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Development of the Black–Scholes Model for Determining Insurance Premiums to Mitigate the Risk of Disaster Losses Using the Principles of Mutual Cooperation and Regional Economic Growth

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Abstract: The frequency and economic damage of natural disasters have increased globally over the last two decades due to climate change. This increase has an impact on the disaster insurance field, particularly in the calculation of premiums. Many regions have a shortcoming in employing insurance because the premium is too high compared with their budget allocation. As one of the solutions, the premium calculation can be developed by applying the cross-subsidies mechanism based on economic growth. Therefore, this research aims to develop premium models of natural disaster insurance that uniquely involve two new variables of an insured region: cross-subsidies and the economic growth rate. Another novelty is the development of the Black–Scholes model, considering the two new variables, and it is used to formulate the premium model. Following the modeling process, this study uses the model to estimate the premiums for natural disaster insurance in each province of Indonesia. The estimation results show that all new variables involved in the model novelties significantly affect the premiums. This research can be used by insurance companies to determine the premium of natural disaster insurance, which involves cross-subsidies and economic growth.

Keywords: natural disaster insurance; premium; cross-subsidies; economic growth; Black–Scholes model

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1. Introduction

An intriguing area of study within actuarial science is mathematical modelling as it relates to disaster insurance. The use of claims reserves, premiums, claims volume, and claim magnitude, which insurance companies are obligated to prepare, can be achieved through the utilization of mathematical modelling (Kalfin et al. 2021; Joyette et al. 2015; Peng et al. 2014; Skees et al. 2008; Linnerooth-Bayer and Amendola 2000). Insurance participants who are exposed to the risk of loss as a result of natural disasters are obligated to pay a premium that is computed using a mathematical model for determining disaster insurance premiums (Ermolieva et al. 2017; Paudel et al. 2015; Philippi and Schiller 2024; Reijnen et al. 2005; Ismail 2016; Tao et al. 2009). When developing insurance premium models, it is customary to account for various risk factors associated with natural disasters, including claim frequency and severity. Insurance companies can establish competitive and suitable premium rates and effectively manage adequate claims reserves by considering the aforementioned factors (Johny et al. 2021; Sukono et al. 2022). In order to mitigate the potential for claim failure, which occurs when an insurance company is unable to remit claims submitted by policyholders, it is imperative that the model used to calculate disaster insurance premiums is precisely established (Chen et al. 2012; Subartini et al. 2018; Supian and Mamat 2022). Sewu et al. (2022) and Kunreuther (1996) list a few examples of the factors that can contribute to the possibility of claim failure: modelling errors, extreme natural disasters, and claims errors. According to Jametti and von Ungern-Sternberg (2004) and Kousky and Michel-Kerjan (2017), the estimated frequency and severity of claims may not be accurate if the model used to represent them does not match the data's properties. This phenomenon may result in insurance companies establishing excessively high or low premiums, thereby reducing their profitability or competitiveness. Extreme natural disasters, on the other hand, transpire infrequently but have a profound effect on the losses incurred by insurance participants (Kunreuther 2015; Kunreuther and Lyster 2016; Chen and Yang 2023). Pandemics, earthquakes, tsunamis, floods, and fires are all instances of extreme events or natural disasters. These incidents may increase claims to the point where the insurance company's ability to pay them is exceeded.

Prior studies in the field of disaster insurance have devised models to ascertain the cost of insurance premiums. Picard (2008) devised a cross-subsidy model to ascertain the premiums for disaster insurance. The devised model distinguishes between locations with high and low potential for natural disasters. Subsidies will be allocated to regions exhibiting a high propensity for disasters, whereas less susceptible areas will be subjected to taxation. Additionally, the study conducted by Subartini et al. (2018) employed the Tsukamoto method, which is a fuzzy inference system (FIS) technique, to ascertain the insurance premium amounts. This study ascertains the most advantageous flood disaster insurance premiums for communities along river coasts. Sukono et al. (2022) devised a collective risk model-based framework to ascertain insurance premiums for natural disasters. Using the pure premium principle, the devised model incorporates the distribution level of natural disaster risks in Indonesia. In addition, Supian and Mamat (2022) constructed a predictive model for natural disaster insurance premiums through the integration of Picard's (2008) findings. The devised model implements a cross-subsidy system and a collective risk model approach in determining insurance premiums. Based on previous research, there are gaps in the models that have been developed. In the insurance premium determination model from the research results (Sukono et al. 2022; Subartini et al. 2018), the model focuses on collective risk and has not considered the natural disaster potential index and economic growth rate. While in other research findings (Supian and Mamat 2022; Picard 2008), the insurance premium determination model has considered collective risk and the natural disaster potential index. However, the insurance premium determination model has not considered the economic growth rate. In the research by Chen and Yang (2023), they used the Black–Scholes model in determining insurance premiums. However, in determining insurance premiums, they focus on environmental pollution. Thus, the insurance premium determination model has not considered collective risk, the disaster potential index, and the economic growth index.

Insurance companies must choose a model that suits the characteristics of their claims data, such as distribution, dependency, and heterogeneity. Insurance companies must also periodically validate and evaluate models to ensure that they are still relevant to current conditions (Kalfin et al. 2022b; Cahyandari et al. 2023). Insurance companies must set premiums that reflect the risks faced by insurance participants, such as the type of natural disaster, location, or characteristics of the insured object. Insurance companies must also consider other factors influencing insurance market demand and supply, such as government regulations, market competition, or consumer preferences. In addition, insurance companies must provide sufficient claims reserves to cover claims that may occur in the future (McAneney et al. 2016; Quan and Valdez 2018). Claim reserves are funds set aside by insurance companies to pay claims that have not been resolved or reported (Suwandani and Purwono 2021; Fang et al. 2016). Insurance companies must estimate claims reserves using appropriate statistical or actuarial methods. One of the efforts that can be made to avoid the risk of default is to diversify and reinsure. Insurance companies must diversify their portfolio by offering different insurance products to different insurance participants (Lee 2020). Of course, each product provides a premium according to field needs. Diversifying the insurance product portfolio can reduce the risk of concentrating on one type of product or one group of insurance participants. Diversification of insurance product portfolios can also increase insurance company revenues and profitability (Lee 2017; Acha and Ukpong 2012). Meanwhile, reinsurance is a process in which the insurance company hands over some of the risks to the reinsurance company in exchange for a premium. Reinsurance can help insurance companies overcome the risk of failure to pay claims due to extreme events or natural disasters. Reinsurance can also help insurance companies increase the capacity and solvency of insurance companies (Powell and Sommer 2005; Aduloju and Ajemunigbohun 2017; Bressan 2018).

This research was driven by the objective of developing a model that could ascertain the premiums for natural disaster insurance, as evidenced by the problem description. In addition to considering the economic growth rate, the devised model formulation employs a cross-subsidy system and a collective risk coefficient. The devised model is novel in that it incorporates the rate of economic expansion in each region when calculating insurance premiums. According to a study by Lima and Barbosa (2019), regions prone to frequent natural disasters will observe a reduction in their economic development. This finding forms the foundation of this innovation. Disproportionate impacts will result from the substantial insurance premiums that must be paid in regions characterized by a high potential index for natural disasters and a sluggish economic growth rate. Consequently, subsidy provision is necessary for regions characterized by low economic growth rates and high natural disaster potential indices. Establishing insurance premiums that are affordable and commensurate with the rate of economic expansion in each region is the objective of devising a model for calculating disaster insurance premiums. Efficient insurance claim reserves can be generated from the insurance premiums through the implementation of the developed model. In addition, historical data from natural disasters are utilized to simulate the developed model. The anticipated outcome of the conducted research is that the findings may furnish the government with a comprehensive outline of policy recommendations pertaining to the implementation of the natural disaster insurance system. Insurance companies, meanwhile, can be consulted for guidance on how to calculate premiums for natural disaster insurance.

