

## Article

# Research on the Optimization Method of Safety Input Structure in Coal Mine Enterprise

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**Abstract:** In order to study the application of the Cobb-Douglas production function on the optimization of safety inputs and further reduce accident losses, two safety input structures of a coal mine enterprise were constructed using literature, and the weight order of each safety input indicator was determined by the entropy weight method (EWM) and the analytical hierarchy process (AHP). The Cobb-Douglas production function was used to calculate the accident loss function of the safety input structure, and the accident loss function was obtained by multiple regression analysis. The optimal configuration of safety inputs was obtained by fitting the accident loss function. Finally, the optimal loss and mean squared error (MSE) of the corresponding functions of the two safety input structures were compared. The results show that the optimal configuration of Safety Input Structure 2 is better than that of Safety Input Structure 1, and the MSE of Safety Input Structure 2 is less than that of Safety Input Structure 1. The research results demonstrate that coal enterprises can find more significant indicators by refining the safety input structure and increasing monetary resources for more crucial indicators of safety input to effectively minimize accident loss and boost economic benefits, and to test the quality of safety input structures' regression function using MSE.



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**Keywords:** safety input; Cobb-Douglas production function; structure optimization; safety input structure; comprehensive empowerment

## 1. Introduction

Due to the continued expansion of China's economy and society, personal safety, particularly the life and health of enterprise workers, has received increasing attention in recent years. Coal, which accounts for more than half of China's total energy supply, will continue to be the primary energy source for a long time. Various safety accidents occur occasionally in large-scale coal mines. According to statistics, the total number of coal mine accident fatalities in China is 1–2 times higher than that of other coal-producing countries. The coal mine fatality rate per million tons in 2017 was 0.06, which is twice that of the United States and 1.5 times that of Germany [1,2]. One of the most essential approaches to managing the frequency of accidents successfully is to ensure sufficient safety input. Because the development of businesses is always focused on lowering costs and increasing benefits, the money businesses invest in safety is limited. Safety input is characterized by latency and invisibility [3]. As a result of limited safety funds, how to enhance the utilization rate of safety funds, maximize the protection of employees' lives and maintain normal corporate production is a subject to be explored.

Numerous researchers have conducted extensive studies of the utilization of safety input funds. Some researchers established a safety input model and used data analysis of the safety input–output function or the safety input–output relative outcomes of each scheme to determine the ideal safety solution. For example, Jiang et al. [2] used a type of entropy-close to ideal solution (TOPSIS) to establish a safe input scheme evaluation model and calculate the safety input's best configuration. Song et al. [4] used the order relation method and expert scoring method to determine the weight, and the

Cobb-Douglas (C-D) production function to construct the safety input–output function relationship. Guo et al. [5] comprehensively worked out the optimal scheme of safety input allocation by using grey correlation degree analysis, partial correlation analysis and the C-D production function model. Based on the literature, Ye et al. [6] identified widely used safety input indicators and used data envelopment analysis to build a safety output efficiency assessment model and evaluate the efficiency methods of various safety inputs and outputs. Various academics have also proposed some strategies for identifying safety input indicators: Wang et al. [7] used cluster analysis to pick the primary indications of safety input. By studying the rationale and accuracy of the assessment indicators of safety input, Li et al. [8] created an evaluation indicator system comprising three Level Two and 16 Level Three indicators. From the standpoint of a safety system, Zhang et al. [9] built a safety input assessment indicator system. Xin [10] converted the safety decision in engineering to a mathematical problem, establishing three robust optimization models using robust optimization technology, thereby providing safety input decision-making models for different enterprise risk scenarios. Chao [11] presented the economic optimization model and relative optimization algorithm based on CBA and CEA for the process industry. Roy and Gupta [12] created a nonlinear optimization model for safety input. Son et al. [13] established a safety input optimization model that aims to minimize the overall safety input expenditure of coal mining enterprises. Compared with foreign countries' research on industry enterprises, many studies of coal mine enterprises' safety input have been conducted in China. Although research on safety input has reached a relatively mature stage in China [14], China lags far behind the United States in terms of coal mine safety [15]; there is currently less relevant study of the influence of various safety input models on safety output.

