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Simulation-Based Approach for Multi-Echelon Inventory System Selection: Case of Distribution Systems

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Abstract: Due to the current complexity of the supply chain, multi-echelon inventory management has become challenging while also being an interesting field of research as it allows efficient control of supply chain interdependencies. It became clear to many researchers that analytical models are no longer effective for addressing the multi-echelon inventory management problem. Simulation can be used to assess and quantify the impact of each inventory strategy on a supply chain performance . Our paper aims to provide a simulation-based approach to guide decision makers select and validate a multi-echelon distribution inventory system. The proposed approach is composed of four major steps that involve characterization of the current supply chain, conceptual modeling of the multi-echelon inventory system alternatives, and finally, simulation modeling using appropriate simulation software to compare and test different options. The approach was also tested and validated through an application to the case of the Moroccan pharmaceutical products supply chain in the public sector. The results of the simulation demonstrated that adopting an installation stock policy at all levels of the supply chain with an allocation of safety stocks in the most downstream stages is the best and most appropriate alternative for the pharmaceutical supply chain under study.

Keywords: supply chain management; multi-echelon inventory management; simulation modeling; pharmaceutical supply chain

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1. Introduction and Problem Statement

Over the past three decades, the supply chain has developed in complexity and become a significant driver of demand and customer loyalty satisfaction [1]. Traditionally, inventories at different facilities in a supply chain were handled separately, with high product quantities serving as a buffer. Increasing competitive pressures and globalization of markets have driven businesses to make greater efforts to minimize inventories while increasing customer service. As a result, researchers and industry practitioners are increasingly focusing on multi-echelon inventory management, which considers the relationships between multiple inventories in a supply chain.

Establishing interdependent relations between decision parameters of different processes of the supply chain cannot be easily modeled analytically. Actually, modeling dynamic networks can be possible and flexible through simulation [2,3]. It provides an effective evaluation tool for supply chain performance and risks. The simulation model can provide details about the dynamic and stochastic inventory system with accuracy [4]

We conducted a literature review in previous work [5] to look into several research studies that implemented simulation modeling for multi-echelon inventory management. Multiple simulation models of multi-echelon inventory systems were categorized using a set of features specified in the literature.

Many research studies built models for simulating inventory control policies in multi-echelon inventory systems [6–10], while others compared replenishment strategies or investigated the relationship between inventory parameters [6,9,11,12], according to our findings.

In very recent research works, Xu et al. [13] proposed a simulation-based optimization model of the multi-echelon inventory system for fresh agricultural products. The authors proved in the simulation outcomes that the suggested simulation model can aid decision-makers in addressing the inventory system's complexity. For a multi-echelon inventory control model for fresh products, Zhang et al. [14] simulated two inventory methods. The authors demonstrated that the findings of the research study could assist fresh product supply chain managers in making inventory management decisions and cutting costs.

Based on our literature review findings ([5] and the outcomes of the most recent research articles), we noted that none of the previous research work has created a general simulation-based approach for comparing, selecting, and validating the appropriate inventory policies for a multi-echelon supply chain for the distribution structure.

Decision-makers are looking for guidelines to determine the appropriate inventory policies for their supply chains. In this context, we intend to create and implement a simulation-based approach to assist decision-makers in selecting and validating the best multi-echelon inventory system for their needs. With the use of simulation, a comparison of different multi-echelon inventory system options for distribution systems will be developed.

The proposed simulation-based approach is composed of four major steps. First, the background to the problem situation should be defined. After that, the simulation study goals and validation parameters are determined. Then, the expected simulation benefits are identified. In the third step, the conceptual modeling of the multi-echelon inventory system alternatives is developed. After that, we run the simulation model using simulation software to compare and test different alternatives. Finally, the decision maker can select the alternative that corresponds to the supply chain specifications and his needs in terms of product availability and inventory costs.

The simulation-based approach suggested in our paper is explained in detail in Section 3. The approach was also tested and validated through an application to the case of the Moroccan pharmaceutical products supply chain in the public sector in Section 5.

2. The Multi-Echelon Inventory System Model Description: Case of Distribution Systems

2.1. Model Description and Notations

There are many different inventory systems, and no single strategy can be used to manage them all [15]. A multi-echelon distribution inventory system includes a variety of network configurations, ranging from the simplest network, in which a single node distributes products directly to end customers, to the most complex network, in which products pass through multiple nodes and transportation routes before arriving at their final destination [16]. The "echelon inventory" is the inventory of a specific installation added to the inventory of all downstream installations [17]. A facility has only one predecessor in the distribution inventory system and one or more successors. In this paper, we focus on a multi-echelon distribution inventory system, which is depicted in Figure 1.

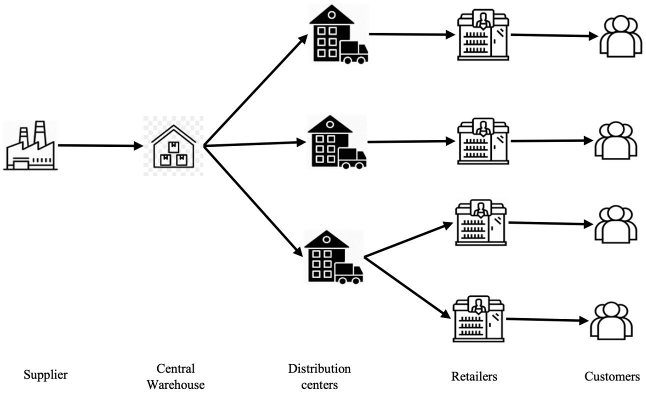

Figure 1. Multi-echelon Distribution Inventory System: an illustration.

Figure 1 illustrates a multi-echelon inventory system that consists of a supplier, a central warehouse, a couple of distribution centers, retailers, and customers that present external demand. When the supplier receives an order, it is expected that it will always have sufficient inventory on hand to fulfill it. The inventory control policy of the supplier is not considered in this paper. Before proceeding to the following sections, we provide in Table 1 the notations to be used later for the inventory policies.

Table 1. Problem Notations.

Symbol/ Notation	**Description/Definition**
$d_i(t)$	Demand at the *i*th retailer at time *t*
L_r	The lead time of the retailer
L_d	The lead time of the distributor
IR_i^r	Installation reorder point of the *i*th retailer
IR_j^d	Installation reorder point of the *j*th Distributor
IR^w	Installation reorder point of the central warehouse
ER_i^r	Echelon reorder point of the *i*th retailer
ER_j^r	Echelon reorder point of the *j*th distributor
ER^w	Echelon reorder point of the central warehouse
Q_i^r	Order quantity of the *i*th retailer
Q_j^d	Order quantity of the *j*th distributor
Q^w	Order quantity of the central warehouse
$OI_i^r(t)$	On – hand inventory at time *t* of the *i*th retailer
$OI_j^d(t)$	On – hand inventory at time *t* of the *j*th distributor
$OI^w(t)$	On–hand inventory at time *t* of the central warehouse
$BO_i^r(t)$	Backorders at the *i*th retailer at time *t*
$BO_j^d(t)$	Backorders at the *j*th distributor at time *t*
$BO^w(t)$	Backorders at the central warehouse at time *t*
$IL_i^r(t)$	Inventory level at time *t* of the *i*th retailer
$IL_j^d(t)$	Inventory level at time *t* of the *j*th distributor
$IL^w(t)$	Inventory level at time *t* of the central warehouse
$IIP_i^r(t)$	Installation inventory position at time *t* of the *i*th retailer
$IIP_j^d(t)$	Installation inventory position at time *t* of the *j*th Distributor
$IIP^w(t)$	Installation inventory position at time *t* of the central warehouse
$EOI_i^r(t)$	Echelon On–hand inventory at time *t* of the *i*th retailer
$EOI_j^d(t)$	Echelon On–hand inventory at time *t* of the *j*th distributor
$EOI^w(t)$	Echelon On–hand inventory at time *t* of the central warehouse
$EIL_i^r(t)$	Echelon inventory level at time *t* of the *i*th retailer
$EIL_j^d(t)$	Echelon inventory level at time *t* of the *j*th distributor
$EIL^w(t)$	Echelon inventory position at time *t* of the central warehouse
$EIP_i^r(t)$	Echelon inventory position of the *i*th retailer at time *t*
$EIP_j^d(t)$	Echelon inventory position of the *j*th retailer at time *t*
$EIP^w(t)$	Echelon inventory position of the central warehouse at time *t*
SS_i^r	Safety stock amount allocated to the *i*th retailer
SS_j^d	Safety stock amount allocated to the *j*th distributor
SS^w	Safety stock amount allocated to the central warehouse
R	Reorder point for (R, Q)Policy
s	Reorder point for (s, S)Policy
S	Order up to level for (s, S)Policy

2.2. Inventory Control Policies

The optimal policy structure for multi-echelon systems is difficult to determine and remains uncertain. In multi-echelon inventory management, the dominant ordering policies are the (R,Q) policy and (s, S) policy [15]. We focus in this paper on the (R, Q) policies for the case of distribution systems.

