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A Renewal Shot Noise Process with Subexponential Shot Marks

Yiqing Chen

College of Business and Public Administration, Drake University, Des Moines, IA 50311, USA; yiqing.chen@drake.edu

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Abstract: We investigate a shot noise process with subexponential shot marks occurring at renewal epochs. Our main result is a precise asymptotic formula for its tail probability. In doing so, some recent results regarding sums of randomly weighted subexponential random variables play a crucial role.

Keywords: shot noise; subexponentiality; tail probability; randomly weighted sum; renewal process

1. Introduction

Consider a shot noise process $\{S_t, t \geq 0\}$ defined by

$$S_t = \sum_{k=1}^{\infty} X_k h(t - \tau_k) 1_{(\tau_k \leq t)}, \qquad t \geq 0, \tag{1}$$

where $X_1, X_2, \ldots$ are shot marks, which successively occur at times $0 \leq \tau_1 \leq \tau_2 \leq \cdots$, and $h(\cdot)$ is a response function non-negative on $[0, \infty)$ and vanishing on $(-\infty, 0)$.

We make the following standard assumptions:

Assumption 1. *In the shot noise process (1):*

(i) the shot marks X_1, $X_2, \ldots$ form a sequence of independent and identically distributed (i.i.d.) real-valued random variables with a generic random variable X and distribution F;

(ii) their arrival times τ_1, $\tau_2, \ldots$ form a sequence of renewal epochs, so that the number of shots by time $t \geq 0$, namely,

$$N_t = \sup \{k \in \mathbb{N} : \tau_k \leq t\}, \tag{2}$$

is an ordinary renewal counting process;

(iii) the two sequences $\{X_1, X_2, \ldots\}$ and $\{\tau_1, \tau_2, \ldots\}$ are mutually independent;

(iv) the response function $h(\cdot)$ is non-increasing on $[0, \infty)$ with $0 < h(0+) < \infty$.

Items (i)–(iii) above equip the shot noise process (1) with a renewal framework. By using the renewal counting process (2), we can rewrite the shot noise process as

$$S_t = \sum_{k=1}^{N_t} X_k h(t - \tau_k), \qquad t \geq 0. \tag{3}$$

The assumption $h(0+) > 0$ in item (iv) above is to avoid triviality; otherwise, the shot noise process $\{S_t, t \geq 0\}$ will simply take values of $\{X_1, X_2, \ldots\}$ at discrete time moments $\{\tau_1, \tau_2, \ldots\}$. Also notice that if $h(s) \equiv 1$ for $s \in [0, \infty)$, then the shot noise process reduces to a compound renewal process.

In this paper, we understand the shot noise process (3) in the context of insurance. We interpret the random shot marks as sizes of insurance claims and interpret the response function $h(s)$ as the proportion of a claim to be settled after s units of time have elapsed; that is, each term $X_k h(t - \tau_k)$ for $t \geq \tau_k$ is the unsettled part at time t of a claim of size X_k occurring at time τ_k. In this way, S_t defined by (3) represents the total amount of unsettled claims at time t.

The focus of this work is on the tail probability of the shot noise process $\{S_t, t \geq 0\}$ for the case of heavy-tailed (precisely, subexponential) shot marks. Our result can be applied, e.g., to calculating the solvency capital requirement under contemporary insurance regulatory frameworks such as the Solvency II Directive 2009. As a referee kindly points out, the asymptotic study has an immediate implication for risk management. There is a vast literature devoted to studies that focus on tail probabilities as an essential risk measurement tool; see, e.g., Asmussen et al. (1999); Yang and Wang (2013); Kelly and Jiang (2014); Landsman et al. (2016); Daouia et al. (2018) and Tang et al. (2019).

We end this introduction with a brief literature review of the study of shot noise processes. In the rest of this paper, we present our main result in Section 2, and after preparing two lemmas in Section 3, we show the proof of our main result in Section 4.

A Brief Literature Review

Shot noise processes were introduced at the beginning of last century, with pioneering works including Campbell (1909) and Schottky (1918). Since then, the topic has been extensively studied and shot noise processes have been used to model a very wide variety of natural phenomena.

Among this huge literature, recent works in the general context of applied probability include Lowen and Teich (1991); Klüppelberg and Mikosch (1995); McCormick (1997); Samorodnitsky (1998); Lund et al. (1999); Brix (1999); Brémaud and Massoulié (2002); Klüppelberg et al. (2003); Møller (2003); Lund et al. (2004); Møller and Torrisi (2005) and Ganesh et al. (2005).

During the recent two decades, there has been another active strand of this literature focusing on applications to insurance and finance; see Basu and Dassios (2002); Dassios and Jang (2003); Jang and Krvavych (2004); Albrecher and Asmussen (2006); Scherer et al. (2012); Jang and Dassios (2013); Weng et al. (2013); Schmidt (2014); Li and Wu (2014) and Liang and Lu (2017).