To sum up, the current studies on the optimization of safety input structure have all established only one indicator structure [2,4–6,10–13] for optimization. In order to better increase the efficiency of safety inputs, this research uses two safety input structures for comparison for the first time. The two safety input structures are obtained by combining references to previous security input construction methods and models [2,16–18]. The research objectives are twofold: in contrast to the current situation, where only one indicator structure has been used, two safety input structures were used to verify that different indicator structures make different optimization assignments for safety input. While the C-D production function was used to construct a function model, the Mean Squared Error (MSE) of the two obtained regression functions were compared for the first time to illustrate that when selecting safety input indicators for modeling, the regression function that minimizes the MSE should be selected. The regression function with the lower MSE is not only closer to reality, but also obtained a better safety input assignment. The research discusses that when the C-D production function is used to model and the accident loss function equation is obtained by multiple linear regression, if the safety input indicator structure can be refined, the optimal solution of the accident loss function is lower and the coal mine enterprise's safety input efficiency also can be raised to a higher extent.

Thus, the analytic hierarchy process and the entropy weight method (AHP-EWM) is used to assign weight comprehensively to each indicator, and the C-D production function is applied to construct the accident loss function model. Finally, the optimal accident losses of functions are compared and the fitness quality of accident loss functions is examined using MSE. While constructing accident loss functions, the preventive safety input is assumed to be constant, minimizing the accident losses of the enterprise by changing the assignment of each safety input in the safety input structure. Simultaneously, two safety input structures of the same coal mine enterprise are constructed according to the various hierarch and the number of indicators of the preventive safety input, and the optimal accident loss and MSE of the two safety input structures are compared under the same preventive safety input expenditure.

## 2. Construction of Safety Input Structure of Coal Mine

### 2.1. Establishment of a Hierarchical Structure of Safety Input in Coal Mine Enterprises

At the moment, there is no consistent definition of safety input, and there are variances in the separation of various elements of safety input. Safety input [8,16] refers to the total amount of money spent by a company to ensure the safety and health of its personnel during the manufacturing process. Safety input [8,19] refers to the total of a country's or enterprise's safety-related costs, such as expenditures on safety measures, personal protective equipment and occupational illness prevention, among other things. The human, material and financial resources invested to manage hazard sources in the manufacturing process, prevent possible accidents and provide safe production conditions are referred to as safety input [8,20]. The term "safety input" [2] refers to the amount of human, material and financial resources expended during the production and operation processes to assure the safe production of enterprises, eliminate possible mishaps and lower the fatality rate. The State Administration of Work Safety has split the safety expenditures of coal enterprises into 10 categories in the Measures for the Administration of Extraction and Use of Enterprise Safety Production Expenses. According to Duan et al. [17], the safety expenditures of coal firms are divided into two categories: safety input and accident loss. In terms of enterprise safety input, coal mining businesses may be classified into a variety of safety input variables based on the current condition. Professor Mei [16] classified safety input into five categories: safety technology, industrial cleanliness, safety education, personal protective equipment, daily management and labor expenses. When researching the logic of safety input structure, Zhao et al. [18] separated safety input into first-level indicators and second-level indicators, with the first-level indicators including personnel, safety technology and safety management input.

In conjunction with existing safety input structure and references, the research constructed the evaluation indicator system of coal mine safety inputs using Jiang et al. [2] and the TOPSIS method based on the entropy weight of coal mine safety input decision analysis. The safety input of coal mining enterprises is divided into five major categories—safety science and technology, safety engineering, safety equipment, safety management and safety education and training—and each category can be further subdivided into one or more safety indicators. Enterprise safety input has an effect on an enterprise's safety output to a certain extent, which can include impairment and value-added output. The former relates to accident economic damage, while the latter refers to coal yield. To clearly demonstrate the link to the resulting safety input structure according to the nature of various kinds of safety inputs, the inputs of safety management and safety education and training were integrated into the input in people, and the input of safety technology, safety engineering and safety equipment were integrated into the input in objects.

