

# Credit Risk Migration and Economic Cycles

Camilla Ferretti <sup>1</sup>, Giampaolo Gabbi <sup>2,\*</sup>, Piero Ganugi <sup>3</sup>, Federica Sist <sup>4</sup> and Pietro Vozzella <sup>5</sup>

<sup>1</sup> Department of Economic and Social Sciences, Università Cattolica del Sacro Cuore, 29122 Piacenza, Italy; Camilla.Ferretti@unicatt.it

<sup>2</sup> SDA Bocconi School of Management, 20136 Milan, Italy

<sup>3</sup> Department of Industrial Engineering, University of Parma, 43100 Parma, Italy; piero.ganugi@unipr.it

<sup>4</sup> Department of Law, Economics, Politics and Modern Languages, LUMSA University, 00193 Rome, Italy; federica.sist@gmail.com

<sup>5</sup> Department of Management and Law, University of Siena, 53100 Siena, Italy; vozzzy71@gmail.com

\* Correspondence: gabbi@sdabocconi.it

Received: 22 September 2019; Accepted: 19 October 2019; Published: 29 October 2019



**Abstract:** The misestimation of rating transition probabilities may lead banks to lend money incoherently with borrowers' default trajectory, causing both a deterioration in asset quality and higher system distress. Applying a Mover-Stayer model to determine the migration risk of small and medium enterprises, we find that banks are over-estimating their credit risk resulting in excessive regulatory capital. This has important macroeconomic implications due to the fact that holding a large capital buffer is costly for banks and this in turn influences their ability to lend in the wider economy. This conclusion is particularly true during economic downturns with the consequence of exacerbating the cyclicity in risk capital that therefore acts to aggravate economic conditions further. We also explain part of the misevaluation of borrowers and the actual relevant weight of non-performing loans within banking portfolios: some of the prudential requirements, at least as regards EMS credit portfolios, cannot be considered effective as envisaged by the regulators who developed the "new" regulation in response to the most recent crisis. The Mover-Stayers approach helps to reduce calculation inaccuracy when analyzing the historical movements of borrowers' ratings and consequently, improves the efficacy of the resource allocation process and banking industry stability.

**Keywords:** credit risk; Markov chains; absorbing state; rating migration

## 1. Introduction

Credit risk transition probabilities are the key to improving forward-looking risk management for investors and commercial banks. This is true for listed bonds whose risk is generally estimated by rating agencies (Lando and Skødeberg 2002; Gabbi and Sironi 2005; D'Amico et al. 2016), and for loans that are more frequently analyzed through banks' internal models. The deterioration of mortgage and loan became a contributing source to different phases of the most recent financial crisis. Unreliable migration metrics can be considered as a serious model risk factor with systemic consequences, particularly due to the fact that most of the rating models are based on Markov chain assumptions to estimate transition matrices.

One of the issues embedded in pure Markov chains is the possible presence of an absorbing state that is a state in which all the individuals will fall sooner or later. In a chain applied to credit risk estimation, the absorbing state means a default. This raises the interesting question: Why should banks trust models based on the assumption that all their counterparts will fail sooner or later? The issue is based on the incoherency affecting credit risk transition matrix estimates when they are based on this pure Markovian assumption, since if the absorbing state is incorporated, then all the borrowers move, at least in the long run, to the default state. A second relevant question relates to the estimation of the

rating migration risk for the banks' economic capital: to be effective, internal rating models should be designed coherently not only with the actual borrowers' standing, but also with their expected assessment pattern (Gabbi et al. 2019). As a matter of fact, companies are characterized by different propensities to change and some of them could be defined as stayers, as opposed to movers. A good transition estimate should be able to differentiate the two clusters and recognize the trajectories of movers. An inconsistent rating transition model inevitably affects the robustness of capital absorption. Finally, we question whether the credit risk approximation due to the migration risk could increase the cyclical and consequent probability of crisis.

Our paper focuses on the issue especially for SMEs that are not listed or issue bonds to improve their funding; SMEs play a significant role as key generators of employment and income, and as drivers of innovation and growth. SMEs constitute about 99% of all euro area firms, and employ around two-thirds of the workforce, generating approximately 60% of value added. Their economic contribution is particularly relevant in the Mediterranean countries, particularly Italy, Spain, and Portugal. In Italy, during the crisis, the uncertainty and the reduced prospects for growth in the real economy reduced firms' profitability worsening their creditworthiness. At the same time, in presence of problems of budget sustainability, Italian banks have become more cautious in granting credit. This occurred mainly from the end of 2008 till the beginning of 2009 and in the last months of 2011, when sovereign debt turmoil has worsened the capitalization of banks in the countries most affected.

Regardless of the historical period, SMEs suffer lower credit availability compared with large companies that usually benefit with the support of skilled experts. As a result, larger firms generally benefit with favorable credit lines (Lin et al. 2011). Conversely, banks usually suffer higher information asymmetries when lend to SMEs (Berger and Udell 1995; Degryse and Van Cayseele 2000). Accounting requirements for SMEs are relatively soft, so their managers have only small incentives to deliver detailed information (Baas and Schrooten 2006). Due to the relatively limited available information about SMEs, banks usually are reluctant to provide credit to SMEs and often they shy away from financing firms directly or indirectly by pricing higher spreads (Ivashina 2009).

Small firms are also vulnerable because of their dependency on financial institutions for external funding. These firms simply do not have access to public capital markets. As a result, shocks to the banking system can have a significant impact on the supply of credit to small businesses (Berger and Udell 2002).

To fill this gap, banks can opt for a relational approach with non-financial firms. The available studies emphasize the advantages of relationship lending in terms of credit availability and explicit lending terms and conditions e.g., (Petersen and Rajan 1994, 1995; Berger and Udell 1995; Cole et al. 2004; Elsas and Krahen 1998; Harhoff and Körting 1998). Boot (2000) defines RL approach as an organizational model of relational nature between banks and customers that meet the following conditions: (i) the bank, in the course of the relationship with customers-firm, makes investments aimed to the acquisition of soft information, that is its exclusive ownership, functional to a more accurate definition of the creditworthiness; (ii) the bank recovers these costs realizing profits through repeated interactions with its customers.

The basic literature on this topic suggests that relationship lending mainly aims to resolve agency problems and informational asymmetries. Economic literature states that a closer relationship reduces agency and asymmetric information problems due to the quality of the information flow that occurs when the relationship is closer. The basic variables generally used in the literature are the duration of the relationship, distance, frequency of contacts exclusivity, cross-selling, and organizational structure of the bank. Our paper presents the originality of comparing all these factors simultaneously in order to assess how they can influence banks' willingness to grant credit even in times of crisis.

To address these issues in depth, we focus our analysis on transition matrices applied to credit risk which show the pattern of changes for different borrowers over time from one rating notch to another. Every row of the matrix can be defined as a set of probabilities describing the likelihood of credit quality staying unchanged or moving to any of the other rating classes over a given time horizon,

conditioned to the starting state. In most applications, matrices are based on a Markov transition probability model.

Nevertheless, the pure Markov approach is unable to model the increasing probability for companies to stay within a rating class once they have been rated for a long period in that notch (Lando and Skødeberg 2002; Altman 1998 and Figlewski et al. 2006). Moreover, if individuals are heterogeneous, migration probabilities may depend on their individual characteristics (Gómez-González and Kiefer 2009). Finally, transition matrices computed within a pure Markovian approach, are affected by the presence of an absorbing state that dictates that sooner or later, rated companies will be attracted by the “black hole” of the default state (Kremer and Weißbach 2013).