2. Materials and Methods

2.1. Materials

The model employed in this study was finalized using sample data pertaining to the frequency of occurrences, incurred losses, and the natural disaster potential index. Aside from that, the economic development index for each region was incorporated into the data. Data on the frequency of occurrence, losses incurred, and the index of potential natural disasters were obtained through the National Disaster Management Agency in Indonesia through the websites http://dibi.bnpb.go.id and http://bnpb.cloud (accessed on 1 May 2024). In addition, the data considered the economic growth index in each region in Indonesia as obtained through the website https://www.bps.go.id (accessed on 1 May 2024). The focal research entities for the purpose of simulating the developed model were the thirty-four provinces of Indonesia. The utilized data pertaining to the frequency of disaster events and losses span the years 2000 to 2019. The collected data were then checked and analyzed. The data analysis was carried out in the form of modeling the frequency data of natural disasters using compound Poisson with jump processes and modeling disaster loss data using normal distribution. In contrast, the data utilized for the calamity potential index and economic development in each province was limited to the year 2019. Using these sample values, the simulation of determining natural disaster insurance premiums was constructed. The purpose of conducting the simulation was to evaluate the applicability of the developed model to natural disaster data in Indonesia.

2.2. Method

A concise description of the research methodologies employed to model the premium of natural disaster insurance is provided, considering the economic growth rate and the cross-subsidy mechanism. The approaches consist of the Black–Scholes model and the compound Poisson process with jumping, normal distribution models, and collective risk models. In determining insurance premiums, there are several factors that affect the amount of premiums. In the insurance premium determination model that was developed, the variables of concern are the frequency of occurrence, economic losses, and the potential disaster index.

2.2.1. Modelling Frequencies of Disaster Events

Several methods and statistical techniques that are tailored to the data's characteristics and intended use can be employed to model disaster event frequency data. Compound Poisson with jump processes were utilized to model it in this research. Capturing random jumps in natural disasters, where the frequency fluctuates arbitrarily between two jumps, is an advantage of the jumping processes method. Equation (1) represents compound Poisson with jump processes $(Y_t)_{t\geq 0}$ via random addition (Gikhman and Skorokhod 2004; Berger 1993):

$$Y_t = Z_1 + Z_2 + \cdots + Z_{N_t} = \sum_{k=1}^{N_t} Z_k : \ t \geq 0, \quad (1)$$

where N_t represents homogeneous Poisson process with intensity $\lambda > 0$, and $(Z_i)_{i=1,2,3,\ldots}$ represents random variable sequences that are independent and identically distributed (i.i.d.). It represents the economic loss jump of i-th natural disasters. The expectation of the compound Poisson process $(Y_t)_{t\geq 0}$ for a given t can be determined by multiplying the mean jump size by the average number of jump times, as given in Equation (2):

$$\mathbb{E}(Y_t) = \mathbb{E}(N_t)\mathbb{E}(Z) = \lambda t \mathbb{E}(Z), \quad (2)$$

where $\mathbb{E}$ represents expectation under the same probability measure. Meanwhile, the variance of Y_t for fixed t is the multiplication between the average number of jump times and the second moment of the size. It is mathematically given in Equation (3) as follows (Osaki 1992):

$$\mathrm{Var}(Y_t) = \mathbb{E}(N_t)\mathbb{E}\left(Z^2\right) = \lambda t \mathbb{E}\left(Z^2\right). \quad (3)$$

The advantage of the jumping processes method is that it can capture random event jumps. While in previous studies, in determining insurance premiums, the standard Poisson process was still used, which only had a jump size of +1 and a constant path between the two jumps. Compound Poisson with jump processes can be used for those with randomly sized jumps. Compound Poisson with jump processes considers the jump process that has a random size, while the standard Poisson process is limited to developing models whose jumps are constant in size. Based on these circumstances, in determining natural disaster insurance premiums, compound Poisson with jump processes is used when the frequency of natural disaster events has a random size between the two jumps and, in accordance with the model developed, by considering random event data in each of its data jumps.

2.2.2. Modeling Disaster Loss

In this study, disaster loss modeling uses a normal distribution. This refers to the Black–Scholes model, for the form of data used is normally distributed. Estimation of the risk of natural disaster losses in this study was deducted by calculating the average value and standard deviation of the available loss data (Li et al. 2022b). In the estimation carried out, the average value indicated the expected level of loss, while the standard deviation indicated how much variation in losses may occur. Using a normal distribution, we calculated the probability of natural disaster losses falling within a certain range (Shi et al. 2020). Determination of the expected value and variance of the normal distribution for natural disaster losses is formulated as Equations (4) and (5) (Bryc 1995; Luis 2015):

$$\mathbb{E}(X) = \lambda_X \tag{4}$$

and

$$\mathrm{Var}(X) = \sigma_X^2, \tag{5}$$

where X is random variable representing economic loss of the natural disaster.

2.2.3. Collective Risk Model

A collective risk model is a mathematical model used to calculate the expected number of claims from a group of mutually independent and identically distributed risks. In addition, this model considers interdependent variables, meaning that they have a relationship or dependency on each other. In the context of a collective risk model, these variables can be connected through some copulas. Copulas are mathematical functions that connect the distributions of several random variables into a joint distribution. Thus, by using copulas, we can model the dependency between the frequency of events and aggregate losses in a collective risk model. This model is often used in the insurance sector, especially life insurance, health insurance, and casualty insurance. This model consists of two components, namely frequency and severity. Frequency is the number of claims that occur in one insurance period, while severity is the number of claims per incident. Mathematically, it is formulated as follows (Dickson 2016; Ma and Ma 2013; Li et al. 2022a; Ma et al. 2017):

$$S = X_1 + X_2 + \ldots + X_{N_t} = \sum_{i=1}^{N_t} X_i, \tag{6}$$

where $(X_i)_{i=1,2,3,\ldots}$ represents i.i.d. random variables representing the economic damage of the i-th natural disasters. The expectation and the variance of collective risk, S, are formulated in Equations (7) and (8) as follows (Syuhada et al. 2024; Kalfin et al. 2022a):

$$\mathbb{E}(S) = \mathbb{E}(Y_t)\mathbb{E}(X) \tag{7}$$

and

$$\mathrm{Var}(S) = \mathbb{E}(Y_t)\mathrm{Var}(X) + \mathrm{Var}(Y)\mathbb{E}^2(X), \tag{8}$$

2.2.4. Black–Scholes Model

In determining the Black–Scholes PDE for a call option on a stock that does not pay dividends with strike K and maturity T, it is assumed that the stock price follows a geometric Brownian motion so that

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t \tag{9}$$

where W_t is the standard Brownian motion. It is assumed that the interest rate is constant so that 1 unit of currency invested in a cash account at time 0 will be worth $B_t := exp(rt)$ at time t. So, it can be stated that $C(S,t)$ is the value of a call option at time t. Furthermore, by referring to the Ito lemma, it is obtained that

$$dC(S,t) = \left(\mu S_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right) dt + \sigma S_t \frac{\partial C}{\partial S}\, dW_t \tag{10}$$

Furthermore, assume that at any time t, we hold x_t units of cash and y_t units of stock. Then P_t, the time t value of this strategy, satisfies

$$P_t = x_t B_t + y_t S_t \tag{11}$$

We choose x_t and y_t such that the strategy reflects the option value. The self-financing assumption implies that

$$dP_t = x_t dB_t + y_t dS_t \tag{12}$$

$$dP_t = rx_t B_t dt + y_t(\mu S_t dt + \sigma S_t dW_t) \tag{13}$$

$$dP_t = (rx_t dB_t + y_t \mu S_t)dt + y_t \sigma S_t dW_t \tag{14}$$

Furthermore, by paying attention to Equation (10), which corresponds to Equation (14), we obtain

$$y_t = \frac{\partial C}{\partial S} \tag{15}$$

$$rx_t dB_t = \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} \tag{16}$$

If we set $C_0 = P_0$, the initial value of the self-financing strategy, then it must be that $C_t = P_t$ for all t because C and P have the same dynamics. By substituting Equations (15) and (16) into Equation (11), we obtain the Black–Scholes PDE.