### 2.2. Construction of Safety Input Structure

Based on the definition of coal mine safety input and safety cost in Duan et al. [17], given the definition of safety cost and the safety standards of the benchmark, the safety related costs incurred include two parts: preventive safety input and accident loss. The cost of safety input (which is also called total safety input) can be divided into preventive safety input and loss of safety input.

Preventive safety input, also known as safety input, refers to the safety input before the occurrence of an accident and the loss of safety input refers to the total loss caused by an enterprise accident, also known as accident loss. Total safety input includes both loss of safety input and preventive safety input. The following two safety input structures were constructed using the above hierarchical structure of safety input. Safety Input Structure 1 is more detailed than Safety Input Structure 2 and has three more safety input indicators, as shown in Figures 1 and 2.

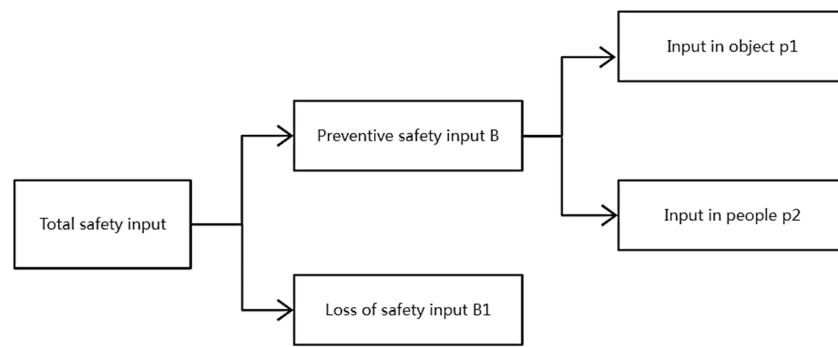


Figure 1. Safety Input Structure 1 of coal mine enterprise.

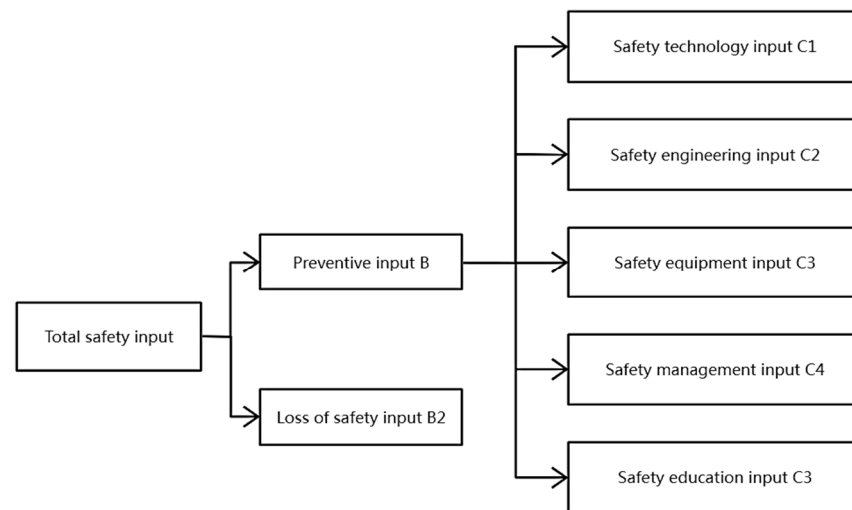


Figure 2. Safety Input Structure 2 of coal mine enterprise.