In connection to the multi-echelon inventory systems and depending on the decision system (decentralized or centralized), we may use two different ordering policies: installation stock ordering policies or echelon stock ordering policies. In an installation stock (*R*, *Q*) policy, each facility is controlled using an (*R*, *Q*) policy. When the inventory position falls to or below *R*, a batch of size *Q* (or even several batches) is ordered, resulting in a new inventory position that is larger than R but less than or equal to *R* + *Q*. The reorder points and batch quantities for different facilities do not have to be the same [15]. A different type of reorder point policy used in connection to multi-echelon inventory distribution systems is the echelon stock (*R*, *Q*) policy. The inventory position is defined in a different method when employing this modified reorder point strategy, and it is not only based on the on-hand installation stock [15].

In the current paper, we consider two types of ordering policies: Installation stock (*R*, *Q*) policy and echelon stock (*R*, *Q*) policy. Based on [15], we provide in Table 2 the inventory control policies equations and parameters for the problem being studied in the current paper.

Table 2. Inventory control parameters notations.

Inventory Parameters	Installations Stock (R, Q) Policy	Echelon Stock (R, Q) Policy
On Hand Inventory	$OI_i^r(t)$, $OI_j^d(t)$ and $OI^w(t)$	$EOI_i^r(t) = OI_i^r(t)$ (1)
		$EOI_j^d(t) = OI_j^d(t) + \sum_{i=1}^{m_j} OI_i^r(t)$ (2)
		$EOI^w(t) = OI^w(t) + \sum_{j=1}^{n} OI_j^d(t)$ (3)
Inventory Level	$IL_i^r(t) = OI_i^r(t) - BO_i^r(t)$ (4)	$EIL_i^r(t) = EOI_i^r(t) - BO_i^r(t)$ (5)
	$IL_j^d(t) = OI_j^d(t) - BO_j^d(t)$ (6)	$EIL_j^d(t) = EOI_j^d(t) - BO_j^d(t)$ (7)
	$IL^w(t) = OI^w(t) - BO^w(t)$ (8)	$EIL^w(t) = EOI^w(t) - BO^w(t)$ (9)
Inventory Position	$IIP_i^r(t) = OI_i^r(t) + Q_i^r - BO_i^r(t)$ (10)	$EIP_i^r(t) = EOI_i^r(t) + Q_i^r - BO_i^r(t)$ (11)
	$IIP_j^d(t) = OI_j^d(t) + Q_j^d - BO_j^d(t)$ (12)	$EIP_j^d(t) = EOI_j^d(t) + Q_j^d - BO_j^d(t)$ (13)
	$IIP^w(t) = OI^w(t) + Q^w - BO^w(t)$ (14)	$EIP^w(t) = EOI^w(t) + Q^w - BO^w(t)$ (15)

2.3. Lot Sizing

As we consider in this paper the multi-echelon distribution inventory system structure, we mention in this section the most used lot sizing techniques in this type of multi-echelon inventory system which are the Deterministic lot sizing method and Roundy's approximation approach.

- ***Deterministic Lot sizing***

The notion of deterministic demand can seem to be unrealistic. There are stochastic changes in demand in the majority of cases. However, it turns out that the deterministic demand assumption is generally fairly reasonable. More significant is the fact that, in cases of stochastic demand, it is frequently possible to adopt deterministic lot sizes. Even in a stochastic case, determining *Q* should essentially include balancing the costs of ordering and holding products. Therefore, it is common practice to first replace the stochastic demand with its mean and then calculate *Q* using a deterministic model. The reorder point *R* is then determined in the following step using a stochastic model given *Q*.

The classical economic order quantity formula may be the most well-known finding in the entire inventory control area. This simple result has had and continues to have a large range of practical applications. The fact that the model has been used for more than a century is fascinating.

We use the Economic order quantity model for determining the Q values for the Installation stock order policy in the application of the simulation-based approach in Section 5. We mention in Appendix A the calculation method for this model.

- ***Roundy's Approximation***

We attempt to seek simple policies with promised high effectiveness since optimal policies are difficult to find and are often extremely complicated. Q-*optimal integer ratio* lot—sizing was defined as a 94% effective method by [18]. The author showed that the technique can be used in a serial system, but the same concept can also be used in assembly and distribution systems. Roundy's approximation suggests mainly that batch quantities for various items should remain constant over time [19].

Considering a multi-echelon distribution system of "m" installations, the batch quantity for item "i" is given as the following [15]:

$$Q_i = 2^{k_i} Q_{i-1};\ i = 2, 3, \ldots m; \tag{16}$$

where

k_i is a positive integer.

Researchers are still investigating alternatives as it is typically quite difficult to determine the optimal lot sizes for multi-echelon inventory systems. The batch quantities for upstream installations in assembly systems are often specified to be integer multiples of the downstream batch quantities. Increasing the batch quantities in serial systems might lead to higher lot sizes. It is challenging to manage the trade-off between getting significant batches due to the multi-echelon network structure and, having small batches to balance the production load [15].

2.4. Reorder Points

This section deals with various techniques for determining to reorder points in multi-echelon inventory systems. Throughout this section, we assume that the batch quantities are given.

The determination of reorder points for multi-level inventory systems is not as simple as for single-echelon systems [15]. Again, this is due to the difficulty to manage various installations separately. Finding the optimal balance between upstream and downstream stock is a fundamental problem with multi-level inventory systems. We shall provide the calculations for reorder points for the two ordering policies: Installation stock (R, Q) policy and Echelon stock (R, Q) policy.

- ***Installation Stock Reorder Point***

This case means we order a batch Q when the inventory level hits exactly the reorder point IR. The batch size is assumed to be given. The reorder point is defined in [15] as the following:

$$IR = SS + Lead\ time\ Demand \tag{17}$$

- ***Echelon Stock Reorder Point***

The two types of inventory policies were compared by Axsater and Rosling in [17], which drew some significant results for serial and assembly systems. They first showed how the two policies may be transformed into one another in mild conditions. For any inventory "i" in a system with "n" inventories, an installation (IR_i, Q_i) policy can always be replaced by an equivalent echelon (ER_i, Q_i) policy with the following equation [15]:

$$ER_1 = IR_1\ and\ ER_i = IR_i + \sum_{k=1}^{n} (IR_k + Q_k) \tag{18}$$

We shall use the Equations (17) and (18) to determine reorder points in the application of the simulation-based approach for multi-echelon inventory system selection for the case study in Section 5.

2.5. Safety Stock Allocation Policies

The system's structure, holding costs, and lead times are only a few of the variables that have an impact on how safety inventories are distributed across the supply chain. In a multi-echelon inventory system, the first research paper that proposed a method for calculating safety stocks was written by [20]. This approach is accurate for serial systems and is regarded as a decomposition method. The ultimate installation confronting external demand comes first (customer demand). The most appropriate strategy for the subsequent upstream facility is then decided after evaluating and adding the extra costs caused by shortages. Assembly systems can also adopt this methodology [21]. The Clark and Scarf technique can be applied for distribution systems, but only with the "balancing" hypothesis mentioned previously.

Another strategy applied to distribution networks is the METRIC approximation of Sherbrooke [22,23]. METRIC approximation was studied by numerous authors as well. In order to find the optimal size and distribution of inventory levels in a general serial or distribution system, Karl [24] presented a process with a base-stock policy. Karl and Minner [25] addressed the issue of choosing safety stock levels in multi-level inventory systems in another research study. The optimization problem for a general inventory system was addressed by the authors. The structure of the multi-level system and the chosen service level metric determine the best policy. In a two-echelon distribution system, Chakravarty and Shtub [26] simulated the allocation of safety stock levels. This study also provides guidelines for managing safety stock effectively in a two-echelon inventory system.

Two strategies for carrying safety inventory in a multi-echelon supply chain were proposed by Chopra [27]. The solutions depend on two factors: the holding costs of the items and the length of waiting time that customers can tolerate. According to the author, carrying safety inventory upstream to enable more aggregation is appropriate if the inventory is expensive to store and customers can accept a delay. The second alternative is to hold safety inventory downstream if the products are cheap to keep and the customers need the product immediately. The guaranteed-service model approach (GSM) also addressed the issue of distributing safety stocks to meet desired service levels at the lowest cost. A thorough analysis of the GSM literature was developed by the authors of [28].

2.6. Multi-Echelon Inventory System Selection Problem: Distribution System Alternatives

We developed in a previous work [29] a process for generating multiple alternatives for multi-echelon inventory systems in the case of the distribution structure. Generating alternatives involved defining the inventory policies and determining the decision nodes.The process for generating alternatives resulted in 8 multi-echelon inventory system options that were combinations of the following inventory policies: Continuous or periodic replenishment policies, Installation or Echelon stock ordering policies, and Safety stock allocation policies. As we chose to adopt a continuous review *(R, Q)* inventory policy in the present paper, we consider the alternatives with a continuous review policy only. Consequently the number of multi-echelon inventory system alternatives will be reduced to 4 options instead of 8..We provide in Table 3 the multi-echelon distribution inventory system alternatives to be considered in our simulation approach.

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Table 3. Multi-echelon inventory system alternatives: Case of distribution systems.