Our paper helps overcome all of these drawbacks, as it applies the mover-stayers model to the estimation of rating migration matrices. Nevertheless the mover-stayer model generalizes the Markov chain model, and it is based upon two types of agents: (a) the “stayers” who are assumed to remain in the same category during the period; (b) the “movers” which are expected to move from a rating class to another with a probability described by a pure Markov chain. The empirical evidence of the existence of stayers allows us to solve the issue of an absorbing default state for borrowers.

The economic issue about the estimations of transition matrices pertains to the structural impact of financial crisis on the estimations, and the implications for regulators and credit policy makers. Since the crisis has potentially changed the patterns of defaults experienced before, all the matrices were biased by different estimates. Moreover, the regulatory response, particularly the introduction of the Incremental Risk Charge for the trading book (Basel Committee on Banking Supervision 2009), was based on the Markov way of estimating the migration risk.

One of the most important goals behind the current debate aimed at revising the Basel Capital Accord (Basel Committee on Banking Supervision 2011) is to replace the existing risk weights with a system which more clearly recognizes the differences in risk of various instruments. It is likely that rating systems will play a larger role in quantifying these differences. This is particularly relevant for small and medium enterprises (SMEs), without an external rating, that are generally rated by banks' internal rating models (Angilella and Mazzù 2019). To capture this factor, we apply a statistical model calibrated to assess SMEs ratings as used by financial institutions.

Our prediction is that when banks accept to lend money incoherently with the borrowers' trajectory, this inevitably contributes to worsen asset quality which leads to a hyper-speculative position and, within an origination-to-distribute model with a large credit risk transfer via securitization, also to a highly distressed system. Moreover, the application of incorrect transition matrices causes over-estimated credit risk, a consequent capital misallocation and an inefficient increase, with the perspective that the loan process will be affected by growing transaction costs and bounded rationality. Our research question is to prove that this scenario particularly affects small and medium enterprises search for credit. Our contribution differs from existing research because it estimates how the estimation of the transition matrices of SMEs in relatively stable periods can be perfected by banks by replacing the pure Markovian method with the movers-stayers approach.

The remainder of this paper is organized as follows. Section 2 explains how we construct the Movers Stayers model compared with pure Markov chains that are applied to the transition matrices. Section 3 describes the database. In Section 4 we present the results. Section 5 concludes.

## 2. Markov Chains and the Movers Stayers Model

Our study has been designed to compare the credit transition approach based upon the pure Markovian model against the Movers-Stayers approach in order to verify whether commercial banks could optimize their credit risk portfolio models when lending to small and medium enterprises. The research design aims to verify the robustness of the Mover-Stayer model (MS) compared to the classical Markov Chain (MC), especially when applied to SMEs judgement.

In the classical Markov Chain (MC), single names are supposed to move from state  $i$  to state  $j$  with a given probability  $p_{ij} = \Pr(X_t = j | X_{t-1} = i)$ , depending only on  $i$  and  $j$ . Individuals are then homogeneous since they move according to only with their starting state  $i$ , and not with some idiosyncratic features.

The Mover-Stayer model (MS) arises from the mixture of two different Markov chains, one of which degenerates with transition probabilities  $p_{ij} = \delta_{ij}$ , for  $i, j = 1, \dots, k$  (the Kronecker Delta, equal to 1 if  $i = j$ , 0 otherwise). Then individuals are actually divided in two subsets: the set of Movers (M), following a classical MC with transition probabilities  $m_{ij}$ , and the set of Stayers (S), constantly living in their initial state. This model is introduced in Blumen et al. (1955) with the aim of including heterogeneity among individuals. Furthermore, MS avoids some problems related to the simple MC, for example the tendency to underestimate the probability  $p_{ii}$  to remain in the same state.

For any individual, we consider the probability  $s_i$  to be a Stayer, given the starting state  $i$ . Note that this probability depends again only on  $i$ , and not on individual factors. The global transition probability is then given by:

$$p_{ij} = s_i \times \delta_{ij} + (1 - s_i) \times m_{ij}, \quad (1)$$

where  $\delta_{ij}$  is the Kronecker's Delta defined before.

The main difference with the MC model relates to the computation of the transition probabilities after  $s$  steps: if  $p_{ij}^{(s)} = \Pr(X_t = j | X_{t-s} = i)$  and  $P^{(s)} = \{p_{ij}^{(s)}\}$  is the corresponding matrix, then:

$$P^{(s)} = P^s \quad (2)$$

in the classical MC, and

$$P^{(s)} = S + (I - S) \times M^s \quad (3)$$

in the MS model (with  $S = \text{diag}\{s_1, \dots, s_k\}$ ).

When banks monitor their borrowers, they collect data which are characterized by a continuous stream over the lending period. Measuring credit transitions in the continuous time has a number of advantages: (i) it allows a rigorous formulation and testing of assumptions 'rating drift' and other non-Markov type behavior (Altman and Kao 1992a, 1992b; Lucas and Lonski 1992; and Carty and Fons 1993); (ii) it permits us to formulate and test the dependence on external covariates, and quantify changes in regimes either due to business cycles (Nickell et al. 2000), or changes in rating policies (Blume et al. 1998); (iii) it leads to better estimates of rating-based term structure modeling by different rating classes, especially when their slopes are un-orthodox (Jarrow et al. 1997; Lando 1998; and Das and Tufano 1996).

In this regard, we note that in the aforementioned models, transitions are allowed to happen only at equispaced instants of time  $t, t + 1, t + 2, \dots, t + n$ . The "continuous-in-time" version of MC (CTMC) instead permits to model the continuous stream of transitions at any point of time  $t$  in  $\mathbb{R}$ . It means that  $p_{ij}$  is actually a continuous function of  $t$ :

$$p_{ij}(t) = \Pr(X_t = j | X_0 = i) \quad (4)$$

for every  $t$  in  $\mathbb{R}$ .

The continuous version of MS (CTMS herein, see for example (Frydman and Kadam 2002) is defined in the same way: Movers may move at any instant  $t$ , and the global transition probabilities are:

$$p_{ij} = s_i \times \delta_{ij} + (1 - s_i) \times m_{ij}(t). \quad (5)$$

The continuous time models are characterized by the existence of a matrix  $Q$  describing the rate of transitions among the states.  $Q$  is this generating matrix and satisfies the following properties:

1.  $q_{ij} \geq 0 \forall i \neq j$  and  $q_{ii} < 0$ ;
2.  $\sum_{j=1}^k q_{ij} = 0 \forall i$ ;
3.  $M(t) = \exp(tQ)$  for all  $t > 0$ , where  $\exp(\cdot)$  stands for the exponential matrix function (Golub and Van Loan 1996).

The MS version of this property will be:

$$P(t) = S + (I - S) \times \exp(tQ). \quad (6)$$

As we will see in the subsequent sections,  $Q$  is beneficial to obtain additional information about the persistence of borrowers (single names) in every state.

