) + J^H(c^H,e) - K - L - P^H(e)]\left(\frac{\partial I^H}{\partial c^H} + \frac{\partial J^H}{\partial c^H}\right) \\ & + (1-\rho^H)(\rho^H\Delta) v^{H\prime}[W_0 + J^H(c^H,e) - K - P^H(e)]\frac{\partial J^H}{\partial c^H} \\ & + \rho^H(1-\rho^H\Delta) v^{H\prime}[W_0 + I^H(c^H,e) - L - P^H(e)]\frac{\partial I^H}{\partial c^H} \\ & + (1-\rho^H)(1-\rho^H\Delta) v^{H\prime}[W_0 - P^H(e)] - 1 = 0; \end{aligned} \tag{2a}$$

and

$$\begin{aligned} \frac{dEU^L}{dc^L} = {} & \rho^L \cdot (\rho^L\Delta) v^{L\prime}[W_0 + I^L(c^L,e) + J^L(c^L,e) - K - L - P^L(e)]\left(\frac{\partial I^L}{\partial c^L} + \frac{\partial J^L}{\partial c^L}\right) \\ & + (1-\rho^L)(\rho^L\Delta) v^{L\prime}[W_0 + J^L(c^L,e) - K - P^L(e)]\frac{\partial J^L}{\partial c^L} \\ & + \rho^L(1-\rho^L\Delta) v^{L\prime}[W_0 + I^H(c^H,e) - L - P^L(e)]\frac{\partial I^L}{\partial c^L} \\ & + (1-\rho^L)(1-\rho^L\Delta) v^{L\prime}[W_0 - P^L(e)] - 1 = 0. \end{aligned} \tag{2b}$$

Note that, unless the derivatives of the $v(\cdot)$, $I(c,e)$, and $J(c,e)$ functions differ substantially between risk types (for which there is no apparent reason), the high-risk types are predicted to undertake more search effort than the low-risk types, which can be seen given $\rho^H > \rho^L$ and

$$\begin{aligned}&\rho^H \cdot (\rho^H\Delta) + (1-\rho^H)(\rho^H\Delta) + \rho^H(1-\rho^H\Delta) + (1-\rho^H)(1-\rho^H\Delta)\\&-\{\rho^L \cdot (\rho^L\Delta) + (1-\rho^L)(\rho^L\Delta) + \rho^L(1-\rho^L\Delta) + (1-\rho^L)(1-\rho^L\Delta)\}\\&= (\rho^{H2}-\rho^{L2})\Delta + (\rho^H-\rho^L)\Delta - (\rho^{H2}-\rho^{L2})\Delta + (\rho^H-\rho^L)\Delta\\&-2(\rho^H-\rho^L)\Delta + (\rho^{H2}-\rho^{L2})\Delta\\&= (\rho^{H2}-\rho^{L2})\Delta > 0,\end{aligned} \tag{3}$$

indicating that the marginal benefit of searching is higher for the high-risk types. This confirms economic intuition and implies that at a given value of selection effort, e, consumer search effort exercised by high-risk types is at least as large as that exercised by low-risk types (see also Figure 1 below). Figure 1 depicts consumers' reaction functions.[9] It can be seen that both of the two type-specific reaction functions have negative slopes.[10]

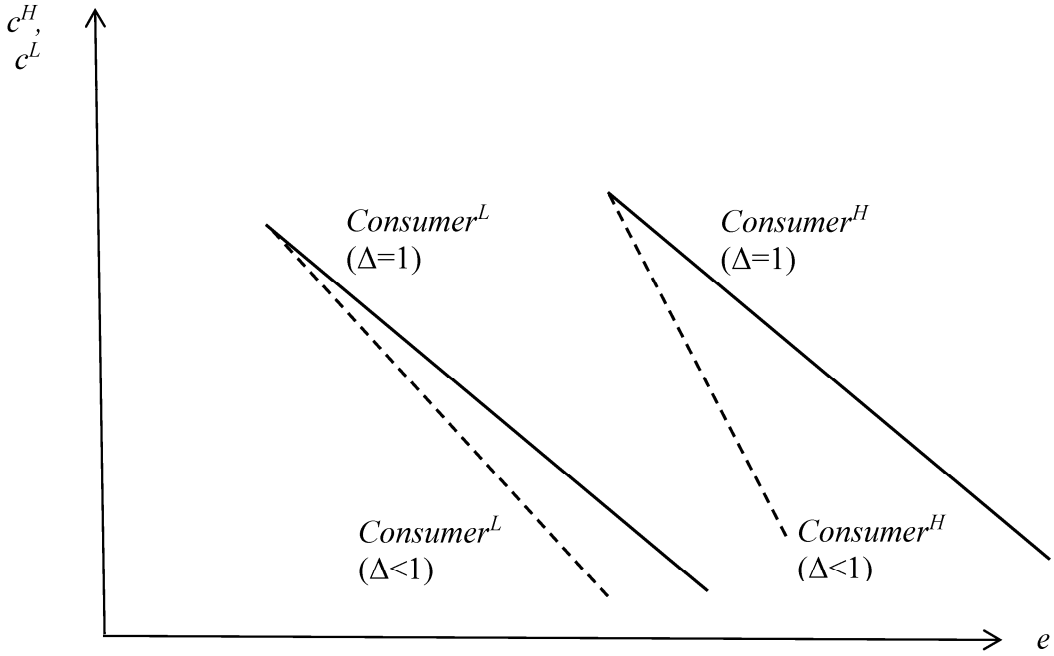

Figure 1. Reaction functions when consumers are "better risks" w.r.t. a second peril.

2.2. Insurers

Insurers choose a single optimal effort level e to maximize their expected profit,

$$\begin{aligned}\max_e E\Pi = \pi(e)\left[P^H(e) - I^H(c^H,e) - J^H(c^H,e)\right]\\+(1-\pi(e))\left[P^L(e) - I^L(c^L,e) - J^L(c^H,e)\right] - e.\end{aligned} \tag{4}$$

Here, $E\Pi$ denotes expected profit, $\pi(e)$ indicates the probability of enrolling a high-risk type depending on risk selection effort e (at a unit cost of one, also comprising administrative expenses for simplicity), with $\frac{\partial\pi}{\partial e} < 0$ and $\frac{\partial\pi^2}{\partial e^2} > 0$ indicating decreasing marginal effectiveness and a premium income from a high-risk type which depends positively on selection effort e through the loading $\lambda^H(e)$, and covered losses are $I^H(c^H,e)$ and $J^H(c^H,e)$, depending positively on consumer search effort but negatively on the insurer's selection effort, as outlined above. Specifically, a reasonable assumption (to be used in Appendix A) is that $\frac{\partial^2 I^H}{\partial c^H \partial e} < 0$, $\frac{\partial^2 I^L}{\partial c^L \partial e} < 0$, indicating that the marginal effectiveness of the consumer search is reduced by the insurer's selection effort. The reason is that selection effort usually

goes along with additional documentation requirements for consumers, which burden them each time they contact an insurer. Analogous specifications apply to the low-risk policyholder type.

