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Solving the Formation and Containment Control Problem of Nonlinear Multi-Boiler Systems Based on Interval Type-2 Takagi–Sugeno Fuzzy Models

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Abstract: An interval type-2 (IT-2) fuzzy control design method is developed to solve the formation and containment problem of nonlinear multi-boiler systems. In most practical industrial systems such as airplanes, vessels, and power plants, the boiler system often exists as more than one piece of equipment. An efficient control theory based on the leader-following multi-agent system is applied to achieve the control purpose of multiple boiler systems simultaneously. Moreover, a faithful mathematical model of the nonlinear boiler system is extended to construct the multi-boiler system so that the dynamic behaviors can be accurately presented. For the control of practical multi-agent systems, the uncertainties problem, which will deteriorate the performance of the whole system greatly, must be considered. Because of this, the IT-2 Takagi–Sugeno (T–S) fuzzy model is developed to represent the nonlinear multi-boiler system with uncertainties more completely. Based on the fuzzy model, the IT-2 fuzzy formation and containment controllers are designed with the imperfect premise matching scheme. Thus, the IT-2 fuzzy control method design can be more flexible for the nonlinear multi-boiler system. Solving the formation problem, a control method without the communication between leaders differs from the previous research. Since leaders achieve the formation objective, the followers can be forced into the specific range formed by leaders. Via the IT-2 fuzzy control method in this paper, not only can the more flexible process of the controller design method be developed to solve the uncertainties problem magnificently, but a more cost-effective control purpose can also be achieved via applying the lower rules of fuzzy controllers. Finally, the simulation results of controlling a nonlinear multi-boiler system with four agents are presented to demonstrate the effectiveness of the proposed IT-2 fuzzy formation and containment control method.

Keywords: nonlinear multi-boiler system; interval type-2 Takagi–Sugeno fuzzy model; imperfect premise matching; formation control; containment control

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1. Introduction

No matter if in an industrial system or a system related to people's daily life, boiler systems play a very important and indispensable role, especially for fossil power plants. The performance of the power unit will be affected by the dynamics of the boiler control system. On ships, the boiler system is also applied to the heating of fuel oil, water, and air [1,2]. Because of this, a better design method for the control of boiler systems, which consist of high nonlinearities, is still the most important issue in the industry [3,4]. To improve the efficiency of a boiler system, one of the most reasonable manners is to promote the control system [5]. However, the control method demands a well-constructed mathematical model to sufficiently represent the boiler system. The difficulties of modeling a boiler system arise not only because of the large range of variates considered for the particular application domain, such as the 300 MW boiler in [5], 200 MW boiler in [6], or 1000 MW boiler in [7], but because the system consists of strong nonlinearities and complex coupling effects between the components [8,9]. From then until now, there still are many researchers who have been devoting their efforts to establishing and identifying more reasonable mathematical models for boiler systems [10,11]. With the more and more appropriate models available, various control methods have recently been developed for boilers [12,13]. The main control objective for the boiler system is to maintain the value of output variables, including steam pressure, drum water level, and so on; even the steam production is changed all the time, corresponding to the demand of the electrical load or operators [14,15]. Nevertheless, the problems of nonlinearities, coupling effects, and uncertainties are inevitable in the control system, which will make the above control objective difficult to achieve.

In most practical industrial systems, there is not only one boiler equipped; frequently, many boilers work at the same time. Nowadays, common headers are the preferred method for boiler systems to achieve their objective [16,17]. The header will collect all the steam generated by each boiler unit and then exhaust the required applications. In other words, boiler systems need to have the ability to control many units to achieve the common objective. Compared to the scheme of controlling each unit individually, a control concept based on the multi-agent system has been proposed to accomplish the common objective of the whole system with multiple units more efficiently [18]. Multi-agent control systems have already been widely applied to practical systems such as robots, autonomous spacecraft, and remote satellites [19–21]. Distinguished by the purpose of a multi-agent system, the control issue can be further divided into containment control, formation control, or cooperative control [22–26]. The main concept of the containment control problem, which is usually solved by the leader-following control method, is to let the followers converge into the convex hull constructed by leaders, while the formation control problem has to do with arranging the agents of the multi-agent system in the desired formation. There are very few papers that focus on the control of multiple boilers system via the control theory based on the multi-agent [27]. However, the multi-agent control system can provide a more efficient and cost-effective method to let the common objective be satisfied with the communication flow between each boiler unit. In this paper, the containment and formation control method are applied to control the dynamics of the nonlinear multi-boiler system simultaneously. As far as we know, this control issue has not been discussed in the existing research.

For the control design problem of nonlinear systems, that is, the boiler systems considered in this paper which consist of strong nonlinearities, a well-known Takagi–Sugeno (T–S) fuzzy model has been developed as a powerful tool [28–30]. The feature of the T–S fuzzy model is that the dynamic behaviors of a nonlinear system can be captured by many linear fuzzy subsystems in the representation of if–then fuzzy rules. Therefore, various valuable linear control theories can be provided to solve the control and analysis problem of practical systems with nonlinearities [31–33]. In [34,35], the fuzzy controller method based on the T–S fuzzy model was also proposed for the nonlinear boiler system, and a good control performance was obtained. However, the type-1 T–S fuzzy model was applied in [34,35], which was not enough to represent the uncertainties in the system parameters or the membership function utilized in the model. Moreover, the uncertainties problem is expected to be more serious in the multi-agent system since the variables need to be exchanged between agents. For the control purpose of nonlinear multi-boiler systems, the output variables must be maintained at a certain magnitude. If there are uncertainties in the dynamics of each boiler unit, the purpose will be difficult to achieve and the performance of whole multi-boiler systems may be deteriorated. Thus, the interval type-2 (IT-2) T–S fuzzy model has been developed with the IT-2 membership function, which can represent the uncertainties in the nonlinear system more completely [36–38]. Based on the IT-2 T–S fuzzy model, the designed IT-2 fuzzy controller can better deal with the uncertainties problem. Moreover, the IT-2 fuzzy control for the nonlinear boiler system issue has not been discussed. In this paper, an IT-2 fuzzy controller design method is developed to solve the containment and formation problem of a nonlinear multi-boiler system.

The framework of this paper is provided as follows. Firstly, the description of the nonlinear multi-boiler system is presented referring to [39], which proposed a faithful mathematical model for the benchmark problem of controlling boiler systems. The stability analysis and IT-2 fuzzy controller design method can be developed via the conversion of state variables from the equilibrium point. Note that the formation problem can also be regarded as the requirement that the variable dynamics of the leader units of a nonlinear multi-boiler system are converged to a certain value. Different from the formation–containment protocol design method such as in [40,41], which can also perform well in the formation and containment problem of a multi-agent system, another perspective to solve the control problem is proposed in this paper. It is known that the leaders are always the agents farthest from each other in containment control problems, which have the highest probability of suffering from communication faults if the communication between leaders accomplishes the formation. On the other hand, via the design method in this paper, leaders can be placed individually to the desired location or formation by the IT-2 fuzzy state feedback controller. The method cannot only avoid the communication problem of leaders but can also reduce the complexity in the process of stability analysis. Since the formation of leaders is accomplished, the IT-2 fuzzy containment controller design method is proposed to let all the followers into the region formed by the leaders. Finally, the simulation results are presented to verify the effectiveness of controlling the multi-boiler system by the proposed IT-2 fuzzy formation and containment control design method.

Improving the fluency of this paper, the parameters utilized in the paper and the corresponding explanations are presented in Table 1. Moreover, the structure of this paper can be provided as follows. In Section 1, introductions of background and motivation are given. In Section 2, the system description of a nonlinear multi-boiler system with uncertainties and the IT-2 T–S fuzzy model is presented. Moreover, the formation and containment control problems for the system are stated. In Section 3, the formation and containment controller design and stability analysis methods are developed based on the IT-2 T–S fuzzy model. In Section 4, the simulation results of the nonlinear multi-boiler system are presented to demonstrate the effectiveness of the proposed control method. In Section 5, based on the simulation results, some conclusions are given.

Table 1. Nomenclature.

Parameters	Descriptions
$1, \ldots, \psi$	Numbers of leader agents (ψ agents in total)
$\psi+1, \ldots, \psi+\vartheta$	Numbers of follower agents (ϑ agents in total)
j_{mn}	Elements of m row and n column in adjacency matrix $\mathfrak{J}$
$\mathbf{I}_Q, \mathbf{I}_G$	Identity matrix with the dimension of leader and follower numbers
α, β	Rule numbers of fuzzy model and fuzzy controller
$\overline{\delta}_{\alpha\beta i_1 q}, \underline{\delta}_{\alpha\beta i_1 q}$	Upper and lower bound membership function-dependent scalars related to fuzzy rules α and β
$\mathbf{P}_\ell, \mathbf{P}_f$	Common positive definite matrix of Lyapunov function for the stability analysis of leader and follower agents
$\mathbf{W}_{\ell\alpha\beta}, \mathbf{W}_{f\alpha\beta}$	Semi-positive definite matrices for the stability analysis of leader and follower agents related to fuzzy rules α and β
$\mathbf{H}_\ell, \mathbf{H}_f$	Common symmetric matrix for the stability analysis of leader and follower agents

2. System Description and Problem Statement of Multi-Boiler Systems

The boiler system is an industrial system that is still widely applied in various fields. Thus, the stability analysis and control problem of the boiler become important issues. However, there is a lack of literature to accurately construct the mathematical model for the boiler. This is because the model of boilers is difficult to identify owing to the complex behaviors caused by nonlinearities, uncertainties, or even unmeasured disturbances. To solve the problem, a faithful control-oriented boiler mathematical model was developed in [39] by combining some previous results [42,43], physical laws, experiments of identification, and some knowledge related to the boiler's behaviors.

By distinctly representing the process responses from controlled input to measured output, the model is also provided as a benchmark problem for various control design methods of nonlinear systems. The outputs of interest for the boiler system are the steam pressure, excess oxygen level, drum water level, and steam flow rate. Moreover, the inputs of interest are considered as the fuel flow rate, air flow rate, and feed water flow rate. The configuration of a nonlinear boiler system is presented in Figure 1, referring to [39].

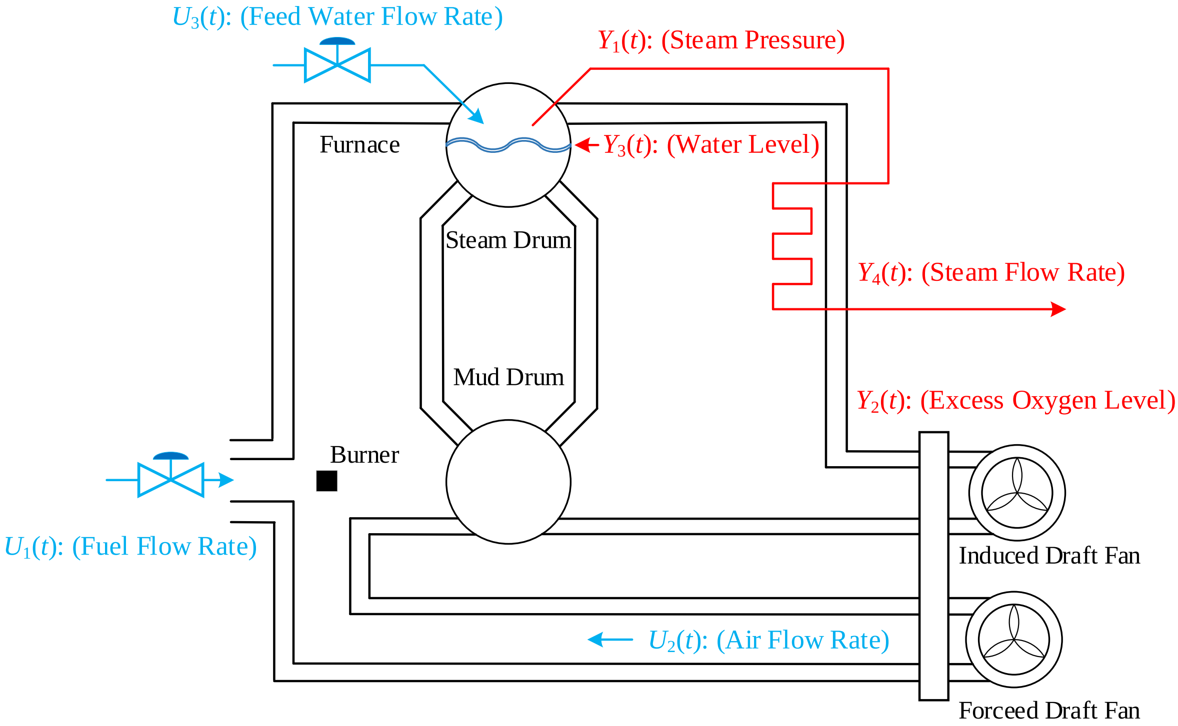

Figure 1. Configuration of nonlinear boiler system.

The control purposes of the boiler system are to guarantee the steam pressure, which is outputted to the header, at a certain value even when a difference exists in the steam flow rate as demanded by manipulators. Furthermore, the drum water level must be kept at the desired magnitude to avoid the dangers of overheating and flooding. Moreover, it is necessary to maintain the desired percentage of excess oxygen for the safety of combustion with fuel and air.

In most practical industrial systems, there is generally more than one boiler system. Moreover, every single boiler will exhaust the steam to a common header to achieve the goal of the boiler system. From this perspective, the control objective mentioned above will become more difficult and complex. To solve the problem, the control theory based on the multi-agent system can provide a powerful tool to control multiple systems simultaneously. Moreover, the leader-following control scheme has been widely applied in the control of multi-agent systems because of the effective cost saving. Because of this, the nonlinear dynamic equations of the multi-boiler system are presented based on the relationship of outputs and inputs to achieve the above control purposes as follows:

$$\dot{X}_{1m}(t) = -0.00478X_{4m}(t)X_{1m}^{9/8}(t) + 0.28U_{1m}(t) - 0.01348U_{3m}(t) \tag{1}$$

$$\dot{X}_{2m}(t) = 0.1540357X_{2m}(t) + \frac{103.5462U_{2m}(t) - 107.4835U_{1m}(t) - 1.9515U_{1m}(t)X_{2m}(t)}{(29.04U_{2m}(t) - 1.824U_{1m}(t))} \tag{2}$$

$$\dot{X}_{3m}(t) = -0.00533176X_{1m}(t) - 0.025195X_{1m}(t)X_{4m}(t) + 0.7317058U_{3m}(t) \tag{3}$$

$$\dot{X}_{4m}(t) = -0.04X_{4m}(t) + 0.029988U_{1m}(t) + 0.018088 \tag{4}$$

where $X_{1m}(t)$, $X_{2m}(t)$, $X_{3m}(t)$, and $X_{4m}(t)$ denote the state of the steam pressure of the drum (kgf/cm^2), the excess oxygen level (percent), the fluid density (kg/m^2), and the exogenous variable related to the load disturbance intensity (0–1); $U_{1m}(t)$, $U_{2m}(t)$, and $U_{3m}(t)$ denote the fuel, air, and feed water flow rate; and $m = 1, \ldots, \psi$ and $m = \psi + 1, \ldots, \psi + \vartheta$ denote the leader and follower numbers of the multi-agent system. For the multi-agent system, the essential information about graph theory is given in the following remark.