Alternative	Description
A1	The multi-echelon distribution inventory system is controlled by an echelon stock *(R, Q)* policy. In other words, the calculation of different inventory parameters is based on the echelon inventory. The safety stock is stored in the central warehouse.
A2	The present alternative allows the decision maker to adopt the same inventory policy for the cycle stock as above while allocating safety stock in the retailers' facilities.
A3	The installation stock *(R, Q)* policy is used for monitoring inventory levels in the supply chain. The safety inventory quantities are allocated to the upstream installation.
A4	The decision maker can choose the Installation stock *(R, Q)* policy in this alternative while storing safety stock in the most downstream facilities.

2.7. Multi-Echelon Inventory Management in Pharmaceutical Products Supply Chains

Pharmaceutical supply chains were once thought of as a tool for delivering items to markets while taking supply security into consideration. Pharmaceutical industries have recently been searching for effective and efficient strategies to offer more advantages. The pharmaceutical supply chain has a large number of players, including major manufacturing facilities, distribution centers and warehouses, wholesalers, hospitals, and many more. The need for advanced supply chain management approaches is required by the interdependencies between those elements and the sensitive nature of pharmaceutical products.

Different researchers have investigated the multi-echelon inventory problem in pharmaceutical products supply chains. We conducted a literature review in a previous work [30] on multiple research works that used multi-echelon inventory management policies in the pharmaceutical sector. The primary challenges that the pharmaceutical supply chain sector must overcome are the difficulty of ensuring responsiveness at the operational stage and the requirement to match future capacity with projected demand at the strategic stage [31]. Many healthcare organizations now give serious consideration to adopting supply chain management principles by using techniques and procedures created for industrial contexts. In the current paper, our goal will be to develop an approach that will assist decision-makers in selecting the multi-echelon inventory system that best meets their demands and the needs of the system as a whole.

3. Proposed Simulation-Based Approach for Multi-Echelon Inventory System Selection: Case of Distribution Systems

Simulation can bring many benefits to the multi-echelon inventory management problem. Relying only on analytical models to make decisions and discuss results cannot be efficient. Due to this matter, simulation can be relevant to model and predict multi-echelon inventory system parameters. We discussed in a previous work [5] the relevance of simulation modeling for multi-echelon inventory management.

The current section aims to propose a simulation-based approach for selecting a suitable distribution inventory system. In the current section, we provide an approach for guiding decision-makers in selecting from a set of various options the best combination of multi-echelon inventory policies that are consistent with the current structure of their supply chain network and meet their preferences.

Because of the uncertainties and non-linearities present in today's multi-level supply chain networks, analyzing performance indicators and decision factors using analytical approaches is getting more difficult [6]. Fortunately, the simulation method can be used to represent systems with complex flows between nodes.

Simulating a multi-echelon inventory system has the goal of determining the network performance parameters that correspond to a certain situation. The nature of supply chain networks can be dealt with through simulation. Interconnected multi-level inventory systems face both variability and complexity.

Simulation models can be used to show a system's variability and interconnection [32]. As a result, using simulation to estimate system performance, compare various network configurations, and quantify the influence of each inventory strategy on total system performance, becomes possible [33].

For the case of distribution systems, the multi-echelon inventory system alternatives will be those established in Table 3. We choose the appropriate alternative based on the DM's supply chain efficiency and responsiveness preferences.

The simulation-based approach presented in Figure 2 is suggested to assist decision-makers in selecting the appropriate multi-echelon inventory system from a range of alternatives while considering their preferences.

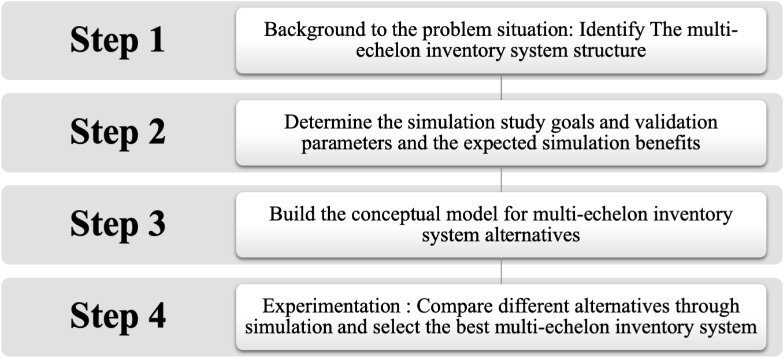

Figure 2. The proposed simulation-based approach for multi-echelon inventory system selection.

3.1. *Background of the Problem Situation*

The multi-echelon supply chain's structure for this case is a divergent/distribution system. This step involves identifying the number of echelons and installations in each stage of the multi-echelon supply chain. In addition, the demand type is specified (stochastic or deterministic, static or non-static demand). The product types held in various facilities are also specified. We illustrated the multi-echelon inventory distribution system considered in the current paper in Figure 1.

3.2. *Simulation Study Goals, Validation Parameters, and Expected Simulation Benefits*

We aim to compare several multi-echelon inventory system options based on a set of criteria that match the preferences of the decision-makers. The multi-echelon inventory system selection criteria to be considered in this approach are product availability, customer service level, and inventory costs.

Three main elements must be identified in this step:

(i) The goal of the decision-maker is to enhance responsiveness-related criteria or decrease cost-related criteria.
(ii) The performance level to be used to measure the simulation's goal: maximize product availability and improve customer service or minimize inventory costs.
(iii) Constraints include transportation modes, tolerance for delays, and so on.

The proposed simulation approach will assist decision-makers in comparing and testing several multi-echelon inventory system alternatives and selecting the best one for their needs. Furthermore, simulation can be used to demonstrate the validity of a chosen multi-echelon inventory management policy and, as a result, to encourage decision-makers to adopt it by visualizing the results using a simulation model.

3.3. Build the Conceptual Model for Multi-Echelon Inventory System Alternatives

A conceptual model for each multi-echelon inventory system alternative must be created before a simulation model can be developed. We develop a conceptual model for our multi-echelon inventory system selection problem using the conceptual modeling framework of [33]. Figure 3 depicts the major steps of the approach that illustrate our problem situation.

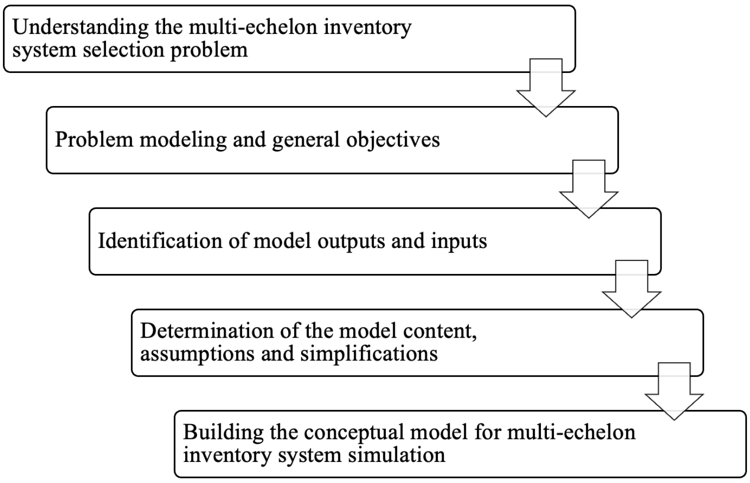

Figure 3. A suggested approach for designing the conceptual model for the multi-echelon inventory system simulation modeling.

First, we perform a thorough understanding of the problem situation. After that, we determine the general objectives of the modeling. We define the outputs of the model and the inputs. Finally, assumptions and simplifications are identified. This step is developed in more detail in Section 4.

3.4. Compare Different Alternatives Through Simulation and Select the Best Multi-Echelon Distribution Inventory System

In the current paper, we intend to compare multiple multi-echelon inventory system alternatives through simulation. We use Flexsim [34] to simulate various inventory strategies for a given multi-echelon supply chain structure. Flexsim is a discrete event system simulation software. It can be used for different industries such as manufacturing, healthcare, warehousing, etc. Simulation optimization is between its main application areas. Simulation model creation, simulation logic implementation, design validation, result output, and simulation analysis are among its main functions. Flexsim has been used by multiple researchers to address a variety of supply chain issues [13,14,35,36]. Because Flexsim's properties make it appropriate for simulating the logistics process connecting warehouse systems at all levels of the supply chain, we chose it to simulate multiple multi-echelon inventory system alternatives for our paper.

The final step in our approach is to choose the best multi-echelon inventory system for the given supply chain structure. After modeling various alternatives with Flexsim Software, the decision-maker can select the best option based on his preferences. In Section 5, we illustrate how the suggested approach works in a real case study of the Moroccan supply chain of pharmaceutical products in the public sector.

4. Conceptual Modeling Framework for Multi-Echelon Inventory System Simulation

4.1. Introduction

We aim in this section to create a conceptual model for multi-echelon inventory system simulation. The suggested methodology for designing the conceptual model is based on the work of [33] and consists of four major steps: (i) understanding of the multi-echelon inventory system simulation problem, (ii) problem modeling and general objectives, (iii) identification of model inputs and outputs, (iv) determination of the model content, assumptions, and simplifications and finally (v) building the conceptual model for multi-echelon inventory system simulation.