$$rS_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC = 0 \tag{17}$$

A mathematical model that is employed to ascertain option prices, spWecifically the right to purchase or sell an asset at a specified time and price, is the Black–Scholes model. The equations representing the Black–Scholes model are (18) and (19) as follows (Pascucci 2011; Bloch 2023; Morales-Bañuelos et al. 2022; Giudice et al. 2015; Shinde and Takale 2012):

$$C(S,t) = Se^{-\delta t}N(d_1) - Ke^{-rt}N(d_2) \tag{18}$$

$$P(S,t) = Ke^{-rt}N(-d_2) - Se^{-\delta t}N(-d_1) \tag{19}$$

where d_1 and d_2 are given in Equations (20) and (21) as follows:

$$d_1 = \frac{ln\left(\frac{Se^{-\delta t}}{Ke^{-rt}}\right) + (0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln\left(\frac{S}{K}\right) + \left(r + \delta + 0.5\sigma^2\right)t}{\sigma\sqrt{t}} \tag{20}$$

and

$$d_2 = \frac{ln\left(\frac{Se^{-\delta t}}{Ke^{-rt}}\right) + (0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln\left(\frac{S}{K}\right) + \left(r - \delta + 0.5\sigma^2\right)t}{\sigma\sqrt{t}}. \tag{21}$$

In detail, $C(S,t)$ is the call option price, $P(S,t)$ is the put option price, S is the initial stock price, K is the option strike price, μ is the deviation rate of S in a year, σ represents the standard deviation of stock movements, t represents time, δ represents the dividend rate (stock profit), $N(x)$ represents the normal cumulative distribution function, and r represents the annual risk-free interest rate. The Black–Scholes model can be applied in the insurance sector to calculate insurance premium prices by assuming that the value of insurance benefits is an option purchased by the policyholder.

3. Results

3.1. The Developed Black–Scholes Model

The equation in the Black–Scholes model is used to determine the price of put and buy options. However, determining the price of a put option is similar to determining the price of insurance premiums. Because there are several similarities between the price of the put option and the determination of the insurance premium, the price (premium) can be determined in the same way as the price of the put option. Of course, insurance premium prices are adjusted to the variables used.

The developed model was designed based on the Black–Scholes formula for put options written in Equations (19) through (21). In insurance against losses due to disasters, the insurance payment value (payout) always depends on the severity of an event that creates a risk of economic loss, which is mathematically designed as a put option as follows:

$$Payout = \begin{cases} K; & R < R_T \\ 0; & Other \end{cases}, \tag{22}$$

where K represents the insurance coverage price for the insured who experienced economic losses due to natural disasters and R_T represents the benchmark value defined as the average annual economic losses due to natural disasters. In this study, the insurance coverage price is calculated based on the average (expected) aggregate loss risk per year based on Equation (7). Hence, we approximate $K = E(S)$. Based on Equation (22), the insurance premium for economic losses due to natural disasters is calculated as follows:

$$P_{[BS]} = Ke^{-\tau t}N(-d_2) = E(S)e^{-\tau t}N(-d_2) \tag{23}$$

where $P_{[BS]}$ premiums are calculated using the Black–Scholes method and the cumulative distribution d_2 is formulated in the equation as Equation (24):

$$d_2 = \frac{\ln\left(\frac{R_0}{R_T}\right) + \left(\tau - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}. \tag{24}$$

In detail, R_0 is the number of losses from the last natural disaster, R_T is the benchmark value for losses from natural disasters, σ is the standard deviation of natural disaster risk, and τ is the unexpected risk level (loading factor). Additionally, if the model is constructed with the potential for natural disasters and cross-subsidies in mind, Equation (25) can be used to formulate the Black–Scholes model for calculating insurance premiums when cross-subsidies are considered, as indicated by Equation (23) as follows:

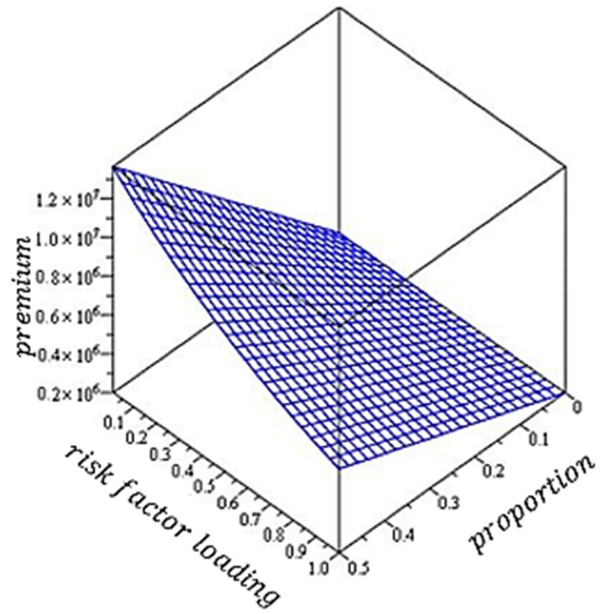
$$P^*_{[BS]i} = \begin{cases} \pi_i E(S)e^{-\tau t}N(-d_2) + \varsigma; & 0 \le \pi_i \le 0.5 \\ \pi_i E(S)e^{-\tau t}N(-d_2) - \varsigma; & 0.5 < \pi_i \le 1 \end{cases}, \tag{25}$$

where ς represents the natural disaster insurance subsidy, the "+" sign represents providing the subsidy, and the "−" sign represents receiving the subsidy. Furthermore, π_i is the disaster potential index in each region. While d_2 is formulated as follows:

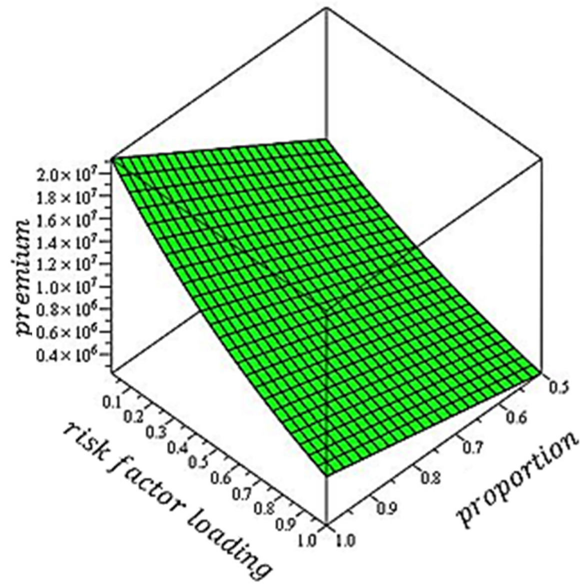
$$d_2 = \frac{\ln\left(\frac{R_0}{R_T}\right) + \left(\tau - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}. \tag{26}$$

Based on Equation (25), numerical simulations were carried out to obtain results as given in Figure 1.

Figure 1 illustrates the distribution of points for natural disaster insurance premiums. Figure 1c depicts the intersection of insurance premiums. This intersection is impacted by the cross-subsidy system, which introduces deductions for receiving subsidies and additions for giving subsidies into the premium calculation. Subsidy-providing regions are indicated by the shading of insurance premium distribution points in blue. Meanwhile, the insurance premium distribution points that are shaded green are areas that receive subsidies.

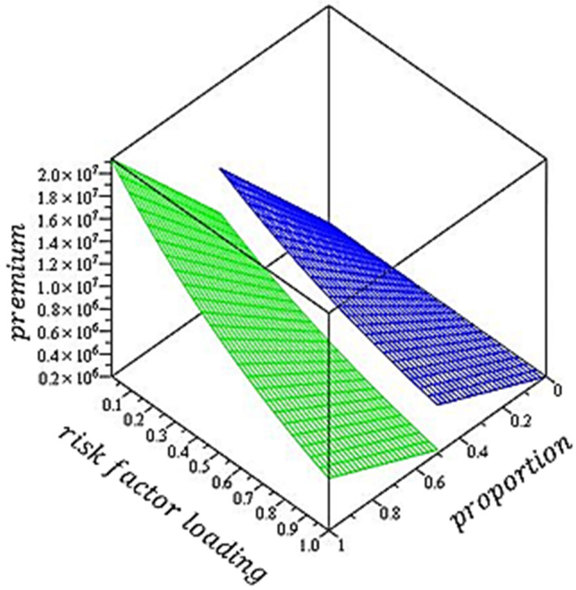


(**a**)



(**b**)

Figure 1. *Cont*.