2.1. Remark 1 (Graph Theory)

An undirected graph Ω, which represents the interaction topology of a multi-agent system, is established by the node set $\mathfrak{N}(\Omega) = \{n_1, n_2, \ldots, n_\vartheta\}$ and the edge set $\mathfrak{T}(\Omega) \subseteq \{(n_m, n_n) : n_m, n_n \in \mathfrak{N}(\Omega)\}$, in which $\widetilde{\mathfrak{N}}(\Omega) = \{n_n \in \mathfrak{N}(\Omega) : (n_m, n_n) \in \mathfrak{T}(\Omega)\}$ denotes the set of the neighbor agent n_n from n_m. Then, an adjacency matrix $\mathfrak{J} = [j_{mn}] \in \mathrm{R}^{\vartheta \times \vartheta}$ is defined. The component $j_{mn} = 1$ denotes that there is a connection between n_m and n_n, that is, $(n_m, n_n) \in \mathfrak{T}(\Omega)$. Otherwise, $(n_m, n_n) \notin \mathfrak{T}(\Omega)$ is denoted by $j_{mn} = 0$. The degree matrix is constructed via $\mathfrak{D} = diag\{d_1, d_2, \ldots, d_m\}$, where $d_m = \sum_{n=1}^{\vartheta} j_{mn}$. Thus, the Laplacian matrix is defined as $\mathbf{L} = \mathfrak{D} - \mathfrak{J}$.

Based on the relationship between the inputs and outputs of a nonlinear boiler system, the outputs of a nonlinear boiler system are presented with the dynamic equations of selected system states (1)–(4) as follows:

$$Y_{1m}(t) = 14.214X_{1m}(t) \tag{5}$$

$$Y_{2m}(t) = X_{2m}(t) \tag{6}$$

$$\begin{aligned} Y_{3m}(t) = {} & -0.1048569X_{1m}(t) + 0.15479X_{3m}(t) + 0.495X_{1m}(t)X_{4m}(t) - 0.2U_{3m}(t) \\ & +1.272U_{1m}(t) - \tfrac{(324212.78X_{1m}(t)+99556.25)(1-0.0012X_{3m}(t))}{(1-0.0012X_{3m}(t))(X_{3m}(t)(X_{1m}(t)-1704.5))} - 103.74 \end{aligned} \tag{7}$$

$$Y_{4m}(t) = (0.85663X_{4m}(t) - 0.18128)X_{1m}(t) \tag{8}$$

where $Y_{1m}(t)$ and $Y_{2m}(t)$ denote the measured output of a drum's steam pressure (PSI) and level of excess oxygen (percent) and $Y_{3m}(t)$ and $Y_{4m}(t)$ denote the water level of the drum (inch) and the steam flow rate (kg/s).

Based on the results of [39], the equilibriums of each state's variables and inputs are selected for the control objectives as follows:

$$\begin{aligned} x_{1m}(t) = X_{1m}(t) - 22.5,\ x_{2m}(t) = X_{2m}(t) - 2.5 \\ x_{3m}(t) = X_{3m}(t) - 621.17,\ x_{4m}(t) = X_{4m}(t) - 0.8374 \end{aligned} \tag{9}$$

$$u_{1m}(t) = U_{1m}(t) - 0.5138,\ u_{2m}(t) = U_{2m}(t) - 0.5064,\ u_{3m}(t) = U_{3m}(t) - 0.8127 \tag{10}$$

Then, by replacing the state variables in (1)–(4) by (9) and (10), the following nonlinear dynamic equations are applied to represent the state dynamics of the boiler system:

$$\dot{x}_{1m}(t) = -0.00478(x_{4m}(t) + 0.8374)(x_{1m}(t) + 22.5)^{\frac{8}{9}} + 0.28(u_{1m}(t) + 0.5138) + \Delta b_{11}(t)u_{1m}(t) \tag{11}$$

$$\begin{aligned} \dot{x}_{2m}(t) = {} & 0.1540357(x_{2m}(t) + 2.5) + \tfrac{103.5462(u_{2m}(t)+0.5064)-107.4835(u_{1m}(t)+0.5138)}{(29.04(u_{2m}(t)+0.5064)-1.824(u_{1m}(t)+0.5138))} \\ & + \tfrac{-1.9515(u_{1m}(t)+0.5138)(x_{2m}(t)+2.5)}{(29.04(u_{2m}(t)+0.5064)-1.824(u_{1m}(t)+0.5138))} \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{x}_{3m}(t) = {} & -0.00533176(x_{1m}(t) + 22.5) - 0.025195(x_{1m}(t) + 22.5)(x_{4m}(t) + 0.8374) \\ & +0.7317058(u_{3m}(t) + 0.8127) \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{x}_{4m}(t) = {} & -0.04(x_{4m}(t) + 0.8374) + 0.029988(u_{1m}(t) + 0.5138) + 0.018088 \\ & +\Delta a_{44}(t)x_{4m}(t) + \Delta b_{41}(t)u_{1m}(t) \end{aligned} \tag{14}$$

where $\Delta a_{44}(t)$ is the uncertainty related to the state and $\Delta b_{11}(t)$ and $\Delta b_{41}(t)$ are uncertainties related to the inputs. Referring to [39], the uncertainties problem can be explained in the following remark for the nonlinear boiler system.

2.2. Remark 2 (Uncertainty Problem of the Boiler System)

In the practical situation of a boiler, the fuel flow rate on all outputs may be affected heavily because of corrosion, wear, the piping's transportation delay, varying dead times caused by stack, and so on. The above reasons will lead to the uncertainties of process parameters. To verify the effectiveness of the IT-2 T–S fuzzy control method in this paper, the uncertainties of the fuel flow rate are considered in the state dynamics in the boiler system (11) and (14), which is related to input $u_{1m}(t)$. Moreover, the state $x_{4m}(t)$ is considered as an exogenous variable which is related to the load disturbance. It also means that the accurate value of this state will be more difficult to obtain, which will cause parameter uncertainty. Thus, the uncertainties of $x_{4m}(t)$ are also considered in Equation (14).

The uncertainties problem of a practical boiler system is also an important control issue, and many researches have already been devoted to developing the control theory for such a robust control problem. However, the effect of uncertainties will become more severe in the multi-boiler system. This is because the uncertainties make the information of state variables transported between each agent not accurate enough. For the nonlinear multi-agent system control issue, the problem may cause the whole system to be unstable or have a weak performance. The IT-2 T–S fuzzy model is applied to represent the nonlinear system with uncertainties more completely to solve the problem. Thus, the IT-2 T–S fuzzy model of the nonlinear multi-boiler system (11)–(14) is constructed as follows.

To apply the fuzzy modelling method [44], the operating points of each fuzzy rule are selected, referring to [35] as

$$x_{m(op1)} = \begin{bmatrix}-22.5 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},\ x_{m(op2)} = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},\ x_{m(op3)} = \begin{bmatrix}22.5 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}} \tag{15}$$

Then, each following fuzzy rule of the IT-2 T-S fuzzy model can be obtained with the application of type-2 fuzzy sets.

Model Rule 1:
If $x_{1m}(t)$ is $\widetilde{\mathrm{M}}_{11}$, then

$$\dot{x}_m(t) = \mathbf{A}_1 x_m(t) + \mathbf{B}_1 u_m(t) \tag{16}$$

Model Rule 2:
If $x_{1m}(t)$ is $\widetilde{\mathrm{M}}_{21}$, then

$$\dot{x}_m(t) = \mathbf{A}_2 x_m(t) + \mathbf{B}_2 u_m(t) \tag{17}$$

Model Rule 3:
If $x_{1m}(t)$ is $\widetilde{\mathrm{M}}_{31}$, then

$$\dot{x}_m(t) = \mathbf{A}_3 x_m(t) + \mathbf{B}_3 u_m(t) \tag{18}$$

where $\widetilde{\mathrm{M}}_{11}$, $\widetilde{\mathrm{M}}_{21}$, and $\widetilde{\mathrm{M}}_{31}$ are IT-2 fuzzy sets, and the model matrices are obtained via the fuzzy modeling method [44] as follows:

$$\mathbf{A}_1 = \begin{bmatrix}-0.0059 & 0 & 0 & 0\\ 0 & 0.0812 & 0 & 0\\ -0.0264 & 0 & 0 & 0\\ 0 & 0 & 0 & -0.04\end{bmatrix},\ \mathbf{A}_2 = \begin{bmatrix}-0.0066 & 0 & 0 & -0.1587\\ 0 & 0.0812 & 0 & 0\\ -0.0264 & 0 & 0 & -0.5669\\ 0 & 0 & 0 & -0.04\end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix}-0.007 & 0 & 0 & -0.3462\\ 0 & 0.0812 & 0 & 0\\ -0.0264 & 0 & 0 & -1.1338\\ 0 & 0 & 0 & -0.04\end{bmatrix},\ \mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix}0.28 & 0 & -0.0135\\ -8.2117 & 8.3317 & 0\\ 0 & 0 & 0.7317\\ 0.03 & 0 & 0\end{bmatrix}$$

The fired strength of the fuzzy model's IT-2 membership function is given in the following interval sets:

$$\mu_{\tilde{M}_{\alpha 1}}(x_{1m}(t)) = \left[\underline{\mu}_{\tilde{M}_{\alpha 1}}(x_{1m}(t)), \overline{\mu}_{\tilde{M}_{\alpha 1}}(x_{1m}(t))\right], \text{ for } \alpha = 1,2,3 \tag{19}$$

where $\underline{\mu}_{\tilde{M}_{\alpha 1}}(x_{1m}(t))$ and $\overline{\mu}_{\tilde{M}_{\alpha 1}}(x_{1m}(t))$ denote the lower and upper bound of the membership function, satisfying the condition $0 \leq \underline{\mu}_{\tilde{M}_{\alpha 1}}(x_{1m}(t)) \leq \overline{\mu}_{\tilde{M}_{\alpha 1}}(x_{1m}(t)) \leq 1$.

In the IT-2 T–S fuzzy model, it is seen that the consequent part of the model is constructed by applying the type-1 T–S fuzzy modeling method. However, the effects of uncertainties in the nonlinear boiler system (11)–(14) is considered by the IT-2 membership function in the model (16)–(18). In view of the magnitude of the state variable and the fuel flow rate potentially being affected by the uncertainties, the IT-2 membership functions are designed with upper and lower bound membership functions of each rule, as shown in Figure 2.

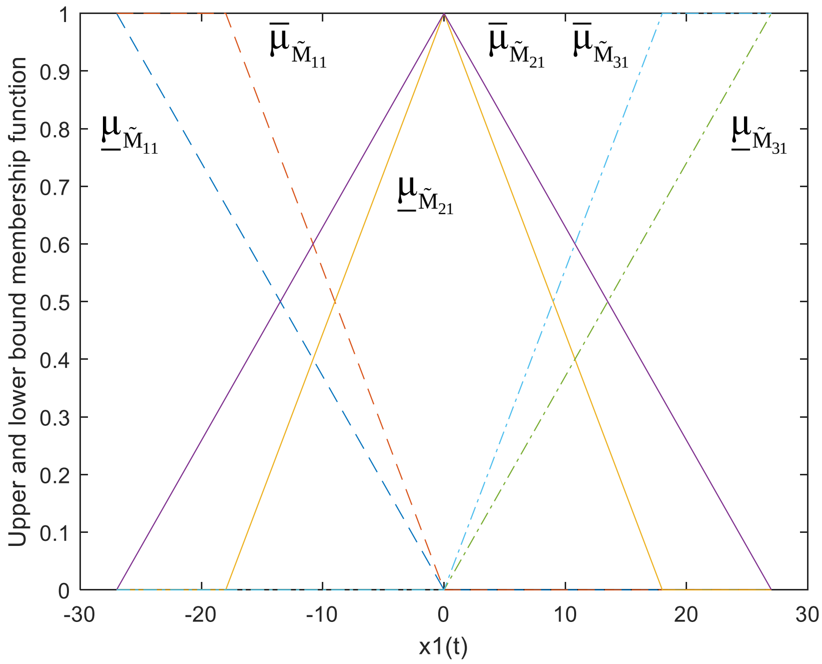

Figure 2. IT-2 membership function of boiler system.

In this paper, a different control scheme is designed to simultaneously achieve the objective of the formation and containment problem in the control of a multi-agent system. In the development of the control protocol design method, considering the problems of communication such as packet dropout, failure, or disturbance is crucial. Many existing papers have developed various control theories to solve or prevent the problem. To achieve the objective of the containment problem, the leader agents are always furthest from each other. That is, it is expected that the communication problem will be the most serious for the leader in the containment control issue. Compared to the formation–containment control method in [40,41], which requires communication between the leaders, the control method in this paper can avoid the communication problem effectively by applying the state feedback controller of the leader itself.

For this purpose, the IT-2 T–S fuzzy model (16)–(18) is represented in the following fuzzy model.

Model Rule 1:
If $x_{1m}(t)$ is $\widetilde{\mathrm{M}}_{11}$, then

$$\begin{cases} \dot{\phi}_m(t) = \mathbf{A}_1\phi_m(t) + \mathbf{B}_1 u_m(t) & for \quad m = 1, \ldots, \psi \\ \dot{x}_m(t) = \mathbf{A}_1 x_m(t) + \mathbf{B}_1 u_m(t) & for \quad m = \psi+1, \ldots, \psi+\vartheta \end{cases} \tag{20}$$

Model Rule 2:
If $x_{1m}(t)$ is $\widetilde{\mathrm{M}}_{21}$, then

$$\begin{cases} \dot{\phi}_m(t) = \mathbf{A}_2\phi_m(t) + \mathbf{B}_2 u_m(t) & for \quad m = 1, \ldots, \psi \\ \dot{x}_m(t) = \mathbf{A}_2 x_m(t) + \mathbf{B}_2 u_m(t) & for \quad m = \psi+1, \ldots, \psi+\vartheta \end{cases} \tag{21}$$

Model Rule 3:
If $x_{1m}(t)$ is $\widetilde{\mathrm{M}}_{31}$, then

$$\begin{cases} \dot{\phi}_m(t) = \mathbf{A}_3\phi_m(t) + \mathbf{B}_3 u_m(t) & for \quad m = 1, \ldots, \psi \\ \dot{x}_m(t) = \mathbf{A}_3 x_m(t) + \mathbf{B}_3 u_m(t) & for \quad m = \psi+1, \ldots, \psi+\vartheta \end{cases} \tag{22}$$

where $\phi_m(t) = x_m(t) - \mathbb{R}_m$ and $\mathbb{R}_m$ is the desired value for the convergence of the leader's states.