Most of existing works on this topic model the shot arrivals by a Poisson process or one of its numerous extensions such as a doubly stochastic Poisson process, also known as a Cox process. A literature search only finds a handful of works on renewal shot noise processes, namely Takács (1956); Iksanov (2013); Iksanov et al. (2014) and Dassios et al. (2015). Our current work helps to fill in this gap .

2. The Main Result

Throughout this paper, all limit relationships are for $x \to \infty$ unless stated otherwise. For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(x) \sim b(x)$ if $\lim a(x)/b(x) = 1$.

A random variable X or its distribution function F with $\overline{F}(x) > 0$ for all x is said to be heavy tailed to the right if $Ee^{\gamma X} = \infty$ for all $\gamma > 0$. One of the most important classes of heavy-tailed distributions is the subexponential class. By definition, a distribution F on $[0, \infty)$ is subexponential, denoted by $F \in \mathcal{S}$, if

$$\lim_{x \to \infty} \frac{\overline{F^{n*}}(x)}{\overline{F}(x)} = n$$

holds for all (or, equivalently, for some) $n = 2, 3, \ldots$, where F^{n*} denotes the n-fold convolution of F. More generally, a distribution F on $(-\infty, \infty)$ is still said to be subexponential to the right if $F^{+}(x) = F(x)1_{(0 \leq x < \infty)}$ is subexponential. The class $\mathcal{S}$ is very broad in the sense that it covers distributions with very different heavy tails ranging from very heavy (such as the Pareto distribution), moderately heavy (such as the lognormal distribution), and mildly heavy (such as the Weibull distribution). The famous class $\mathcal{R}$, as a subclass of $\mathcal{S}$, covers very heavy-tailed distributions. By definition, for a distribution function F on $\mathbb{R}$, we write $F \in \mathcal{R}_{-\alpha}$ for some $0 < \alpha < \infty$ if its right tail is regularly varying with index $-\alpha$, that is,

$$\lim_{x \to \infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha}, \qquad y > 0.$$

The reader is referred to Bingham et al. (1987); Resnick (1987); Embrechts et al. (1997) and Foss et al. (2011) for textbook treatments of heavy-tailed distributions with applications to insurance and finance.

Now we are ready to state our main result.

Theorem 1. *Consider the shot noise process (3). If F is subexponential, then for any $t > 0$ such that $P(\tau_1 \leq t) > 0$, we have*

$$P\left(S_t > x\right) \sim \int_0^t P\left(Xh(t-s) > x\right) dEN_s. \tag{4}$$

Some immediate refinements of Theorem 1 follow. First, if $\{N_t, t \geq 0\}$ is a homogeneous Poisson process with rate $\lambda > 0$, then plugging in $EN_s = \lambda s$ into (4) yields

$$P\left(S_t > x\right) \sim \lambda \int_0^t P\left(Xh(s) > x\right) ds.$$

This corresponds to Theorem 2.1 of Tang (2006) with $\gamma = 0$.

Second, if $F \in \mathcal{R}_{-\alpha}$ for some $0 < \alpha < \infty$, then subject to a standard argument based on Potter's bounds (see Proposition 2.2.3 of Bingham et al. (1987)), the asymptotic formula (4) becomes

$$\lim_{x \to \infty} \frac{P\left(S_t > x\right)}{\overline{F}(x)} = \int_0^t h^{\alpha}(t-s) dEN_s. \tag{5}$$

Actually, by Assumption 1(iv), the response function $h(\cdot)$ is non-increasing on $[0, \infty)$ and bounded by $0 < h(0+) < \infty$. Thus, as x becomes large, so does $x/h(t-s)$ for all $s \in (0, t]$, where $x/0$ is understood as ∞ by convention. For arbitrarily fixed small $\varepsilon > 0$, by Potter's bounds, it holds for all large x and all $s \in (0, t]$ that

$$\begin{aligned} & (1-\varepsilon) \min\left\{h^{\alpha+\varepsilon}(t-s), h^{\alpha-\varepsilon}(t-s)\right\} \\ \leq \quad & \frac{P\left(Xh(t-s) > x\right)}{\overline{F}(x)} \\ \leq \quad & (1+\varepsilon) \max\left\{h^{\alpha+\varepsilon}(t-s), h^{\alpha-\varepsilon}(t-s)\right\}. \end{aligned}$$

Plugging these bounds into the right-hand side of (4) yields

$$\begin{aligned} & (1-\varepsilon)\int_0^t \min\left\{h^{\alpha+\varepsilon}(t-s), h^{\alpha-\varepsilon}(t-s)\right\} dEN_s \\ \leq & \frac{P\left(S_t > x\right)}{\overline{F}(x)} \\ \leq & (1+\varepsilon)\int_0^t \max\left\{h^{\alpha+\varepsilon}(t-s), h^{\alpha-\varepsilon}(t-s)\right\} dEN_s. \end{aligned}$$

Letting $\varepsilon \downarrow 0$, by the dominated convergence theorem, the two bounds above coincide with each other and equal to $\int_0^t h^\alpha(t-s)dEN_s$. This verifies (5).