(**c**)

Figure 1. Insurance premiums with the formula $\pi_i E(S) e^{-\tau t} N(-d_2) + \varsigma;\ 0 \leq \pi_i \leq 0.5$ (**a**), $\pi_i E(S) e^{-\tau t} N(-d_2) - \varsigma;\ 0.5 < \pi_i \leq 1$ (**b**), and combine conditions $\pi_i E(S) e^{-\tau t} N(-d_2) + \varsigma;\ 0 \leq \pi_i \leq 0.5$ and $\pi_i E(S) e^{-\tau t} N(-d_2) - \varsigma;\ 0.5 < \pi_i \leq 1$ (**c**).

Assume that each area's economic growth rate (γ_i) is considered while developing the model. Under such circumstances, Equation (27) can be used to build the Black–Scholes model for calculating insurance premiums using the rate of economic growth as follows:

$$P^*_{[BS]i} = \begin{cases} \pi_i E(S) e^{-\tau t} N(-d_2) + \gamma_i \varsigma; & 0 \leq \pi_i \leq 0.5;\ \gamma_i \geq 5\% \\ \pi_i E(S) e^{-\tau t} N(-d_2); & \begin{array}{l} 0.5 < \pi_i \leq 1;\ \gamma_i \geq 5\% \\ \text{atau } 0 \leq \pi_i \leq 0.5; \gamma_i < 5\% \end{array} \\ \pi_i E(S) e^{-\tau t} N(-d_2) - \gamma_i \varsigma; & 0.5 < \pi_i \leq 1; \gamma_i < 5\% \end{cases} \tag{27}$$

where d_2 is formulated as follows:

$$d_2 = \frac{\ln\left(\frac{R_0}{R_T}\right) + \left(\tau - \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}. \tag{28}$$

Based on Equation (27), collective risk expectations E(S) are related to natural disaster conditions. This includes factors such as the frequency and losses of natural disasters. This collective risk cannot be avoided, so it needs to be taken into account in determining insurance premiums. Furthermore, based on the premium model of natural disaster insurance in Equation (27), a numerical simulation is carried out to show the point distribution of disaster insurance premiums. The results of the numerical simulation of model Equation (27) are given in Figure 2.

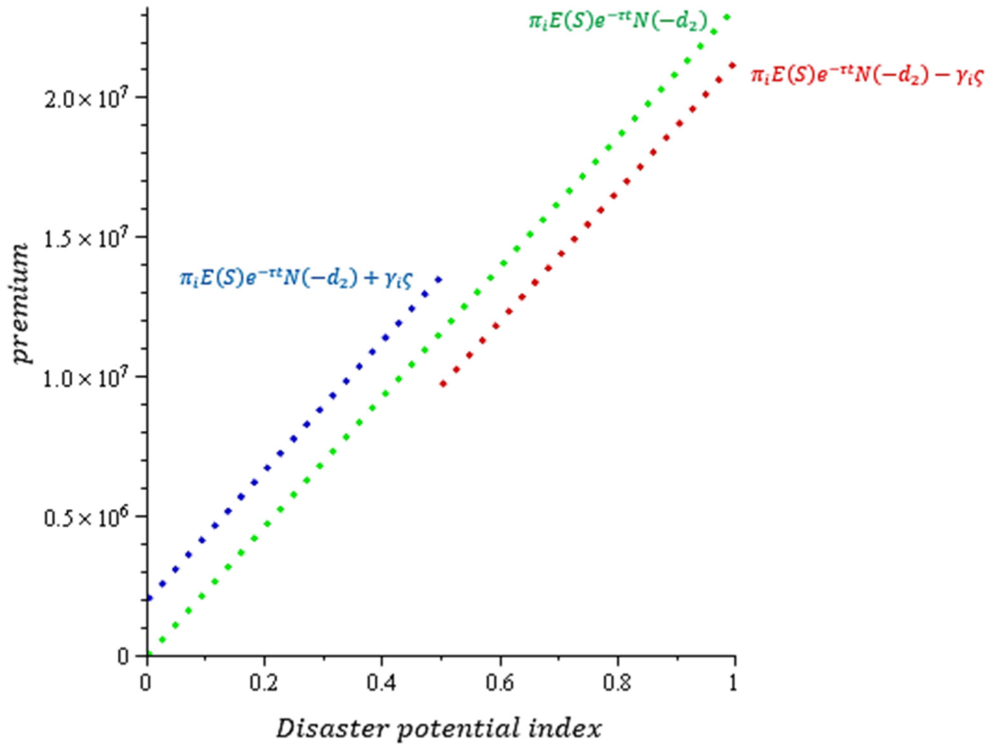


Figure 2. Spread of insurance premiums from the Black–Scholes model in Equation (27).

Three sections make up the distribution points of insurance premiums, as shown in Figure 2. The places at the top of the distribution points for natural disaster insurance premiums (blue) are considered subsidy-producing zones because they have a high natural disaster potential index with a low economic growth rate. Places with a low risk of natural disasters and slow rates of economic development or places with a high risk of natural disasters and rapid economic growth rates are the middle points of the natural disaster insurance premium distribution (green). One may alternatively characterize this situation as one in which no subsidies are given or received. Subsidized regions are those with poor rates of economic development and a high probability of natural disasters; the red areas on the distribution curve of natural disaster insurance premiums show these areas. It is evident from Figure 2 that the likelihood of natural disasters influences disaster insurance rates. An increased insurance premium for natural disasters with a higher potential index is required.

Subsequently, development is conducted in accordance with Equation (27) by incorporating the loss distribution model and transitioning from natural disaster occurrences. Three conditions must be met for the insurance premium to be calculated correctly. These are that events must follow a Poisson process with jumping events and that economic losses must be estimated using a normal distribution model that takes economic growth into account. The aforementioned are (1) $0 \leq \pi_i \leq 0.5;\ \gamma_i \geq 5\%$; (2) $0.5 < \pi_i \leq 1;\ \gamma_i < 5\%$; and (3) $0.5 < \pi_i \leq 1;\ \gamma_i \geq 5\%$ or $0 \leq \pi_i \leq 0.5;\ \gamma_i < 5\%$.

1. The conditions $0 \leq \pi_i \leq 0.5$ and $\gamma_i \geq 5\%$.

For this condition, the following equation applies:

$$P^*_{[BS]i} = \pi_i E(S)e^{-\tau t}N(-d_2) + \gamma_i\varsigma. \quad (29)$$

Employing Equation (7), as well as for the subsidy $\varsigma = \pi_i E(X)$ in Equation (29), this equation can be rewritten into Equation (30) as follows:

$$P^*_{[BS]i} = \pi_i E(Y)E(X)e^{-\tau t}N(-d_2) + \gamma_i\pi_i E(X), \quad (30)$$

where E(X) is the risk of loss due to natural disasters. Furthermore, by using Equation (2) in Equation (30), it can be reformulated into (31) as follows:

$$\begin{aligned} P^{*}_{[BS]i} &= e^{-\tau t}N(-d_2)\pi_i(\lambda_N\lambda_Z t)(\lambda_X) + \pi_i\gamma_i(\lambda_X), \\ &= \pi_i\big(\big(e^{-\tau t}N(-d_2)\big)(\lambda_N\lambda_Z t)(\lambda_X) + (\gamma_i\lambda_X)\big), \\ &= \pi_i\lambda_X\big[\big(\lambda_N\lambda_Z t e^{-\tau t}N(-d_2)\big) + \gamma_i\big]. \end{aligned} \tag{31}$$