Then, the overall fuzzy model of the boiler system can be inferred from the IT-2 T–S fuzzy model (20)–(22) as follows:

$$\dot{\phi}_m(t) = \sum_{\alpha=1}^{3} \mu_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1m}(t))\{\mathbf{A}_\alpha\phi_m(t) + \mathbf{B}_\alpha u_m(t)\},\ for \quad m = 1, \ldots, \psi \tag{23}$$

$$\dot{x}_m(t) = \sum_{\alpha=1}^{3} \mu_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1m}(t))\{\mathbf{A}_\alpha x_m(t) + \mathbf{B}_\alpha u_m(t)\},\ for \quad m = \psi+1, \ldots, \psi+\vartheta \tag{24}$$

where $\mu_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1m}(t)) = \overline{\mu}_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1m}(t))\overline{\tau}_\alpha(x_{1m}(t)) + \underline{\mu}_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1m}(t))\underline{\tau}_\alpha(x_{1m}(t)) \geq 0$, $\sum\limits_{\alpha=1}^{3} \mu_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1m}(t)) = 1$, $\overline{\tau}_\alpha(x_{1m}(t))$ and $\underline{\tau}_\alpha(x_{1m}(t))$ are the nonlinear functions satisfying conditions $1 \geq \overline{\tau}_\alpha(x_{1m}(t)) \geq \underline{\tau}_\alpha(x_{1m}(t)) \geq 0$ and $\overline{\tau}_\alpha(x_{1m}(t)) + \underline{\tau}_\alpha(x_{1m}(t)) = 1$. Note that these nonlinear functions may be dependent on the parameter uncertainties of the nonlinear system and it is not necessary that they be known.

Before designing the IT-2 fuzzy control method, the containment problem of a multi-agent system is introduced in the following definition.

Definition 1. *The IT-2 T–S fuzzy multi-agent system (22) and (23) is said to achieve containment if there exists a nonnegative constant γ_{mn}, where $n \in \{1, 2, \ldots, \psi\}$, satisfying $\sum\limits_{n=1}^{\psi} \gamma_{mn} = 1$ such that the following condition is satisfied with any given bounded initial states.*

$$\lim_{t \to \infty} \left(x_m(t) - \sum_{n=1}^{\psi} \gamma_{mn} x_n(t) \right) = 0,\ \text{for any } m \in \{\psi+1, \ldots, \psi+\vartheta\} \tag{25}$$

Via satisfying condition (25) for the multi-agent system, the follower agents are driven into the convex hull set up by leader agents. Extending the concept, it also means that the follower's states can converge into the interval value specified by the leaders. Due to the state $x_{4m}(t)$ being the variable which is related to load disturbance, it is not simple to control it at the corrected value for all boiler agents in the system. For this reason, the containment problem of boiler system (11)–(14) can be regarded as designing an interval value, which is tolerable for the boiler, for the state $x_{4m}(t)$ of followers to converge in.

To achieve the containment control objective, the IT-2 fuzzy control method is designed for the IT-2 T–S fuzzy model (20)–(22) as follows.

Controller Rule β:

If $x_{1m}(t)$ is $\widetilde{\mathrm{N}}_{\beta 1}$, then

$$u_m(t) = \mathbf{F}_\beta \phi_m(t) \qquad for\ m = 1,\ldots,\psi \tag{26}$$

Controller Rule β:

If $x_{1m}(t)$ is $\widetilde{\mathrm{N}}_{\beta 1}$, then

$$u_m(t) = \mathbf{K}_\beta \sum_{n\in\widetilde{\mathfrak{N}}(\Omega)} j_{mn}(x_m(t) - x_n(t)) \qquad for \quad m = \psi+1,\ldots,\psi+\vartheta \tag{27}$$

where $\widetilde{\mathrm{N}}_{\beta 1}$ is the IT-2 fuzzy set of the fuzzy controller, $\mathbf{F}_\beta$ and $\mathbf{K}_\beta$ are the feedback gains of leaders and followers, and $\beta = 1,2,\ldots,v$ is the rule numbers of the fuzzy controller.

Via the fuzzy controller design method in this paper, the containment objective can be achieved effectively by designing the proper gains $\mathbf{K}_\beta$. Moreover, the formation problem can also be solved by the feedback controller and gains $\mathbf{F}_\beta$ for each leader itself individually. Without the communication flow between the leaders, the risk of the transmission failure is reduced greatly. It is worth noticing that the fuzzy controller design method of (26) and (27) applies the IT-2 fuzzy set, which does not need to be designed the same as the fuzzy model (20)–(22). This also means the stability analysis and fuzzy controller design method become more relaxed and flexible.

Then, the overall IT-2 fuzzy controller is referred to (26) and (27) as follows:

$$u_m(t) = \sum_{\beta=1}^{v} \mu_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1m}(t))\{\mathbf{F}_\beta \phi_m(t)\},\ for \quad m = 1,\ldots,\psi \tag{28}$$

$$u_m(t) = \sum_{\beta=1}^{v} \mu_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1m}(t))\left\{\mathbf{K}_\beta \sum_{n\in\widetilde{\mathfrak{N}}(\Omega)} j_{mn}(x_m(t) - x_n(t))\right\},\ for \quad m = \psi+1,\ldots,\psi+\vartheta \tag{29}$$

where $\mu_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1m}(t)) = \dfrac{\overline{\mu}_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1m}(t))\overline{\varepsilon}_\beta(x_{1m}(t)) + \underline{\mu}_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1m}(t))\underline{\varepsilon}_\beta(x_{1m}(t))}{\sum_{\lambda=1}^{v}\left\{\overline{\mu}_{\widetilde{\mathrm{N}}_{\lambda 1}}(x_{1m}(t))\overline{\varepsilon}_\lambda(x_{1m}(t)) + \underline{\mu}_{\widetilde{\mathrm{N}}_{\lambda 1}}(x_{1m}(t))\underline{\varepsilon}_\lambda(x_{1m}(t))\right\}} \geq 0$, $\sum\limits_{\beta=1}^{v} \underline{\mu}_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1m}(t)) = 1$, $\overline{\varepsilon}_\beta(x_{1m}(t))$ and $\underline{\varepsilon}_\beta(x_{1m}(t))$ are the predefined functions satisfying conditions $1 \geq \overline{\varepsilon}_\beta(x_{1m}(t)) \geq \underline{\varepsilon}_\beta(x_{1m}(t)) \geq 0$ and $\overline{\varepsilon}_\beta(x_{1m}(t)) + \underline{\varepsilon}_\beta(x_{1m}(t)) = 1$.

Therefore, the following closed-loop IT-2 multi-agent system can be obtained by substituting IT-2 fuzzy controller (28) and (29) into the IT-2 fuzzy multi-agent system (23) and (24), respectively:

$$\dot{\phi}_\ell(t) = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \mu_{\widetilde{\mathrm{M}}_{\alpha 1}}(x_{1\ell}(t))\mu_{\widetilde{\mathrm{N}}_{\beta 1}}(x_{1\ell}(t))\{(\mathbf{I}_Q \otimes (\mathbf{A}_\alpha + \mathbf{B}_\alpha \mathbf{F}_\beta))\phi_\ell(t)\} \tag{30}$$

$$\dot{x}_f(t) = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \mu_{\widetilde{\mathrm{M}}_{\alpha 1}}\Big(x_{1f}(t)\Big)\mu_{\widetilde{\mathrm{N}}_{\beta 1}}\Big(x_{1f}(t)\Big)\Big\{(\mathbf{I}_G \otimes \mathbf{A}_\alpha + \mathbf{L}_1 \otimes \mathbf{B}_\alpha \mathbf{K}_\beta)x_f(t) + (\mathbf{L}_2 \otimes \mathbf{B}_\alpha \mathbf{K}_\beta)x_\ell(t)\Big\} \tag{31}$$

where $\phi_\ell(t) = \begin{bmatrix} \phi_1^{\mathrm{T}}(t) & \phi_2^{\mathrm{T}}(t) & \cdots & \phi_\psi^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ denotes the vector of the new state of leaders, $x_\ell(t) = \begin{bmatrix} x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t) & \cdots & x_\psi^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ and $x_f(t) = \begin{bmatrix} x_{\psi+1}^{\mathrm{T}}(t) & x_{\psi+2}^{\mathrm{T}}(t) & \cdots & x_{\psi+\vartheta}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ denote the state vector of all leaders and followers, $x_{1\ell}(t) = \begin{bmatrix} x_{11}^{\mathrm{T}}(t) & x_{12}^{\mathrm{T}}(t) & \cdots & x_{1\psi}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ and $x_{1f}(t) = \begin{bmatrix} x_{1(\psi+1)}^{\mathrm{T}}(t) & x_{1(\psi+2)}^{\mathrm{T}}(t) & \cdots & x_{1(\psi+\vartheta)}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ denote the vector of the premise variables $x_1(t)$ of all leaders and followers, $\mathbf{I}_Q$ and $\mathbf{I}_G$ denote the identity matrix with the dimensions related to the quantity of leaders and followers, and the symbol $\otimes$ denotes the Kronecker product.

The Laplacian matrix for the IT-2 T–S fuzzy multi-agent system (31) is constructed as follows:

$$\mathbf{L} = \begin{bmatrix} 0 & 0 \\ \mathbf{L}_2 & \mathbf{L}_1 \end{bmatrix} \tag{32}$$

where matrix $\mathbf{L}_1 \in \mathrm{R}^{\vartheta\times\vartheta}$ denotes the interaction topology between the followers and $\mathbf{L}_2 \in \mathrm{R}^{\vartheta\times\psi}$ denotes the interaction topology from the leaders to the followers.

To develop the IT-2 fuzzy containment controller design method, an essential lemma should be given for the Laplacian matrix, which is presented as follows.

Lemma 1 [45]. *Assuming there is at least one leader who has the directed path of interaction with the followers, every component of $-\mathbf{L}_1^{-1}\mathbf{L}_2$ is nonnegative, and the sum of its individual row is equal to one. Moreover, for the matrix $\mathbf{L}_1$, all of the eigenvalues have a positive real part.*

Thus, the containment control problem can be solved efficiently via the proper design method for the IT-2 fuzzy controller (28) and (29). Moreover, the formation objective of leader agents in the system can also be satisfied without the communication between leaders. In the next section, a different kind of formation–containment control method is proposed for the nonlinear multi-boiler system.

3. IT-2 Fuzzy Formation and Containment Controller Design for the Multi-Boiler System

This section develops the IT-2 T–S fuzzy formation and containment controller design method for the nonlinear multi-boiler system (1)–(8) to achieve the control objective. Moreover, the IT-2 membership function is applied to represent the uncertainty problem in the practical system. Thus, the design method based on the IT-2 T–S fuzzy multi-agent system (30) and (31) can be presented in the following theorems. Firstly, the stability analysis and formation control design method are presented.

Theorem 1. *Given positive scalars $\overline{\delta}_{\alpha\beta i_1 q}$ and $\underline{\delta}_{\alpha\beta i_1 q}$, if there exists a common positive definite matrix $\mathbf{Q}_\ell$, the matrices $\mathbf{G}_\beta$, the semi-positive definite matrices $\mathbf{W}_{\ell\alpha\beta}$, and a common symmetric matrix $\mathbf{H}_\ell$, such that the following sufficient conditions are satisfied, the dynamics of all leaders of the IT-2 T–S fuzzy multi-agent (30) and (31) are ensured to be asymptotically stable. Additionally, leader agents can all be forced to the desired values to complete the formation objective.*

$$\mathbf{Q}_\ell > 0 \tag{33}$$

$$\mathbf{W}_{\ell\alpha\beta} \geq 0, \ for \quad \alpha = 1,2,3 \tag{34}$$

$$\mathbf{A}_\alpha\mathbf{Q}_\ell + \mathbf{Q}_\ell\mathbf{A}_\alpha^{\mathrm{T}} + \mathbf{B}_\alpha\mathbf{G}_\beta + \mathbf{G}_\beta^{\mathrm{T}}\mathbf{B}_\alpha^{\mathrm{T}} + \mathbf{W}_{\ell\alpha\beta} + \mathbf{H}_\ell > 0, \ for \quad \alpha = 1,2,3 \tag{35}$$

$$\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\left(\overline{\delta}_{\alpha\beta i_1 q}\left(\mathbf{A}_\alpha\mathbf{Q}_\ell + \mathbf{Q}_\ell\mathbf{A}_\alpha^{\mathrm{T}} + \mathbf{B}_\alpha\mathbf{G}_\beta + \mathbf{G}_\beta^{\mathrm{T}}\mathbf{B}_\alpha^{\mathrm{T}}\right) - \left(\underline{\delta}_{\alpha\beta i_1 q} - \overline{\delta}_{\alpha\beta i_1 q}\right)\mathbf{W}_{\ell\alpha\beta} + \overline{\delta}_{\alpha\beta i_1 q}\mathbf{H}_\ell\right) - \mathbf{H}_\ell < 0 \tag{36}$$

where $\mathbf{G}_\beta = \mathbf{F}_\beta\mathbf{Q}_\ell$ and $\mathbf{Q}_\ell = \mathbf{P}_\ell^{-1}$.

Proof 1. In the method of Theorem 1, it is worth noticing that the stability analysis and IT-2 fuzzy controller design method just need to be considered for one leader. Based on the feature of a homogeneous multi-agent system, the other leaders can also achieve the asymptotic stability and converge to the desired state via the designed IT-2 fuzzy controller. Thus, to develop the stability analysis with the Lyapunov theory, the candidate function is considered with leader 1 as follows.

$$\mathrm{V}_1(\phi_1(t)) = \phi_1^{\mathrm{T}}(t)\mathbf{P}_\ell\phi_1(t) \tag{37}$$

where $\mathbf{P}_\ell$ is the common positive definite matrix. Taking the derivative to Lyapunov function (37), one can obtain

$$\dot{V}_1(\phi_1(t)) = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \mu_{\widetilde{M}_{\alpha 1}}(x_{1\ell}(t))\mu_{\widetilde{N}_{\beta 1}}(x_{1\ell}(t))\Big\{\phi_1^{\mathrm{T}}(t)\Big(\big(\mathbf{A}_\alpha + \mathbf{B}_\alpha\mathbf{F}_\beta\big)^{\mathrm{T}}\mathbf{P}_\ell + \mathbf{P}_\ell\big(\mathbf{A}_\alpha + \mathbf{B}_\alpha\mathbf{F}_\beta\big)\Big)\phi_1(t)\Big\} \tag{38}$$

Obviously, the asymptotic stability can be achieved by $\dot{V}_1(\phi_1(t)) < 0$ if the following condition is satisfied:

$$\mathbf{\Theta}_1 = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \widetilde{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\Big\{\big(\mathbf{A}_\alpha + \mathbf{B}_\alpha\mathbf{F}_\beta\big)^{T}\mathbf{P}_\ell + \mathbf{P}_\ell\big(\mathbf{A}_\alpha + \mathbf{B}_\alpha\mathbf{F}_\beta\big)\Big\} < 0 \tag{39}$$

where $\widetilde{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) = \mu_{\widetilde{M}_{\alpha 1}}(x_{1\ell}(t))\mu_{\widetilde{N}_{\beta 1}}(x_{1\ell}(t))$ and it can be constructed as $\widetilde{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) = \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\overline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) + \underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\underline{\sigma}_{\alpha\beta}(x_{1\ell}(t))$, and $\overline{\sigma}_{\alpha\beta}(x_{1\ell}(t))$ and $\underline{\sigma}_{\alpha\beta}(x_{1\ell}(t))$, which are satisfying the condition $\overline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) + \underline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) = 1$, are the functions that don't need to be known.