Multi-echelon inventory management problems are commonly studied using the simulation method. We conducted in a previous work [5] a detailed literature review on different simulation models established by authors for the multi-echelon inventory management problem. The simulation models performed were developed to simulate an inventory policy [6,9,37], to analyze the relationships between different performance parameters such as customer service level and inventory costs [12], or to compare inventory transshipment methods or replenishment policies [8,38]. As a result of our findings, we concluded that the most difficult aspect of creating a multi-echelon inventory system simulation model is the conceptual modeling that necessitates a high level of information for each stage to be modeled. Variables that impact performance measurement should be considered in the model as well.

The process of abstracting a model from a real or suggested system is known as conceptual modeling. It is, without a doubt, the most crucial part of a simulation project. The model's design has an impact on every area of the study, including the information needs, the speed of the model development, the model's accuracy, the speed of running experiments, and the correctness of the model's results. A well-designed model increases the chances of a successful simulation study [39].

It's relatively difficult to find research work that focuses on developing a conceptual model for the multi-echelon inventory management problem. Although some articles give simulation models for numerous multi-echelon inventory systems, none of them suggest or create a generic conceptual model for the simulation of multi-echelon inventory systems. We aim in the next sections to propose a general conceptual model for multi-echelon inventory system simulation.

The overall inventory allocation in a multi-echelon inventory system can be driven by a range of factors such as demand fluctuations, unit costs, and transit time. Carrying significant amounts of inventory at an upstream facility (warehouse) or downstream installations (retailers) is always a trade-off [15].

In the research paper [39], The author developed a framework that lays out the steps that should be followed to create a conceptual model. The author conducted a discussion on how a modeler might approach each of the steps proposed, with recommendations and techniques provided. We adopt the revised and updated framework proposed by [33] in his most recent work.

A conceptual model for the multi-echelon inventory system simulation must be created before a simulation model can be developed. We develop a conceptual model for the multi-echelon inventory system simulation problem. Figure 3 depicts the major steps of the framework.

4.2. Understanding the Multi-Echelon Inventory System Simulation Modeling Problem

The simulation study will be performed to assist in the simulation of a multi-echelon inventory system. A model that describes the real world must be used to acquire a good knowledge of the problem situation. This step entails a thorough discussion between the modeler and the decision-maker to achieve a good grasp of the problem situation.

Validation of the conceptual model is required as it is developed. As areas of limited knowledge and comprehension of the problem circumstances exist, assumptions about these areas must be drawn. A modeler must verify his understanding by giving clear and detailed descriptions of different problem scenarios and circumstances. The problem situation dealt with in this research paper is the following:

- Multi-echelon inventory management: The problem situation:
  Many decision-makers seem to find it difficult to determine inventory strategies that are compliant with their supply chain design and meet their needs in terms of costs and product availability. Furthermore, contemporary managers may face several

trade-offs. Using single–echelon solutions for each facility is easy, but it excludes the inventory status of other installations. As a result, some locations may have stock-outs, while others may have extra inventory. Consequently, different inventory policies need to be modeled and tested before effective implementation. Multi-echelon inventory control policies include replenishment policies, ordering policies, and safety stock allocation policies. A typical illustration of a general multi-echelon inventory system is presented in Figure 1.

4.3. Problem Modeling and General Objectives

The modeling objectives are the metrics against which the study's performance is assessed [40]. Because the model is employed to assist decision-making, the study's goal isn't just to build the model itself. "By the end of this study, what do we aim to achieve?" is a good point to consider while setting objectives. Besides, the following questions are asked:

- What does the decision-maker want to achieve? Increasing throughput, lowering costs, or enhancing product availability.
- What is the desired level of performance? Performance targets for each objective should be determined. This is going to be only possible if the objective can be quantified [33].
- What limitations/constraints do the decision-makers and modelers must work with?

After considering the previous questions and aspects we determine the modeling objectives for the multi-echelon inventory system simulation problem as the following:

- The multi-echelon inventory system simulation: Overall project objectives.
  - The flexibility of the model: Limited. Changes on a large scale are unexpected.
  - Run speed of the experiment: several experiments to be carried out.
  - Visual display/ presentation of the model: Simple 3D animation
  - Ease of use: the modeler is the only user.

4.4. Identification of Model Outputs and Inputs

The first step in abstracting a conceptual model is to determine what the model's outputs are and then inputs.

- ***Identifying the multi-echelon inventory system simulation model outputs (responses)***

In general, the model's outputs serve a dual purpose. First, to determine whether the objectives are met. After that, point out the reasons why the aims were not fulfilled. Another factor to examine is how the data is presented. Numerical data such as maximum, minimum, standard deviation, and so on can be used to present the outputs. Graphical data can be used to represent the responses as well such as frequency diagrams, pie charts, and so on. Constant collaboration between the simulation modeler and the decision-maker, each bringing their expertise to light, should be used to select appropriate outputs and reporting techniques.

The model outputs for the multi-echelon inventory system simulation are the following:

- The Multi-echelon inventory system simulation: Model Outputs.
  The outputs of simulating the multi-echelon inventory system will provide information on the following:
  - Total inventory costs: Inventory costs for each supply chain node are to be recorded and illustrated in a table.
  - Fill rate for each downstream facility to be recorded and presented in a table. The fill rate is defined as the proportion or fraction of orders that can be filled instantly. When a customer makes an order (and the products ordered are already in production), some available inventory can be used to immediately fill the order.
  - Inventory levels at different nodes of the system.

We chose the order fill rate to measure the product availability and the customer service level criteria in downstream installations [41]. The decision-maker will have the opportunity to evaluate the multi-echelon inventory system in terms of the defined outputs.

- ***Identifying the multi-echelon inventory system simulation model inputs (experimental factors)***

The model inputs can be defined as the mechanism by which the modeling objectives are supposed to be met. Inputs or experimental factors might be both qualitative and quantitative [39].

To determine the multi-echelon inventory system simulation inputs, inventory policies, and parameters need to be identified in this step. We defined in Section 2 the main inventory policies used in multi-echelon inventory management. We present the model inputs for the multi-echelon inventory system simulation problem as the following:

- The multi-echelon inventory system simulation: Model inputs:
    - Ordering costs
    - Holding costs
    - Shortage costs
    - Lead time for each node
    - Demand type and structure at each retailer
    - Demand arrival rate
    - Reorder point and order quantity for each node
    - Maximum capacity for each node
    - Inventory policy-related parameters to be used in each node.

4.5. Determination of the Model Content, Assumptions, and Simplifications

- ***The model scope and level of detail***

The model scope defines the model's boundaries or perimeter and the level of detail determines the model's depth [39]. The scope of a model must be obtained by identifying the entities, activities or processes, queues, and resources that will be part of the model. The modeler, decision-makers, and experts can go over the specifications for each element in the model scope, deciding whether the detail should be included or not, as well as how each detail should be simulated. We develop in Tables 4 and 5 the model and the level of detail for the multi-echelon inventory system simulation modeling respectively.

Table 4. The multi-echelon inventory system simulation modeling: Model scope.

Component of the Model	Explanation and Justification
Entities	
- Products	Output: Inventory levels of products
- Demand arrival	Model Input (experimental factor)
Activities	
- Supplier manufacturing site	An infinite source of inventory should be available at the most upstream stage of the system to ensure the availability of products.
Queues	
- A central warehouse	Products are stored in these facilities which will perform the processing of experimental factors.
- Distribution centers	
- Retailers	
Resources	
Operators, machines, and trucks	Trucks will be included to model product movement and distribution from one facility to another. A transportation time will be assigned for each truck.

- ***Assumptions and simplifications identification***

When there are uncertainties about the real system to be modeled, assumptions are made. Simplifications are introduced into a model to allow for faster model creation and application [33]. Identification of potential simplifications is generally the concern of the modeler's experience, though communication among the modeler, clients, and experts can also generate ideas for simplification. it's important to refer to a set of conventional simplifications. There are several approaches for simplifying models, including combining model components, substituting components with random variables, and eliminating infrequent events [39].

Table 5. The multi-echelon inventory system modeling: the level of detail.

Component of the Model	Detail	Description
Entities		
- Products	Types	Single type products or multiple types of products
	Information	Specific information related to the products (size, cost . . .)
- Demand arrival	Arrival Pattern	Distribution of customer demand: deterministic or stochastic
Activities		
- Supplier manufacturing facility	Arrival pattern	How products enter the model
	Quantity	Quantity of products produced by the supplier at a given period
Queues		
- Central warehouse, Distribution centers, Retailers	Quantity	Number of queues in each echelon
	Capacity	Maximum capacity for each queue
	Queue discipline and routing	Inventory policy adopted by each facility (sequence of products into and out of the queue)
Resources		
- Operators, machines, and trucks	Quantity	Number of Resources (Trucks) needed for each echelon .
	Transportation time	Each truck will have a transportation time.

4.6. Building the Conceptual Model for Multi-Echelon Inventory System Simulation

This step consists of constructing the model for multi-echelon inventory system simulation. All the previous steps are based on the ability to describe the conceptual model in a way that can be shared and understood by all participants in the simulation project. The tables created in the above-mentioned conceptual modeling framework are used to describe the conceptual model.

As a communicating tool, graphical/diagrammatic representations of the model are similarly valuable and probably more effective. The conceptual model might, of course, be represented using the simulation software's visual display features without the requirement to code the model's detail. We provide in Section 5 an application of the framework suggested using the Flexsim software [34].