The first-order condition for an interior optimum reads

$$\begin{aligned}\frac{dE\Pi}{de} &= \frac{\partial\pi}{\partial e}\cdot\left[P^H(e)-I^H-J^H\right]+\pi(e)\left[\frac{\partial P^H}{\partial e}-\frac{\partial I^H(c^H,e)}{\partial e}-\frac{\partial J^H(c^H,e)}{\partial e}\right]\\ &-\frac{\partial\pi}{\partial e}\cdot\left[P^L(e)-I^L-J^L\right]+(1-\pi(e))\left[\frac{\partial P^L}{\partial e}-\frac{\partial I^L(c^L,e)}{\partial e}-\frac{\partial J^L(c^L,e)}{\partial e}\right]-1\\ &= 0.\end{aligned} \tag{5}$$

The first term is positive due to $\frac{\partial\pi}{\partial e} < 0$ combined with the fact that the benefits paid exceed the current premium; the second term is positive as well (because $\partial P^H(e)/\partial e = (\partial P/\partial\lambda)\cdot\left(\frac{\partial\lambda}{\partial e}\right) > 0$, $\frac{\partial I^H(c^H,e)}{\partial e} < 0$, $\frac{\partial J^H(c^H,e)}{\partial e} < 0$); the third term is negative but smaller in absolute value than the first, given $P^H(e) > P^L(e)$ for similar losses; and the fourth term is again positive. In sum, the overall marginal benefit almost certainly covers the marginal cost, ensuring an interior solution exists.

The derivation of the insurer's reaction functions is relegated to Appendix B. Note that while there is only one decision variable, e, in principle the functions depend on whether the insurer is dealing with a high-risk or a low-risk type. However, the pertinent Equations (A9) and (A10) cannot be distinguished without further assumptions; therefore, only one insurer's reaction function is shown in Figure 2. This has the consequence that there are only two rather than four Nash equilibria in the picture.

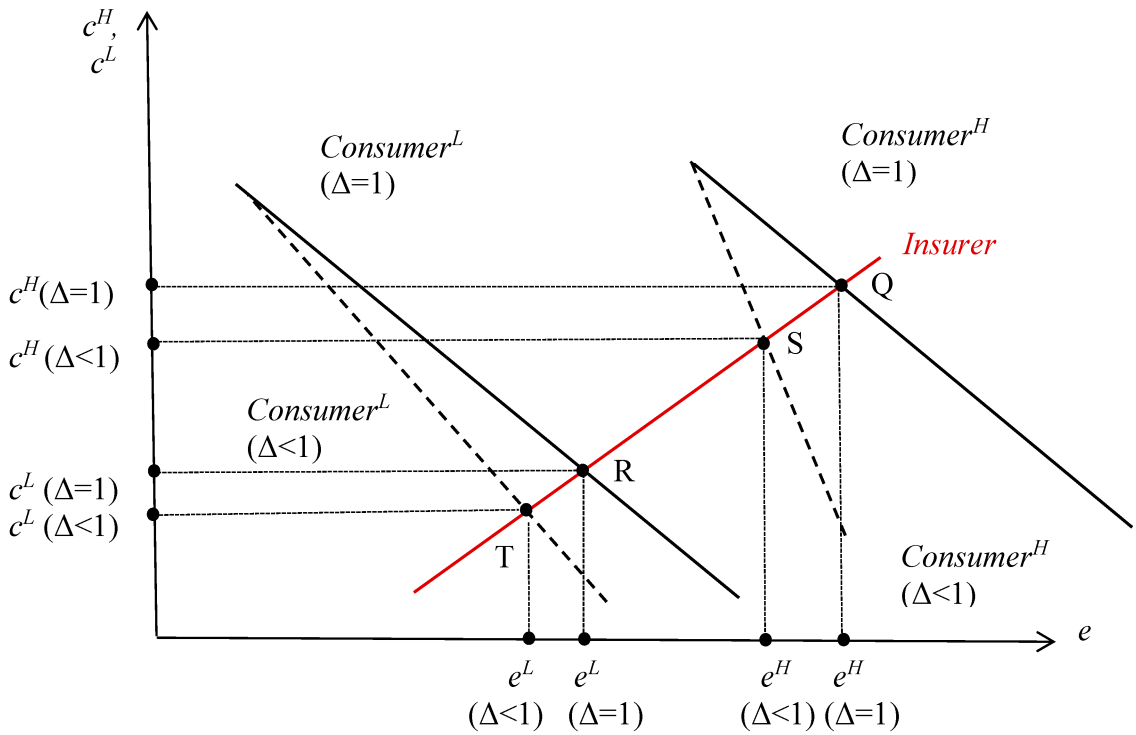

Figure 2. Reaction functions of consumers for one insurer and separating equilibria.

Nash equilibrium Q results if the insurer is confronted with a high-risk type w.r.t. both perils (Δ = 1), who spends a great deal of search effort (c^H). The resulting risk selection effort e^H is comparatively high, which is intuitive. In equilibrium R, the low-risk consumer spends little search effort. Since search effort is observable by assumption, the difference between e^H and e^L reveals the risk type to the insurer in equilibrium. Accordingly, a low-risk type is confronted with a lower selection effort e^L than the high-risk type, still with

$\Delta = 1$. Note that for the non-existence of a separating equilibrium, Q and R would have to coincide and hence cause the two type-specific consumer reaction functions to intersect. Yet, in Equation (3), the marginal benefit of searching for the high-risk type exceeds that of the low-risk type, resulting in a higher search effort throughout (as in Figure 1). This precludes an intersection of the two reaction functions and hence assures the existence of a separating equilibrium.

With $\Delta < 1$, the equilibrium for the high-risk type moves to point S, with a lower search effort as well as risk selection effort. The same holds for the low-risk type, with a transition from R to T. However, the reduction in risk selection effort is greater for the high-risk type [from e^H ($\Delta = 1$) to e^H $\Delta < 1$)] than for the low-risk one [from e^L ($\Delta = 1$) to e^L $\Delta < 1$)], which is because, as Equation (A7) in Appendix A demonstrates, a decrease in Δ causes the reaction function of the high-risk type to rotate inwards more strongly than that of the low-risk type. Therefore, the difference between the two levels [e^H ($\Delta < 1$) vs. e^L $\Delta < 1$)] diminishes (see Appendix C). If only the high-risk type were a "better" risk as a driver, the two Nash equilibria would be represented by points R and S in Figure 2, with even less of a difference between the selection effort levels e^L ($\Delta = 1$) and e^H ($\Delta < 1$).