Thus, the purpose of the stability analysis in Theorem 1 is to satisfy the condition (39). For this reason, the following two slack matrices can be introduced if the condition (34) is satisfied by Theorem 1:

$$\left(\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\overline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) + \underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\underline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) - 1\right)\mathbf{H}_\ell = 0 \tag{40}$$

$$-\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\Big(1 - \underline{\sigma}_{\alpha\beta}(x_{1\ell}(t))\Big)\Big(\underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) - \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\Big)\mathbf{W}_{\ell\alpha\beta} \geq 0 \tag{41}$$

Multiplying $\mathbf{P}_\ell^{-1} = \mathbf{Q}_\ell$ at the left-hand and right-hand sides of (39) and combining with (40) and (41), the following relationship can be obtained:

$$\begin{aligned}\mathbf{Q}_\ell\mathbf{\Theta}_1\mathbf{Q}_\ell \leq \mathbf{Q}_\ell\mathbf{\Theta}_1\mathbf{Q}_\ell &+ \left(\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\overline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) + \underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\underline{\sigma}_{\alpha\beta}(x_{1\ell}(t)) - 1\right)\mathbf{H}_\ell \\ &- \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\Big(1 - \underline{\sigma}_{\alpha\beta}(x_{1\ell}(t))\Big)\Big(\underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) - \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\Big)\mathbf{W}_{\ell\alpha\beta}\end{aligned} \tag{42}$$

Then, the right-hand side of inequality (42) can also be rewritten as

$$\begin{aligned}&\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\Big(\underline{\sigma}_{\alpha\beta}(x_{1\ell}(t))\Big(\underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) - \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\Big)\Big)\Big\{\mathbf{A}_\alpha\mathbf{Q}_\ell + \mathbf{Q}_\ell\mathbf{A}_\alpha^{\mathrm{T}} + \mathbf{B}_\alpha\mathbf{G}_\beta + \mathbf{G}_\beta^{\mathrm{T}}\mathbf{B}_\alpha^{\mathrm{T}} + \mathbf{W}_{\ell\alpha\beta} + \mathbf{H}_\ell\Big\} \\ &\quad + \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\Big\{\overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\Big(\mathbf{A}_\alpha\mathbf{Q}_\ell + \mathbf{Q}_\ell\mathbf{A}_\alpha^{\mathrm{T}} + \mathbf{B}_\alpha\mathbf{G}_\beta + \mathbf{G}_\beta^{\mathrm{T}}\mathbf{B}_\alpha^{\mathrm{T}}\Big) \\ &\quad - \Big(\underline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) - \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\Big)\mathbf{W}_{\ell\alpha\beta} + \overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t))\mathbf{H}_\ell\Big\} - \mathbf{H}_\ell\end{aligned} \tag{43}$$

In [46], the membership function dependent stability analysis method has been proposed to combine the property of the IT-2 membership function of the fuzzy system into the analysis process. Because the IT-2 membership function, which can represent the uncertainties factor, is the main feature of the IT-2 T–S fuzzy model, the membership function is also constructed as follows, referring to [46]:

$$\overline{\Phi}_{\alpha\beta 1}(x_{1\ell}(t)) = \sum_{q=1}^{\rho}\sum_{i_1=1}^{2} \varsigma_{1 i_1 q}(x_1(t))\overline{\delta}_{\alpha\beta i_1 q} \tag{44}$$

$$\underline{\Phi}_{\alpha\beta1}(x_{1\ell}(t)) = \sum_{q=1}^{\rho}\sum_{i_1=1}^{2}\varsigma_{1i_1q}(x_1(t))\overline{\delta}_{\alpha\beta i_1q} \tag{45}$$

where $q = 1, 2, \ldots, \rho$ is the number of the substate space and $\overline{\delta}_{\alpha\beta i_1q}$ and $\underline{\delta}_{\alpha\beta i_1q}$ are the designed constant scalars related the substate space which satisfies $1 \geq \overline{\delta}_{\alpha\beta i_1q} \geq \underline{\delta}_{\alpha\beta i_1q} \geq 0$. Moreover, $\varsigma_{1i_1q}(x_1(t))$ is the cross term independent of fuzzy rules, satisfying the conditions $0 \leq \varsigma_{1i_1q}(x_1(t)) \leq 1$ and $\varsigma_{11q}(x_1(t)) + \varsigma_{12q}(x_1(t)) = 1$. The details of the method can be referred to in [46].

Based on the construction of IT-2 fuzzy membership function (44) and (45), (43) is represented in the following form:

$$\begin{aligned}
&\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\left(\underline{\sigma}_{\alpha\beta}(x_{1\ell}(t))\left(\underline{\Phi}_{\alpha\beta1}(x_{1\ell}(t)) - \overline{\Phi}_{\alpha\beta1}(x_{1\ell}(t))\right)\right)\left\{\mathbf{A}_\alpha\mathbf{Q}_\ell + \mathbf{Q}_\ell\mathbf{A}_\alpha^{\mathrm{T}} + \mathbf{B}_\alpha\mathbf{G}_\beta + \mathbf{G}_\beta^{\mathrm{T}}\mathbf{B}_\alpha^{\mathrm{T}} + \mathbf{W}_{\ell\alpha\beta} + \mathbf{H}_\ell\right\}\\
&\quad + \sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\sum_{i_1=1}^{2}\varsigma_{1i_1q}(x_1(t))\left\{\overline{\delta}_{\alpha\beta i_1q}\left(\mathbf{A}_\alpha\mathbf{Q}_\ell + \mathbf{Q}_\ell\mathbf{A}_\alpha^{\mathrm{T}} + \mathbf{B}_\alpha\mathbf{G}_\beta + \mathbf{G}_\beta^{\mathrm{T}}\mathbf{B}_\alpha^{\mathrm{T}}\right)\right.\\
&\qquad \left. - \left(\underline{\delta}_{\alpha\beta i_1q} - \overline{\delta}_{\alpha\beta i_1q}\right)\mathbf{W}_{\ell\alpha\beta} + \overline{\delta}_{\alpha\beta i_1q}\mathbf{H}_\ell\right\} - \mathbf{H}_\ell
\end{aligned} \tag{46}$$

Obviously, if the conditions (35) and (36) can be achieved by Theorem 1, then the item of (46) is negative definite because of the property $0 \leq \underline{\Phi}_{\alpha\beta1}(x_{1\ell}(t)) \leq \overline{\Phi}_{\alpha\beta1}(x_{1\ell}(t)) \leq 1$. That is, the item (43) is also negative definite. Accordingly, the condition (39) is satisfied via the relationship of (42), which also means that $\dot{V}_1(\phi_1(t)) < 0$ in (38) is achieved. It can be said that the leader 1 agent of the closed-loop IT-2 T–S fuzzy multi-agent system (30) is asymptotically stable by satisfying the conditions (33)–(36) by the fuzzy controller design method of Theorem 1. Thus, the proof is completed. □

Via the IT-2 fuzzy controller design method of Theorem 1, the new state variable of leader 1, $\phi_1(t)$, can converge to zero, which also means that the state variable can be forced to the desired value $\mathbb{R}_1$ by $\phi_1(t) = x_1(t) - \mathbb{R}_1$. The formation problem of the purpose of driving the agent states to the desired place or maintaining the system in a specific form is solved effectively by Theorem 1.

In Theorem 1, it is known that the stability analysis and IT-2 fuzzy controller design method for the leader's multi-agent system just need to develop for one leader only. Because of the characteristics of a homogeneous type multi-agent system, the designed IT-2 fuzzy controller can also be applied to all the other leaders to achieve the formation objective. Thus, the advantage of Theorem 1 is not only to avoid the communication required between leaders but also to reduce the complexity and difficulty in the process of the controller design method.

Since the formation and the stability of leaders in the multi-boiler system (1)–(8) can be achieved by Theorem 1, the containment control method is proposed in the following theorem based on the IT-2 T–S fuzzy multi-agent system (30) and (31).

Theorem 2. *Given positive scalars $\overline{\delta}_{\alpha\beta i_1q}$ and $\underline{\delta}_{\alpha\beta i_1q}$, if the conditions (33)–(36) are satisfied by Theorem 1, and if there exists a common positive definite matrix $\mathbf{Q}_f$, the matrices $\mathbf{T}_\beta$, the semi-positive definite matrices $\mathbf{W}_{f\alpha\beta}$, and a common symmetric matrix $\mathbf{H}_f$, such that the following sufficient conditions are satisfied, all the followers of IT-2 T–S fuzzy multi-agent system (30) and (31) can be driven into the convex hull formed by the leaders and be said to achieve the containment control objective.*

$$\mathbf{Q}_f > 0 \tag{47}$$

$$\mathbf{W}_{f\alpha\beta} \geq 0,\ for \quad \alpha = 1, 2, 3 \tag{48}$$

$$\begin{gathered}
\left(\left(\mathbf{I}_G \otimes \mathbf{A}_\alpha\mathbf{Q}_f + \mathbf{L}_1 \otimes \mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right) + \left(\mathbf{I}_G \otimes \mathbf{A}_\alpha\mathbf{Q}_f + \mathbf{L}_1 \otimes \mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right)^{\mathrm{T}}\right) + \mathbf{W}_{f\alpha\beta} + \mathbf{H}_f > 0,\\
for \quad \alpha = 1, 2, 3
\end{gathered} \tag{49}$$

$$\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\left(\overline{\delta}_{\alpha\beta i_1 q}\left(\left(\mathbf{I}_G\otimes\mathbf{A}_\alpha\mathbf{Q}_f+\mathbf{L}_1\otimes\mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right)+\left(\mathbf{I}_G\otimes\mathbf{A}_\alpha\mathbf{Q}_f+\mathbf{L}_1\otimes\mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right)^{\mathrm{T}}\right)\right.$$
$$\left.-\left(\underline{\delta}_{\alpha\beta i_1 q}-\overline{\delta}_{\alpha\beta i_1 q}\right)\mathbf{W}_{f\alpha\beta}+\overline{\delta}_{\alpha\beta i_1 q}\mathbf{H}_f\right)-\mathbf{H}_f<0 \tag{50}$$

where $\mathbf{T}_\beta=\mathbf{K}_\beta\mathbf{Q}_f$ and $\mathbf{Q}_f=\mathbf{P}_f^{-1}$.

Proof 2. Firstly, the error between the followers and the neighbor agents is defined as $e_m(t)=\sum_{n\in\widetilde{\mathfrak{N}}(\Omega)} j_{mn}(x_m(t)-x_n(t))$ for $m=\psi+1,\psi+2,\ldots,\psi+\vartheta$. Then, considering the error vector of all followers $e_f(t)=\begin{bmatrix} e_{\psi+1}^{\mathrm{T}}(t) & e_{\psi+2}^{\mathrm{T}}(t) & \ldots & e_{\psi+\vartheta}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}$, the following equation can be obtained.

$$e_f(t)=(\mathbf{L}_1\otimes\mathbf{I}_4)x_f(t)+(\mathbf{L}_2\otimes\mathbf{I}_4)x_\ell(t) \tag{51}$$

Via the property of the Laplacian matrix in Lemma 1, (51) can be derived as

$$x_f(t)=\left(\mathbf{L}_1^{-1}\otimes\mathbf{I}_4\right)e_f(t)-\left(\mathbf{L}_1^{-1}\mathbf{L}_2\otimes\mathbf{I}_4\right)x_\ell(t) \tag{52}$$

With the definition $\phi_m(t)-\mathbb{R}_m$ for the leader agent, $m=1,2,\ldots,\psi$, and taking the derivative for (51), one can obtain

$$\dot{e}_f(t)=(\mathbf{L}_1\otimes\mathbf{I}_4)\dot{x}_f(t)+(\mathbf{L}_2\otimes\mathbf{I}_4)\dot{\phi}_\ell(t) \tag{53}$$

Because of the requirement of the control precision in the practical industrial system nowadays, the item related to $\mathbb{R}_m$, which is relatively small in most situations, can be neglected in the derivation of the containment control design method. That is, for the nonlinear boiler system (11)–(14), the state $x_4(t)$ is more difficult to control to a certain value because it is related to the disturbance. However, its dynamic still should be ensured in a tolerable range which is generally small. Thus, (53) can be derived into the following dynamic equation by applying (30), (31), and (52):

$$\dot{e}_f(t)=\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\widetilde{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\left(\mathbf{I}_G\otimes\mathbf{A}_\alpha+\mathbf{L}_1\otimes\mathbf{B}_\alpha\mathbf{K}_\beta\right)e_f(t) \tag{54}$$

Similar to the stability analysis method for leaders in Theorem 1, the process of containment analysis can be provided as follows. Firstly, the candidate for the Lyapunov function is given as

$$\mathrm{V}_f\left(e_f(t)\right)=e_f^{\mathrm{T}}(t)\left(\mathbf{I}_G\otimes\mathbf{P}_f\right)e_f(t) \tag{55}$$

where $\mathbf{P}_f$ is the common positive definite matrix. The derivative of (55) can be obtained as follows:

$$\dot{\mathrm{V}}_f\left(e_f(t)\right)=\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\widetilde{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\left\{e_f^{\mathrm{T}}(t)\mathbf{\Lambda}e_f(t)\right\} \tag{56}$$

where $\mathbf{\Lambda}=\left(\mathbf{I}_G\otimes\mathbf{A}_\alpha+\mathbf{L}_1\otimes\mathbf{B}_\alpha\mathbf{K}_\beta\right)^{\mathrm{T}}\left(\mathbf{I}_G\otimes\mathbf{P}_f\right)+\left(\mathbf{I}_G\otimes\mathbf{P}_f\right)\left(\mathbf{I}_G\otimes\mathbf{A}_\alpha+\mathbf{L}_1\otimes\mathbf{B}_\alpha\mathbf{K}_\beta\right)$.