2. The conditions $0.5 < \pi_i \leq 1$ and $\gamma_i < 5\%$.

   For this condition, the following equation applies:

$$P^{*}_{[BS]i} = \pi_i E(S)e^{-\tau t}N(-d_2) - \gamma_i\varsigma. \tag{32}$$

Employing Equation (7), as well as for the subsidy $\varsigma = \pi_i E(X)$ in Equation (32), this equation can be rewritten into Equation (33) as follows:

$$P^{*}_{[BS]i} = \pi_i E(Y)E(X)e^{-\tau t}N(-d_2) - \gamma_i\pi_i E(X). \tag{33}$$

Furthermore, by using Equation (2) in Equation (33), it can be reformulated into (34) as follows:

$$\begin{aligned} P^{*}_{[BS]i} &= e^{-\tau t}N(-d_2)\pi_i(\lambda_N\lambda_Z t)(\lambda_X) - \pi_i\gamma_i(\lambda_X), \\ &= \pi_i\big(\big(e^{-\tau t}N(-d_2)\big)(\lambda_N\lambda_Z t)(\lambda_X) - (\gamma_i\lambda_X)\big), \\ &= \pi_i\lambda_X\big[\big(\lambda_N\lambda_Z t e^{-\tau t}N(-d_2)\big) - \gamma_i\big]. \end{aligned} \tag{34}$$

3. The conditions $0.5 < \pi_i \leq 1$ and $\gamma_i \geq 5\%$ or $0 \leq \pi_i \leq 0.5$ and $\gamma_i < 5\%$.

   For this condition, the following equation applies:

$$P^{*}_{[BS]i} = \pi_i E(S)e^{-\tau t}N(-d_2). \tag{35}$$

Employing Equation (7), as well as for the subsidy $\varsigma = \pi_i E(X)$ in Equation (35), this equation can be rewritten into Equation (36) as follows:

$$P^{*}_{[BS]i} = \pi_i E(Y)E(X)e^{-\tau t}N(-d_2). \tag{36}$$

Furthermore, by using Equation (2) in Equation (36), it can be reformulated into (37) as follows:

$$P^{*}_{[BS]i} = \pi_i\lambda_N\lambda_Z t\lambda_X e^{-\tau t}N(-d_2). \tag{37}$$

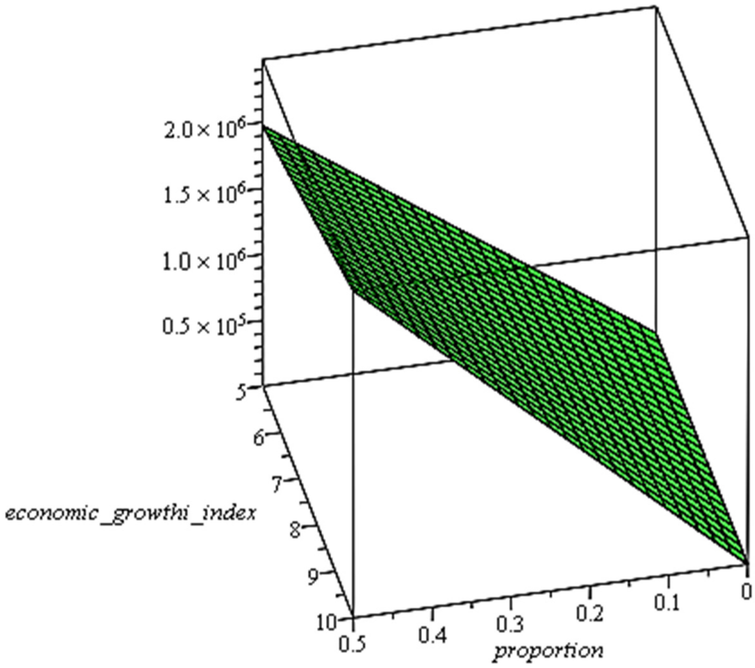
Based on these three conditions, we obtained the premium model of natural disaster insurance given in Equation (38) as follows:

$$P^{*}_{[BS]i} = \begin{cases} \pi_i\lambda_X\big[\big(\lambda_N\lambda_Z t e^{-\tau t}N(-d_2)\big) + \gamma_i\big]; & 0 \leq \pi_i \leq 0.5;\ \gamma_i \geq 5\% \\ \pi_i\lambda_N\lambda_Z t\lambda_X e^{-\tau t}N(-d_2); & \begin{matrix} 0.5 < \pi_i \leq 1;\ \gamma_i \geq 5\% \\ \text{or } 0 \leq \pi_i \leq 0.5; \gamma_i < 5\% \end{matrix} \\ \pi_i\lambda_X\big[\big(\lambda_N\lambda_Z t e^{-\tau t}N(-d_2)\big) - \gamma_i\big]; & 0.5 < \pi_i \leq 1; \gamma_i < 5\% \end{cases}, \tag{38}$$

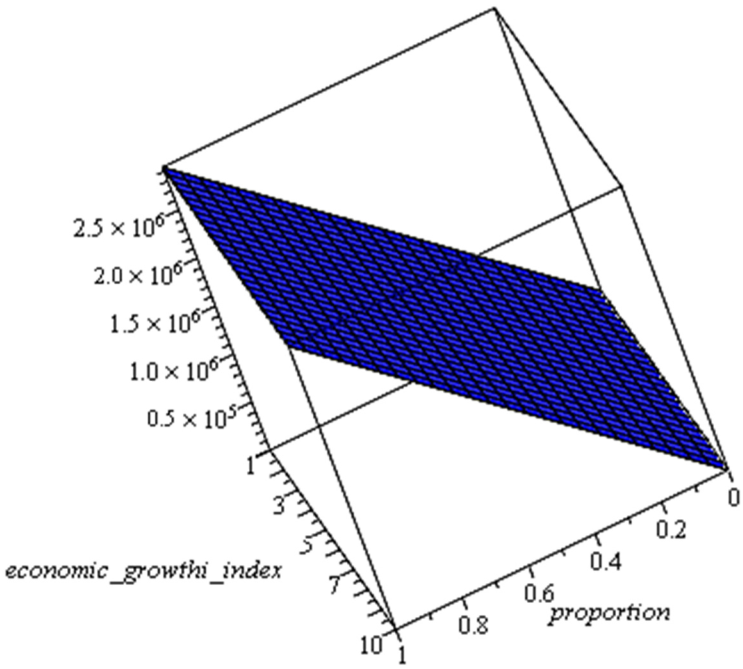
where

$$d_2 = \frac{ln\left(\frac{R_0}{R_T}\right) + \left(\tau - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}. \tag{39}$$

Based on the equation function of the premium model of natural disaster insurance in Equation (38), a numerical simulation is carried out to show the area of points that form the premium of natural disaster insurance. The numerical simulations are based on the variables of the proportion of economic growth and the natural disaster indices, and the other variables are considered constant. Figure 3 illustrates the results of the numerical simulations carried out.

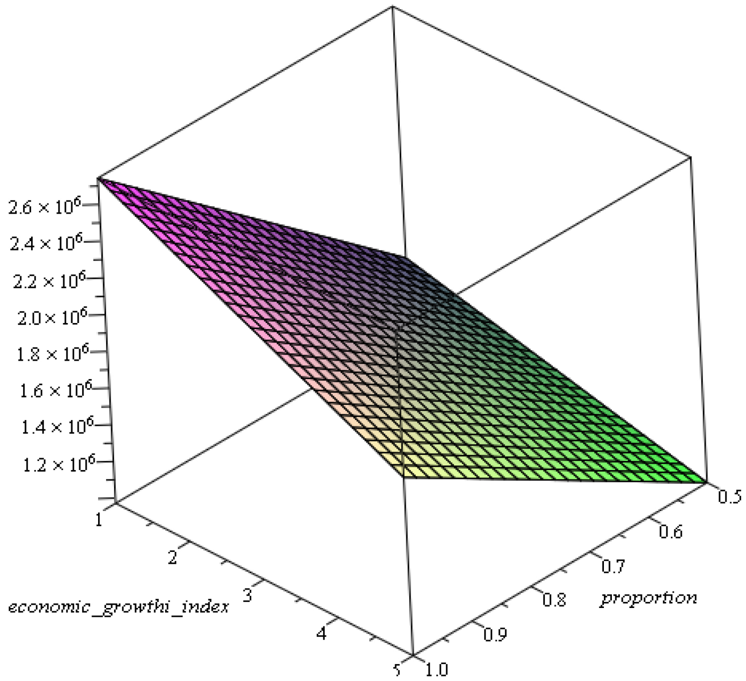


(**a**)