Conclusion 1. *Introducing search effort for consumers and selection effort for insurers into insurance markets under adverse selection makes a separating equilibrium very likely. In particular, the interaction of consumers searching for maximum coverage given the market premium and risk-selecting insurers is predicted to result in a separating Nash equilibrium characterized by a high consumer search and selection effort if the insurer is confronted with a high-risk type and a low consumer search and selection effort if the insurer is confronted with a low-risk type. These differences (in risk selection effort in particular, benefiting the high-risk type) are predicted to be lowered in the case of a multi-peril policy, where both consumer types are "better risks" with regard to one peril.*

3. Projecting Results from Efforts Space into Wealth Levels Space

The Nash equilibria in Figure 2 can be projected into a conventional (W_1, W_2)-space.[11] Building on Conclusion 1, the insurer deploys relatively high risk selection effort when being confronted with a high-risk type. This results in a high premium loading that causes a reduction in optimal coverage; on the other hand, high-risk types are particularly keen to obtain high coverage. In Figure 3, the location of their optimum C^{*H} depends on the parameters appearing in Equation (2a). These efforts may even drive up the loading to such a high value that the endowment point A_0 dominates all points on the insurance line, labeled λ^H [$e^H(\Delta = 1)$], causing high-risk types to go without insurance coverage altogether. This outcome is an extreme case of a separating equilibrium, not to be studied any further. In Figure 2, both the high-risk type and the insurer are shown as exerting comparatively high search and risk selection effort, respectively. In Figure 3, this has two effects. First, the origin of the insurance line $I^H = 0$ ($\Delta = 1$) shifts far away from the endowment point A_0 because the search cost c^H has to be borne in both the loss and the no-loss state. Second, the loading $\lambda^H(e^H)$ is low, causing the insurance line $\lambda^H[e^H(\Delta = 1)]$ to run rather flat. Now, let there be a second peril where both consumer types are "better risks" ($\Delta < 1$). This has three effects. First, since both types become "better risks" with a lowered overall probability of loss, their indifference curves run steeper (dashed). Second, according to Figure 2, a $\Delta < 1$ is associated with a lower amount of consumer search c^H; therefore, the insurance line $I^H = 0$ ($\Delta < 1$) starts closer to point A_0 in Figure 3. Third, it has a steeper slope since the insurer's selection effort is also lower, resulting in reduced proportional loading $\lambda^H[e^H(\Delta < 1)$. In all cases, the high-risk type benefits from an increase in insurance coverage, indicated by the transition from $C^{*H}(\Delta = 1)$ to $C^{*H}(\Delta < 1)$.

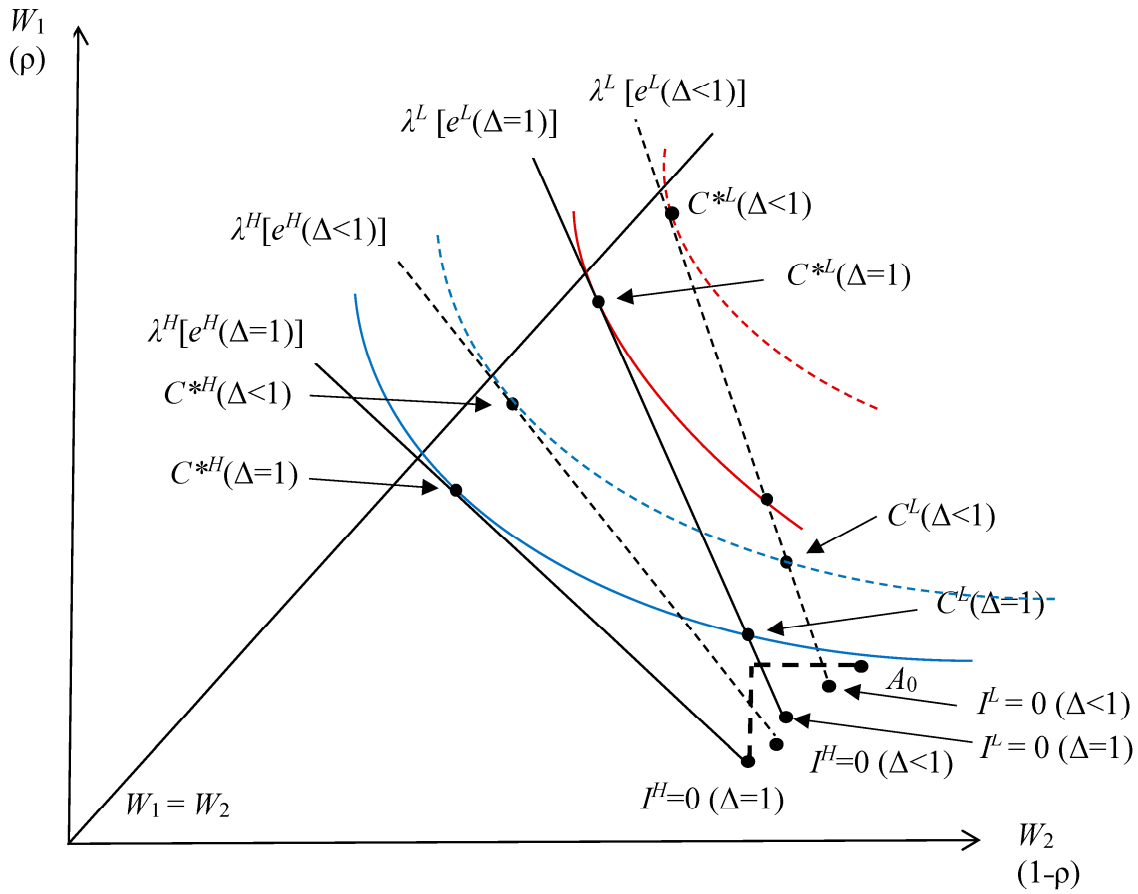

Figure 3. Projecting Nash equilibria into (W_1, W_2)—space.

Turning to the low-risk type, Figure 2 indicates that the multi-peril policy is associated with a reduced amount of consumer search, as well. In the particular case shown, this reduction is less marked than for the high-risk type. Accordingly, the origin of the pertinent insurance line $I^L = 0$ (Δ = 1) from the endowment point A_0 in Figure 3 is less marked. Also, its slope does not increase as much [from λ^L [e^L(Δ = 1) to λ^L [e^L(Δ < 1) in Figure 3] as for the high-risk type because the reduction in the insurer's selection effort is not as great. Now, the consumer optimum of the low-risk type moves from C^{*L}(Δ = 1) to C^{*L}(Δ < 1) in principle; however, the relaxation of the rationing constraint permits it to shift from C^L(Δ = 1) to C^L(Δ < 1) only. Still, the existence of a multi-peril policy where both consumer types are "better risk" w.r.t. the second peril allows both of them to attain an increased amount of coverage, resulting in a Pareto improvement.