In our research we focus on the continuous version of MC and MS. In this sense it is worth noting two different problems affecting such models: embeddability and aliasing (Fougère and Kamionka 2003). Indeed, from the estimated one-year transition matrix  $\hat{M}$  (for the Movers) we obtain the estimated  $\hat{Q}$  as a solution of  $M = \exp(Q)$ . Two cases may happen:

1. Embeddability: there is no solution  $\hat{Q}$  which also satisfies the aforementioned properties of a generating matrix. This drawback arises because not every discrete time Markov chain can be realized as a discretized continuous-time chain (Israel et al. 2001). Hence, it may be impossible from a one-year transition matrix (for example, if it contains zeros in some of the non-default rows) to structure a continuous-time chain which has the one-year transition matrix as its “marginal”.
2. Aliasing: more than one continuous time chain exists, from which the discrete process arises. This drawback is caused by the fact that the equation  $M = \exp(Q)$  may have different solutions.

If one of the aforementioned problems occurs, we cannot find the right estimates for MS parameters: in the first case the underlying continuous-in-time model does not exist, and in the second case we have to choose among several models, all of them fitting the observed data.

In Fougère and Kamionka (2003), a method is proposed to check if embeddability or aliasing arise from our data. In particular to estimate the parameters  $s$ ,  $M$  and  $Q$  they proposed a Bayesian framework, later re-elaborated in Cipollini et al. (2013), which is based on the Gibbs Sampling algorithm. Shortly, given the starting values  $M_0$ ,  $S_0$ , and defining  $Z$  as the random variable describing the number of Stayers in every state, at the  $l$ -th iteration the algorithm is based on the following steps more details are in (Cipollini et al. 2013):

1. it randomly draws  $Z_l$  from a binomial distribution depending on  $S_{l-1}$ ,  $M_{l-1}$ ;
2. it updates  $S_{l-1}$ ,  $M_{l-1}$  to  $S_l$ ,  $M_l$  randomly drawing from a probability density function which is a sort of multivariate Dirichlet distribution, with parameters depending on  $Z_l$  and on the observed starting distribution and the total observed number of transition from  $i$  to  $j$ , for every couple  $i$  and  $j$ .

The algorithm allows us to estimate also the standard error for  $\hat{S}$  and  $\hat{M}$ , and the matrix  $\hat{Q}$ . The same procedure permits us to estimate the probability that an underlying continuous model exists (if it is  $<1$  then embeddability may occur) and also the number of existing models (if it is  $>1$  then aliasing occurs). We will see in the following section that neither embeddability nor aliasing affect our estimates.

### 3. Data Description

We study the Italian loan market, as it is peculiar for three reasons: only a small number of very large companies are rated by rating agencies; Italian GDP is strongly dependent on the output of SMEs; finally, the high bank dependent liability rate of Italian firms, which enforces credit institutions to improve their internal rating models.

The database is composed of the balance sheets and income statements recorded for a large set of Italian corporate firms collected from Aida-Bureau van Dijk. The sample contains 44,192 firms over the 11-year period from 1999 to 2010. We do not incorporate new companies into the sample that are not built present at the starting period: in this way, we do not allow the cross-section to vary over time. One of the original contributions of our research is that for the first time, to our knowledge, the issue of pure Markovian weaknesses has been addressed specifically with regard to small and medium sized companies that are without an external rating and any bond issuance. This allows us to compare our sample to





Most of the companies have been rated within the A, B, or C rating notches whereas a lower percentage falls within the speculative or non-investment grades (i.e., D or below). This fact depends on the logic applied to fit the rating model. First, the probability of default estimated for firms is drawn by a model calibrated on this sample and does not come from rating agencies or large banking institutions, as shown in previous studies<sup>1</sup>. Moreover, in our study, we refer to not listed companies that contrast to firms issuing publicly traded debt that are usually rated by the large international rating agencies.

A company's score provided by the model cannot be used directly as a probability of default. For this purpose, we applied cluster analysis and 15 clusters were obtained. We then computed the expected default frequency (EDF), i.e., the ratio between the number of failed enterprises and the number of total enterprises within each cluster and, in order to make it easier to reading, we grouped clusters into a granular rating system with six rating notches by mapping the S&P scale. Mapping is done looking at the ranges of expected default frequencies derived from our credit risk model and assigning each of them to one of the classes of S&P master scale. Doing a mechanical comparison between the S&P scale and the one derived from our analysis, we observe that even the best firms within of the data set will receive only a BBB+ rating.

As a result of the mapping process, the probabilities of default of our class A do not match with that of the international rating agencies. Indeed, the lowest probability of default drawn from our model (0.26 percent) corresponds, approximately, to a BBB+/BBB notch. Similarly, the B risk class includes more than one notch where the highest probability to default is close to 1 percent, corresponding to a BB/BB− notch. This fact explains why more than 50% of firms fall within the best two classes. It is worth highlighting that the distribution of our sample is in line with the real distribution of Italian small and medium enterprises.

Table 2 reports the evolution of firms leaving their risk rating states each year, those never moving from the starting state, and defaults over the sample period 2000–2010. Table 2 Panel 1 confirms the expected monotonic pattern of defaults by rating notches. The dynamic is recorded at year 11 (2010) for the bankruptcy procedures length, even though corporate failures significantly decrease. This problem does not corrupt the purpose of our analysis, that is unconditioned by the probability of default. Panel 2 shows the share of firms that at time ( $t + 1$ ) recorded a change (upgrading or downgrading) compared to time  $t$ . Unsurprisingly, the percentage of shifts across rating notches is not only high throughout the observed period but it increases as credit quality deteriorates. Finally, Panel 3 shows the percentage of firms for every state that have never changed their rating over the sample period (potential stayers). The introduction of this category of firms suggests that, in contrast to the pure Markov chains that implicitly assume the default as an absorbing state, a non-pure Markovian behaviour in ratings migrations occurs. Modeling the existence of this kind of companies allows us to manage most of the pure Markovian assumptions implied in the transition matrices usually computed within external and internal credit or bond portfolio models.

---

<sup>1</sup> In Fei et al. (2012) data are derived from the S&P CreditPro 7.7 database. In Dietsch and Petey (2004), relating to the empirical studies on asset correlation, data were collected from the internal rating system of Coface for France and Creditform for Germany. Similarly, in Bandyopadhyay et al. (2007), results were based on CRISIL's (a lending credit rating agency in India) annual ratings of long-term bonds. For more details on the process we applied to compute PD, see (Gabbi and Vozzella 2013).