The simulation-based approach proposed in the current chapter will be applied and validated through a case study of the Moroccan pharmaceutical products supply chain in the public sector in Section 5.

5. Application of the Simulation-Based Approach to the Case of the Moroccan Pharmaceutical Products Supply Chain in the Public Sector

To illustrate the proposed simulation approach for the multi-echelon inventory system selection problem, we provide an application through the comparison of the 4 alternatives defined in Table 3 for the case of the Moroccan pharmaceutical products supply chain in the public sector. The model parameters are based on data provided by the procurement division of the Ministry of health. In the current section, we present an application of the steps of our simulation-based approach. We design the conceptual model of the multi-echelon inventory system alternatives by applying the framework for conceptual modeling presented in Section 4. We start with an illustration of the simulation model and objectives. After that, we identify the model inputs and outputs based on Section 4. Then, The model scope and simulation layout are developed for the Moroccan pharmaceutical products supply chain in the public sector using Flexsim software [34] and following Tables 4 and 5. Finally, we run the experimentation on Flexsim software [34] and we discuss the simulation outcomes and results.

5.1. Simulation Model and Objectives

The deliveries by the Procurement Division of the Moroccan Ministry of Health are made in a planned manner with generally 4 deliveries per year to the following warehouses: Central Warehouse of Berrechid, Central pharmacy, warehouse of Beausejour, and Warehouse of Derb Ghalef. The four warehouses deliver pharmaceutical products to the 12 regions of Morocco grouped as shown in Figure 4.

Morocco has adopted a new territorial division. It now has 12 Regions according to Decree No. 2.15.10 of 20 February 2015, fixing the number of regions, their names, their capitals, and the Prefectures and Provinces composing them, published in Official Bulletin No. 6340 of March 5, 2015. The list of the 12 regions is as follows: (1) Tanger-Tétouan-Al Hoceima, (2) Oriental, (3) Fez-Meknès, (4) Rabat- Salé-Kénitra, (5) Beni Mellal -Khnifra, (6) Casablanca-Settat,(7) Marrakesh-Safi,(8) Draa-Tafilalt, (9) Souss-Massa, (10) Guelmim Oued Noun, (11) Laayoune Sakia al Hamra, (12) Dakhla-Oued Eddahab [42].

The model to be studied in the current section will be a single-product multi-echelon inventory management problem in a two-echelon distribution system as illustrated in Figure 4. The distribution system structure is composed of two echelons as the following: Echelon 1: The central warehouse and 3 other secondary warehouses. Echelon 2: Regional warehouses, regional pharmacies, provincial pharmacies, and hospital pharmacies in 4 major groups of the 12 regions of Morocco. Following the work of [15], the 12 Moroccan regions were grouped such that all regions in a group order from the same warehouse and have approximately the same reorder point.

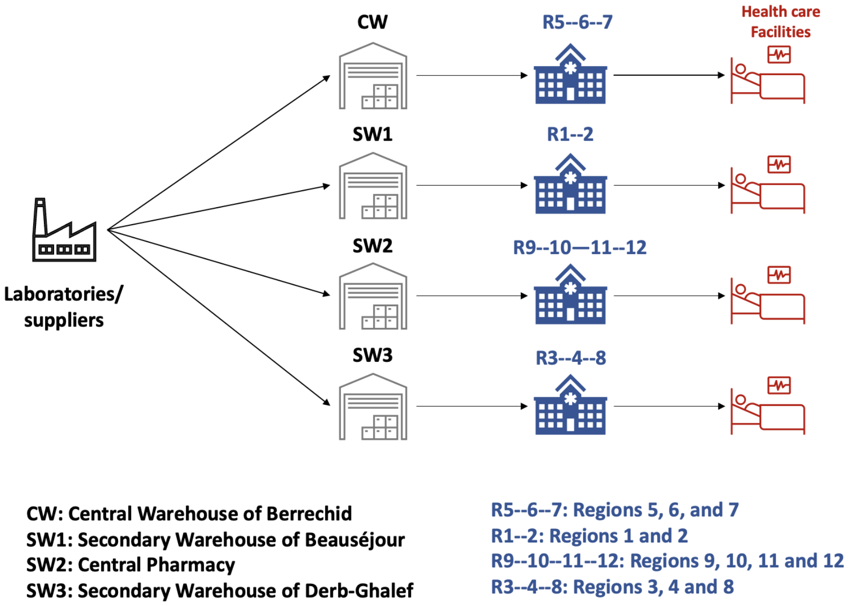

Figure 4. Illustration of the Moroccan pharmaceutical products multi-echelon inventory distribution system.

The system is composed of one supplier presenting different pharmaceutical laboratories and suppliers, the central warehouse (CW) of Berrechid, the secondary warehouse (SW1) of Beauséjour, the central pharmacy (SW2), and the secondary warehouse of Derb-Ghalef (SW3) and four major "retailers" that present the four major regions of Morocco. Each node replenishes items from a designated location at the next upstream echelon. When the upstream facility has sufficient inventory, the next location receives the order after a stochastic lead time. Thus, demand is fulfilled at the downstream installations. Patients and basic healthcare facilities are considered willing to wait and demand is back-ordered if it is not fulfilled. The four regions in the downstream stage of the studied supply chain will wait for the demand to be fulfilled if the warehouses of the upstream stage have a stock-out. The regions face external demand with a stochastic arrival time. We assume demand has a stochastic distribution. We assume that the highest echelon (the supplier) has an infinite source of supply. The (R, Q) inventory control policy is used in all nodes of the system. We recall that the installation stock (R, Q) ordering policy is an ordering method that consists of ordering a fixed quantity Q when the inventory level at a certain installation fall below the reorder point R. In this case, each facility uses its inventory position while in the echelon stock ordering policy, the inventory position of a certain installation is the installation inventory added to all downstream inventory positions [15].

The objective of this simulation model is to compare the implementation of four different scenarios presenting four multi-echelon inventory system alternatives described in Table 6. The major preference for the Ministry of Health regarding the decision problem is the level of product availability at the most downstream stages of the supply chain. Thus, we aim in the current section to compare inventory levels at each node of the system to illustrate the product availability at different stages of the supply chain for the four scenarios/alternatives.

Table 6. The four simulation scenarios for the multi-echelon inventory system selection: Case of Moroccan pharmaceutical products supply Chain in the public sector.

Alternative	Description
PA1	The multi-echelon distribution inventory system is controlled by an echelon stock *(R, Q)* policy. The Moroccan supply chain needs to purchase/ own an information system for supply chain management that provides data about inventory status at all facilities to be able to adopt a continuous inventory policy. If the management decides to have a centralized decision system, product quantities are determined by the supply division of the ministry of health. All facilities are going to adopt echelon stock inventory positions for determining to reorder points. In this alternative, the pharmaceutical products' safety inventory is stored in the central warehouse and the three secondary warehouses.
PA2	The present alternative allows the decision maker to adopt the same inventory policy for the cycle stock as above while allocating safety stock close to patients in hospital pharmacies and basic healthcare systems that are present in the 4 major regions.
PA3	If a centralization of inventory management is not possible, a decentralized decision system is adopted, and the quantities of the product are decided by each node. Consequently, an installation stock *(R, Q)* ordering policy is implemented, and each facility determines the ordering quantities based on its inventory position. The safety inventory quantities are allocated to the upstream installations at the central warehouse and the secondary warehouses.
PA4	The decision maker can choose the Installation stock *(R, Q)* policy in this alternative while storing safety stock in the most downstream facilities.

5.2. Model Inputs and Outputs

We define in this step the outputs of simulating the Moroccan pharmaceutical products multi-echelon distribution inventory system. We aim to present the inventory levels for each node of the system and that for the four alternatives mentioned in Table 6.

- PA1 and PA2 outputs: echelon inventory level at the warehouse, the secondary warehouses, and the four regions at the supply chain downstream stage. The amount of demand fulfilled at each region.
- PA3 and PA4 outputs: installation inventory level at the warehouse, the secondary warehouses, and the four regions at the supply chain downstream stage. The amount of demand fulfilled at each region.

We compare and analyze the four alternative inventory policies for the multi-echelon inventory system studied. The calculation of the inventory parameters are based on equations previously defined in Table 2. We show in Table 7 different parameters (inputs) to be used for the considered multi-echelon inventory system. Different Data was provided by the procurement division of the ministry of health. Due to confidentiality matters regarding data, different inventory information and storage information were coded in the simulation software. Different inventory parameters presented in Table 7 were coded for each entity of the model.

The calculation of the order quantity and reorder points using different inventory policies was done following equations previously defined in Table 2. The safety stock is estimated by the procurement division as 3 months of inventory. Another constraint that was mentioned by the procurement division was that the central warehouse and the secondary warehouses make orders from suppliers 4 times per year only. This was taken into consideration as well while calculating different initial inventory amounts and order quantities for the upstream installations.

Table 7. The Moroccan pharmaceutical products multi-echelon inventory system selection: Simulation model parameters.