### 3. Method of Weight Determination of Safety Input Index

#### 3.1. Weight Determination by the Analytic Hierarchy Process

##### (1) Construction of a safety input structure comparison matrix

The safety input fund allocation generates countless solutions, and each indicator of safety input is greater than 0. The evaluation indicators of safety input were determined by safety input hierarchy. For each evaluation indicator, quantitative comparison to the expert scoring method was used to establish comparison matrixes, using mutual comparisons of two indicators to determine the relative degree of importance of each evaluation indicator with a structure comparison matrix of  $n \times n$ . The relative importance value is shown in Table 1.

$$a_{ij} = i\text{'s relative importance} / j\text{'s relative importance}, \tag{1}$$

Table 1. Relative importance table.

Scale $a_{ij}$	Implication
1	factor i is as important as factor j
3	factor i is slightly more important than factor j
5	factor i is significantly more important than factor j
7	factor i is mightily more important than factor j
9	factor i is extremely more important than factor j
2, 4, 7, 8	the median of the above two adjacent judgments
Reciprocal value	when scale $a_{ij}$ is n, scale $a_{ji}$ is $1/n$

## (2) Indicator weight and consistency test

The normalization of the feature vector of the comparison matrix's maximum characteristic root  $\lambda_{\max}$  is designated as  $W$ , and the element  $W$  is the ordering weight of the elements at the same level to the relative importance of a component at the next level. This is known as hierarchical single ordering. The consistency indicator is calculated using the following formula:

$$CI = (\lambda_{\max} - n) / (n - 1), \quad (2)$$

where  $n$  denotes the number of matrix evaluation indicators. Based on the number of indicators in the judgment matrix, look for the average random consistency  $RI$  in Table 2.

**Table 2.** Average stochastic consistency indicator  $RI$ .

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49	1.52	1.54	1.56	1.58	1.59

The consistency ratio  $CR$  is calculated by the following formula:

$$CR = CI / RI, \quad (3)$$

It is generally believed that when the consistency ratio is:

$$CR < 0.1, \quad (4)$$

it indicates standard consistency, and its normalized feature vector can be used as the weight vector through the consistency test. Otherwise, the comparison matrix should be reconstructed and the value of  $a_{ij}$  adjusted.

### 3.2. Weight Determination by the Entropy Weight Method

The entropy weight method (EWM) is a thorough, objective approach to weight assignment that determines the indicator weight based on current data while minimizing the variation caused by subjective assignment. Entropy is a measure of uncertainty in information. The lower the entropy number, the more information and weight there is. The specific steps of this model are as follows:

(1) Data standardization. Standardized processing of data into dimensionless data.

Positive indicators:

$$x_{ij} = \frac{x_{ij} - \min\{x_{1j} \cdots x_{mj}\}}{\max\{x_{1j} \cdots x_{mj}\} - \min\{x_{1j} \cdots x_{mj}\}} \quad (5)$$

Matrix:

$$\begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix}$$

where,  $x_{ij}$  is the value of the  $j$  indicator of the  $i$  sample,  $x'_{ij}$  is the value of the  $j$  indicator of the standardized  $i$  sample,  $n$  is the number of indicators and  $m$  is the number of samples.

(2) Calculate the entropy value of the  $j$  indicator.

$$P_{ij} = \frac{x'_{ij}}{\sum_{i=1}^m x'_{ij}} \quad (6)$$

$$e_j = -k \sum_{i=1}^m P_{ij} \ln(P_{ij}) \quad (7)$$

$$j = 1, \dots, n$$

In the formula,  $P_{ij}$  is the proportion of  $x'_{ij}$  in  $i$  sample and

$$K = 1/\ln(n). \quad (8)$$

- (3) Calculate the information entropy of  $j$  indicator.

$$d_j = 1 - e_j \quad (9)$$

$$j = 1, \dots, n$$

In the formula,  $d_j$  is the information entropy of  $j$  indicator.

- (4) Determine the weight of each indicator  $\beta_j$ .

$$\beta_j = \frac{d_j}{\sum_{j=1}^n d_j} \quad (10)$$

$$j = 1, \dots, n$$

In the formula,  $\beta_j$  is the weight of  $j$  indicator.