Table 3. Cont.

probability of embeddability = 1 number of possible models ( <i>aliasing</i> ) = 1							
Panel 2: Estimated generating matrix and probability to be a Stayers for the MS model							
	A	B	C	D	E	F	Default
<b>A</b>	−0.2978	0.2223	0.0300	0.0346	0.0034	0.0060	0.0016
<b>Se</b>	0.0023	0.0021	0.0011	0.0008	0.0004	0.0004	0.0002
<b>B</b>	0.2721	−0.5661	0.2818	0.0000	0.0065	0.0027	0.0030
<b>Se</b>	0.0022	0.0033	0.0024	0.0000	0.0005	0.0004	0.0003
<b>C</b>	0.0071	0.3015	−0.5828	0.2550	0.0072	0.0048	0.0072
<b>Se</b>	0.0010	0.0026	0.0033	0.0023	0.0009	0.0005	0.0004
<b>D</b>	0.0203	0.0047	0.3075	−0.4824	0.1093	0.0125	0.0280
<b>Se</b>	0.0008	0.0013	0.0029	0.0034	0.0020	0.0009	0.0008
<b>E</b>	0.0297	0.0372	0.0209	0.7006	−1.0165	0.1611	0.0670
<b>Se</b>	0.0034	0.0041	0.0063	0.0131	0.0141	0.0075	0.0039
<b>F</b>	0.1876	0.0855	0.1392	0.2122	0.3971	−1.3175	0.2959
<b>Se</b>	0.0122	0.0110	0.0138	0.0190	0.0221	0.0305	0.0135
<b>Default</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	−0.0002
<b>Se</b>	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
<b>Stayers</b>	0.2625	0.0115	0.0167	0.0424	0.0318	0.0018	0.5015
<b>Se</b>	0.0036	0.0014	0.0016	0.0027	0.0056	0.0019	0.2884

probability of embeddability = 1  
number of possible models (*aliasing*) = 1

#### 4.2. The Estimated Annual Transition Matrix

The second step of the analysis is the estimation of the annual transition matrix given the existence of the continuous-time Markov chain (Table 4, panel 1) and the annual global matrix obtained from the Movers-Stayers model (Table 4, panel 2).

Table 4. Estimated annual transition matrices.

Panel 1: Estimated annual transition matrix $\hat{P}$ for the MC							
	A	B	C	D	E	F	Default
<b>A</b>	84.46%	10.02%	2.89%	1.94%	0.27%	0.22%	0.19%
<b>B</b>	18.13%	61.68%	16.67%	2.43%	0.49%	0.22%	0.40%
<b>C</b>	3.24%	17.52%	61.28%	15.53%	1.15%	0.35%	0.93%
<b>D</b>	1.92%	3.24%	18.23%	67.70%	5.36%	0.91%	2.64%
<b>E</b>	3.02%	3.12%	7.22%	34.67%	40.79%	5.22%	5.96%
<b>F</b>	10.14%	6.17%	8.93%	15.97%	13.32%	27.92%	17.54%
<b>Default</b>	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Panel 2: Estimated annual transition matrix $\hat{P}$ for the MS							
	A	B	C	D	E	F	Default
<b>A</b>	82.62%	11.19%	3.22%	2.19%	0.30%	0.25%	0.22%
<b>B</b>	18.04%	61.63%	16.62%	2.61%	0.49%	0.22%	0.40%
<b>C</b>	3.23%	17.48%	61.33%	15.54%	1.15%	0.35%	0.93%
<b>D</b>	1.92%	3.24%	18.19%	67.76%	5.36%	0.91%	2.63%
<b>E</b>	3.01%	3.11%	7.21%	34.56%	40.98%	5.20%	5.93%
<b>F</b>	10.11%	6.16%	8.92%	15.95%	13.29%	28.06%	17.51%
<b>Default</b>	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

As expected, both credit migration matrices are diagonally dominant implying a relatively large ratings stability over a one-year horizon. However, the diagonal entries are smaller for speculative grade ratings than for investment grade ones, confirming that low ratings are more volatile. Unsurprisingly,

the probability of default increases monotonically as credit quality deteriorates. Also, in the case of SMEs, our estimates confirm the stylized row monotonicity in rating migrations, as frequently observed for ratings assigned by rating agencies (Nickell et al. 2000; Bangia et al. 2002; Lando and Skødeberg 2002; and Fuertes and Kalotychou 2007). However, contrary to the values generally observed in the transition matrices of international rating agencies for listed companies, within both our estimated matrices, with the exception of the F notch the immediate off-diagonal elements are generally larger for upgrades than downgrades. Moreover, both estimated annual transition matrices MC and MS, suggest a non-zero probability to default for “A” firms however this is approximately 0.20 percent higher than of that commonly found by rating agencies. This result is consistent with the fact that our notch “A” is PD-equivalent to BBB in S&P’s scale where the transition probability to default is substantially similar.

### 4.3. Comparison between the Markov Chain and Movers Stayers Models

Using  $\hat{Q}$  and  $\hat{S}$  we are allowed to estimate the annual rating distribution for firms over the period 1999–2010. The comparison between estimated and observed distributions (Table 5) allows us to choose the suitable model to adjust the pure Markovian limits. The estimated distribution for MC and MS is respectively given by:

$$\begin{aligned} MC : \hat{d}_t &= d_{1999} \times \hat{P}(t)_{MC}, \\ MS : \hat{d}_t &= d_{1999} \times (\hat{S} + (I - \hat{S}) \times \hat{M}(t)_{MS}), \end{aligned} \tag{7}$$

where  $\hat{d}_t$  and  $d_{1999}$  are respectively the estimated distribution for any time  $t$  and the observed distribution in the year 1999 (which corresponds to the starting distribution, being 1999 the first observed year).

Table 5. Estimated distribution for the MC and MS models.

Panel 1: Estimated distribution for the MC model											
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
<b>A</b>	41.14%	39.32%	37.91%	36.77%	35.82%	35.01%	34.31%	33.68%	33.11%	32.58%	32.10%
<b>B</b>	19.24%	19.97%	20.27%	20.35%	20.30%	20.18%	20.02%	19.83%	19.64%	19.43%	19.23%
<b>C</b>	19.00%	19.26%	19.45%	19.57%	19.61%	19.59%	19.53%	19.43%	19.30%	19.16%	19.01%
<b>D</b>	16.30%	16.21%	16.15%	16.11%	16.07%	16.03%	15.97%	15.90%	15.82%	15.72%	15.61%
<b>E</b>	2.37%	2.38%	2.36%	2.34%	2.33%	2.32%	2.31%	2.30%	2.29%	2.27%	2.26%
<b>F</b>	0.82%	0.70%	0.67%	0.65%	0.65%	0.64%	0.64%	0.63%	0.63%	0.62%	0.62%
<b>Default</b>	1.12%	2.17%	3.19%	4.21%	5.22%	6.23%	7.23%	8.23%	9.22%	10.21%	11.18%
Panel 2: Estimated distribution for the MS model											
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
<b>A</b>	40.32%	38.17%	36.68%	35.58%	34.74%	34.06%	33.50%	33.01%	32.59%	32.21%	31.85%
<b>B</b>	19.73%	20.54%	20.77%	20.72%	20.55%	20.32%	20.07%	19.80%	19.54%	19.27%	19.01%
<b>C</b>	19.14%	19.52%	19.79%	19.92%	19.94%	19.88%	19.77%	19.62%	19.44%	19.25%	19.04%
<b>D</b>	16.45%	16.46%	16.46%	16.46%	16.44%	16.40%	16.33%	16.24%	16.12%	16.00%	15.85%
<b>E</b>	2.39%	2.40%	2.39%	2.38%	2.37%	2.36%	2.35%	2.34%	2.32%	2.30%	2.28%
<b>F</b>	0.84%	0.72%	0.68%	0.67%	0.66%	0.65%	0.65%	0.64%	0.63%	0.63%	0.62%
<b>Default</b>	1.13%	2.19%	3.23%	4.27%	5.30%	6.32%	7.34%	8.35%	9.35%	10.35%	11.33%

Re-proposing (and re-elaborating) the method introduced in Frydman et al. (1985), we use the estimated distributions to choose between MC and MS. Considering the 1999 observed distribution among rating notches as starting probability distribution, and setting iteratively  $t = 2000, 2001, \dots, 2010$ , we are able to calculate the expected distribution of firms provided by both MC and MS for every year 2000,  $\dots$ , 2010, and to compare it with the corresponding empirical distribution. Indeed, we are able to evaluate the error affecting our estimates by means of the following formula:

$$err_t = \frac{\|\hat{d}_t - d_t^{obs}\|}{\|d_t^{obs}\|} \tag{8}$$

where the symbol  $\|\cdot\|$  stands for the vectorial norm defined by  $\|v\| = \sqrt{v_1^2 + \dots + v_k^2}$  for every  $k$ -coordinates vector  $v = (v_1, \dots, v_k)$ .  $err_t$  represents a sort of distance between the observed and the estimated distributions for every year or, equivalently, the percentage error caused by using the estimated distribution at time  $t$  ( $\hat{d}_t$ ) instead the observed distribution at the same time ( $d_t^{obs}$ ).