For the containment analysis process of Theorem 2, the following two slack matrices are also introduced based on the IT-2 membership function and condition (48):

$$\left(\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\overline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\overline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right)+\underline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\underline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right)-1\right)\mathbf{H}_f=0 \tag{57}$$

$$-\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\left(1-\underline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right)\right)\left(\underline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)-\overline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\right)\mathbf{W}_{f\alpha\beta}\geq 0 \tag{58}$$

Then, the inequality for (56) can also be obtained by combining (57) and (58) and multiplying $\left(\mathbf{I}_G \otimes \mathbf{Q}_f\right)$ as follows:

$$\begin{aligned}&\left(\mathbf{I}_G \otimes \mathbf{Q}_f\right)\mathbf{\Lambda}\left(\mathbf{I}_G \otimes \mathbf{Q}_f\right) \leq \left(\mathbf{I}_G \otimes \mathbf{Q}_f\right)\mathbf{\Lambda}\left(\mathbf{I}_G \otimes \mathbf{Q}_f\right)\\ &+\left(\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v} \overline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\overline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right) + \underline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\underline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right) - 1\right)\mathbf{H}_f\\ &-\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\left(1-\underline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right)\right)\left(\underline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right) - \overline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\right)\mathbf{W}_{f\alpha\beta}\end{aligned} \tag{59}$$

Based on the representation of the IT-2 membership function (44) and (45), the right-hand side of the inequality (59) can be rewritten as

$$\begin{aligned}&\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\left(\underline{\sigma}_{\alpha\beta}\left(x_{1f}(t)\right)\left(\underline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right) - \overline{\Phi}_{\alpha\beta 1}\left(x_{1f}(t)\right)\right)\right)\\ &\times\left(\left(\mathbf{I}_G \otimes \mathbf{A}_\alpha\mathbf{Q}_f + \mathbf{L}_1 \otimes \mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right) + \left(\mathbf{I}_G \otimes \mathbf{A}_\alpha\mathbf{Q}_f + \mathbf{L}_1 \otimes \mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right)^{\mathrm{T}} + \mathbf{W}_{f\alpha\beta} + \mathbf{H}_f\right)\\ &+\sum_{\alpha=1}^{3}\sum_{\beta=1}^{v}\sum_{i_1=1}^{2}\varsigma_{1i_1q}\left(x_{1f}(t)\right)\left(\overline{\delta}_{\alpha\beta i_1q}\left(\left(\mathbf{I}_G \otimes \mathbf{A}_\alpha\mathbf{Q}_f + \mathbf{L}_1 \otimes \mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right) + \left(\mathbf{I}_G \otimes \mathbf{A}_\alpha\mathbf{Q}_f + \mathbf{L}_1 \otimes \mathbf{B}_\alpha\mathbf{T}_\beta\mathbf{Q}_f\right)^{\mathrm{T}}\right)\right.\\ &\left.-\left(\underline{\delta}_{\alpha\beta i_1q} - \overline{\delta}_{\alpha\beta i_1q}\right)\mathbf{W}_{f\alpha\beta} + \overline{\delta}_{\alpha\beta i_1q}\mathbf{H}_f\right) - \mathbf{H}_f\end{aligned} \tag{60}$$

Then, if the conditions (49) and (50) can be satisfied by Theorem 2, (60) is negative definite, which also means that $\dot{\mathrm{V}}_f\left(e_f(t)\right) < 0$ in the form of (56) can be achieved because of the relationship (59). Moreover, the following equation can be obtained since the dynamics of error $e_f(t)$ are all converged to zero by the stability analysis:

$$\lim_{t\to\infty}\left(x_f(t) - \left(-\mathbf{L}_1^{-1}\mathbf{L}_2\right)x_\ell(t)\right) = 0 \tag{61}$$

Via Lemma 1, the condition of containment problem (25) can be satisfied with (61). Therefore, it is said that the containment problem of the nonlinear multi-boiler system can be solved by the IT-2 fuzzy control design method in Theorem 2. Thus, the proof is accomplished. □

In this section, the leader's formation problem of nonlinear multi-boiler system (1)–(8) is solved with states dynamic system (11)–(14) by Theorem 1. Via applying the IT-2 fuzzy state feedback controller for each leader individually, not only is the required communication of leaders, whose distance is always the farthest from each other, avoided, but Theorem 1 also provides a more straightforward stability analysis and IT-2 fuzzy controller design method for leaders in a multi-agent system. After the leaders form a convex hull by placing the followers individually at the desired place or value of states, the containment control problem to force the followers into the hull is solved by Theorem 2. In the next section, the simulation results of the nonlinear multi-boiler system (1)–(8) are presented via the design method in Theorem 1 and Theorem 2.

4. Simulation Results

This section presents the simulation results of nonlinear multi-boiler system (1)–(8) by applying the proposed IT-2 fuzzy formation and containment control method. To demonstrate the effectiveness of the proposed method, the simulation results of the essential type-1 fuzzy control and robust control methods are also provided for comparison. To begin with the simulation, the interaction topology for the agents in the system is considered as in Figure 3, which consists of two leaders and two followers, that is, $m = 1, 2, 3, 4$ and $\psi = 2$, $\vartheta = 2$.

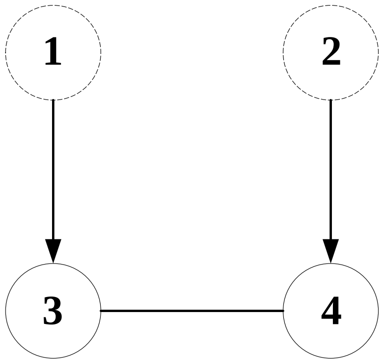

Figure 3. Structure of the multi-boiler system.

The agents 1 and 2 denoted in the dot circle are the leader agents, and agents 3 and 4 are followers. For the nonlinear multi-boiler system (1)–(8), the state dynamics of the system are transferred into (11)–(14) firstly. Then, based on Figure 3, the simulation results can be presented as follows.

4.1. Simulation of IT-2 Fuzzy Control Method

In the first part, the proposed IT-2 fuzzy control method is considered to solve the formation and containment problem of a nonlinear multi-boiler system. Representing the nonlinear system with uncertainty factors, the IT-2 T–S fuzzy model is constructed in (20)–(24), and the membership function of the model is selected as in Figure 2. Via the fuzzy model, the IT-2 fuzzy formation and containment control method is designed in (23), (24), (28), and (29). In order to develop the IT-2 fuzzy controller of a nonlinear multi-boiler system, the membership function of the fuzzy controller is designed in Figure 4. In addition, the mathematical form of Figure 4 can also be presented as follows:

$$\begin{aligned}&\underline{\mu}_{\tilde{N}_{11}}(x_{1m}(t)) = 1/\left(e^{(-x_{1m}(t)+27)/24}\right),\ \underline{\mu}_{\tilde{N}_{11}}(x_{1m}(t)) = 1/\left(e^{(-x_{1m}(t)+27)/12}\right)\\&\overline{\mu}_{\tilde{N}_{21}}(x_{1m}(t)) = 1/\left(e^{(x_{1m}(t)+27)/24}\right),\text{ and }\underline{\mu}_{\tilde{N}_{21}}(x_{1m}(t)) = 1/\left(e^{(x_{1m}(t)+27)/12}\right)\end{aligned} \tag{62}$$

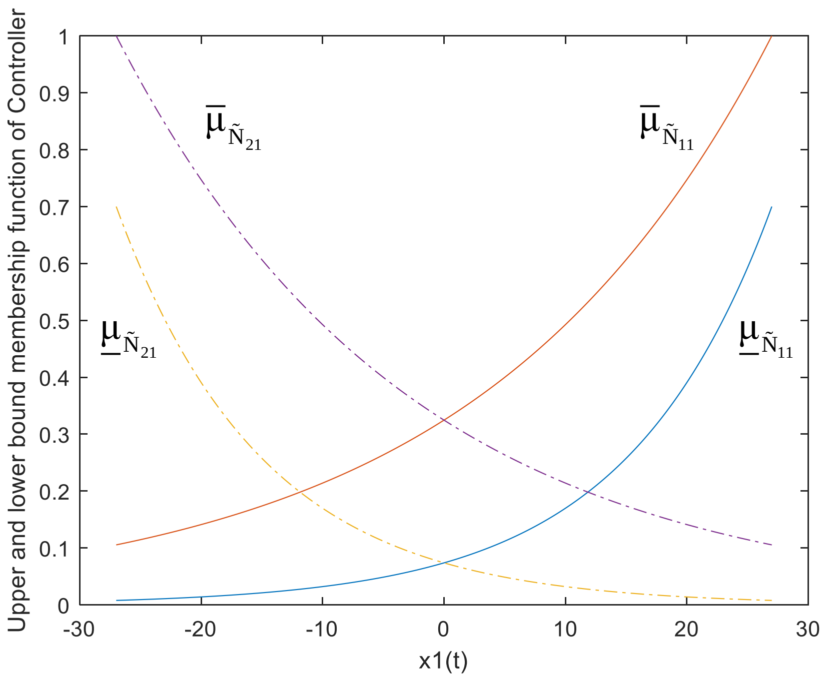

Figure 4. IT-2 membership functions of fuzzy controller.

In order to apply the fuzzy controller design method of Theorem 1 and Theorem 2, the state space is divided into four subspaces. Thus, the membership function can be constructed in the form of (44) and (45) with the cross term $\varsigma_{11q}(x_{1m}(t)) = 1 - \left(\left(x_{1m}(t) - \underline{x}_{1mq}\right) / \left(\underline{x}_{1mq} - \overline{x}_{1mq}\right)\right)$, $\varsigma_{12q}(x_1(t)) = 1 - \varsigma_{11q}(x_1(t))$, and the scalars $\overline{\delta}_{\alpha\beta 1q} = \overline{\mu}_{\widetilde{M}_{\alpha 1}}\left(\underline{x}_{1mq}\right)\overline{\mu}_{\widetilde{N}_{\beta 1}}\left(\underline{x}_{1mq}\right)$, $\overline{\delta}_{\alpha\beta 2q} = \overline{\mu}_{\widetilde{M}_{\alpha 1}}(\overline{x}_{1mq})\overline{\mu}_{\widetilde{N}_{\beta 1}}(\overline{x}_{1mq})$, $\underline{\delta}_{\alpha\beta 1q} = \underline{\mu}_{\widetilde{M}_{\alpha 1}}\left(\underline{x}_{1mq}\right)\underline{\mu}_{\widetilde{N}_{\beta 1}}\left(\underline{x}_{1mq}\right)$, and $\underline{\delta}_{\alpha\beta 2q} = \underline{\mu}_{\widetilde{M}_{\alpha 1}}(\overline{x}_{1mq})\underline{\mu}_{\widetilde{N}_{\beta 1}}(\overline{x}_{1mq})$. In this paper, the convex optimization algorithm of MATLAB is applied to solve the IT-2 fuzzy controller design problem of Theorem 1 and Theorem 2 for the formation and containment. Considering the range of the first state as $x_{1m}(t) \in \begin{bmatrix}-28 & 28\end{bmatrix}^{\mathrm{T}}$, the following feasible solutions can be obtained by solving the LMI form sufficient conditions (33)–(36) and (47)–(50). Note that the gains $\mathbf{F}_\beta$ and $\mathbf{K}_\beta$ are represented as $\mathbf{F}_\beta^{\mathrm{T2}}$ and $\mathbf{K}_\beta^{\mathrm{T2}}$ to be distinguished from the gains of the type-1 fuzzy controller.

For leaders, one has

$$\mathbf{F}_1^{\mathrm{T2}} = \begin{bmatrix} -0.4204 & 0.0381 & -0.1128 & 0.8505 \\ -0.4144 & -0.1023 & -0.1106 & 0.8393 \\ 0.0417 & -0.0062 & -1.0259 & 1.6758 \end{bmatrix}, \mathbf{F}_2^{\mathrm{T2}} = \begin{bmatrix} -0.4816 & 0.0421 & -0.0877 & -0.4160 \\ -0.4748 & -0.0983 & -0.0858 & -0.4092 \\ -0.0796 & -0.0055 & -1.5861 & 0.4771 \end{bmatrix},$$
$$\mathbf{P}_\ell = \begin{bmatrix} 0.3125 & 0.048 & 0.0157 & 0.1868 \\ 0.0048 & 0.8259 & -0.0001 & -0.0015 \\ 0.0157 & -0.0001 & 0.8162 & -0.0354 \\ 0.01868 & -0.0015 & -0.0354 & 1.9064 \end{bmatrix} \tag{63}$$

For followers, one has

$$\mathbf{K}_1^{\mathrm{T2}} = \begin{bmatrix} -0.0876 & 0.0251 & 0.1828 & 0.0244 \\ -0.0865 & -0.0749 & 0.1809 & 0.0241 \\ 0.9178 & -0.0037 & -4.4679 & 0.6972 \end{bmatrix}, \mathbf{K}_1^{\mathrm{T2}} = \begin{bmatrix} -0.0975 & 0.0226 & 0.2271 & -0.1781 \\ -0.0963 & -0.0774 & 0.2245 & -0.1754 \\ 0.8168 & -0.0024 & -3.9624 & 0.5688 \end{bmatrix},$$
$$\mathbf{P}_f = \begin{bmatrix} 0.2928 & 0.0028 & -0.7489 & -0.1787 \\ 0.0028 & 3.8116 & -0.0024 & -0.0001 \\ -0.7489 & -0.0024 & 3.5479 & -0.1105 \\ -0.1787 & -0.0001 & -0.1105 & 9.9860 \end{bmatrix} \tag{64}$$

Applying the IT-2 fuzzy controller (63) and (64) via the form of (28) and (29) to control the nonlinear multi-boiler system, the responses of the system states are obtained with the following initial conditions of each agent.

Leader 1′s initial condition is considered as $X_1(0) = \begin{bmatrix}26 & 2.7 & 560 & 0.9\end{bmatrix}^{\mathrm{T}}$.

Leader 2′s initial condition is considered as $X_2(0) = \begin{bmatrix}20 & 2.3 & 540 & 0.7\end{bmatrix}^{\mathrm{T}}$.

Follower 3′s initial condition is considered as $X_3(0) = \begin{bmatrix}29 & 2.9 & 570 & 1\end{bmatrix}^{\mathrm{T}}$.

Follower 4′s initial condition is considered as $X_4(0) = \begin{bmatrix}17 & 2.1 & 530 & 0.6\end{bmatrix}^{\mathrm{T}}$.

To present the results of the IT-2 fuzzy containment control method more clearly, the state dynamics of boiler system (11)–(14) are first given in Figures 5–8. As seen from the responses of Figures 5–8, the dynamics of all states can converge to zero smoothly. In other words, the uncertainties problem of $\Delta a_{44}(t)$, $\Delta b_{11}(t)$, and $\Delta b_{41}(t)$ is solved effectively by the fuzzy controller design method based on the IT-2 T–S fuzzy model. In this simulation, the uncertainties are considered as follows:

$$\Delta a_{44}(t) = -0.004sin(t),\ \Delta b_{11}(t) = 0.028sin(t),\ \text{and } \Delta b_{41}(t) = 0.003sin(t) \tag{65}$$

In Figure 8, the state variable related to the loading disturbance of the leaders can also be placed at the desired values $\mathbb{R}_1 = \begin{bmatrix}0 & 0 & 0 & 0.02\end{bmatrix}^{\mathrm{T}}$ and $\mathbb{R}_2 = \begin{bmatrix}0 & 0 & 0 & -0.02\end{bmatrix}^{\mathrm{T}}$. Thus, it can be said that the formation objective of the leaders can be achieved effectively via the IT-2 fuzzy controller designed by Theorem 1. Meanwhile, via the IT-2 fuzzy containment controller design of Theorem 2, all the followers can also be driven into the interval formed by the leaders. Based on the results of Figures 5–8, the output responses of the nonlinear multi-boiler system (5)–(8) are presented in Figures 9–12.