Conclusion 2. *Introducing two-peril (or even multi-peril) policies into insurance markets under adverse selection can lead to a Pareto improvement over single-peril policies. The projection of Nash equilibria from the effort space into the wealth level space shows that due to a reduced consumer search effort and the insurer's risk selection effort, both risk types attain a higher degree of insurance coverage if they are "better risks" w.r.t. one peril and if a two-peril policy is available. In addition to this Pareto improvement, the high-risk types benefit from a reduced risk selection to an even greater extent than the low-risk types.*

4. Discussion and Conclusions

The point of departure of this paper is an intuition: if consumers who are a "better risk" with regard to at least one peril were able to purchase a multi-peril policy, this could possibly mitigate the adverse selection problem for the insurer. In pursuing this intuition, a two-risk model is developed in which both high-risk and low-risk types deploy costly

search effort to find the policy offering as much coverage as possible for the given premium. In turn, the insurance company deploys costly effort designed to stave off high-risk types while attracting low-risk ones. If it exists, the separating Nash equilibrium in the effort space is associated with a high amount of consumer search effort combined with a high amount of risk selection effort if the uninformed insurer is confronted with a high-risk type. It combines a low amount of consumer search and selection effort if the company is dealing with a low-risk type. In addition, if both risk types are "better risks" with regard to one peril, the Nash equilibrium shifts towards lower consumer search and risk selection efforts. Interestingly, the degree of reduction is especially marked if the insurer is confronted with a high-risk type. The reason is that the same reduction in the probability of loss for the second peril has a higher impact on the high-risk Nash equilibrium because of the higher overall probability of loss characterizing the high-risk type (Conclusion 1).

Next, these findings are projected into the more familiar wealth level space, which permits the depiction of the rationing constraint imposed on low-risk types for ensuring the sustainability of separating contracts. Here, the fact that both risk types are "better risks" with regard to one peril has three consequences. First, the slope of their indifference curves increases; second, the origin of their insurance lines does not move as far away from the endowment point, reflecting less search effort; and third, the insurance lines have a higher slope, reflecting a reduced loading due to less risk selection effort. Most importantly, Conclusion 1 is confirmed in that the second and the third changes are more marked if the insurer is confronted with a high-risk type, who therefore benefits to a particularly high degree from the existence of a multi-peril policy. However, the low-risk type benefits as well thanks to a relaxation of the rationing constraint; therefore, multi-peril policies hold the promise of a Pareto improvement (Conclusion 2).

There are several limitations to this analysis. First, using expected utility as the criterion governing choices under risk has been met with criticism, in particular from empiricists. Concerning the demand for insurance, however, Bleichrodt and Schmidt (2009) have found that most predictions tend to carry over from expected utility to its main alternatives, confirming Machina's (1995) robustness result. Of course, certain rationality assumptions are necessary for the results obtained, as in the prominent literature on adverse selection. Second, risk types may differ not only with respect to their probability of loss (and size of loss) but also with regard to other characteristics, in particular risk aversion. Arguably, individuals become higher risks with age—at least in health and life insurance, as shown by Halek and Eisenhauer (2001). Risk aversion thus correlates positively with high-risk status. Yet, this would accentuate the finding that high-risk types, who feature a high value of $v^{H''}(.)$ in Equation (A7), would be exposed to even less risk selection effort than predicted in the context of a multi-peril policy. Third, the one-period model developed here cannot accommodate learning on both sides of the market. Experimental evidence suggests that consumers re-estimate the probability of loss in the wake of a loss occurrence (Dumm et al. 2020). Insurers update their risk assessment as well, applying experience rating (e.g., in auto insurance). But this is unlikely to modify the findings presented here as long as consumers continue to be "better risks" with regard to at least one additional peril covered by a multi-peril policy. Fourth, as is known from the classical Rothschild and Stiglitz (1976) model, a separating equilibrium may not always exist and depends on the proportion of high- and low-risk types within the population. Another limitation is that—unlike the contribution of Crocker and Snow (2011)—the present analysis does not contain a proof that the Nash equilibrium derived in the effort space always exists and is sustainable; the pertinent condition is imposed as part of the transition to the wealth level space.

Finally, the analysis of this paper is purely theoretical, thus lacking supporting empirical evidence. Yet, statistical inference is confronted with the challenge that direct measurements of consumer search effort and insurer selection effort are hardly available. This caused Rowell and Zweifel (2024) to turn to Structural Equation Modelling, which allows the two decision variables to be imperfectly reflected by multiple indicators. In the

case of the Australian auto insurance market, high-risk types are indeed found to deploy more search effort and to be confronted with more selection effort by insurers than low-risk ones, as predicted in the present work.

The core conclusion is that a multi-peril policy may have the potential for a Pareto improvement on the side of consumers, regardless of their risk type, as long as they present a "better risk" with regard to another covered peril. If this finding should hold true, the European regulation mandating the separation of life from nonlife insurance (EIOPA 2020) may not necessarily have a favorable benefit–cost ratio. Whereas social insurance schemes almost by definition concern themselves with one risk exclusively, the separation of lines in private insurance markets dates from an era when regulators were afraid that reserves accumulated in life insurance would be misspent on nonlife insurance. Theoretically, any two classes of insurance policies could be combined, but under the condition of representing i.i.d. events, that is, identically and independently distributed risks to ensure that the law of large numbers still guarantees risk pooling benefits for the insurer. In the case of positively correlated risks, the effect could hinder risk pooling and then our analysis could be biased. However, available evidence of correlation between losses across the lines of insurance is limited to lines of nonlife insurance. Avanzi et al. (2015, Table 2.1) cite estimates for Australia. The Pearson correlation coefficients range from a low 0.10 for auto property damage and fire to an amazingly high 0.75 for auto property damage and homeowners. The line most closely related to health is workers' compensation, where the estimated correlation coefficient with auto property damage is zero, providing a degree of justification for the pairing in this paper. However, today's capital markets can serve as institutions providing market discipline [see e.g., Deng et al. (2017) for banks; Epermanis and Harrington (2006); Halek and Eckles (2010); Eling and Schmit (2012) for insurers]. An inefficient allocation of reserves likely would be reflected in the share price and/or in the market share of listed companies. Therefore, it may be time to reconsider regulations imposing the separation of lines which hamper the development of multi-peril policies.

These considerations give rise to two questions. First, what would be the price (reflected by the loading) of an "umbrella" policy that covers all important risks of an individual? Also, is there a (lack of) incentive on the part of insurers to launch multi-peril policies of the type discussed here rather than continue to bundle closely related risks, as, e.g., in homeowners' policies? These are questions for future research beyond the scope of this contribution.

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Appendix A. Consumers' Reaction Functions

In this Appendix, the reaction functions shown in Figure 1 are derived. Let there be an exogenous shock $d\alpha > 0$, with $d\alpha$ symbolizing one of the changes to be specified below.