Table 6 shows the estimation errors for the estimated distributions of firms among the rating notches, computed with the Markov chain assumption and with the Movers-Stayers model.

**Table 6.** Errors evaluated on the estimated frequency distributions among rating notches, provided by Markov Chain (MC) and Movers-Stayers (MS) models.

Transition Matrix Estimation Model	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
MC	3.58	25.49	18.21	15.36	12.80	10.62	10.36	6.93	12.94	16.22	14.75
MS	4.07	23.14	15.64	12.85	10.48	8.57	8.59	5.51	14.07	16.99	15.29
Error gap	-0.49	2.35	2.57	2.51	2.32	2.05	1.77	1.42	-1.13	-0.77	-0.54

The mean error is 13.39 for MC and 12.29 for MS. The latter’s dominance deteriorates after the beginning of the crisis (from 2007 to 2010), when MS errors are 0.255 larger than MC. This is due to the increased instability of the ‘stayers’ structure because of the crisis shock and the unexpected increase of default probabilities. Nevertheless, we observe that the error gap was particularly high in 2008 (1.13) and that this reduces progressively when the MS model appears to become more reliable. Since MS, like all the Markov models, suffers when there is a huge shock in credit markets and a turning point in credit cycles given the lack of temporal homogeneity, their application in these cases could deviate from picturing the transition risk as is.

#### 4.4. The Equilibrium Distribution

When time  $t$  tends to infinity, individuals reach the equilibrium distribution among the states, given that there exists the limit for  $t \rightarrow \infty$  in Equation (7). Table 7 enlightens the implications of the absorbing state, which in the case of MC matrices attracts all the existing firms to a default state.

**Table 7.** Estimated equilibrium distributions (percentages). The table shows the probability to be attracted in the default state. In the case of the MS model, we observe that stayer companies remain stable within their rating notch.

	A	B	C	D	E	F	Default
MC	0.00	0.00	0.00	0.00	0.00	0.00	100.00
MS	11.78	0.52	0.64	0.97	0.11	0.02	85.96

The equilibrium distribution highlights an important drawback of the MC model: since “default” is by definition an absorbing state (at least for companies), estimating transition matrices with the pure MC model, any firm will default sooner or later. On the other hand, the MS model facilitates the appraisal of a cluster of firms (the Stayers), which are unlikely to fail. The same fact is supported by Table 8, containing the last column of the transition matrix for different values of  $t$ . The distribution of stayers is determined by the credit mapping by rating notch, which is highly concentrated within the A-class. Since Table 8 is directly generated by Equation (5) when  $t \rightarrow \infty$ , this means that the real credit state as observed over the period is coherently represented in the long run.

When the equilibrium distribution is run for all the rating classes, with  $t$  ranging from 1 to 1000 periods (Table 8), we can appreciate how the deterioration process, particularly for the best quality borrowers, is differently designed. The adoption of MC models implies that lenders should be very careful to maintain long-term investments, and mortgages (say between 20 and 50 years) are scarcely “rational”, even when supplied to very good firms.

**Table 8.** Estimated transition probabilities to default. Comparison of MC and MS models with  $1 < t < 1000$  (% values).

	Time	1	2	3	4	5	10	20	50	100	500	1000
A	MC	0.19	0.53	0.98	1.53	2.15	6.1	15.33	39.28	65.18	99.59	100
	MS	0.22	0.58	1.05	1.6	2.23	5.94	13.94	33.24	52.61	73.64	73.75
B	MC	0.4	0.96	1.66	2.47	3.34	8.15	17.69	41.03	66.19	99.6	100
	MS	0.4	0.99	1.75	2.63	3.6	9.01	19.8	45.33	70.92	98.7	98.85
C	MC	0.93	2.11	3.4	4.71	6.01	11.91	21.62	43.91	67.84	99.62	100
	MS	0.93	2.13	3.44	4.79	6.13	12.35	23.01	47.36	71.73	98.19	98.33
D	MC	2.63	5.08	7.27	9.2	10.93	17.6	27.07	47.86	70.1	99.65	100
	MS	2.63	5.05	7.2	9.1	10.79	17.44	27.39	49.51	71.63	95.63	95.77
E	MC	5.96	10.3	13.36	15.64	17.47	23.88	32.65	51.85	72.39	99.68	100
	MS	5.93	10.2	13.19	15.42	17.23	23.67	32.99	53.63	74.28	96.68	96.81
F	MC	17.53	23.77	26.65	28.42	29.75	34.54	41.8	58.36	76.12	99.72	100
	MS	17.51	23.79	26.73	28.55	29.94	35.07	43.15	61.47	79.81	99.71	99.82

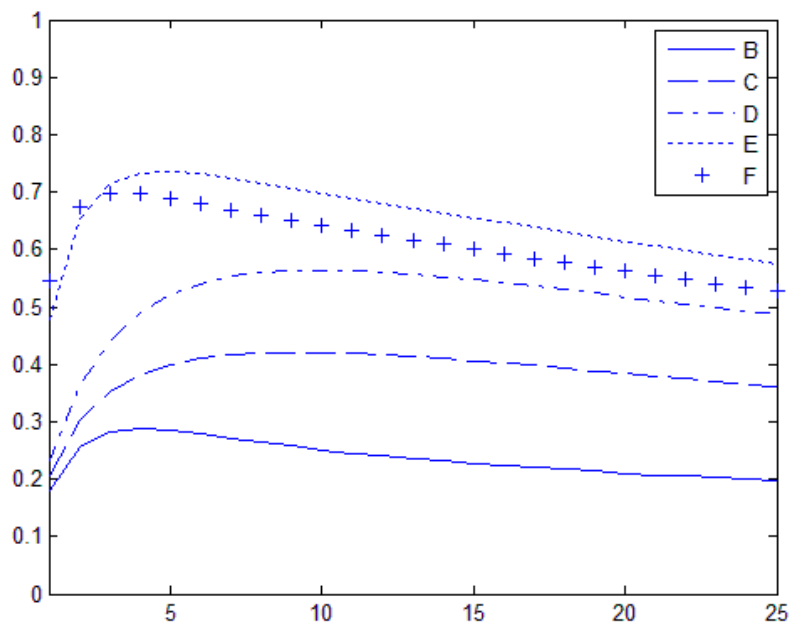
#### 4.5. Time Persistence

One of the assumptions of the pure MC approach is that rating transitions in  $t$  only depend on the  $t - 1$  state which presumes that there is no path dependence over the long period. In our study we find out whether, to the contrary, ratings follow a process whose direction could be differently explained with the purpose to optimize the probability estimates of transition changes.