If, moreover, $\{N_t, t \geq 0\}$ is a homogeneous Poisson process with rate $\lambda > 0$, then the asymptotic formula (5) is further refined to

$$\lim_{x\to\infty} \frac{P\left(S_t > x\right)}{\overline{F}(x)} = \lambda \int_0^t h^\alpha(s)ds.$$

3. Lemmas

This section recalls two important lemmas regarding the tail probabilities of sums of randomly weighted subexponential random variables. The following first lemma is a restatement of Theorem 1 of Tang and Yuan (2014):

Lemma 1. *Let $X_1, \ldots, X_n$ be n i.i.d. random variables with common distribution function $F \in \mathcal{S}$, and let $\theta_1, \ldots, \theta_n$ be n non-negative, bounded, and not-degenerate-at-zero random variables independent of $X_1, \ldots, X_n$. Then*

$$P\left(\sum_{k=1}^n \theta_k X_k > x\right) \sim \sum_{k=1}^n P\left(\theta_k X_k > x\right).$$

The next lemma, attributed to a recent work of the author in Chen (2019), establishes a Kesten-type upper bound for the tail probabilities of the sums of randomly weighted subexponential random variables:

Lemma 2. *Let $\{X_1, X_2, \ldots\}$ be a sequence of i.i.d. and real-valued random variables with common distribution function $F \in \mathcal{S}$, let $\{\theta_1, \theta_2, \ldots\}$ be another sequence of non-negative and uniformly bounded random variables independent of $\{X_1, X_2, \ldots\}$. Then for any $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that*

$$P\left(\sum_{k=1}^n \theta_k X_k > x\right) \leq C_\varepsilon (1+\varepsilon)^n \sum_{k=1}^n P\left(\theta_k X_k > x\right)$$

holds for all $n \in \mathbb{N}$ and all $x \geq 0$.

4. Proof of Theorem 1

For an arbitrarily fixed $M \in \mathbb{N}$, we expand the tail probability of S_t as

$$\begin{aligned} P\left(S_t > x\right) &= \left(\sum_{n=1}^M + \sum_{n=M+1}^\infty\right) P\left(\sum_{k=1}^n X_k h(t-\tau_k) > x, N_t = n\right) \\ &= I_1 + I_2. \end{aligned} \tag{6}$$

We apply Lemma 1 to deal with I_1. For each $n = 1, \ldots, M$, we condition each tail probability

$$P\left(\sum_{k=1}^{n} X_k h(t-\tau_k) > x, N_t = n\right)$$

in I_1 on $(N_t = n)$ and then interpret each conditional random variable $h(t-\tau_k) |\, (N_t = n)$ as a random weight θ_k. To be strict, such a random variable θ_k involves both arguments t and n, but this does not matter; key requirements for the applicability of Lemma 1 are that the random weight $\theta_1, \ldots, \theta_n$ are bounded and independent of the shot marks $X_1, \ldots, X_n$. Thus, by Lemma 1 we obtain

$$P\left(\sum_{k=1}^{n} X_k h(t-\tau_k) > x, N_t = n\right) \sim \sum_{k=1}^{n} P\left(X_k h(t-\tau_k) > x, N_t = n\right).$$

It follows that

$$\begin{aligned} I_1 &\sim \sum_{n=1}^{M}\sum_{k=1}^{n} P\left(X_k h(t-\tau_k) > x, N_t = n\right) \\ &= \left(\sum_{n=1}^{\infty}\sum_{k=1}^{n} - \sum_{n=M+1}^{\infty}\sum_{k=1}^{n}\right) P\left(X_k h(t-\tau_k) > x, N_t = n\right) \\ &= I_{11} + I_{12}. \end{aligned} \tag{7}$$