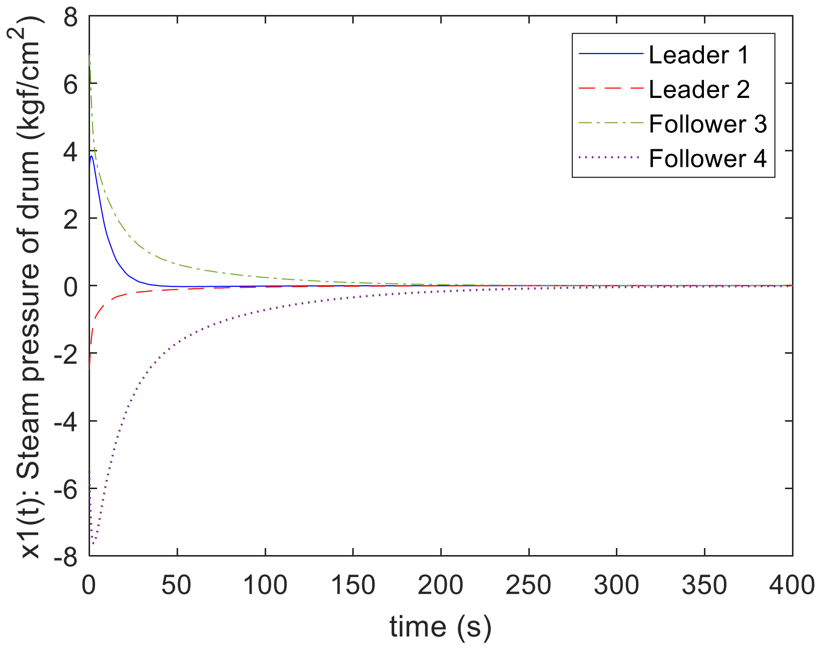

Figure 5. Responses of state $x_1(t)$ with IT-2 fuzzy control.

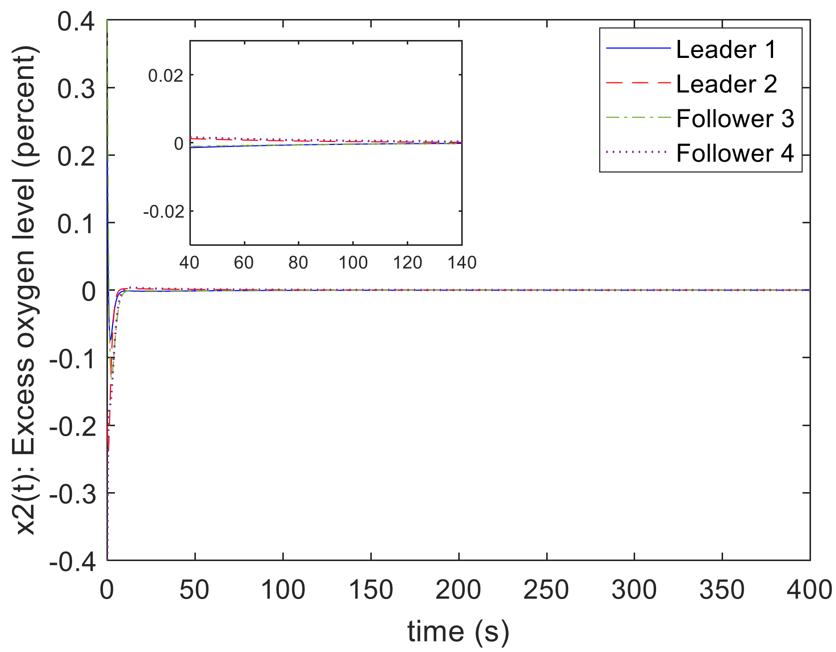

Figure 6. Responses of state $x_2(t)$ with IT-2 fuzzy control.

From Figures 9–11, the output dynamics of the nonlinear multi-boiler system (5)–(8) can be controlled to the desired value by the IT-2 fuzzy formation and containment design method in this paper. In Figure 12, the outputs of the steam flow rate of all followers also converge into the small interval generated by the leader dynamics in Figure 8. With the simulation results, the steam pressure of the drum and the excess oxygen level can be maintained at 320 psi and 2.5 percent, respectively, to satisfy the operating requirement of a nonlinear multi-boiler system. More, the steam flow rate of all followers and leaders can also be maintained in the required range.

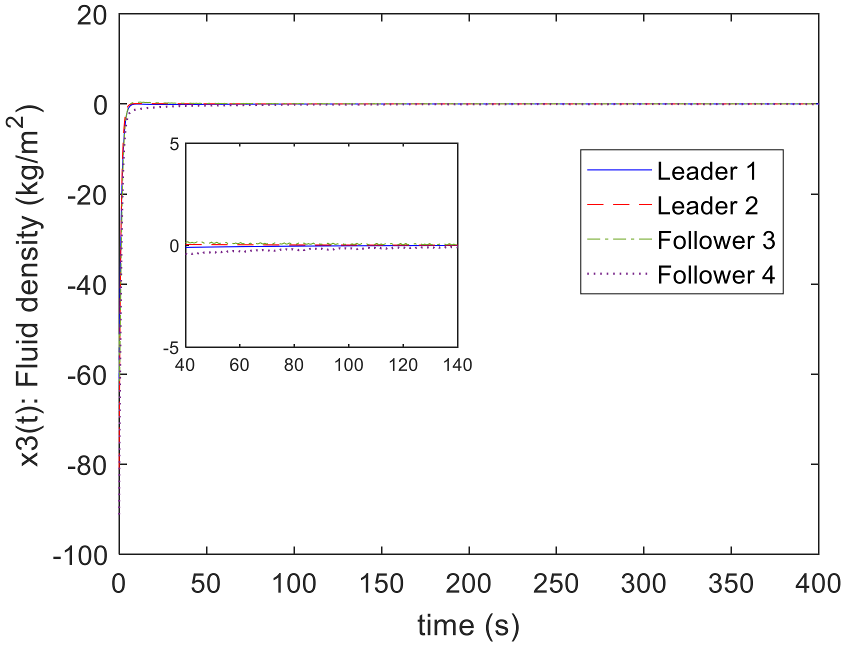

Figure 7. Responses of state $x_3(t)$ with IT-2 fuzzy control.

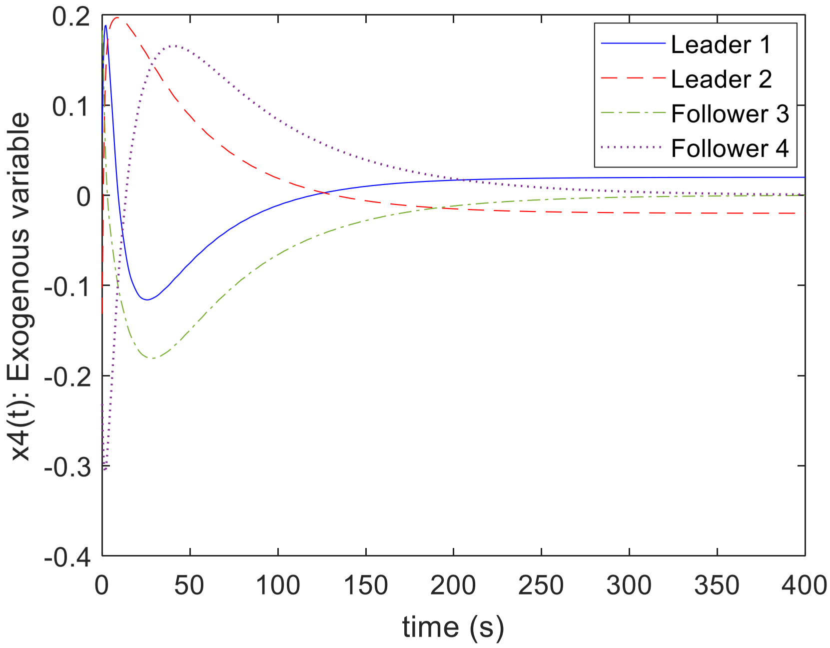

Figure 8. Responses of state $x_4(t)$ with IT-2 fuzzy control.

To demonstrate the effectiveness of the proposed design method for the control of the nonlinear multi-boiler system (1)–(8), which consists of uncertainties in every agent, the essential fuzzy control and robust fuzzy control based on the type-1 T–S fuzzy model are presented as follows for the comparisons.

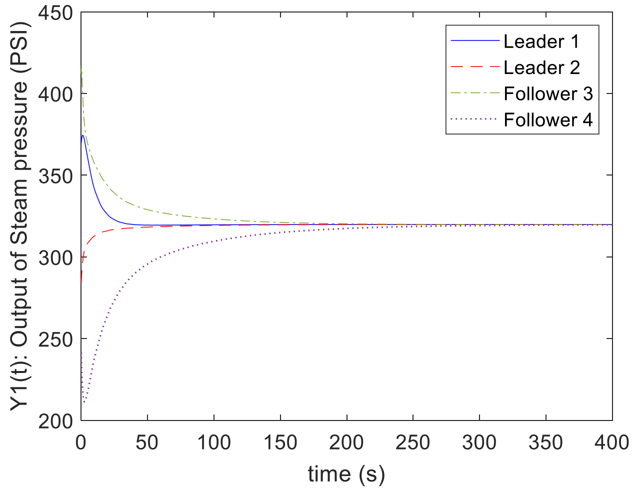

Figure 9. Responses of output $Y_1(t)$ with IT-2 fuzzy control.

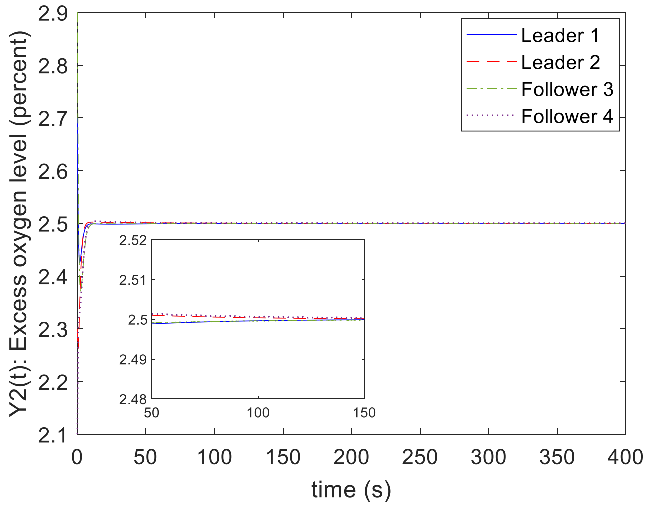

Figure 10. Responses of output $Y_2(t)$ with IT-2 fuzzy control.

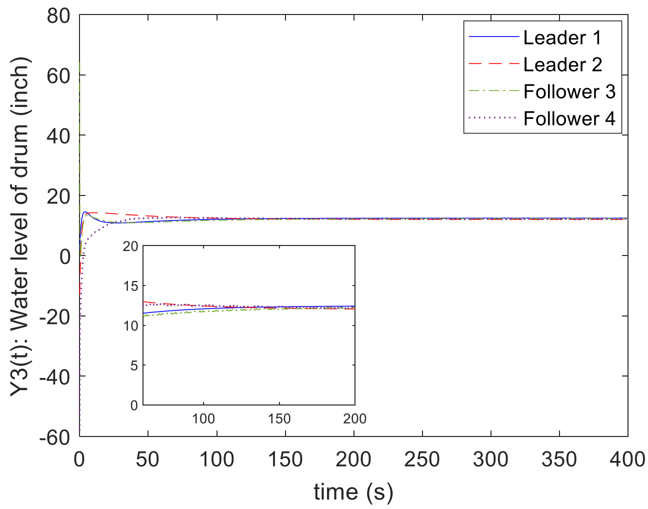

Figure 11. Responses of output $Y_3(t)$ with IT-2 fuzzy control.

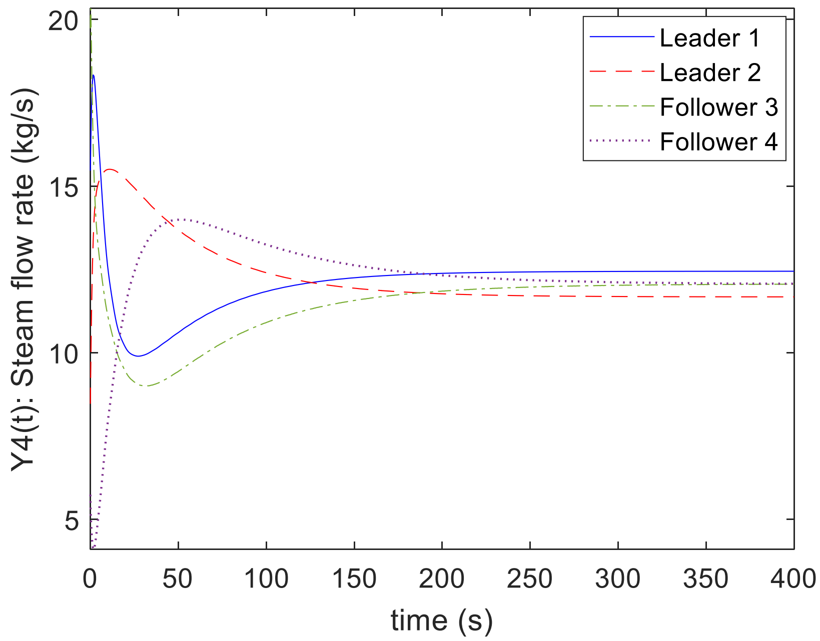

Figure 12. Responses of output $Y_4(t)$ with IT-2 fuzzy control.

4.2. Simulation

Firstly, referring to [35], the membership functions shown in Figure 13 are selected for the type-1 T–S fuzzy model.