### 3.3. AHP-EWM for Comprehensive Weight Assignment

The analytic hierarchy process (AHP) is highly subjective and based on specialists' practical experience. In contrast, EWM is based only on objective data, which has objective advantages but cannot be applied to the actual situation. To compensate for the lack of single weighting, the AHP-EWM are coupled to assure the dependability of evaluation outcomes. The comprehensive weight of AHP-EWM is calculated as:

$$W_j = \frac{\alpha_j \beta_j}{\sum_{j=1}^n \alpha_j \beta_j} \quad (11)$$

In the formula,  $\alpha_j$  is the weight of the  $j$  indicator that was calculated by the analytic hierarchy process;  $\beta_j$  is the weight of the  $j$  indicator that was calculated by the entropy weight method.

## 4. Establishment of the Accident Loss Model Based on the C-D Production Function

### 4.1. Cobb-Douglas Production Function

The C-D production function, proposed by American mathematician C.W. Cobb and American economist Paul H. Douglas in the 1930s, is widely used in economic quantitative analysis and can be used to analyze the relationship between input and output under certain conditions of time.

The original form of the C-D production function is as follows:

$$Y = AK^\alpha L^\beta \quad (12)$$

In the formula,  $Y$  is the total output,  $K$  is the total capital input,  $L$  is the total labor input,  $A$  is the constant determined by the technical conditions of production in a certain era,  $\alpha$  is the capital elasticity coefficient and  $\beta$  is the labor elasticity coefficient.

#### 4.2. Establishment of Accident Loss Models

When preventive safety input remains constant in a safety input model, total safety input depends on loss of safety input, and there is a certain functional relation between loss of safety input and each preventive safety input's safety input indicator. The word "model" means a set of relationships between two or more variables. These relationships can be expressed in terms of mathematical equations (34). The C-D production function [4,21,22] is used to simulate the functional relationship between safety input indicators and loss of safety input.

- (1) According to the Safety Input Structure 1 of coal mine enterprises in Figure 1 and C-D production function, the accident loss multiple regression model  $S_1$  was constructed [4,21]:

$$B_1 = F(p_1, p_2) = A_1 p_1^{\alpha_1} p_2^{\alpha_2} \quad (13)$$

In the above formula,  $p_1$  was the input in people and  $p_2$  was the input in objects;  $\alpha_1$  and  $\alpha_2$  were the capital elasticity coefficient corresponding to  $p_1$  and  $p_2$ , respectively; and  $B_1$  was the loss of safety input (accident loss) of Structure 1. At this point, Safety Input Structure 1's total safety input was  $B + B_1$ .

- (2) According to the Safety Input Structure 2 of coal mine enterprises in Figure 2 and C-D production function, the accident loss multiple regression model  $S_2$  was constructed [4,21]:

$$B_2 = F(x_1, x_2, x_3, x_4, x_5) = A_2 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} x_4^{\beta_4} x_5^{\beta_5} \quad (14)$$

In the formula above,  $x_1$  represented the input of safety technology,  $x_2$  represented the input of safety engineering,  $x_3$  represented the input of safety equipment,  $x_4$  represented the input of safety management and  $x_5$  represented the input of safety education.  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ ,  $\beta_4$  and  $\beta_5$  corresponded to the capital elasticity coefficient of safety science and technology input, safety engineering input, safety equipment input, safety management input and safety education input, respectively.  $B_2$  was the loss of safety input (accident loss) of Structure 2. At this time, the total safety input of Structure 2 was  $B + B_2$ .