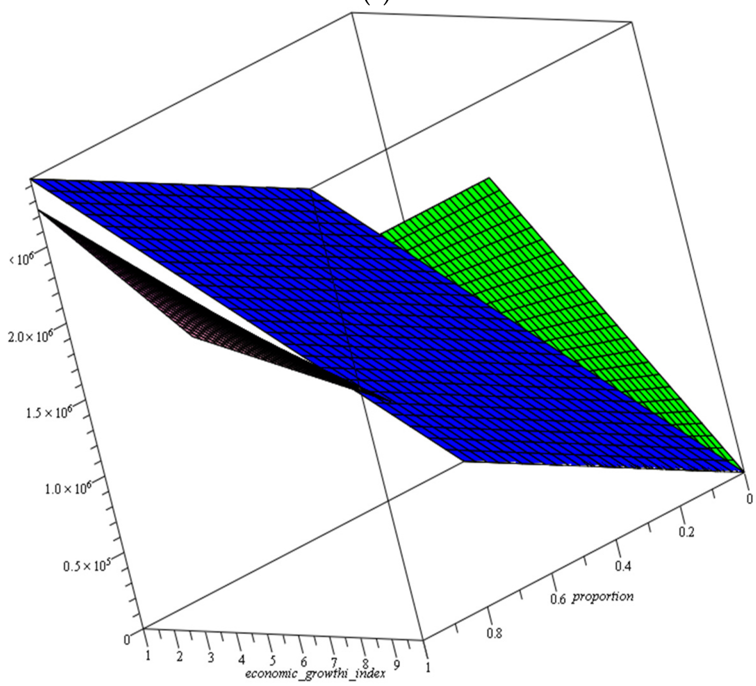


(**b**)

Figure 3. *Cont*.



(**c**)



(**d**)

Figure 3. Insurance premiums with the formula $\pi_i \lambda_X \left[\left(\lambda_N \lambda_Z t e^{-\tau t} N(-d_2) \right) + \gamma_i \right];\ 0 \le \pi_i \le 0.5;\ \gamma_i \ge 5\%$ (**a**), $\pi_i \lambda_N \lambda_Z t \lambda_X e^{-\tau t} N(-d_2);\ 0.5 < \pi_i \le 1;\ \gamma_i \ge 5\%$ *or* $0 \le \pi_i \le 0.5;\ \gamma_i < 5\%$ (**b**), $\pi_i \lambda_X \left[\left(\lambda_N \lambda_Z t e^{-\tau t} N(-d_2) \right) - \gamma_i \right];\ 0.5 < \pi_i \le 1;\ \gamma_i < 5\%$ (**c**), and combine conditions (**a**–**c**) of the insurance premium formula (**d**).

In accordance with three conditions, Figure 3 partitions the overall distribution of points for disaster insurance premiums. The magnitude of the disaster proportion index and the economic growth index exert an influence on the allocation of premium points for disaster insurance. An area's mandatory insurance premiums will increase proportionately with the disaster proportion index. A rise in the regional economic development index may have a comparable effect on mitigating the impact of the mandatory insurance premium increase.

3.2. Black–Scholes Model Simulation on the Natural Disaster and Economic Growth Rate Data

The model was simulated using the data specified in Equation (38), which comprised natural disaster occurrences and losses, natural disaster potential indices, and economic growth rates for 34 provinces in Indonesia. Table 1 provides a concise summary of the average estimation results derived from the data utilized in all provinces of Indonesia.

Table 1. Parameter estimation results for premium model of natural disaster insurance in all provinces in Indonesia.

Region	γ_i	π_i	λ_N	λ_Z	λ_X (IDR)
Provinces with a high economic growth rate and low potential disaster index					
DKI Jakarta	5.46	0.2256048	21.7	0.6	30,106,909,810.95
Central Java	5.17	0.4686533	598.2	16.0	4,530,651,960.62
Central Kalimantan	6.53	0.4676314	27.9	0.7	38,729,763,312.62
North Sulawesi	5.24	0.4914887	25.4	0.7	35,368,311,283.38
Gorontalo	7.23	0.4462761	20.4	0.5	28,353,108,132.67
Provinces with a low economic growth rate and low potential disaster index or provinces with a high economic growth rate and high potential disaster index					
Jambi	4.46	0.4885638	46.6	1.2	64,744,471,582.52
South Sumatera	4.11	0.4906782	65.6	1.8	91,197,630,272.00
Riau Island	−0.08	0.4101906	13.5	0.4	18,707,206,026.95
DI Yogyakarta	4.78	0.4965984	43.5	1.2	60,506,118,714.15
East Java	4.88	0.4735869	358.2	9.6	16,222,654,836.86
Banten	5.06	0.5457579	51.9	1.4	72,198,125,152.80
Bali	3.90	0.456108	31.8	0.9	44,283,463,877.29
West Southeast Nusa	1.10	0.4512449	45.4	1.2	63,136,821,231.57
East Southeast Nusa	3.98	0.4964927	62.0	1.7	86,228,527,891.81
West Kalimantan	5.54	0.5427625	67.6	1.8	93,974,480,554.34
East Kalimantan	10.09	0.5413529	3.3	0.1	831,739,925,812.43
North Kalimantan	4.17	0.4880352	27.5	0.7	38,291,312,893.05
Central Sulawesi	7.70	0.5108353	25.8	0.7	35,806,761,702.95
South Sulawesi	6.33	0.5620386	96.2	2.6	133,727,296,033.42
Southeast Sulawesi	6.10	0.5558012	44.5	1.2	61,821,469,972.86
West Sulawesi	5.27	0.5867065	14.0	0.4	19,437,955,538.77
Maluku	5.72	0.5657388	14.6	0.4	20,314,856,377.91
North Maluku	5.99	0.5131964	11.7	0.4	498,079,366,701.85
Papua	−16.36	0.4330964	13.6	1.4	18,853,357,354.28
Provinces with a low economic growth rate and high potential disaster index					
Aceh	3.45	0.5412119	2.3	2.7	142,496,297,300.00
North Sumatera	3.61	0.5116106	84.5	2.3	117,504,637,634.15
West Sumatera	3.14	0.5269398	82.1	2.2	114,143,189,167.33
Riau	2.51	0.5189757	33.1	0.9	46,037,265,555.57
Bengkulu	4.49	0.5708838	16.1	0.4	22,360,957,148.43
Lampung	4.18	0.5172489	43.6	1.2	60,652,270,041.48
Bangka Belitung Islands	3.95	0.5692628	10.0	3.0	13,884,254,974.09
West Java	4.30	0.5138307	391.7	10.5	5,407,552,799.76
South Kalimantan	3.26	0.5108	63.9	1.7	88,859,230,409.23
West Papua	−0.13	0.5107965	3.9	0.1	544,701,230,441.91

Table 1 maps the 34 provinces of Indonesia into three categories: First, regions with strong economic development and low risk of natural disasters, which will provide subsidies ($0 \leq \pi_i \leq 0.5$; $\gamma_i \geq 5\%$). Second, subsidies will not be given or received in places with low economic growth and low potential for natural disasters or in areas with strong economic growth potential and high potential for natural disasters ($0.5 < \pi_i \leq 1;\ \gamma_i \geq 5\%$). Third, subsidies will be given to locations ($0 \leq \pi_i \leq 0.5;\ \gamma_i < 5\%$) that have a high probability of natural disasters and poor rates of economic development. According to Table 1's requirements, up to 19 out of Indonesia's 34 provinces use a system for figuring insurance rates that neither provides nor accepts subsidies. In addition, 10 provinces have put a system in place where insurance rates are decided upon via the receipt of subsidies. The insurance rates in the other five provinces are set by means of subsidies.