In the case of Equation (2a) for example, this gives rise to the following comparative static equation (applying the implicit function theorem),

$$\frac{\partial^2 EU^H}{\partial c^{H2}} dc^H + \frac{\partial^2 EU^H}{\partial c^H \partial \alpha} d\alpha = 0,$$

which can be rewritten to become

$$\frac{dc^H}{d\alpha} = -\frac{\frac{\partial^2 EU^H}{\partial c^H \partial \alpha}}{\frac{\partial^2 EU^H}{\partial c^{H2}}} \tag{A1}$$

Since $\frac{\partial^2 EU^H}{\partial c^{H2}} < 0$ in a maximum, the sign of $\frac{dc^H}{d\alpha}$ is determined by the sign of the mixed second-order derivative $\frac{\partial^2 EU^H}{\partial c^H \partial \alpha}$. In deriving the predictions below, any impact on $\frac{\partial^2 EU^H}{\partial c^{H2}}$ in Equation (A1) is neglected because it must be minor (lest $\frac{\partial^2 EU^H}{\partial c^{H2}}$ changes signs, turning a maximum into a minimum). The first shock to be considered is an increase in the insurer's general selection effort such that $d\alpha := de$. Using Equation (A1), one obtains from Equation (2a),

$$\begin{aligned}\frac{dc}{de}^H \quad &\propto \frac{\partial^2 EU^H}{\partial c^H \partial e} = \rho^{H2}\Delta\left[v^{H''}\left(\frac{\partial I^H}{\partial e} + \frac{\partial J^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\left(\frac{\partial I^H}{\partial c^H} + \frac{\partial J^H}{\partial c^H}\right) + v^{H'}\left(\frac{\partial^2 I^H}{\partial c^H \partial e} + \frac{\partial^2 J^H}{\partial c^H \partial e}\right)\right] \\ &+(1-\rho^H)\left(\rho^H\Delta\right)\left[v^{H''}\left(\frac{\partial J^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\frac{\partial J^H}{\partial c^H} + v^{H'}\frac{\partial^2 J^H}{\partial c^H \partial e}\right] \\ &+\rho^H(1-\rho^H\Delta)\left[v^{H''}\left(\frac{\partial I^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\frac{\partial I^H}{\partial c^H} + v^{H'}\frac{\partial^2 I^H}{\partial c^H \partial e}\right] \\ &+(1-\rho^H)(1-\rho^H\Delta)v^{H''}\left(-\frac{\partial P^H}{\partial e}\right).\end{aligned} \tag{A2}$$

The sign of Equation (A2) can be determined as follows. While its fourth term is positive, it is dominated by the negative first three. For instance, the term in the bracket of the first one can be made comparable to the rest by factoring out $v^{H''}$ and noting that the coefficient of absolute risk aversion is defined by $RA^H = -\frac{v^{H''}}{v^{H'}} > 0$,

$$\begin{aligned}&v^{H''}\left(\frac{\partial I^H}{\partial e} + \frac{\partial J^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\left(\frac{\partial I^H}{\partial c^H} + \frac{\partial J^H}{\partial c^H}\right) + v^{H'}\left(\frac{\partial^2 I^H}{\partial c^H \partial e} + \frac{\partial^2 J^H}{\partial c^H \partial e}\right) \\ &= v^{H''}\left\{\left(\frac{\partial I^H}{\partial e} + \frac{\partial J^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\left(\frac{\partial I^H}{\partial c^H} + \frac{\partial J^H}{\partial c^H}\right) - \frac{1}{RA^H}\left(\frac{\partial^2 I^H}{\partial c^H \partial e} + \frac{\partial^2 J^H}{\partial c^H \partial e}\right)\right\} < 0.\end{aligned} \tag{A3}$$

Estimates of RA are rare, but Friedman (1974) arrives at a value of $3 \cdot 10^{-3}$, derived from health insurance choices, and Wilson and Eidman at no more than $1.2 \cdot 10^{-3}$, derived from decisions of swine producers, estimates which render the positive second term in the bracket dominant (recall that $\frac{\partial^2 I^H}{\partial c^H \partial e}$ and $\frac{\partial^2 J^H}{\partial c^H \partial e}$ are negative negative). The same argument applies to the second and third terms of Equation (A2), with RA^H taking on different values. Now Guiso and Paiella (2008) find evidence confirming the expectation that RA decreases with wealth; however, insurance coverage is designed to stabilize wealth. Therefore, the RA^H values are unlikely to differ so much as to make the terms involving $\frac{1}{RA^H}$ nondominant. Finally, all probability multipliers are in the (0,1) interval. In sum, it can be concluded that

$$\frac{dc}{de}^H < 0 \tag{A4}$$

Developments for the low-risk type are the same, with a few modifications which become evident in the transformation of the analogous first term of Equation (A2),

$$\begin{aligned}&v^{L''}\left(\frac{\partial I^L}{\partial e} + \frac{\partial J^L}{\partial e} - \frac{\partial P^L}{\partial e}\right)\left(\frac{\partial I^L}{\partial c^L} + \frac{\partial J^L}{\partial c^L}\right) + v^{L'}\left(\frac{\partial^2 I^L}{\partial c^L \partial e} + \frac{\partial^2 J^L}{\partial c^L \partial e}\right) \\ &= v^{L''}\left\{\left(\frac{\partial I^L}{\partial e} + \frac{\partial J^L}{\partial e} - \frac{\partial P^L}{\partial e}\right)\left(\frac{\partial I^L}{\partial c^L} + \frac{\partial J^L}{\partial c^L}\right) - \frac{1}{RA^L}\left(\frac{\partial^2 I^L}{\partial c^L \partial e} + \frac{\partial^2 J^L}{\partial c^L \partial e}\right)\right\} < 0.\end{aligned} \tag{A5}$$

There is no apparent reason why the parameters relating to the marginal effectiveness of consumer search effort and the impact of insurer selection effort on them should differ

between low- and high-risk types; however, RA^L is likely lower than RA^H in view of the findings of Halek and Eisenhauer (2001) that individuals become more risk-averse with age (arguably an indicator of risk)—at least in health and life insurance. On the other hand, the probability multipliers have lower values than those appearing in Equation (A2). In sum, one is led to postulate

$$\frac{dc^L}{de} = \frac{dc^H}{de} < 0, \tag{A6}$$

as drawn in Figure 1. Note, however, that this equality does not preclude a differential effect of a change in Δ. Indeed, for the high-risk type, one obtains from Equation (A2)

$$\begin{aligned} \frac{d}{d\Delta}\left[\frac{dc}{de}^H\right] \quad &\propto \rho^{H2}\left[v^{H''}\left(\frac{\partial I^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\left(\frac{\partial I^H}{\partial c^H} + \frac{\partial J^H}{\partial c^H}\right) + v^{H'}\frac{\partial^2 I^H}{\partial c^H \partial e}\left(\frac{\partial I^H}{\partial c^H} + \frac{\partial J^H}{\partial c^H}\right)\right] \\ &-\rho^{H2}\left[v^{H''}\left(\frac{\partial J^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\frac{\partial J^H}{\partial c^H} + v^{H'}\frac{\partial^2 I^H}{\partial c^H \partial e}\right] \\ &-\rho^{H2}\left[v^{H''}\left(\frac{\partial I^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\frac{\partial I^H}{\partial c^H} + v^{H'}\frac{\partial^2 I^H}{\partial c^H \partial e}\right] \\ &-\left(\rho^H - \rho^{H2}\right)\left[v^{H''}\left(-\frac{\partial P^H}{\partial e}\right)\right]. \end{aligned} \tag{A7}$$