By interchanging the order of the two sums in I_{11}, we have

$$\begin{aligned} I_{11} &= \sum_{k=1}^{\infty}\sum_{n=k}^{\infty} P\left(X_k h(t-\tau_k) > x, N_t = n\right) \\ &= \sum_{k=1}^{\infty} P\left(X_k h(t-\tau_k) > x, N_t \geq k\right) \\ &= \sum_{k=1}^{\infty} P\left(X_k h(t-\tau_k) > x, \tau_k \leq t\right) \\ &= \sum_{k=1}^{\infty} \int_0^t P\left(X_k h(t-s) > x\right) P\left(\tau_k \in ds\right) \\ &= \int_0^t P\left(X h(t-s) > x\right) dEN_s, \end{aligned} \tag{8}$$

where the last step is due to $\sum_{k=1}^{\infty} P\left(\tau_k \leq s\right) = EN_s$. For I_{12}, by the monotonicity of the response function $h(\cdot)$, we derive

$$\begin{aligned} I_{12} &\leq \sum_{n=M+1}^{\infty}\sum_{k=1}^{n} P\left(X_k h(t-\tau_1) > x, N_t = n\right) \\ &= \sum_{n=M+1}^{\infty} n \int_0^t P\left(X h(t-s) > x\right) P\left(N_{t-s} = n-1\right) P\left(\tau_1 \in ds\right) \\ &= \int_0^t P\left(X h(t-s) > x\right) E\left[\left(N_{t-s}+1\right) 1_{(N_{t-s} \geq M)}\right] P\left(\tau_1 \in ds\right) \\ &\leq E\left[\left(N_t+1\right) 1_{(N_t \geq M)}\right] \int_0^t P\left(X h(t-s) > x\right) P\left(\tau_1 \in ds\right). \end{aligned}$$

Thus, for arbitrarily fixed $\delta > 0$, due to $E[N_t] < \infty$, we can choose M large enough such that

$$E\left[(N_t+1)\,1_{(N_t\geq M)}\right] \leq \delta.$$

It follows that

$$I_{12} \leq \delta \int_0^t P\left(Xh(t-s) > x\right) P\left(\tau_1 \in ds\right) \leq \delta \int_0^t P\left(Xh(t-s) > x\right) dEN_s. \tag{9}$$

We apply Lemma 2 to deal with I_2 in (6). As in dealing with I_1, for each $n \geq M+1$ we condition each tail probability

$$P\left(\sum_{k=1}^{n} X_k h(t-\tau_k) > x, N_t = n\right)$$

in I_2 on $(N_t = n)$ and then interpret each conditional random variable $h(t-\tau_k)|\,(N_t = n)$ as a random weight θ_k. As explained above, these random weights $\theta_1, \theta_2, \ldots$, though involving both arguments t and n, are uniformly bounded and independent of the shot marks $X_1, X_2, \ldots$. This justifies the applicability of Lemma 2. Thus, for any $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that, for all $n \in \mathbb{N}$ and $x \geq 0$,

$$P\left(\sum_{k=1}^{n} X_k h(t-\tau_k) > x, N_t = n\right) \leq C_\varepsilon (1+\varepsilon)^n \sum_{k=1}^{n} P\left(X_k h(t-\tau_k) > x, N_t = n\right).$$

It follows that

$$\begin{aligned}
I_2 &\leq C_\varepsilon \sum_{n=M+1}^{\infty} (1+\varepsilon)^n \sum_{k=1}^{n} P\left(X_k h(t-\tau_k) > x, N_t = n\right) \\
&\leq C_\varepsilon \sum_{n=M+1}^{\infty} (1+\varepsilon)^n n P\left(Xh(t-\tau_1) > x, N_t = n\right) \\
&= C_\varepsilon \sum_{n=M+1}^{\infty} (1+\varepsilon)^n n \int_0^t P\left(Xh(t-s) > x\right) P\left(N_{t-s} = n-1\right) P\left(\tau_1 \in ds\right) \\
&= C_\varepsilon \int_0^t P\left(Xh(t-s) > x\right) E\left[(1+\varepsilon)^{N_{t-s}+1}\left(N_{t-s}+1\right) 1_{(N_{t-s}\geq M)}\right] P\left(\tau_1 \in ds\right) \\
&\leq C_\varepsilon E\left[(1+\varepsilon)^{N_t+1}\left(N_t+1\right) 1_{(N_t\geq M)}\right] \int_0^t P\left(Xh(t-s) > x\right) P\left(\tau_1 \in ds\right).
\end{aligned}$$

It is easy to verify that for a general renewal counting process $\{N_t, t \geq 0\}$, regardless of $E\tau_1 < \infty$, there is always $b > 1$ such that $Eb^{N_t} < \infty$; see also Theorem 1 of Kočetova et al. (2009). Thus, for arbitrarily fixed $\delta > 0$, we can choose some small $\varepsilon > 0$ and large $M > 0$ such that

$$C_\varepsilon E\left[(1+\varepsilon)^{N_t+1}\left(N_t+1\right) 1_{(N_t\geq M)}\right] \leq \delta.$$

It follows that

$$I_2 \leq \delta \int_0^t P\left(Xh(t-s) > x\right) P\left(\tau_1 \in ds\right) \leq \delta \int_0^t P\left(Xh(t-s) > x\right) dEN_s. \tag{10}$$

Finally, simply combining (6)–(10) together and making use of the arbitrariness of $\delta > 0$, we prove the asymptotic relation (4).

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