#### 4.3. Optimization of Safety Input Fund Assignment

First, the data of the enterprise safety input and loss of safety input were processed into logarithmic form using Excel, and then multiple linear regression was performed using Matlab to determine the accident loss models  $S_1$  and  $S_2$  corresponding to Safety Input Structure 1 and 2. The functional relationship between each safety input indicator and the accident loss was obtained by multiple linear regression analysis [23]. The accident loss models  $S_1$  and  $S_2$  were obtained using the same statistics and modelling method. Between the two models, model  $S_2$  had more specific indicators than model  $S_1$ . We then took the mean value of the preventive safety input of the coal mine enterprise and inserted it in the obtained function to calculate the minimum accident loss under constraints, to determine the optimal assignment of safety input and to compare the minimum values of accident loss.

#### 4.4. Comparison of Accident Loss Regression Functions by MSE

Few studies have been conducted to validate the obtained functions for modeling using the C-D production function. MSE is a common metric for testing regression functions, and its value provides a visual indication of how well the function fits the actual data.

The mean squared error is the expected value of the squared difference between the parameter estimate and the true value of the parameter, denoted as MSE.

The MSE is calculated by the following formula:

$$MSE = \frac{1}{N} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (15)$$



## 5. Application Analysis of Method

### 5.1. Safety Input Data of a Coal Mine Mnterprise

Simulated and analyzed using the safety input details of a large state-owned coal mine enterprise, Table 3 [24] shows the detailed statistical data of different safety element inputs and the accident economic loss of the state-owned coal mine enterprise from 2001 to 2010.

**Table 3.** Safety input and safety output statistics of a coal mine enterprise from 2001 to 2010.

Year	Input in Safety Education and Training/ 10,000 RMB	Input in Safety Science and Technology/ 10,000 RMB	Input in Safety Management/ 10,000 RMB	Input in Safety Equipment/ 10,000 RMB	Input in Things/ 10,000 RMB	Input in People/ 10,000 RMB	Accident Economic Loss/ 10,000 RMB
2001	203.12	168.08	1257.04	1687.67	2166.21	1460.16	983.88
2002	223.58	186.78	1400.71	1596.82	2127.84	1624.29	962.44
2003	310.16	205.32	1270.13	1430.32	1989.37	1580.29	1066.96
2004	362.38	311.73	1390.75	1073.97	1841.85	1753.13	1016.27
2005	385.37	344.61	1645.56	1032.21	1838.2	2030.93	985.58
2006	391.85	358.95	1859.71	1010.93	1856.83	2251.56	889.35
2007	428.67	473.06	1768.85	1052.52	2015.16	2197.52	923.68
2008	496.03	489.33	1730.91	1349.57	2356.77	2226.94	880.27
2009	502.31	490.23	1859.95	1513.58	2523.33	2362.26	913.81
2010	509.38	495.95	1895.63	1965.58	2983.92	2405.01	882.39

### 5.2. Weight Determination of the Safety Input Indicators

- (1) According to the Safety Input Structure 1 and Safety Input Structure 2 of coal enterprises, the expert scoring method was adopted to establish the comparison matrix  $B_1-P_j$  and  $B_2-C_j$ . The obtained results are shown in Tables 4 and 5, and the consistency test was conducted. According to the expert scoring results, the weight was calculated by the geometric evaluation method.

**Table 4.** Comparison matrix of  $B_1-P_j$ .

$B_1$	$P_1$	$P_2$	$W_P$
$P_1$	1	2	2/3
$P_2$	1/2	1	1/3

**Table 5.** Comparison matrix of  $B_2-C_j$ .

$B_2$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$W_C$
$C_1$	1	1/3	1/5	1/7	1/2	0.0529
$C_2$	3	1	1/2	1/3	2	0.1547
$C_3$	5	2	1	1/2	3	0.2659
$C_4$	7	3	2	1	4	0.4322
$C_5$	2	1/2	1/3	1/4	1	0.0942

The consistency test revealed that:

$$CR_1 = 0, CR_2 = 0.008. \quad (16)$$

The consistency ratio  $CR_1$  and  $CR_2$  were less than 0.1, hence the comparison matrixes were consistent and the feature vectors equaled the weight vectors.