Neglecting the fact that the utility terms depend on the value of their arguments, it can be seen that the first two terms of Equation (A8) cancel out, while the third and fourth boil down to

$$\begin{aligned} &-\rho^{H2}v^{H''}\left\{\left(\frac{\partial I^H}{\partial e} - \frac{\partial P^H}{\partial e}\right)\frac{\partial I^H}{\partial c^H} - \frac{1}{RA^H}\frac{\partial^2 I^H}{\partial c^H \partial e}\right\} \\ &-\left(\rho^H - \rho^{H2}\right)v^{H''}\left(-\frac{\partial P^H}{\partial e}\right) < 0. \end{aligned} \tag{A8}$$

The first term is negative because its bracket is negative by Equation (A3), and the second is negative since $\rho^H - \rho^{H2} > 0$ and $\frac{\partial P^H}{\partial e} > 0$. Therefore, with increasing Δ, the slope of the reaction function pertaining to the high-risk type goes towards zero. Conversely, with decreasing Δ this slope becomes more markedly negative. This holds also for the low-risk type, as shown in Figure 1, but because $\rho^H > \rho^L$, a decrease from $\Delta = 1$ causes the reaction function to rotate inwards *more strongly*.

Appendix B. The Insurer's Reaction Function

This Appendix is devoted to deriving the insurer's reaction function, shown in Figure 2. Let the optimum of Equation (5) be disturbed by a marginal increase in consumers' search effort, an observable quantity. Setting first $d\alpha := dc^H$, one obtains the solution to the comparative static equation, in analogy to Equation (A1),

$$\frac{de}{dc^H} \propto \frac{\partial^2 E\Pi}{\partial e \partial c^H} = \partial\pi/\partial e \cdot \left[-\frac{\partial I^H}{\partial c^H} - \frac{\partial J^H}{\partial c^H}\right] + \pi(e)\left[-\frac{\partial^2 I^H}{\partial e \partial c^H} - \frac{\partial^2 J^H}{\partial e \partial c^H}\right] > 0 \tag{A9}$$

because $\frac{\partial^2 I^H}{\partial e \partial c^H} = \frac{\partial^2 I^H}{\partial c^H \partial e} < 0$, $\frac{\partial^2 J^H}{\partial e \partial c^H} = \frac{\partial^2 J^H}{\partial c^H \partial e} < 0$ (recalling that the insurer's selection effort lowers the effectiveness of consumer search effort). Therefore, the reaction function has a positive slope when the insurer is interacting with a high-risk type (which is revealed in the Nash equilibrium, see Figure 2).

Next, with $d\alpha := dc^L$ one obtains

$$\frac{de}{dc^L} \propto \frac{\partial^2 E\Pi}{\partial e \partial c^L} = \partial\pi/\partial e \cdot \left[-\frac{\partial I^L}{\partial c^L} - \frac{\partial J^L}{\partial c^L}\right] + \pi(e)\left[-\frac{\partial^2 I^L}{\partial e \partial c^L} - \frac{\partial^2 J^L}{\partial e \partial c^L}\right] > 0; \tag{A10}$$

and thus the reaction function for a low-risk type has a positive slope as well. Since nothing beyond their signs can be said about the bracketed terms in Equations (A9) and (A10) without further assumptions, only one reaction function is drawn in Figure 2. However, due to the type-specific consumer reaction functions derived in Appendix A, there are two Nash equilibria, giving rise to two levels of risk selection effort, e^H and e^L, respectively.

Appendix C. Changes in Insurer's Selection Effort in Response to a Lower Δ

In this Appendix, the effects of a decrease in Δ on the insurer's levels of risk selection effort, e^H and e^L, are derived. The starting point are the two Nash equilibria defined by the FOCs (where $e^H \neq e^L$ in general, as a result of the interaction with the two consumer types),

$$\frac{dEU^H}{dc^H} - \frac{dE\Pi}{de^H} = 0, \frac{dEU^L}{dc^L} - \frac{dE\Pi}{de^L} = 0. \tag{A11}$$

Recalling that the FOCs in Equations (2a) and (2b) depend on the insurer's selection effort and changing notation slightly, one obtains the two comparative static equations

$$\begin{bmatrix} \frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}} & \frac{\partial^2 EU^H}{\partial e^H \partial e^L} - \frac{\partial^2 E\Pi}{\partial e^H \partial e^L} \\ \frac{\partial^2 EU^L}{\partial e^L \partial e^H} - \frac{\partial^2 E\Pi}{\partial e^L \partial e^H} & \frac{\partial^2 EU^L}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}} \end{bmatrix} \begin{bmatrix} de^H \\ de^L \end{bmatrix} - \begin{bmatrix} \frac{\partial^2 EU^H}{\partial e^H \partial \Delta} - \frac{\partial^2 E\Pi}{\partial e^H \partial \Delta} \\ \frac{\partial^2 EU^L}{\partial e^L \partial \Delta} - \frac{\partial^2 E\Pi}{\partial e^L \partial \Delta} \end{bmatrix} d\Delta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{A12}$$

With $\Omega > 0$ denoting the determinant of the negative definite Hessian of rank two (both agents are maximizing), Cramer's rule yields the solutions

$$\frac{de^H}{d\Delta} = \frac{1}{\Omega} \begin{vmatrix} \left(\frac{\partial^2 EU^H}{\partial e^H \partial \Delta} - \frac{\partial^2 E\Pi}{\partial e^H \partial \Delta}\right) & \frac{\partial^2 EU^H}{\partial e^H \partial e^L} - \frac{\partial^2 E\Pi}{\partial e^H \partial e^L} \\ \left(\frac{\partial^2 EU^L}{\partial e^L \partial \Delta} - \frac{\partial^2 E\Pi}{\partial e^L \partial \Delta}\right) & \frac{\partial^2 EU^H}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}} \end{vmatrix} \tag{A13}$$

and

$$\frac{de^L}{d\Delta} = \frac{1}{\Omega} \begin{vmatrix} \frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}} & \left(\frac{\partial^2 EU^H}{\partial e^H \partial \Delta} - \frac{\partial^2 E\Pi}{\partial e^H \partial \Delta}\right) \\ \frac{\partial^2 EU^L}{\partial e^L \partial e^H} - \frac{\partial^2 E\Pi}{\partial e^L \partial e^H} & \left(\frac{\partial^2 EU^L}{\partial e^L \partial \Delta} - \frac{\partial^2 E\Pi}{\partial e^L \partial \Delta}\right) \end{vmatrix} \tag{A14}$$