The analysis is based on the notion of persistence, defined as the random variable describing the time that individuals spend in every state. It is possible to prove that a single company following a pure MC process and being in  $i$  at time  $t$ , will leave such state after a random time  $T$ , which is distributed as an exponential  $r, v$ , with parameter  $-q_{ii}$  (Grimmett et al. 1992). This means that the probability to leave  $i$  after a time  $T > s$  is given by  $\exp(-q_{ii}s)$ . From the properties of the exponential distribution, the mean persistence time in  $i$  is given by  $-\frac{1}{q_{ii}}$ .

The difference of time persistence within rating notches gives rise to the question about the existence of a path which is designed over time which contradicts the pure Markovian assumption that any rating movement does not depend on time, but on the previous observation. We evaluate the upgrading probability. In other terms, we estimate the probability for firms being in a rating lower than A in 1999, to change their credit quality and to become classified in better rating notches (Figure 1).

Unsurprisingly, the upgrading probability is much higher the lower the start rating is. The figure suggests that, starting from rating E or F, the probability of a firm to reach a better state is around 70 and 75 percent after five years respectively, upon which it tends to be smooth. This is a non-trivial finding. It means that firms have a higher probability to improve their credit quality in the first phases of their life. In particular, companies rated E and F are expected to improve with a high probability (around 70%) within their 4–5 first years. Afterwards, the probability declines. This result underlines the issue about the pure Markovian hypothesis, which is, in this case, adequately solved applying the MS approach particularly for the existence of the absorbing state.



**Figure 1.** Upgrading probability by time. The vertical axis shows the probability, the horizontal axis shows time in years. The five curves compare the upgrade path probability to a better rating notch.

#### 4.6. Mobility Measurements

In Section 4.5, we showed that the way ratings change over time is not adequately captured by the pure MC model. In order to measure this more fittingly, first the wavering of single firms and, second the prevailing trajectory towards an upgrade or a downgrade rating notch, we introduce mobility indices, defined as functions assigning to every transition matrix  $P$  a real value  $I(P)$ . This addresses the rating dynamics which transition matrices should exhibit. The aim is to obtain a univariate measure allowing us to compare two different matrices. In this sense, the lower is  $I(P)$ , the lower the mobility of individuals is which move according to  $P$ .

We apply two mobility indices: the trace index, proposed in Shorrocks (1978) divided by  $\frac{k-1}{k}$  to obtain values in  $[0, 1]$ , defined in Equation (10):

$$I_{tr}(P) = \frac{1}{k} \left( k - \sum_{i=1}^k p_{ii} \right) \tag{9}$$

and the directional index, introduced in Ferretti and Ganugi (2013) as in Equation (11):

$$I_{dir}(P) = \frac{1}{Z} \sum_{i=1}^k \omega_i \sum_{j=1}^k p_{ij} \cdot \text{sign}(j - i) \cdot v(|j - i|) \tag{10}$$

where  $(\omega_1, \dots, \omega_k)$  is a vector of weights to be attributed to the states (labeled with  $1, \dots, k$ ),  $\text{sign}(x)$  is the sign function, equal to  $-1$  if  $x < 0$ ,  $+1$  if  $x > 0$  and  $0$  if  $x = 0$ , and  $v$  is a function to measure the magnitude of the jumps from the  $i$ -th to the  $j$ -th state,  $Z$  is a normalizing constant to have values among  $-1$  and  $+1$ . Since the directional index is defined supposing that states are ordered from the worst to the best ones, while ratings are in the opposite order, we have changed the sign of the index.

The trace index and the directional index measure two different features of mobility: the former one assumes values in  $[0, 1]$  and describes the turbulence of individuals, whereas the latter also measures the prevailing direction towards an upgrade or a downgrade of firm ratings.



Another important feature of both the aforementioned indices is that they can be decomposed as a sum:

$$I(P) = \sum_{i=1}^k \omega_i I_i(P) \quad (11)$$

where  $I_i(P)$  measures the mobility of individuals starting from the  $i$ -th state (and  $\omega_i = 1/k$  for the trace index), providing then additional information about the whole mobility.

In Table 9 we show the 1-year (panel 1) and 10-year (panel 2) transition matrices for both the Movers and the whole set of firms estimated with the MS model. We also exhibit the results of the global mobility indices. Firstly, we evaluate the mobility for every state, and the same indices on the whole matrices (setting  $(\omega_1, \dots, \omega_k) = d_{1999}$  and  $v(|j-i|) = |j-i|$  for the directional index).

**Table 9.** Mobility indices by starting rating and global mobility indices.

<b>Panel 1: Mobility indices by starting rating (and global), for the annual estimated transition matrices <math>\hat{M}</math> and <math>\hat{P}</math> (percentages)</b>							
<b>Rating</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>Default</b>
$I_{dir}(\hat{M})$	−6.32	−1.65	0.68	5.35	13.76	25.98	0
$I_{tr}(\hat{M})$	23.56	38.82	39.33	33.66	60.96	72.07	0
$I_{dir}(\hat{P})$	−4.66	−1.63	0.67	5.12	13.32	25.93	0
$I_{tr}(\hat{P})$	17.38	38.37	38.68	32.24	59.03	71.94	0
Global Mobility Indices		$I_{dir}(\hat{M})$ −2.54	$I_{tr}(\hat{M})$ 38.34	$I_{dir}(\hat{P})$ −1.67	$I_{tr}(\hat{P})$ 36.8		
<b>Panel 2: Mobility indices by starting rating (and global), for the 10-years estimated transition matrices <math>\hat{M}_{10}</math> and <math>\hat{P}_{10}</math> (percentages)</b>							
<b>Rating</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>Default</b>
$I_{dir}(\hat{M})$	−29.36	−17.42	−3.11	18.43	31.89	37.23	0
$I_{tr}(\hat{M})$	72.20	77.06	77.56	80.74	97.32	99.42	0
$I_{dir}(\hat{P})$	−21.65	−17.21	−3.06	17.65	30.88	37.17	0
$I_{tr}(\hat{P})$	53.25	76.17	76.28	77.32	94.24	99.25	0
Global Mobility Indices		$I_{dir}(\hat{M}_{10})$ −16.71	$I_{tr}(\hat{M}_{10})$ 72.04	$I_{dir}(\hat{P}_{10})$ −12.56	$I_{tr}(\hat{P}_{10})$ 68.07		

The mobility of Movers is always higher (in absolute value) than the whole mobility (since Stayers tend to slow down the movements). We also note that the turbulence in the dynamics of firms is quite high, but the global directional mobility tends to be negative. This means that, on average, companies are more likely to be downgraded. This is true not only for A-rated companies, but also for firms that were originally B- and C-rated.