The simulation using the Black–Scholes model for determining the premiums of natural disaster insurance refers to Equation (38). Meanwhile, the data makes use of Table 1, which are estimated data. The risk factor loading level used in this simulation is 1%. The premium burden incurred by the relevant party increases with the value of the risk loading factor. Therefore, the risk factor loading must be maintained so that it does not exceed the specified tolerance limit, for which determination is based on the Indonesian government's regulations which are in the range of 0.1–1%. Table 2 presents insurance premiums from 34 Indonesian provinces based on the analysis's findings.

Table 2. Estimation results for premium of natural disaster insurance in all provinces in Indonesia.

Province	Insurance Premium (IDR)	$N(-d_2)$
Insurance premiums for provinces that provide subsidies		
DKI Jakarta	105,116,814,732.06	0.7787
Central Java	15,678,312,345,661.90	0.7787
Central Kalimantan	390,515,637,170.99	0.7787
North Sulawesi	329,712,274,681.62	0.7787
Gorontalo	190,943,855,566.55	0.7787
Insurance premiums for provinces that do not provide or receive subsidies		
Jambi	1,362,646,116,638.97	0.7787
South Sumatera	4,072,973,023,288.89	0.7787
Riau Islang	31,837,484,097.23	0.7787
DI Yogyakarta	1,209,651,830,714.36	0.7787
East Java	20,368,852,873,367.90	0.7787
Banten	2,208,284,749,461.81	0.7787
Bali	446,342,810,943.86	0.7787
West Southeast Nusa	1,196,834,661,482.21	0.7787
East Southeast Nusa	3,479,677,094,766.09	0.7787
West Kalimantan	277,731,330,156.91	0.7787
East Kalimantan	4,783,848,320,785.64	0.7787
North Kalimantan	113,110,747,892.49	0.7787
Central Sulawesi	254,205,077,805.33	0.7787
South Sulawesi	14,489,547,848,051.90	0.7787
Southeast Sulawesi	1,413,365,803,622.65	0.7787
West Sulawesi	49,165,135,473.62	0.7787
Maluku	51,781,982,865.65	0.7787
North Maluku	919,685,291,617.10	0.7787
Papua	119,499,255,347.17	0.7787
Insurance premiums for provinces that receive subsidies		
Aceh	16,185,736,296,486.30	0.7787
North Sumatera	8,791,480,517,072.46	0.7787
West Sumatera	8,185,640,344,595.48	0.7787
Riau	488,918,532,061.06	0.7787
Bengkulu	5,991,611,415.18	0.7787
Lampung	1,134,911,652,300.06	0.7787
Bangka Belitung Islands	151,318,494,167.84	0.7787
West Java	8,799,426,281,983.76	0.7787
South Kalimantan	3,653,750,980,135.41	0.7787
West Papua	119,592,397,240.00	0.7787

Each Indonesian province has a different disaster insurance premium calculation, which is based on the data in Table 2. The kind and frequency of disasters that strike each province have an impact on them. The insurance company bears a greater risk of loss in the event of more frequent and severe disasters. As a result, the rates for disaster insurance will be modified based on the degree of risk in each province. In this simulation, insurance premiums for natural disasters that affect each province in Indonesia are determined using three categories: regions that provide subsidies, regions that do not provide or receive subsidies, and regions that do receive subsidies.

4. Discussion

The Black–Scholes model is applicable to the calculation of insurance premiums due to the comparable characteristics exhibited by insurance and options. A contract in which the buyer is granted the right but not the obligation to purchase or sell an asset at a predetermined price is known as an option. Insurance is a contract that grants the policyholder the right to receive insurance funds in the event of a disaster occurrence, such as a natural disaster. By employing the Black–Scholes model to calculate insurance premiums, it is anticipated that more optimal insurance premiums can be generated in consideration of field conditions. A multitude of variable factors that impact the calculation of insurance premiums are incorporated into the developed model. In developing the model, factors including the disaster potential index, the frequency of natural disaster occurrences and losses, and the economic growth rate in 34 provinces of Indonesia are taken into account. The developed model also incorporates a cross-subsidy mechanism. The insurance premium determination model's cross-subsidy system can assist regions with a low economic growth rate and a high potential risk index for natural disasters in obtaining insurance coverage at a lower cost. The cross-subsidy system reduces insurance premiums in regions with high economic growth and a low natural disaster risk potential index by utilizing a portion of insurance premiums from regions with a low natural disaster risk potential index and high economic growth.

The Black–Scholes model that has been developed has advantages over other insurance premium determination models. The developed model has considered jumping processes so that it can accommodate extreme events against natural disasters that often occur in Indonesia. In addition, the premium determination model has also implemented a cross-subsidy system, which considers the conditions in the field. The cross-subsidy system means that areas that have a high risk of natural disasters and low economic growth will receive a reduction (receive a subsidy) for the insurance premium to be paid. Furthermore, areas that have a low risk of natural disasters and have high economic growth will be taxed (provide a subsidy) for insurance premiums. Of course, this developed model can help regions with the burden of large insurance premiums due to the risk of natural disasters being high. In general, the model developed in this study has advantages, first in determining premiums considering random jumps, which denote natural disaster events in Indonesia. Second, in terms of economy, for areas with high natural disaster cases, while the economic growth rate is low, it is necessary to receive subsidies from other areas with high economic growth while the risk of natural disasters is low. The determination of natural disaster insurance premiums by considering the level of economic growth in each region has not been carried out by previous researchers. As depicted in Figure 4, it was determined, through the execution of simulations, that the insurance premiums for each province varied.