Facilities/ Nodes	Order Quantity	Installation Stock Reorder Point	Echelon Stock Reorder Point	Installation Stock Initial Inventory	Echelon Stock Initial Inventory	Inter-Arrival Time (Days)	Demand Quantity Per Month	Lead Time (Days)
CW	30	30	67	59	96	———-	———-	———-
SW1	12	12	29	24	42	———-	———-	———-
SW2	14	14	34	28	48	———-	———-	———-
SW3	8	8	21	17	29	———-	———-	———-
R5-6-7	8	30	30	37	37	E(3,04)	10	Tri(0,029;0,058;0,087)
R1-2	5	12	12	17	17	E(7,37)	4	Tri(0,075;0,151;0,227)
R9-10-11-12	5	14	14	19	19	E(6,40)	5	Tri(0,388;0,776;1,164)
R3-4-8	4	8	8	12	12	E(10,83)	3	Tri(0,085;0,170;0,255)

5.3. Model Scope, Level of Detail and Simulation Layout

We present in Table 8 the model scope and level of detail for the Moroccan pharmaceutical multi-echelon distribution inventory system studied in the current section. We use Flexsim software to build the simulation model in our paper. Flexsim is a discrete event system simulation software. It can be used for different industries such as manufacturing, healthcare, warehousing, etc. Simulation optimization is between its main application areas [34].

Simulation model creation, simulation logic implementation, design validation, result output, and simulation analysis are among its main functions. Flexsim has been used by multiple researchers to address a variety of supply chain issues [13,14,36]. Because Flexsim's properties make it appropriate for simulating the logistic process connecting warehouse systems at all levels of the supply chain, we chose it to simulate multiple multi-echelon inventory system alternatives.

The "Queue" is used for each node in the system in the Flexsim layout. The "Queue" is dedicated to storing the products which are modeled as "Items". The patients/Basic healthcare facilities are represented by a "source" that creates demand and consumes products. The infinite source of inventory is simulated by a "Source" that creates "Items". We use "Sink" to absorb delivered and consumed products. To keep a record of all inventory parameters adopted in the model, we use the "Global Table". We illustrate in Figure 5. the layout of the simulation model.

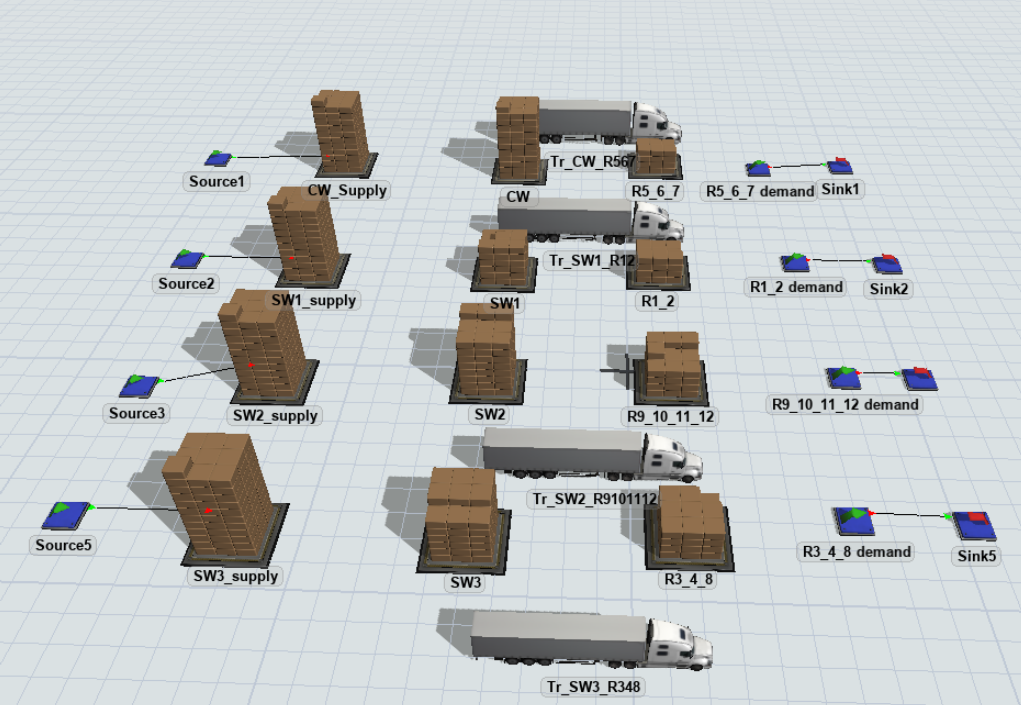

Figure 5. The layout of the simulation model of the Moroccan multi-echelon distribution inventory system model.

5.4. Simulation Outcomes and Discussion

The four simulation experiments were run for 1 year. We simulate 1 year of replenishment and inventory control across the Moroccan pharmaceutical products supply chain stages. Comparing the four multi-echelon inventory system alternatives, we analyze the simulation results.

We present in Figures 6–9 the Content Vs.Time of the 4 scenarios and for each node of the multi-echelon distribution inventory system. The content vs. time graphs provided by the Flexsim simulation software allow for depicting the changes in inventory quantities in each facility denoted as "content", over a period of time (in this case 1 year from October

2022 to October 2023). We provide the Average Content for each node of the system for the four alternatives in Figure 10. The Content Vs. Time illustrates the inventory levels for each node of the supply chain. The Average Content presents the average inventory of the pharmaceutical product considered in the current case study for each installation/node of the multi-echelon inventory system for a certain scenario.

Based on the Content vs. time and average content graphs for the four alternatives and comparing both the installation stock ordering policy and echelon stock ordering policy, the results show that the average inventory level at the downstream stages (the hospitals/healthcare facilities of the 4 major regions) is relatively the same for both policies. The average inventory for the warehouses and secondary warehouses is larger for the echelon stock policy. We can see also that the smaller the values of R, the smaller are inventory amounts at the region's hospitals and healthcare facilities. In other words, the small values of R related to the installation stock policy led to reducing the level of inventories held in the warehouses.

One advantage of an installation stock policy is that once the reorder points are set, all that is needed to control replenishment is local information. We'll need the installation inventory position as well as the inventory positions of all downstream installations to apply an echelon stock policy. An alternative is to have information about the initial echelon stock inventory position and be able to monitor final customer demand. In practice, however, determining the echelon stock from these data is often challenging due to different changes in inventory positions, such as damage and obsolescence. This was also the case when dealing with such data provided by the Ministry of Health procurement division.

For the same ordering policy (installation stock (R, Q) policy or Echelon stock (R, Q) policy), and comparing either PA1 with PA2 or PA3 with PA4, it is clear that inventory levels are higher at the most downstream installations (the four major regions) for PA2 and PA4. This is due to the allocation policy of safety stocks to the downstream facilities. On one hand, this will provide a secure level of product availability (products will be near customers and at appropriate amounts). On the other hand, this means added inventory costs at the lower level of the supply chain.

Table 8. The Moroccan pharmaceutical products multi-echelon inventory system: Scope and level of detail.

Components of the Model	**Detail**	**Description**
Entities		
Products	Types	Single type product
Demand arrival	Arrival pattern	The final/external demand is stochastic following parameters mentioned in Table 7. The patients/basic health care facilities order product quantities mentioned in Table 7.
Activities	Arrival pattern	The suppliers have an infinite source of inventory. Products are always available on the suppliers' sites.
Laboratories/ Suppliers	Quantity	
Queues	Quantity	1 central warehouse, 3 secondary warehouses, and 4 regions groups.
Central warehouse, secondary warehouses, Regions	Capacity	10,000 products capacity.
	Queue discipline and routing	Each facility adopts an (R, Q) ordering policy

Comparing the four simulated scenarios, and starting with an analysis of PA1 results, we can see that PA1 has the highest average inventory in the upstream facilities (CW, SW1, SW2, and SW3) followed by PA2, PA3, and then PA4. For PA1, this is explained by the calculation of inventories that takes into consideration the echelon inventory which is the installation inventory of the facility added to all downstream stages inventory. Moreover, The safety stock allocation for PA1 was for upstream stages. This implies a high amount of average inventory compared to other alternatives. Thus, inventory holding costs will be the highest for the upstream facilities for PA1, but the ordering costs will be lower since not many orders are placed until April 2023 in the simulation model. The average inventory for the four major regions for PA1 is slightly lower than other alternatives using the installation stock policy. This will have a slight impact on product availability compared to other scenarios PA3 and PA4 that prove a higher level of average inventory at the lowest level of the supply chain.

The simulation provided us with an opportunity to test the four alternatives and compare their performance in terms of product availability and inventory costs. This was done through the outputs regarding inventory levels at different installations of the pharmaceutical supply chain studied. Each scenario gives visibility on the average level of inventory as well as the inventory amount ordered and consumed over time by different actors of the multi-echelon inventory system under study. This will guide the decision makers of the procurement division of the Moroccan pharmaceutical product supply chain to understand the pattern behavior of the inventory dynamics for each scenario and be able to take the right and appropriate decision on which option to choose.

Consequently, if we would like to classify the alternatives PA1, PA2, PA3, and PA4 in terms of either the level of product availability or inventory holding costs, we will end up with the ranking provided in Tables 9 and 10 that provide insights to the decision-makers in the procurement division to choose the scenario that corresponds to their preferences. Table 9 presents the total average inventory holding costs for the four alternatives. The values were obtained by multiplying the average inventory costs by the holding cost of the product under study. Table 10 illustrates a ranking of alternatives according to the product availability in the four major regions (R5-6-7, R1-2, R9-10-11-12, and R3-4-8). This table was formed by comparing different average inventory levels in different downstream facilities for the four scenarios simulated.