## 5. Conclusions

Through conducting the research in this paper, we reach two relevant conclusions. First, we find that the rating trajectory cannot be estimated with a pure Markov chain without incurring the risk of an absorbing state which stands for a bankruptcy. Therefore, banks are over-estimating their credit risk resulting in excessive regulatory capital. This may have important macroeconomic implications since holding a large capital buffer is costly for banks and this in turn influences their ability to lend. This conclusion is particularly true during economic downturns with the consequence of exacerbating the

cyclicality in risk capital which therefore further aggravates economic conditions. These implications are confirmed also when we compared the pure Markov chain with the Mover-Stayer model. Our analysis shows that not only is the default not an absorbing state, as assumed in the pure Markov chain model, but also that the equilibrium distribution could be more adequately estimated through the MS approach. Our analysis supports the idea that the MS approach, on average, is more efficient because it provides a lower error than the Markov chain method. The results suggest that separate transition matrices should be applied for small and medium enterprises, and that credit risk is statistically and economically overestimated by the pure MC approach relative to the MS one. This implies that, if employed for SMEs, the capital charges prescribed by the Markov chain approach are higher than that drawn from the MS model, suggesting an increase of cyclicality especially during downturns. Promoting the estimation of transition matrices through the MS method should be encouraged, in order to adjust the procyclicality induced by the use of point-in-time ratings. In this light, we note that the relevant results we obtain through the MS model are a first step in this research field and they are not exhaustive. We indeed recognize that personal attributes such as the economic sectors in which firms operate may be relevant in explaining, for example, the probability to be a Stayer. Nevertheless, the resulting model is not trivial from a methodological point of view, and it has been only recently solved by [Frydman and Matuszyk \(2018\)](#) and [Frydman et al. \(2019\)](#). Further research on this topic will be relevant for future developments.

Second, we show that the immediate off-diagonal elements of the transition matrices for SMEs confirm the ineffectiveness of the naïve transition matrix to estimate credit migration when we refer to small and medium firms. Indeed, upgrades are generally larger than downgrades, suggesting greater capital requirements in a one-year horizon when naïve transition matrices are applied. The mobility indexes outcomes show that in the case of SMEs, the trace index increases along with the probability of default, both when we take into consideration only movers and when we add stayers. These findings confirm that asset correlations increase in the worst rating notches ([Gabbi and Vozzella 2013](#)). When designed by regulators, the supervisory formula for concentration risk aimed to combat pro-cyclicality with a negative link between asset correlations and probabilities of default. Our paper shows that this mis-calibration could increase the pro-cyclical impact of the use of the wrong transition matrices. The policy implication of our study is that regulators should take into consideration alternative approaches to pure Markovian solutions for portfolio models.

Our findings also explain part of the misevaluation of borrowers and the actual relevant weight of non-performing loans within banking portfolios: in stable periods, the approximation of pure-Markov transition matrices increases the transaction costs and information costs involved with the lending activity. Transaction costs include imperfect foresights, bounded rationality and their economic consequences. The Mover-Stayers approach helps to reduce the rough calculation of historical movements of borrowers' rating and, consequently, the efficacy of their allocative process, and the expected industry stability with a better capital absorption ([Gabbi and Levich 2019](#)). It remains to be analyzed how MS behave in shock phases. Our results show that a structural shock in the credit market can alter the predictive capacity of the migration of MS models companies, with the risk of reducing the quality of the portfolio models that adopt them. As proved by [Fei et al. \(2012\)](#) and [Boreiko et al. \(2018\)](#), estimations through the cycles can provide interesting outcomes.

**Author Contributions:** Conceptualization, P.G.; methodology, C.F.; software, P.V.; validation, F.S.; formal analysis, G.G.; investigation, P.V.; resources, P.V.; data curation, P.V.; writing—original draft preparation, C.F.; writing—review and editing, P.V.; visualization, C.F.; supervision, G.G. and P.G.

**Funding:** This research received no external funding.

**Conflicts of Interest:** The authors declare no conflict of interest.

## References

- Altman, Edward I. 1998. The importance and subtlety of credit rating migration. *Journal of Banking and Finance* 22: 1231–47. [\[CrossRef\]](#)
- Altman, Edward, and Duen Li Kao. 1992a. Rating Drift of High Yield Bonds. *Journal of Fixed Income* 14: 15–20. [\[CrossRef\]](#)
- Altman, Edward, and Duen Li Kao. 1992b. The Implications of Corporate Bond Ratings Drift. *Financial Analysts Journal* 48: 64–75. [\[CrossRef\]](#)
- Altman, Edward I., Robert G. Haldeman, and Paul Narayanan. 1977. Zeta-analysis. A new model to identify bankruptcy risk of corporations. *Journal of Banking and Finance* 1: 29–54. [\[CrossRef\]](#)
- Angilella, Silvia, and Sebastiano Mazzù. 2019. A credit risk model with an automatic override for innovative small and medium-sized enterprises. *Journal of the Operational Research Society* 70: 1784–800. [\[CrossRef\]](#)
- Baas, Timo, and Mechthild Schrooten. 2006. Relationship banking and SMEs: A theoretical analysis. *Small Business Economics* 27: 127–37. [\[CrossRef\]](#)
- Bandyopadhyay, Arindam, Tasneem Chherawala, and Asish Saha. 2007. Calibrating asset correlation for Indian corporate exposures. Implications for regulatory capital. *The Journal of Risk Finance* 8: 330–48. [\[CrossRef\]](#)
- Bangia, Anil, Francis X. Diebold, and Til Schuermann. 2002. Ratings migration and the business cycle, with application to credit portfolio stress testing. *Journal of Banking and Finance* 26: 445–74. [\[CrossRef\]](#)
- Basel Committee on Banking Supervision. 2009. *Guidelines for Computing Capital for Incremental Risk in the Trading Book*. Basel: Bank of International Settlements.
- Basel Committee on Banking Supervision. 2011. *Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems—Revised Version*. Basel: Bank of International Settlements.
- Berger, Allen N., and Gregory F. Udell. 1995. Relationship lending and lines of credit in small firm finance. *Journal of Business* 1995: 351–81. [\[CrossRef\]](#)
- Berger, Allen N., and Gregory F. Udell. 2002. Small Business Credit Availability and Relationship Lending: The Importance of Bank Organizational Structure. *The Economic Journal* 112: 32–53. [\[CrossRef\]](#)
- Blume, Marshall E., Felix Lim, and A. Craig Mackinlay. 1998. The declining credit quality of U.S. corporate debt: Myth or reality. *Journal of Finance* 53: 1389–1413. [\[CrossRef\]](#)
- Blumen, Isadore, Marvin Kogan, and Philip McCarthy J. 1955. *The Industrial Mobility of Labor as a Probability Process*. Ithaca: Cornell University Press.
- Boot, Arnoud. 2000. Relationship lending: What do we know? *Journal of Financial Intermediation* 9: 7–25. [\[CrossRef\]](#)
- Boreiko, Dmitri, Serguei Kaniovski, Yuri Kaniovski, and Georg C. Pflug. 2018. Business Cycles and Conditional Credit-Rating Migration Matrices. *Quarterly Journal of Finance* 8: 1–19. [\[CrossRef\]](#)
- Carty, Lea V. 1997. *Moody's Rating Migration and Credit Quality Correlation, 1920–1996*. New York: Moody's Investor Service.
- Carty, Lea V., and Jerome S. Fons. 1993. *Measuring Changes in Corporate Credit Quality*. New York: Moody's Investor Service.
- Cipollini, Fabrizio, Camilla Ferretti, Piero Ganugi, and Mario Mezzanzanica. 2013. *A Continuous Time Mover Stayer Model for Labor Market in a Northern Italian Area, Classification and Data Mining*. Edited by Giusti Antonio, Ritter Gunter and Vichi Maurizio. Berlin and Heidelberg: Springer, pp. 181–88.
- Cole, Rebel A., Lawrence G. Goldberg, and Lawrence J. White. 2004. Cookie cutter vs. character: The micro-structure of small business lending by large and small banks. *Journal of Financial and Quantitative Analysis* 39: 227–51. [\[CrossRef\]](#)
- D'Amico, Guglielmo, Jacques Janssen, and Raimondo Manca. 2016. Downward migration credit risk problem: A non-homogeneous backward semi-Markov reliability approach. *Journal of the Operational Research Society* 67: 393–401. [\[CrossRef\]](#)
- Das, Sanjiv, and Peter Tufano. 1996. Pricing credit sensitive debt when interest rates, credit ratings and credit spreads are stochastic. *The Journal of Financial Engineering* 5: 161–98.
- Degryse, Hans, and Patrick Van Cayseele. 2000. Relationship lending within a bank-based system: Evidence from European small business data. *Journal of Financial Intermediation* 9: 90–109. [\[CrossRef\]](#)
- Dietsch, Michel, and Joël Petey. 2004. Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs. *Journal of Banking & Finance* 28: 773–88.