The signs of the terms appearing in Equations (A13) and (A14) can be determined as follows.[12]

$$\frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}} < 0, \quad \frac{\partial^2 EU^L}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}} < 0 \quad \text{(C.5)} \tag{A15}$$

in view of the negative definiteness of Ω. Differentiating now Equation (2a), one obtains

$$\begin{aligned} \frac{\partial^2 EU^H}{\partial e^{H2}} &= \rho^{H2}\Delta \cdot v^{H''}\left[W_0 + I^H + J^H - K - L - P^H(e)\right]\left(\frac{\partial^2 I^H}{\partial e^{H2}} + \frac{\partial^2 J^H}{\partial e^{H2}} - \frac{\partial^2 P^H}{\partial e^{H2}}\right) \\ &+ \left(\rho^H - \rho^{H2}\Delta\right) v^{H''}\left[W_0 + J^H - L - P^H(e)\right]\left(\frac{\partial^2 J^H}{\partial e^{H2}} - \frac{\partial^2 P^H}{\partial e^{H2}}\right) \\ &+ \left(\rho^H - \rho^{H2}\Delta\right) v^{H''}\left[W_0 + I^H - L - P^H(e)\right]\left(\frac{\partial^2 I^H}{\partial e^{H2}} - \frac{\partial^2 P^H}{\partial e^{H2}}\right) \\ &+ \left(1 - \rho^H - \rho^H\Delta + \rho^{H2}\Delta\right) v^{H''}\left[W_0 - P^H(e)\right]\left(-\frac{\partial^2 P^H}{\partial e^{H2}}\right) > 0. \end{aligned} \tag{A16}$$

Here, $\frac{\partial^2 I^H}{\partial e^{H2}} < 0$ reflects a decreasing marginal effectiveness of effort while $\frac{\partial^2 P^H}{\partial e^{H2}} > 0$ follows from Equation (1a) if $\frac{\partial^2 \lambda^H}{\partial e^2} > 0$, reflecting an increasing marginal cost. Thus, the first three terms are positive and together almost certainly dominate the negative fourth one.

Similarly, differentiating Equation (2b) yields

$$\begin{aligned} \frac{\partial^2 EU^L}{\partial e^{L2}} &= \rho^{L2}\Delta \cdot v^{L''}\left[W_0 + I^L + J^L - K - L - P^L(e)\right]\left(\frac{\partial^2 I^L}{\partial e^{L2}} + \frac{\partial^2 J^L}{\partial e^{L2}} - \frac{\partial^2 P^L}{\partial e^{L2}}\right) \\ &+ \left(\rho^L - \rho^{L2}\Delta\right) v^{L''}\left[W_0 + J^L - L - P^L(e)\right]\left(\frac{\partial^2 I^L}{\partial e^{L2}} - \frac{\partial^2 P^L}{\partial e^{L2}}\right) \\ &+ \left(\rho^L - \rho^{L2}\Delta\right) v^{L''}\left[W_0 + I^H - L - P^L(e)\right]\left(\frac{\partial^2 I^L}{\partial e^{L2}} - \frac{\partial^2 P^L}{\partial e^{L2}}\right) \\ &+ \left(1 - \rho^L - \rho^L\Delta + \rho^{L2}\Delta\right) v^{L''}\left[W_0 - P^L(e)\right]\left(-\frac{\partial^2 P^L}{\partial e^{L2}}\right) > 0. \end{aligned} \tag{A17}$$

Next, from Equation (2a), one also obtains

$$\begin{aligned}\frac{\partial^2 EU^H}{\partial e^H \partial \Delta} &= \rho^{H2} v^{H\prime}\left[W_0 + I^H + J^H - K - P^H(e)\right]\left(\frac{\partial I^H}{\partial e^H} + \frac{\partial J^H}{\partial e^H} - \frac{\partial P^H}{\partial e^H}\right)\\ &+ \left(\rho^H - \rho^{H2}\right) v^{H\prime}\left[W_0 + J^H - K - P^H(e)\right]\left(\frac{\partial J^H}{\partial e^H} - \frac{\partial P^H}{\partial e^H}\right)\\ &+ \left(\rho^H - \rho^{H2}\right) v^{H\prime}\left[W_0 + I^H - L - P^H(e)\right]\left(\frac{\partial I^H}{\partial e^H} - \frac{\partial P^H}{\partial e^H}\right)\\ &+ \left(-\rho^H + \rho^{H2}\right) v^{H\prime}\left[W_0 - P^H(e)\right]\left(-\frac{\partial P^H}{\partial e^H}\right) \qquad < 0.\end{aligned} \tag{A18}$$

With $\rho^H > \rho^{H2}$, the first three terms are negative, and together they almost certainly dominate the positive fourth. From Equation (2b), one similarly obtains

$$\begin{aligned}\frac{\partial^2 EU^L}{\partial e^L \partial \Delta} &= \rho^{L2} \cdot v^{L\prime}\left[W_0 + I^L + J^L - K - L - P^L(e)\right]\left(\frac{\partial I^L}{\partial e^L} + \frac{\partial J^L}{\partial e^L} - \frac{\partial P^L}{\partial e^L}\right)\\ &+ \left(\rho^L - \rho^{L2}\right) v^{L\prime}\left[W_0 + J^L - K - P^L(e)\right]\left(\frac{\partial J^L}{\partial e^L} - \frac{\partial P^L}{\partial e^L}\right)\\ &+ \left(\rho^L - \rho^{L2}\right) v^{H\prime}\left[W_0 + I^L - L - P^L(e)\right]\left(\frac{\partial I^L}{\partial e^L} - \frac{\partial P^L}{\partial e^L}\right)\\ &+ \left(-\rho^L + \rho^{L2}\right) v^{L\prime}\left[W_0 - P^L(e)\right]\left(-\frac{\partial P^L}{\partial e^L}\right) < 0\end{aligned} \tag{A19}$$

since once again the fourth term almost certainly is dominated by the sum of the three first ones.

Next, from Equation (5) one has, noting that $\frac{\partial^2 \pi}{\partial e^H \partial e^L} = \frac{\partial^2 \pi}{\partial e^L \partial e^H} = \frac{\partial^2 P^H}{\partial e^H \partial e^L} = \frac{\partial^2 P^L}{\partial e^L \partial e^P} = \frac{\partial^2 I^H}{\partial e^H \partial e^L} = \frac{\partial^2 I^L}{\partial e^L \partial e^P} = 0$ constitute reasonable assumptions,

$$\frac{\partial^2 E\Pi}{\partial e^{H2}} = \frac{\partial^2 \pi}{\partial e^{H2}} \cdot \left[\frac{\partial P^H}{\partial e^H} - \frac{\partial I^H}{\partial e^H} - \frac{\partial J^H}{\partial e^H}\right] + \frac{\partial \pi}{\partial e^H} \cdot \left[\frac{\partial^2 P^H}{\partial e^{H2}} - \frac{\partial^2 I^H}{\partial e^{H2}} - \frac{\partial^2 J^H}{\partial e^{H2}}\right] < 0 \tag{A20}$$

and

$$\frac{\partial^2 E\Pi}{\partial e^{L2}} = \frac{\partial^2 \pi}{\partial e^{L2}} \cdot \left[\frac{\partial P^L}{\partial e^L} - \frac{\partial I^L}{\partial e^L} - \frac{\partial J^L}{\partial e^L}\right] + \frac{\partial \pi}{\partial e^L} \cdot \left[\frac{\partial^2 P^L}{\partial e^{L2}} - \frac{\partial^2 I^H}{\partial e^{H2}} - \frac{\partial^2 J^L}{\partial e^{L^2}}\right] < 0, \tag{A21}$$

respectively, in view of the negative definiteness of the Hessian pertaining to the insurer's optimum.