By analyzing Table 9, we can see that option PA2 is the highest alternative in terms of average holding costs followed by PA1, PA4, and PA3. We can see in Table 10 that PA2 and PA4 provide a high level of product availability in the most downstream stages of the pharmaceutical supply chain. According to the procurement division of the Ministry of Health, a high level of product availability is more important for the decision-makers than the inventory-related cost criteria. By considering this preference, we can conclude that the alternative PA4 which is characterized by adopting an installation stock (R, Q) policy in all supply chain nodes and allocating safety stock in downstream facilities (the four major regions) is the suitable alternative for the case of the Moroccan pharmaceutical products supply chain in the public sector. It provides not only a high level of product availability but implies less holding costs compared to PA2 as both of these options provide the same level of product availability.

The results obtained by the simulation study provide concrete and clear guidelines to the decision makers to choose and select the best scenario that suits their needs and preferences. Different graphs and tables presented in the current section were a clear illustration of the levels of inventories across the whole supply chain. They describe patterns of inventory consumption and supply for 1 year.

The best alternative for the Moroccan pharmaceutical supply chain in the public sector that resulted from the simulation study in the current section was implementing an installation stock (R, Q) policy with an allocation of safety stocks in downstream stages close to basic healthcare facilities. The same option was proven to be the most appropriate for the supply chain under study in the application of the MCDM-based approach for multi-echelon inventory system selection in a previous work [29]. Thus, both suggested approaches resulted in the same multi-echelon inventory system alternative and this presents a strong guideline to the decision-makers of the pharmaceutical supply chain to proceed with such an option.

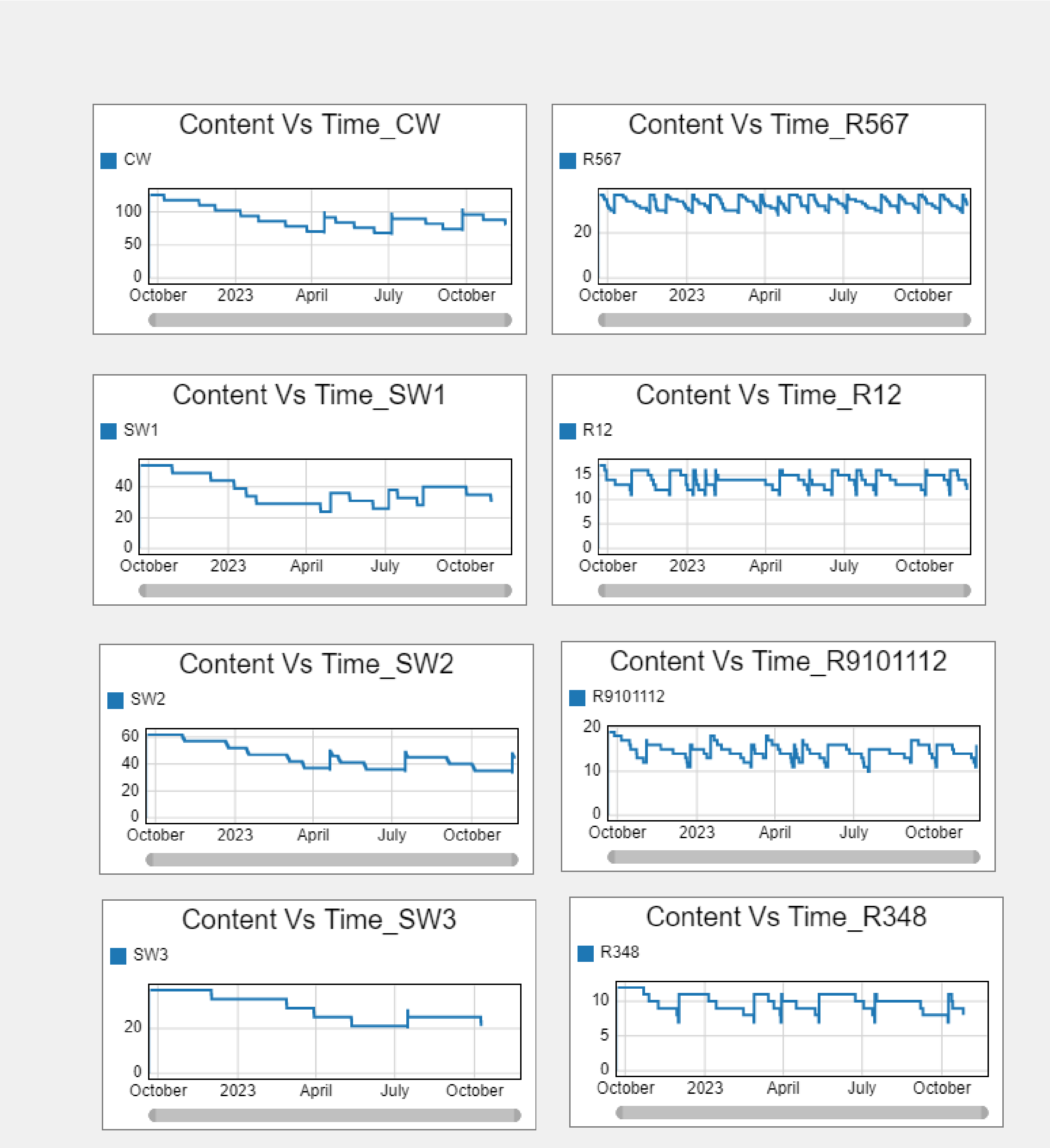

Figure 6. Content vs. time for each node of the system for scenario PA1.

Our paper adds to the existing literature a general simulation-based approach for multi-echelon inventory system selection. By following the suggested approach, decision-makers will have the opportunity to choose and validate the inventory policies that meet their needs in terms of supply chain responsiveness and efficiency. Our research work fits into the existing literature as well by providing guidelines for supply chain managers that can be tested and validated through simulation. Previous studies related to our topic dealt with simulating multi-echelon inventory control policies. Xu et al. [13] developed a simulation-based optimization model of the multi-echelon inventory system for fresh agricultural items in recent research work. The authors demonstrated through the simulation results that the suggested simulation model can help decision-makers cope with the complexity of the inventory system. Zhang et al. [14] simulated two inventory strategies for a multi-echelon inventory control model for fresh goods. The research study's findings, according to the authors, could help managers of supply chains for fresh products make judgments on inventory management and reduce expenses. The application of the simulation-based approach performed in our paper in Section 5 is a valuable and novel contribution to the multi-echelon inventory management literature in the pharmaceutical products supply chain sector.

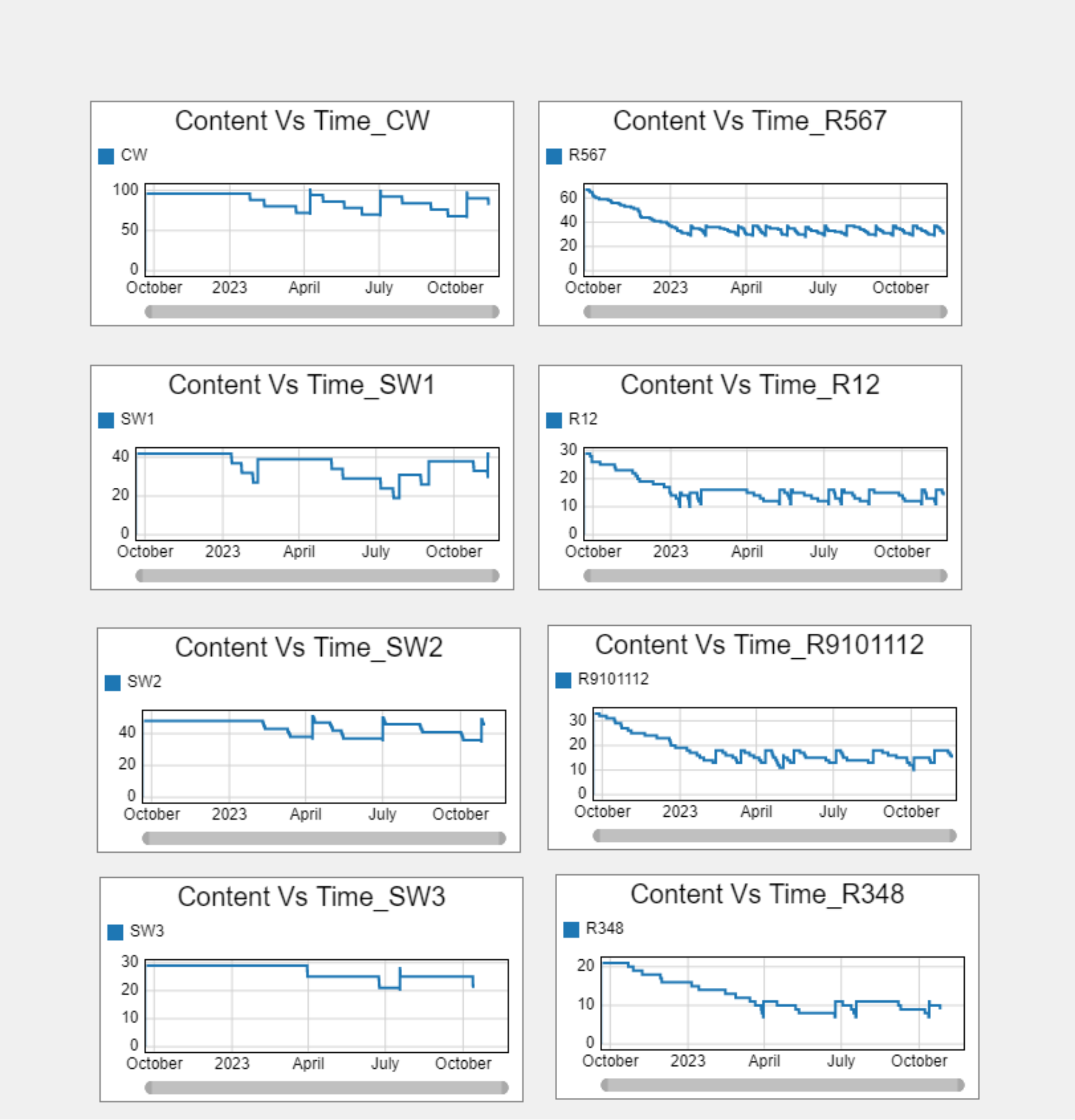

Figure 7. Content vs. time for each node of the system for scenario PA2.