- Elsas, Ralf, and Jan Pieter Krahen. 1998. Is relationship lending special? Evidence from credit-file data in Germany. *Journal of Banking & Finance* 22: 1283–316.
- Fei, Fei, Ana-Maria Fuertes, and Elena Kalotychou. 2012. Credit Rating Migration Risk and Business Cycles. *Journal of Business Finance and Accounting* 39: 229–63. [CrossRef]
- Ferretti, Camilla, and Piero Ganugi. 2013. A new mobility index for transition matrices. *Statistical Methods and Applications* 2: 403–25. [CrossRef]
- Figlewski, Stephen, Halina Frydman, and Weijian Liang. 2006. Modeling the Effect of Macroeconomic Factors on Corporate Default and Credit Rating Transitions. Available online: <http://dx.doi.org/10.2139/ssrn.934438> (accessed on 29 October 2018).
- Fougère, Denis, and Thierry Kamionka. 2003. Bayesian inference for the Mover-Stayer model of continuous time. *Journal of Applied Economics* 18: 697–723. [CrossRef]
- Frydman, Halina, and Ashay Kadam. 2002. Estimation in the continuous time Mover Stayer model with an application to bond rating migration. *Applied Stochastic Models in Business and Industry* 20: 155–70. [CrossRef]
- Frydman, Halina, and Anna Matuszyk. 2018. Estimation and status prediction in a discrete Mover-Stayer model with covariate effects on stayers probability. *Applied Stochastic Models in Business and Industry* 34: 196–205. [CrossRef]
- Frydman, Halina, Jarl G. Kallberg, and Duen-Li Kao. 1985. Testing the adequacy of Markov Chain and Mover-Stayer models as representation of credit behavior. *Operations Research* 33: 1203–14. [CrossRef]
- Frydman, Halina, Anna Matuszyk, Chang Li, and Weicheng Zhu. 2019. Mover Stayer model with covariate effects on stayer's probability and mover's transitions. *Applied Stochastic Models in Business and Industry* 35: 1171–84. [CrossRef]
- Fuertes, Ana-Maria, and Elena Kalotychou. 2007. On sovereign credit migration: A study of alternative estimators and rating dynamics. *Computational Statistics & Data Analysis* 51: 3448–69.
- Gabbi, Giampaolo, and Richard Levich. 2019. Controlling risks to ensure financial stability and reducing volatility. *Journal of International Financial Management and Accounting* 30: 183–87. [CrossRef]
- Gabbi, Giampaolo, and Andrea Sironi. 2005. Which factors affect corporate bonds pricing? Empirical evidence from eurobonds primary market spreads. *The European Journal of Finance* 11: 59–74. [CrossRef]
- Gabbi, Giampaolo, and Pietro Vozzella. 2013. Asset Correlation and Bank Capital Adequacy. *European Journal of Finance* 19: 55–74. [CrossRef]
- Gabbi, Giampaolo, Matthias Massimo, and Giammarino Michele. 2019. Modeling Hard and Soft Facts for SMEs. Some International Evidence. *Journal of International Financial Management and Accounting* 30: 203–22.
- Golub, Gene, and Charles F. Van Loan. 1996. *Matrix Computation*, 3rd ed. Baltimore: Johns Hopkins University Press.
- Gómez-González, José E., and Nicholas M. Kiefer. 2009. Evidence of non-Markovian behavior in the process of bank rating migrations. *Cuadernos de Economía Latin American Journal of Economics* 46: 33–50. [CrossRef]
- Grimmett, Geoffrey, Geoffrey R. Grimmett, and David Stirzaker. 1992. *Probability and random Processes*. Oxford: Clarendon Press.
- Harhoff, Dietmar, and Timm Körting. 1998. Lending relationships in Germany—Empirical evidence from survey data. *Journal of Banking & Finance* 22: 1317–53.
- Israel, Robert, Jeffrey S. Rosenthal, and Jason Z. Wei. 2001. Finding generators for Markov Chains via empirical transition matrices, with application to credit ratings. *Mathematical Finance* 11: 245–65. [CrossRef]
- Ivashina, Victoria. 2009. Asymmetric information effects on loan spreads. *Journal of Financial Economics* 92: 300–19. [CrossRef]
- Jarrow, Robert A., David Lando, and Stuart M. Turnbull. 1997. A Markov model for the term structure of credit risk spreads. *Review of Financial Studies* 10: 481–523. [CrossRef]
- Kremer, Alexander, and Rafael Weißbach. 2013. Consistent estimation for discretely observed Markov jump processes with an absorbing state. *Statistical Papers* 54: 993–1007. [CrossRef]
- Lando, David. 1998. On Cox processes and credit risky securities. *Review of Derivatives Research* 2: 99–120. [CrossRef]
- Lando, David, and Torben M. Skødeberg. 2002. Analyzing rating transitions and rating drift with continuous observations. *Journal of Banking and Finance* 26: 423–44. [CrossRef]
- Lin, Chen, Yue Ma, Paul Malatesta, and Yuhai Xuan. 2011. Ownership structure and the cost of corporate borrowing. *Journal of Financial Economics* 100: 1–23. [CrossRef]

- Lucas, Douglas J., and John G. Lonski. 1992. Changes in corporate credit quality 1970–1990. *The Journal of Fixed Income* 1: 7–14. [[CrossRef](#)]
- Nickell, Pamela, William Perraudin, and Simone Varotto. 2000. Stability of ratings transitions. *Journal of Banking and Finance* 24: 203–27. [[CrossRef](#)]
- Petersen, Mitchell A., and Raghuram G. Rajan. 1994. The benefits of lending relationships: Evidence from small business data. *The Journal of Finance* 49: 3–37. [[CrossRef](#)]
- Petersen, Mitchell A., and Raghuram G. Rajan. 1995. The effect of credit market competition on lending relationships. *The Quarterly Journal of Economics* 110: 407–43. [[CrossRef](#)]
- Shorrocks, Anthony F. 1978. The measurement of Mobility. *Econometrica* 46: 1013–24. [[CrossRef](#)]



© 2019 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).