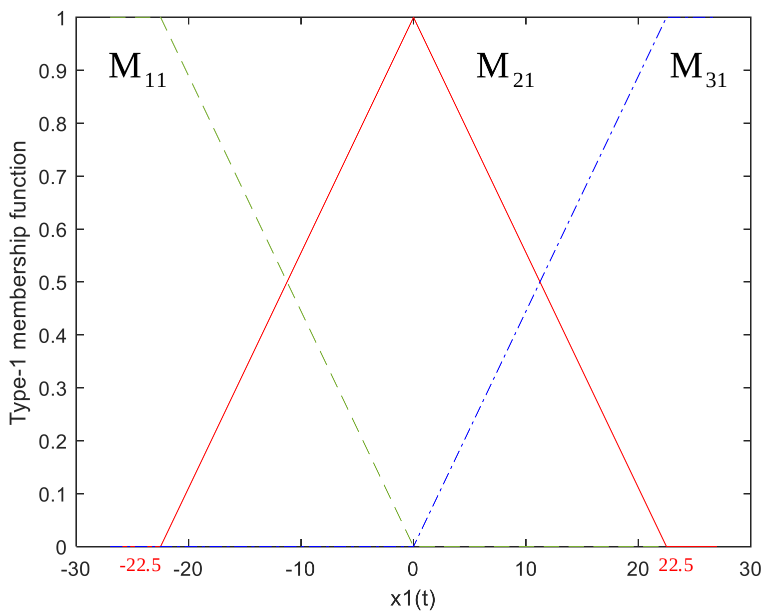

Figure 13. Type-1 membership functions of boiler system.

Different from the membership function designed in Figure 2, the factor of uncertainties is not considered in the type-1 fuzzy modeling. Referring to the IT-2 T–S fuzzy model (20)–(22), the type-1 T–S fuzzy model of the nonlinear multi-boiler system (11)–(14) can be expressed as follows based on the membership function in Figure 13.

Model Rule α:
If $x_{1m}(t)$ is $\mathrm{M}_{\alpha 1}$, then

$$\begin{cases} \dot{\phi}_m(t) = \mathbf{A}_\alpha \phi_m(t) + \mathbf{B}_\alpha u_m(t) & for \quad m = 1,2 \\ \dot{x}_m(t) = \mathbf{A}_\alpha x_m(t) + \mathbf{B}_\alpha u_m(t) & for \quad m = 3,4 \end{cases} \tag{66}$$

where $\alpha = 1,2,3$ and matrices $\mathbf{A}_1$ to $\mathbf{A}_3$, $\mathbf{B}_1$ to $\mathbf{B}_3$ are the same as the model (16)–(18)

Based on parallel distributed compensation (PDC) design method, the fuzzy controller of the type-1 T–S fuzzy model can be designed by sharing the same antecedent part of model (66) as follows.

Controller Rule α:
If $x_{1m}(t)$ is $\mathrm{M}_{\alpha 1}$, then

$$\begin{cases} u_m(t) = \mathbf{F}_\alpha^{\mathrm{T1}} \phi_m(t) & for \quad m = 1,2 \\ u_m(t) = \mathbf{K}_\alpha^{\mathrm{T1}} \sum\limits_{n \in \tilde{\mathfrak{N}}(\Omega)} j_{mn}(x_m(t) - x_n(t)) \ , & for \quad m = 3,4 \end{cases} \tag{67}$$

Then, via the process of defuzzification, the overall type-1 T–S fuzzy controller can be provided in the following form:

$$u_m(t) = \sum_{\alpha=1}^{3} \mu_{\mathrm{M}_{\alpha 1}}(x_{1m}(t)) \left\{ \mathbf{F}_\alpha^{\mathrm{T1}} \phi_m(t) \right\}, \ for \quad m = 1,2 \tag{68}$$

$$u_m(t) = \sum_{\alpha=1}^{3} \mu_{\mathrm{M}_{\alpha 1}}(x_{1m}(t)) \left\{ \mathbf{K}_\alpha^{\mathrm{T1}} \sum_{n \in \tilde{\mathfrak{N}}(\Omega)} j_{mn}(x_m(t) - x_n(t)) \right\}, \ for \quad m = 3,4 \tag{69}$$

where $\mu_{\mathrm{M}_{\alpha 1}}(x_{1m}(t)) = \mathrm{M}_{\alpha 1}(x_{1m}(t)) / \sum\limits_{\alpha=1}^{3} \mathrm{M}_{\alpha 1}(x_{1m}(t))$ and $\mathrm{M}_{\alpha 1}(x_{1m}(t))$ for $\alpha = 1,2,3$ is the membership function presented in Figure 13 corresponding to each fuzzy rule.

In [28], the type-1 fuzzy controller design and stability analysis have been presented systematically and completely. To save space in this paper, only the following feasible solutions of feedback gains obtained by the fuzzy controller design method in [28] are presented in the simulation.

For leaders, one has

$$\begin{aligned} \mathbf{F}_1^{\mathrm{T1}} &= \begin{bmatrix} -0.9244 & 0.0226 & 0.0095 & 0.0798 \\ -0.9108 & -0.0620 & 0.0093 & 0.0793 \\ 0.0558 & 0.0011 & -0.7228 & 0.1053 \end{bmatrix}, \ \mathbf{F}_2^{\mathrm{T1}} = \begin{bmatrix} -0.9819 & 0.0197 & -0.0347 & 0.6010 \\ -0.9675 & -0.0648 & -0.0341 & 0.5928 \\ 0.0317 & -0.0018 & -0.7460 & 0.8650 \end{bmatrix}, \\ \mathbf{F}_3^{\mathrm{T1}} &= \begin{bmatrix} -1.0742 & 0.0154 & -0.1136 & 1.3592 \\ -1.0585 & -0.0691 & -0.1116 & 1.3401 \\ 0.0073 & -0.0049 & -0.7738 & 1.6533 \end{bmatrix} \end{aligned} \tag{70}$$

For followers, one has

$$\begin{aligned} \mathbf{K}_1^{\mathrm{T1}} &= \begin{bmatrix} -0.7658 & 1.1033 & 0.0788 & -3.4321 \\ -0.7561 & -1.1843 & 0.0781 & -3.2534 \\ 0.0704 & 0.0053 & -1.8028 & 0.8506 \end{bmatrix}, \ \mathbf{K}_2^{\mathrm{T1}} = \begin{bmatrix} -0.7453 & 1.1016 & 0.0484 & -2.9902 \\ -0.7359 & -1.1860 & 0.0485 & -2.8178 \\ 0.0724 & 0.0025 & -1.8513 & 1.2626 \end{bmatrix}, \\ \mathbf{K}_3^{\mathrm{T1}} &= \begin{bmatrix} -0.8587 & 1.0930 & -0.1413 & -2.8947 \\ -0.8470 & -1.1945 & -0.1354 & -2.7227 \\ -0.3738 & -0.0199 & -2.8872 & 0.2684 \end{bmatrix} \end{aligned} \tag{71}$$

Then, applying the type-1 formation and containment fuzzy controller (68) and (69) with the gains (70) and (71), the output responses of the nonlinear multi-boiler system (5)–(8) can be presented in Figures 14–17.

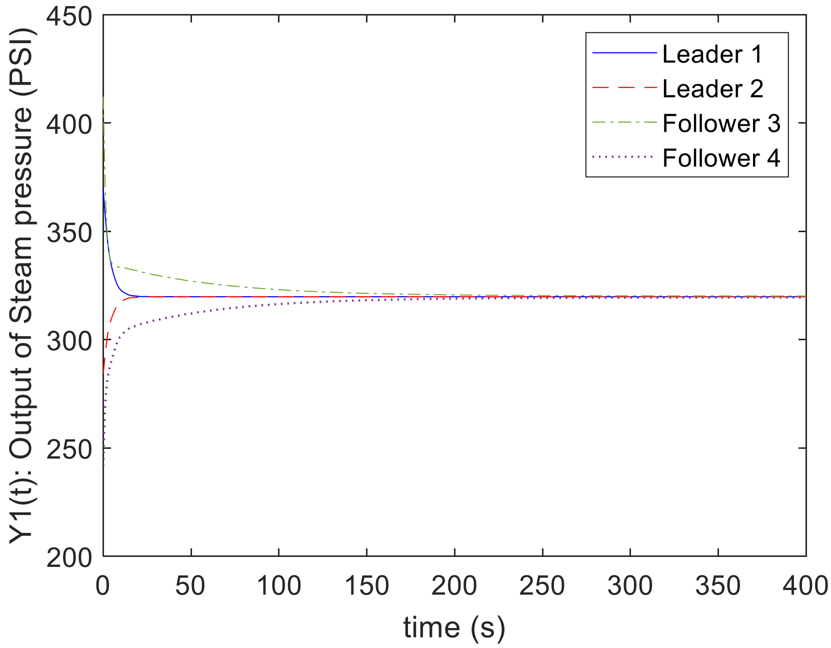

Figure 14. Responses of output $Y_1(t)$ with type-1 fuzzy control.

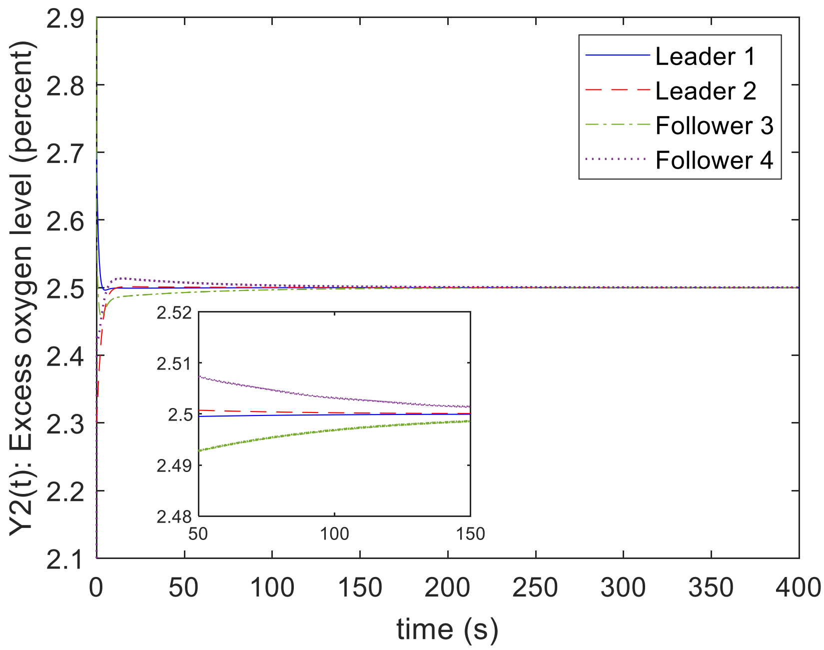

Figure 15. Responses of output $Y_2(t)$ with type-1 fuzzy control.

From Figures 14–17, it is obvious that the overshoots of each output are bigger than the responses obtained by the proposed IT-2 fuzzy control method in Figures 9–12. Moreover, there are a non-smooth process of followers when they are forced to the desired place by leaders in Figure 14. The convergent rate of $Y_2(t)$ obtained in Figure 15 is also much slower than the responses obtained in Figure 10. According to the above comparison results, the difference between the responses obtained by the type-1 fuzzy and IT-2 fuzzy control method is caused by the uncertainties in the nonlinear multi-boiler system (1)–(8). For the multi-agent system, the problem of uncertainties might be much more serious than that of a single system. When the information of states communicated between agents is affected by uncertainties, the error of the accepted value from neighbor agents will become larger and larger. If uncertainties are not considered in the control method design in practical applications, the performance of the multi-agent system will be deteriorated and even be made unstable. Thus, uncertainties are necessary to consider in the control method design of the nonlinear multi-boiler system. In the following simulation, the type-1 robust fuzzy control is applied to solve the uncertainties problem.

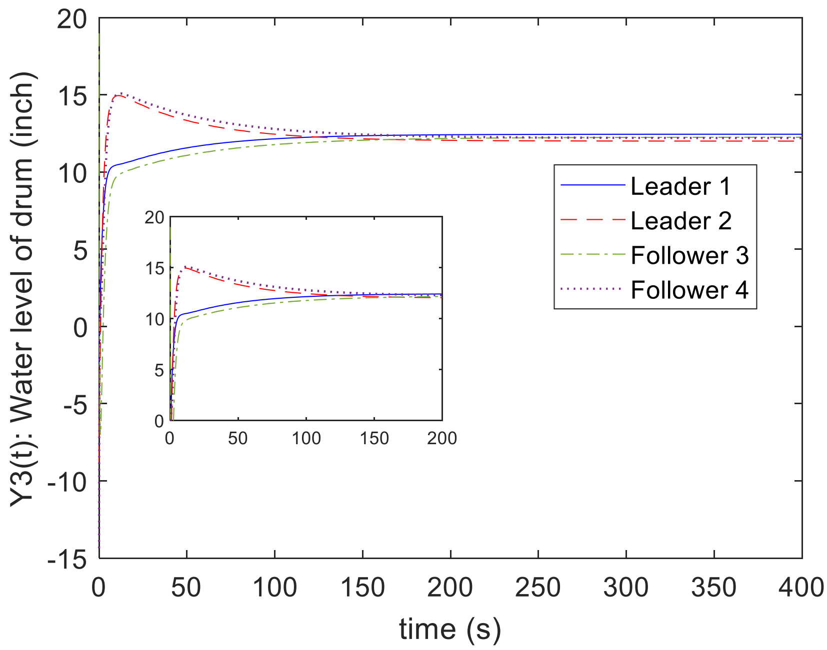

Figure 16. Responses of output $Y_3(t)$ with type-1 fuzzy control.

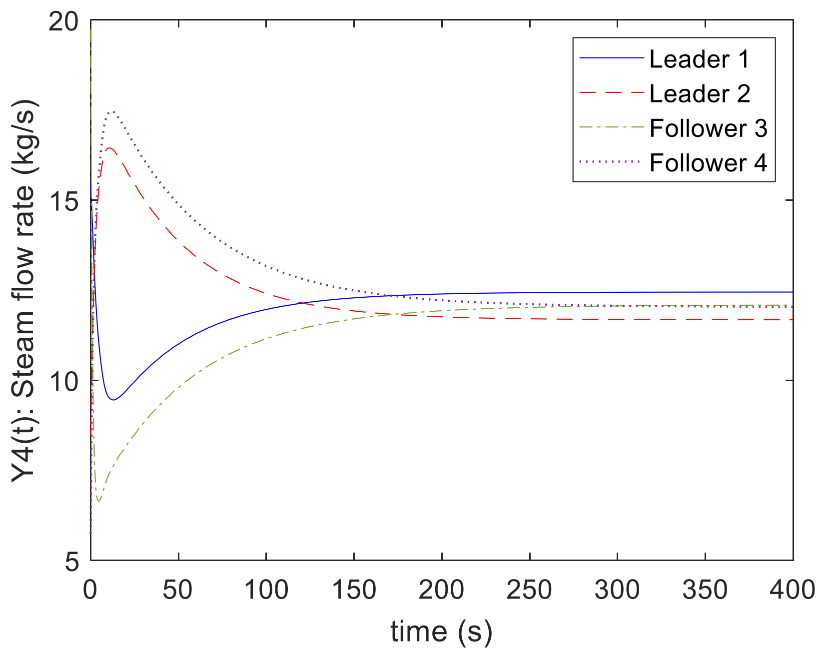

Figure 17. Responses of output $Y_4(t)$ with type-1 fuzzy control.