Moreover, Equation (5) also implies

$$\frac{\partial^2 E\Pi}{\partial e^H \partial \Delta} = \frac{\partial^2 E\Pi}{\partial e^L \partial \Delta} = 0. \tag{A22}$$

Finally,

$$\frac{\partial^2 EU^H}{\partial e^H \partial e^L} = \frac{\partial^2 EU^L}{\partial e^L \partial e^H} = 0. \tag{A23}$$

Altogether, Equation (A13) boils down to recall that $\Omega > 0$, and in view of Equation (A15)

$$\frac{de^H}{d\Delta} = \frac{1}{\Omega}\begin{vmatrix} \frac{\partial^2 EU^H}{\partial e^H \partial \Delta} < 0 & 0 \\ \frac{\partial^2 EU^L}{\partial e^L \partial \Delta} < 0 & -\left(\frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}}\right) > 0 \end{vmatrix} = \frac{1}{\Omega}\left(\frac{\partial^2 EU^H}{\partial e^H \partial \Delta}\right)\left(\frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}}\right) \tag{A24}$$

Given that $\frac{\partial^2 EU^H}{\partial e^H \partial \Delta}$ is negative and $\frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}}$ is also negative from (A15), it can be concluded that

$$\frac{de^H}{d\Delta} < 0,$$

which implies that with the change in Δ defined to be negative in Equation (A11), an increase in this negative change, i.e., a decrease in Δ away from $\Delta = 1$, is found to reduce the insurer's selection effort when confronted with a high-risk type, as shown in Figure 2.

Turning now to the low-risk type, Equation (A14) becomes

$$\frac{de^L}{d\Delta} = \frac{1}{\Omega}\begin{vmatrix} \frac{\partial^2 EU^L}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}} < 0 & -\frac{\partial^2 EU^L}{\partial e^L \partial \Delta} > 0 \\ 0 & -\frac{\partial^2 EU^L}{\partial e^L \partial \Delta} > 0 \end{vmatrix} = \frac{1}{\Omega}\left(\frac{\partial^2 EU^L}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}}\right)\left(-\frac{\partial^2 EU^L}{\partial e^L \partial \Delta}\right) \tag{A25}$$

It is known that $\frac{\partial^2 EU^L}{\partial e^L \partial \Delta}$ is negative from (A19) and $\frac{\partial^2 EU^L}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}}$ is negative from (A15), it can thus be concluded that

$$\frac{de^L}{d\Delta} < 0,$$

which implies that a *decrease* in Δ away from $\Delta = 1$ is again found to decrease the insurer's selection effort when confronted with a low-risk type, as shown in Figure 2.

The final issue is which of the two levels of risk selection effort is affected more strongly. First, comparing Equations (A16) and (A17), one sees that

$$\left|\frac{\partial^2 EU^H}{\partial e^{H2}}\right| > \left|\frac{\partial^2 EU^L}{\partial e^{L2}}\right| \text{almost certainly because} \rho^H > \rho^L. \tag{A26}$$

Second, it is difficult to discern any systematic difference in the terms appearing in Equations (A20) and (A21) since the insurer's selection effort does not have a differential effect on the share of high-risk types π ex ante, resulting in

$$\frac{\partial^2 E\Pi}{\partial e^{H2}} \cong \frac{\partial^2 E\Pi}{\partial e^{L2}} \tag{A27}$$

as a reasonable guess and hence

$$\left|\frac{\partial^2 EU^H}{\partial e^{H2}} - \frac{\partial^2 E\Pi}{\partial e^{H2}}\right| > \left|\frac{\partial^2 EU^L}{\partial e^{L2}} - \frac{\partial^2 E\Pi}{\partial e^{L2}}\right| \tag{A28}$$

Third, Equations (A18) and (A19) imply

$$\left|\frac{\partial^2 EU^H}{\partial e^H \partial \Delta}\right| > \left|\frac{\partial^2 EU^L}{\partial e^L \partial \Delta}\right| \text{almost certainly, again because} \rho^H > \rho^L. \tag{A29}$$

Using Equations (A26) through (A29), a comparison of Equations (A24) and (A25) yields

$$\left|\frac{de^H}{d\Delta}\right| > \left|\frac{de^L}{d\Delta}\right|, \tag{A30}$$

as shown in Figure 2.

Notes

1 Among the ten leading US insurance companies, all have at least two lines of business (typically, auto and homeowners'); Geico (owned by Berkshire Hathaway) even features no fewer than 12. (https://www.thetruthaboutinsurance.com/ and the pertinent company websites, accessed on 26 August 2023).

2 (e.g., https://insuranceadviser.net/consumer-advice, https://car-advice/a26553500/how-to-reduce-car-insurance/, accessed on 27 August 2023).

3 See e.g., https://bizfluent.com/about-6629092-role-marketing-insurance.html, accessed on 12 May 2024.

4 Note that prices are the outcome of the Nash equilibria to be determined below.

5 It is assumed that the common value of $\Delta < 1$, (which is public information) leaves their ordering unchanged in the sense of a single-crossing property. If only the high-risk types were to benefit from a $\Delta < 1$, this information would be private, and a consumer showing interest in covering a second peril would be classified as a high-risk type, rendering a two-peril policy unattractive to begin with.

6 Selection effort e is set at a common value for both high- and low-risk types, reflecting the assumption that the insurer cannot distinguish between them prior to the establishment of the separating Nash equilibrium.

[7] Note that the two losses are assumed to have consequences in terms of wealth levels only, obviating any need to introduce state-dependent VNM utility functions.

[8] The condition for an interior optimum is satisfied because the first four terms of Equations (2a) and (2b) are positive, and as a result, they will balance the marginal cost of search of one. However, it is important to note that the sum of the positive terms cannot exceed one unless the optimal search effort is unrealistically high, a restriction used in Equation (A3) of Appendix A.

[9] The derivation of the reaction functions displayed in Figure 1 is relegated to Appendix A. As argued following Equation (A5) in Appendix A, there is no reason for them to have differing slopes. Also note that "more marked" in the text below Equation (A7) means that a decrease in Δ causes the reaction function of the high-risk type to rotate inwards more strongly than the one of the low-risk type.

[10] Note that the reaction functions in Figure 1 are drawn as straight lines here as an example because nothing can be said about the third derivatives of the functions $I^H(c^H, e), J^H(c^H, e)$, and $I^L(c^L, e), J^L(c^L, e)$.

[11] For simplicity, the insurance lines pertaining to fair premiums are not shown in Figure 3 since the insurer's risk selection effort calls for a loading at any rate.

[12] Note that a definite matrix can be diagonalized so that its characteristic roots show up in the diagonal. Their product equals the determinant of the matrix. If these diagonal elements are all negative, one obtains a negative determinant for a negative definite matrix of rank 1, a positive one for a matrix of rank two, a negative one for a matrix of rank 3, etc.

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