- (2) The weight was determined by the entropy weight method. The results of the entropy weight method assignment obtained by Python programming calculations are shown in Tables 6 and 7:



**Table 6.** Model 1 Entropy weight method assignment results.

	Input in Objects	Input in People
The weight of entropy weight method	0.6764	0.3236

**Table 7.** Model 2 Entropy weight method assignment results.

	Safety Science and Technology	Safety Engineering	Safety Equipment	Safety Management	Safety Education
The weight of entropy weight method	0.2009	0.1448	0.2911	0.2085	0.1547

- (3) AHP-EWM comprehensive empowerment: Python programming was used to calculate the final weights of each safety input indicator. The results are shown in Tables 8 and 9:

**Table 8.** Comprehensive empowerment results of Model 1.

	Input in Objects	Input in People
The comprehensive weight	0.8070	0.1930

**Table 9.** Comprehensive empowerment results of Model 2.

	Safety Science and Technology	Safety Engineering	Safety Equipment	Safety Management	Safety Education
The comprehensive weight	0.0494	0.1042	0.3598	0.4188	0.0677

In Safety Input Structure 1, the ranking of the relative importance of the comprehensive weight assignment was  $p_1 > p_2$ .

In Safety Input Structure 2, the ranking of the relative importance of the comprehensive weight assignment was safety management  $C_4 >$  safety equipment  $C_3 >$  safety engineering  $C_2 >$  safety education  $C_5 >$  safety science and technology  $C_1$ .

### 5.3. Analysis of the Extreme Value of the Accident Loss Function by Matlab Software

- (1) Before multivariate regression of the accident loss function, logarithms of Formulas (13) and (14) corresponding to Safety Input Structure 1 and Safety Input Structure 2, respectively, were taken and the results were as follows:

Model  $S_1$ :

$$\ln B_1 = \ln A_1 + \beta_1 \ln P_1 + \beta_2 \ln P_2 \quad (17)$$

Model  $S_2$ :

$$\ln B_2 = \ln A_2 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 \ln X_3 + \beta_4 \ln X_4 + \beta_5 \ln X_5 \quad (18)$$

- (2) The datasets from 2001 to 2010 were sorted into pairs by Excel, and multiple linear regression was performed on Equations (17) and (18) by Matlab programming [25]. After obtaining the parameters, the accident loss functions were obtained as follows:

Model  $S_1$ :

$$B_1 = 14338.64P_1^{-0.2496}P_2^{-0.107368} \quad (19)$$

Model  $S_2$ :

$$B_2 = 24247.7x_1^{-0.081823}x_2^{-0.1618}x_3^{-0.2544}x_4^{-0.3363}x_5^{-0.1191} \quad (20)$$

- (3) Taking accident loss minimization as the goal, the average safety input of 4159.157 million RMB over the 10-year period 2001–2010 and the minimum economic loss of

950.463 million RMB as the condition, the Matlab software was used to calculate the extreme values of Equations (19) and (20). The minimum values and corresponding safety input indicators values of the accident loss functions were determined. The results of the two models were calculated as follows:

Minimum value of  $B_1$  was 910.894 million RMB,  $P_1$  was 1251.200 million RMB and  $P_2$  was 2907.900 million RMB.

Minimum value of  $B_2$  was 679.770 million RMB,  $x_1$  was 356.00 million RMB,  $x_2$  was 729.400 million RMB,  $x_3$  was 111.140 million RMB,  $x_4$  was 144.557 million RMB and  $x_5$  was 516.600 million RMB.

The results indicated that  $B_1$ 's minimum value  $>$   $B_2$ 's minimum value.

The optimal results of comparing the two types of safety input structure indicates that the optimal accident loss of Safety Input Structure 2 was significantly less than the optimal accident loss of Safety Input Structure 1. Although these two optimization results were the optimal solution of two safety input structures that were modeled by the C-D production function, they were not necessarily the optimal solution of safety input in coal enterprises.