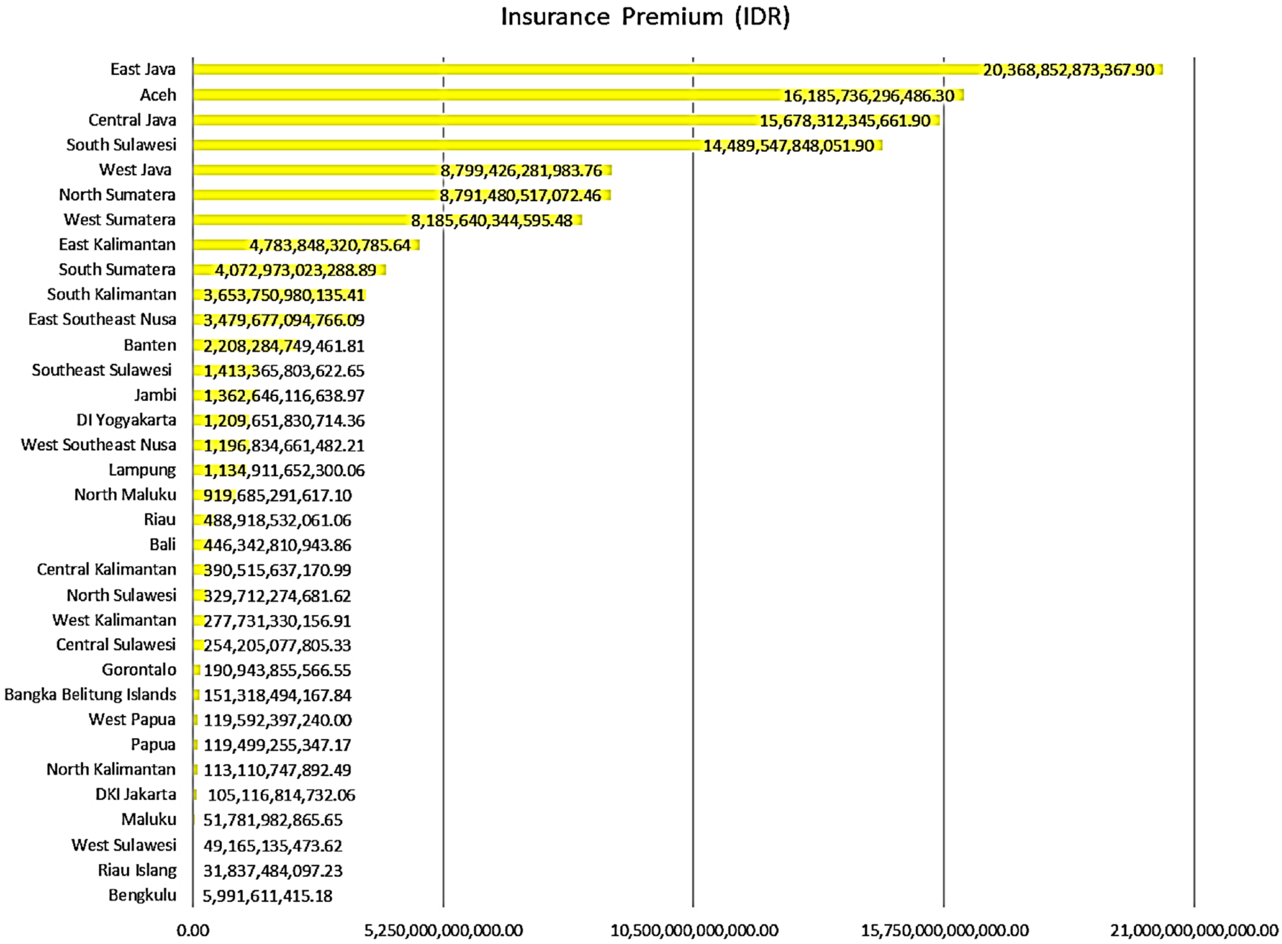


Figure 4. Insurance premiums in all provinces in Indonesia using the Black–Scholes model.

As seen in Figure 4, there are significant differences in the insurance premium comparisons across all Indonesian provinces. A number of provinces have relatively high insurance premiums, while a number of other provinces have relatively low insurance premiums. After analyzing the data, it is clear that East Java pays the highest insurance premium in comparison to other provinces, with an IDR 20,368,852,873,367.90 required for payment. Comparing other provinces with a premium of IDR 5,991,611,415.18 that needs to be paid, Bengkulu province has the lowest premium. The primary factor influencing the amount of insurance premiums for natural disasters is the frequency of events and the extent of loss resulting from localized natural disasters. The insurance premiums that Indonesian provinces are required to pay differ. The most critical factor in determining insurance premium amounts is the frequency of disaster events and losses. The province is required to pay an insurance premium that increases with the frequency of natural disasters and losses.

Based on Figure 5, it can be seen that the four provinces with the highest frequency of disaster are the provinces of Central Java, West Java, East Java, and Aceh. Furthermore, compared to Figure 4, it can be seen that the provinces with the largest insurance premiums are the provinces of East Java, Aceh, Central Java, and South Sulawesi. It can be said that the frequency of natural disasters certainly has a significant impact on the calculation of natural disaster insurance premiums.

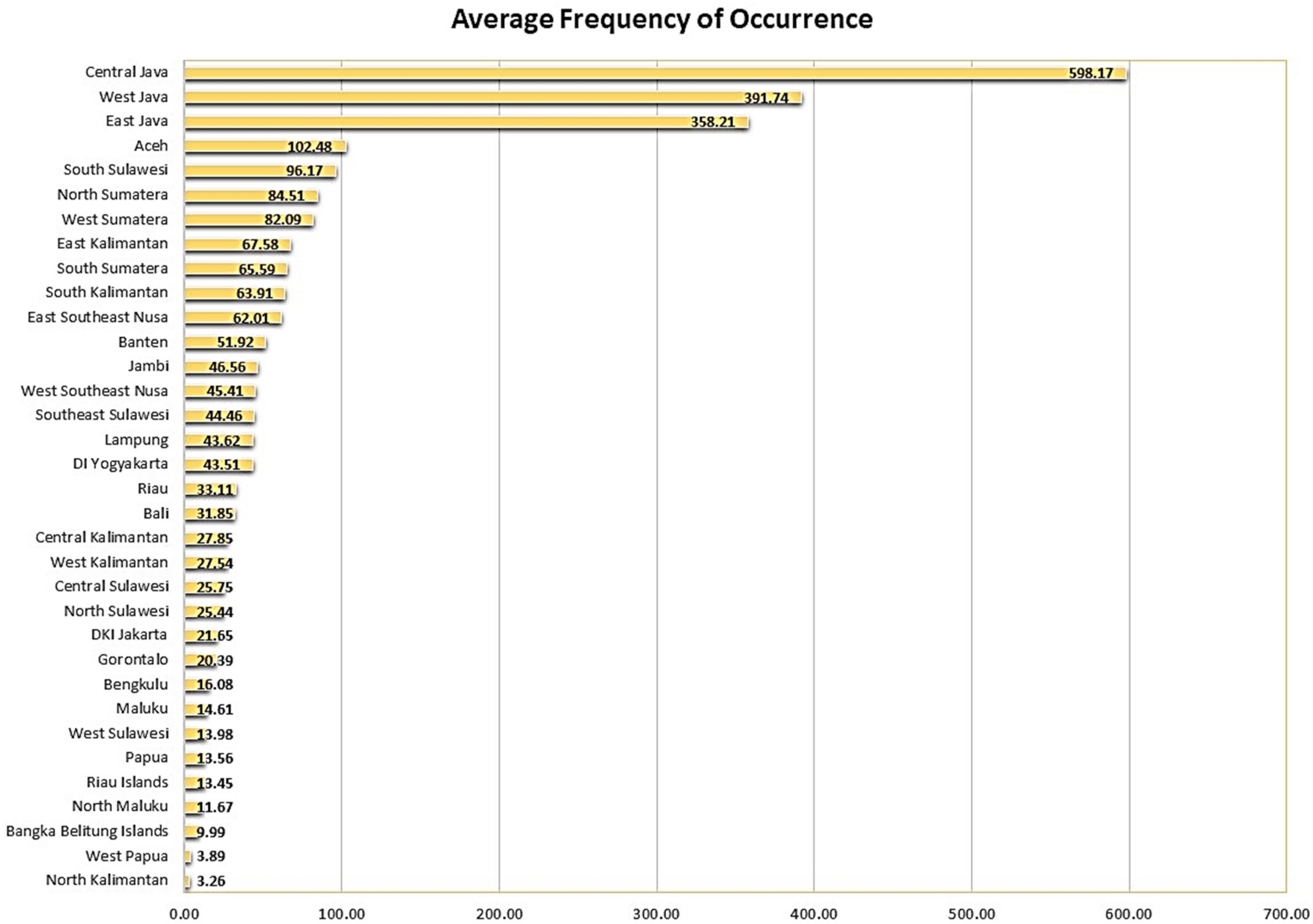


Figure 5. Average frequency of disaster events in each province.

5. Conclusions

The development of research in formulating models for determining disaster insurance premiums has become an exciting and relevant topic. Seeing that climate change is occurring, the potential for natural disasters has increased. Natural disasters can cause significant losses to areas affected by the disaster. The premium determination model is an essential component in natural disaster insurance because it determines how much benefit the company can receive if a claim occurs. Insurance premiums must also be set relatively and in accordance with the level of risk faced by the customer. In modelling disaster insurance premiums, this research uses a mathematical and statistical model to describe the relationship between variables that influence the amount of insurance premiums. The Black–Scholes model considers factors such as the frequency and intensity of natural disasters, the value of economic losses, the economic growth rate, and geographic conditions. Apart from that, the model developed uses a cross-subsidy system. In this model, the mapping is based on economic growth and potential disasters in each region. The model developed is very suitable for countries where the mapping of potential natural disasters is divided based on each region. Based on the simulation results on natural disaster data in Indonesia, insurance premiums from 34 provinces in Indonesia vary. This phenomenon is, of course, influenced by the frequency of events and economic losses due to natural disasters that occur in each province. For future research, it is recommended to include a sensitivity analysis of model parameters to verify the robustness of the model. Of course, with a sensitivity analysis of model parameters, we can identify risks and uncertainties in the model. By evaluating how changes in input variables affect the results, we can anticipate potential risks.

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