4.3. Simulation

Different from the proposed IT-2 fuzzy control method, a robust fuzzy control method based on the type-1 T–S fuzzy model is also considered to solve the uncertainties problem. In the simulation, the type-1 robust fuzzy control method from [47] without considering stochastic behaviors is applied for the comparison. To develop the fuzzy control method, uncertainties in the nonlinear multi-boiler system (11)–(14) are constructed in the following form:

$$\Delta \mathbf{A}_\alpha(t) = \widetilde{\mathbf{M}}\Phi_\Delta(t)\widetilde{\mathbf{J}} \text{ and } \Delta \mathbf{B}_\alpha(t) = \widetilde{\mathbf{M}}\Phi_\Delta(t)\widetilde{\mathbf{O}}, \text{ for } \alpha = 1,2,3 \tag{72}$$

where $\widetilde{\mathbf{M}} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$, $\widetilde{\mathbf{J}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.04 \end{bmatrix}$, $\widetilde{\mathbf{O}} = \begin{bmatrix} 0.28 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.03 & 0 & 0 \end{bmatrix}$, and $\Phi_\Delta(t) = sin(t)$ is the time-varying item of uncertainties.

Obviously, the uncertainties (72) of the type-1 T-S fuzzy model can be derived from $\Delta a_{44}(t)$, $\Delta b_{11}(t)$, and $\Delta b_{41}(t)$ in (65) of the nonlinear system (11)–(14). To save space in this paper, the details of the robust fuzzy controller design and stability analysis method from [47] are not provided. Applying the design method, the following feedback gains can be obtained for the leaders and followers to solve the formation and containment control problem of the nonlinear multi-boiler system (1)–(8) with uncertainties.

For leaders, one has

$$\begin{gathered}\mathbf{F}_1^{T1} = \begin{bmatrix} -0.6450 & 0.7315 & -0.0077 & -0.5602 \\ -0.6286 & -0.7808 & -0.0052 & -0.5244 \\ -0.0182 & 0.0028 & -1.1170 & 0.3916 \end{bmatrix}, \mathbf{F}_2^{T1} = \begin{bmatrix} -0.6260 & 0.7272 & -0.1176 & 0.0319 \\ -0.6097 & -0.7851 & -0.1130 & 0.0592 \\ 0.0148 & -0.0031 & -1.1997 & 1.1812 \end{bmatrix}, \\ \mathbf{F}_3^{T1} = \begin{bmatrix} -0.5791 & 0.7214 & -0.2663 & 0.9057 \\ -0.5633 & -0.7907 & -0.2591 & 0.9206 \\ 0.0659 & -0.0090 & -1.2915 & 2.0478 \end{bmatrix}\end{gathered} \tag{73}$$

For followers, one has

$$\begin{gathered}\mathbf{K}_1^{T1} = \begin{bmatrix} -0.1668 & 0.8951 & 0.0762 & -1.5961 \\ -0.1737 & -0.9658 & 0.0733 & -1.5369 \\ 0.0873 & 0.0067 & -1.7120 & 0.1677 \end{bmatrix}, \mathbf{K}_2^{T1} = \begin{bmatrix} -0.1599 & 0.8942 & 0.0588 & -1.2706 \\ -0.1670 & -0.9666 & 0.0567 & -1.2150 \\ 0.0923 & 0.0045 & -1.7378 & 0.5807 \end{bmatrix}, \\ \mathbf{K}_3^{T1} = \begin{bmatrix} -0.2020 & 0.8730 & -0.6628 & -1.3508 \\ -0.2087 & -0.9873 & -0.6321 & -1.2896 \\ -0.0171 & -0.0683 & -4.7397 & -0.5382 \end{bmatrix}\end{gathered} \tag{74}$$

Then, with the gains (73) and (74), applying the robust type-1 fuzzy controller in the form of (68) and (69), the output responses of the nonlinear multi-boiler system (5)–(8) can also be obtained in Figures 18–21.

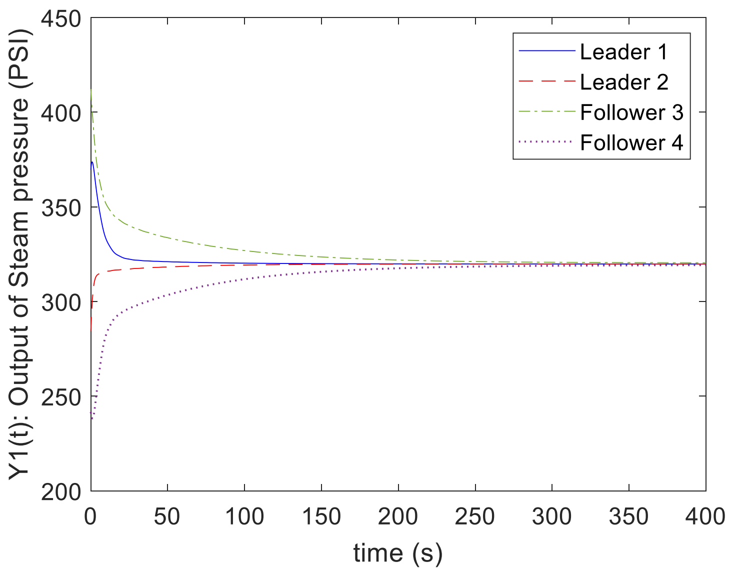

Figure 18. Responses of output $Y_1(t)$ with type-1 robust fuzzy control.

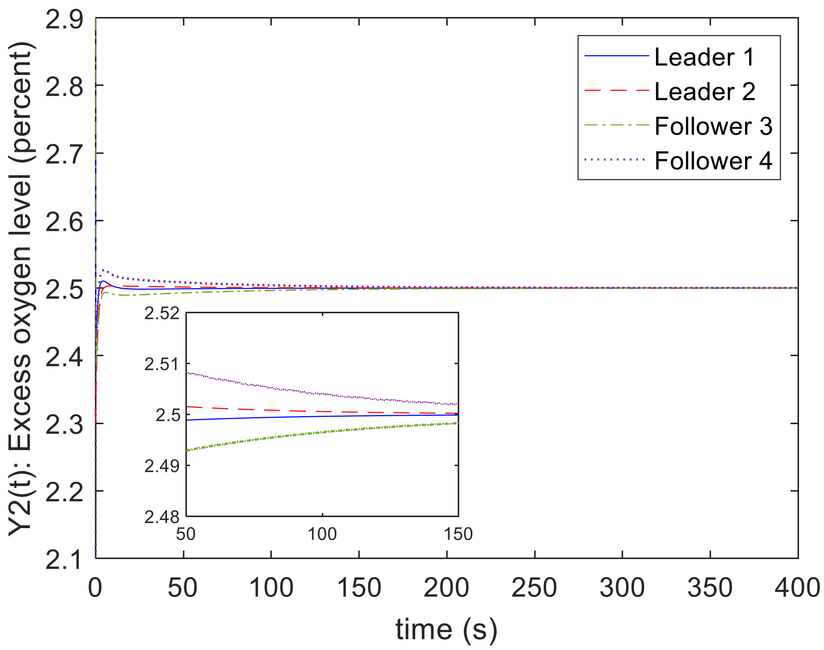

Figure 19. Responses of output $Y_2(t)$ with type-1 robust fuzzy control.

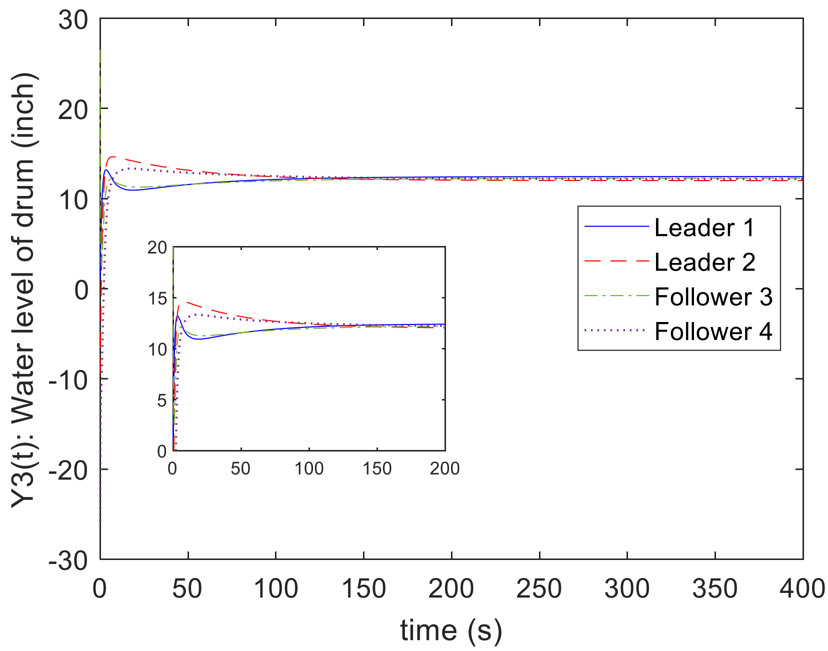

Figure 20. Responses of output $Y_3(t)$ with type-1 robust fuzzy control.

Compared with the simulation results in Figures 14–17, via the type-1 robust fuzzy control method, the smoother responses of each output can be obtained in Figures 18–21. Moreover, the converge rate for all agents is improved in Figure 20. The overshoot of the followers when they are forced to the specific range formed by the leaders is reduced greatly in Figure 21. In the control of the multi-agent system, if the control is not precise enough for all agents, the errors will become bigger and bigger when the signals are communicated between agents continuously. Obviously, despite the instability of the system not being caused, the uncertainties problem will deteriorate the performance of the whole multi-agent system greatly. From the above simulation results, it can be said that the type-1 robust fuzzy control can perform much better than essential type-1 fuzzy control for solving the formation and containment problem of the nonlinear multi-boiler system (1)–(8) with the effect of uncertainties.

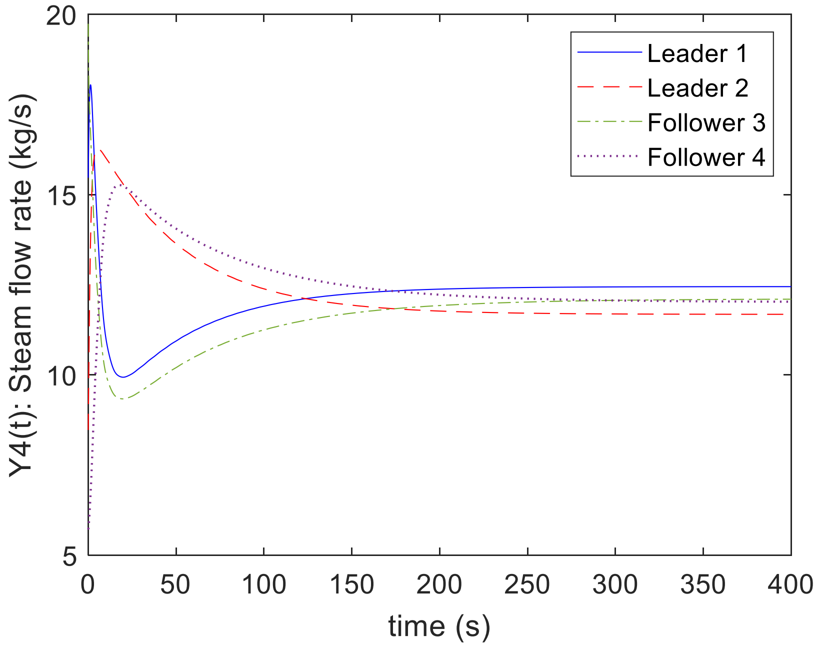

Figure 21. Responses of output $Y_4(t)$ with type-1 robust fuzzy control.

However, applying the IT-2 fuzzy control method proposed in this paper, much smoother responses can be achieved than responses of Figures 18–21 obtained by the type-1 robust fuzzy control method. The converge rate for all outputs is also much faster, especially in the responses of $Y_2(t)$ in Figure 10. For the control purpose of the nonlinear multi-boiler system (1)–(8), the variable of the followers corresponding to the disturbance is required to be constrained in the interval designed by the leader agents under the effect of uncertainties. Thus, from the responses of $Y_4(t)$ obtained by the three control methods, from Figures 12, 17 and 21, one can see that the followers are forced into the specific range by the leaders much more smoothly via the proposed IT-2 T–S fuzzy control method.

It is worth noticing that the IT-2 T–S fuzzy model can represent the factor of uncertainties in the nonlinear system more completely. Based on the fuzzy model, the IT-2 fuzzy controller can also be developed to solve the control problem with uncertainties. Especially for the control design method of the multi-agent, the problem of uncertainties is unavoidable, which may be caused by perturbations, modeling errors, a disturbance of communications between agents, and so on. Moreover, different from the PDC-based fuzzy controller in the essential type-1 and robust type-1 fuzzy control method, via the imperfect premise matching IT-2 fuzzy controller design method, only two rules of the fuzzy controller are required in the simulation. This advantage makes the IT-2 fuzzy controller designed for the practical industrial system more cost-effective and the design process more straightforward. Therefore, from the above simulation results in three cases, the proposed IT-2 fuzzy control method can provide a more proper scheme for the nonlinear multi-boiler system (1)–(8) to achieve better formation and containment performances with the uncertainties problem.

5. Conclusions

The formation and containment control problem of nonlinear multi-boiler systems with the IT-2 fuzzy control method is discussed in this paper. The contributions and novelty of this paper can be summarized as follows. In industrial systems such as vessels or power plants, there is more than one boiler system in most applications. Solving the control problem of multiple boiler systems, the control theory based on the multi-agent is applied as a powerful and effective tool to control all the boiler systems simultaneously. Additionally, from the perspective of practical control systems, it is important to solve the uncertainties problem to improve the control performance, especially for the multi-agent system. Thus, considering the uncertainties problem, the IT-2 membership function is applied to include the factor of uncertainties more entirely into the T–S fuzzy modeling method of the nonlinear multi-boiler system in this paper. Based on the IT-2 T–S fuzzy model, a different formation control method where the communication between leaders is not required is also proposed to avoid the signal transmission problem. Moreover, the complexity of the stability analysis and IT-2 fuzzy controller design method for leaders can reduce efficiency since they only need to be developed for one leader. While the formation of leaders is accomplished, the containment control objective for followers can also be achieved. It is worth noting that the IT-2 fuzzy control method developed in this paper depends on the membership function. For the control issue of practical applications, the more system information is involved in the fuzzy controller design, the more proper control benefits can be obtained. Additionally, in the proposed method, the IT-2 membership function of the fuzzy controller can be designed in a different form with the fuzzy model via the imperfect premise matching scheme. The advantages of the method are that it can more flexibly design the fuzzy controller for the various requirements of formation and containment control problems and can save the control cost effectively. Via the verification of simulation results, the effectiveness of the proposed IT-2 fuzzy control method can also be presented. Thus, the IT-2 T–S fuzzy model-based formation and containment fuzzy controller design method can be provided as a better control scheme for a nonlinear multi-boiler system with uncertainties.

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