#### 5.4. MSE of Two Accident Loss Functions

The MSE of the two accident loss functions was calculated separately by the MSE formula and the results were as follows:

$$\text{Accident Loss Function 1's MSE of Model } S_1: \text{MSE}_1 = 1763.681538$$

$$\text{Accident Loss Function 2's MSE of Model } S_2: \text{MSE}_2 = 392.7999477$$

The  $\text{MSE}_1$  of function 1 was found to be significantly larger than that of function 2. If the MSE is less, the more the function reflects the input–output situation of production; that is, the production function should minimize the MSE. Therefore, the accident loss function 2 should be chosen, and the corresponding safety input indicator structure 2.

#### 5.5. Discussion

In the article, two safety input structures were constructed for the same coal mine, modeled using the C-D production function, and the analytic formula of the accident loss function was obtained by multiple regression analysis. The optimal outputs (least loss) and MSEs of the two accident loss functions were obtained and compared, respectively. The comparison reveals that the MSE of the loss function of the higher output (less loss) Safety Input Structure 2 is also smaller.

Generally, when constructing an accident loss function, we expect the extreme value of this function to be the least possible, which means that the company can save money on accident losses. The optimal accident loss for Safety Input Structure 2 was 679.770 million RMB and the optimal accident loss for Safety Input Structure 1 was 910.894 million RMB. When optimizing the safety input allocation of the company, we can give preference to Safety Input Structure 2. The  $\text{MSE}_2$  of Accident Loss Function 2 was 392.7999477 and the  $\text{MSE}_1$  of Accident Loss Function 1 was 1763.681538, yielding  $\text{MSE}_2 < \text{MSE}_1$ . The fitness of Accident Loss Function 2 was better than that of Accident Loss Function 1, and this result was consistent with the comparison of optimal accident losses.

To summarize: (1) When modeling with the C-D production function, the calculated optimal configuration of safety inputs will be different because of the different safety input structures constructed; (2) The quality of the fitness of the function can be checked by calculating the MSE of the function, and the function with a lower MSE value has a better fit and a better calculated optimal solution; and (3) At present, when using the C-D production function for the optimal solution, the safety input structure construction is considered less, and in the future, when using the C-D production function for modeling, it can be combined with structure optimization.

## 6. Conclusions

In this paper, the models  $S_1$  and  $S_2$  are compared. The two models use data from a coal mine enterprise to simulate the enterprise's accident loss function. In addition, AHP-EWM is employed to assign the weight of each safety input indicator of safety input structure, the C-D production function is utilized to simulate the coal enterprise accident loss function model and Matlab software is used for multiple regression analysis and solution. Due to the varied degrees of refinement of preventive safety input, the optimum solution of the accident loss model of safety input structure presents different results, hence the main conclusions can be summarized as follows.

Comparing safety input structure and two functions of the structure illustrates the more specific indicators, and more refined structures are better when modeling using the Cobb-Douglas model with limited data.

By comparing the MSE of two safety input structures and the optimal configuration of the corresponding accident loss function, we selected a better structure, which is safety input indicator structure 2. This indicates that Safety Input Structure 2 is an effective structure, so we can apply this method to identify the quality of the constructed structure.

Compared to modeling using only the C-D production function, the MSE can be used as a supplement to determine the degree of consistency of the regression function with regard to the actual situation, which provides a reference for the use of the C-D production function.

In the digital era in the future, enterprises can obtain specific internal safety input-related data and determine the best method of allocating safety funds through programming and modeling. Currently, technical conditions cannot be upgraded to improve the utilization of safety input funds effectively in order to help reduce accident losses.

The optimization of the coal firms' safety input structure still has to be strengthened. An in-depth study of the safety input structure of coal enterprises requires the use of more detailed safety input-related data. The statistics from a longer period of years can simulate more accurate accident loss function. In the future, coal enterprises can apply the refined safety input structure and the more accurate function to provide a scientific basis for the allocation and decision-making of safety input funds.

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