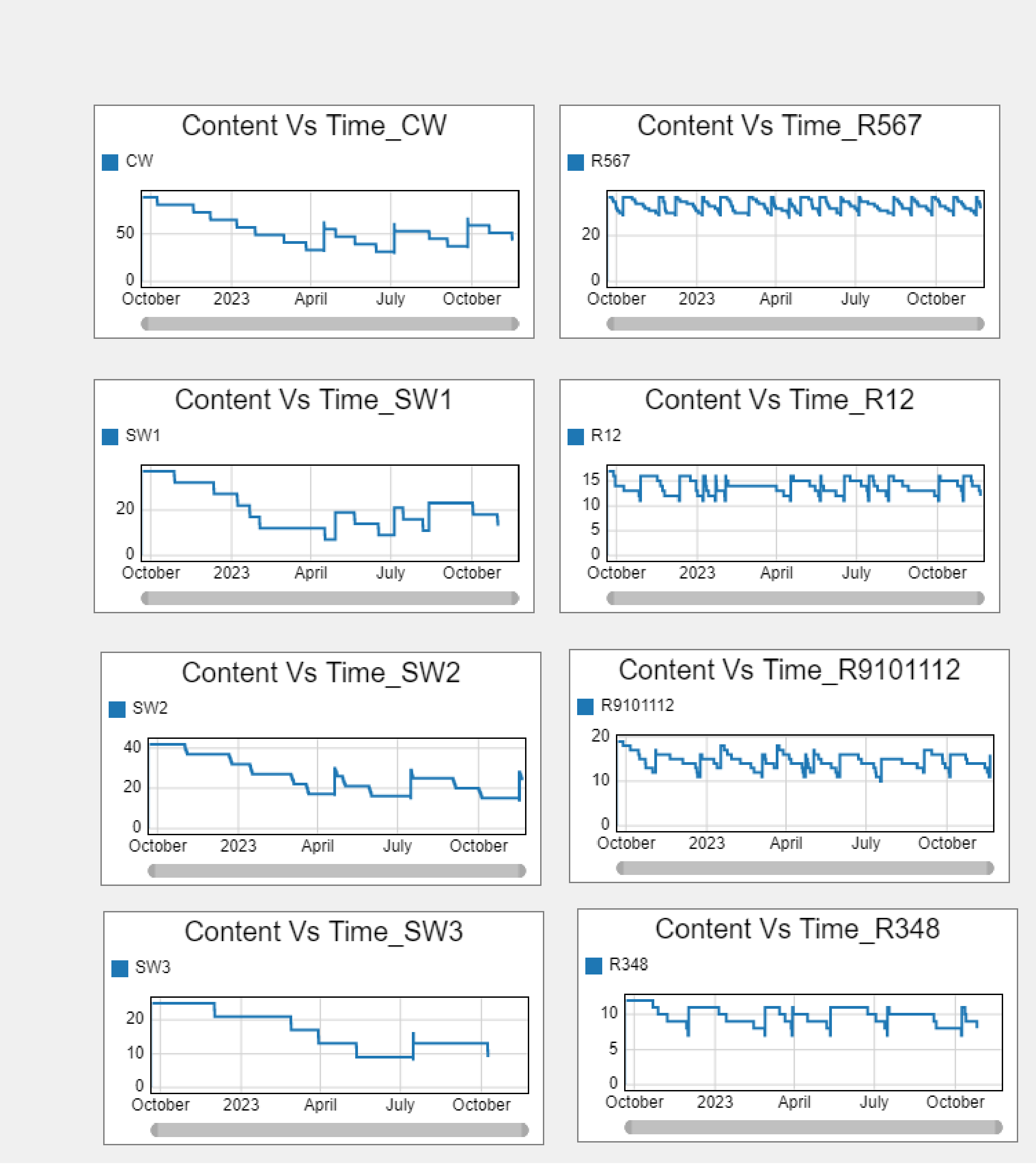

Figure 8. Content vs. time for each node of the system for scenario PA3.

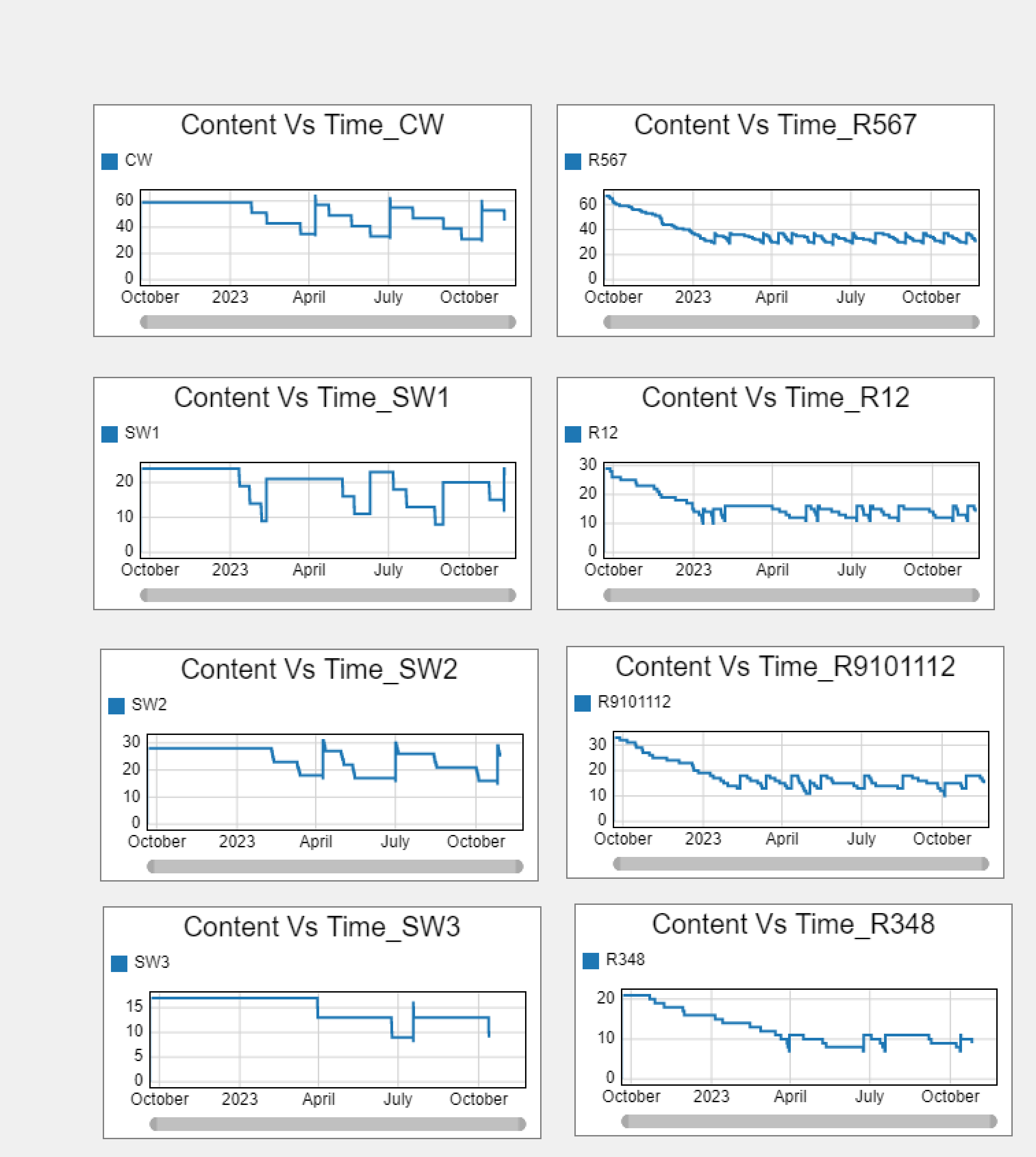

Figure 9. Content vs. time for each node of the system for scenario PA4.

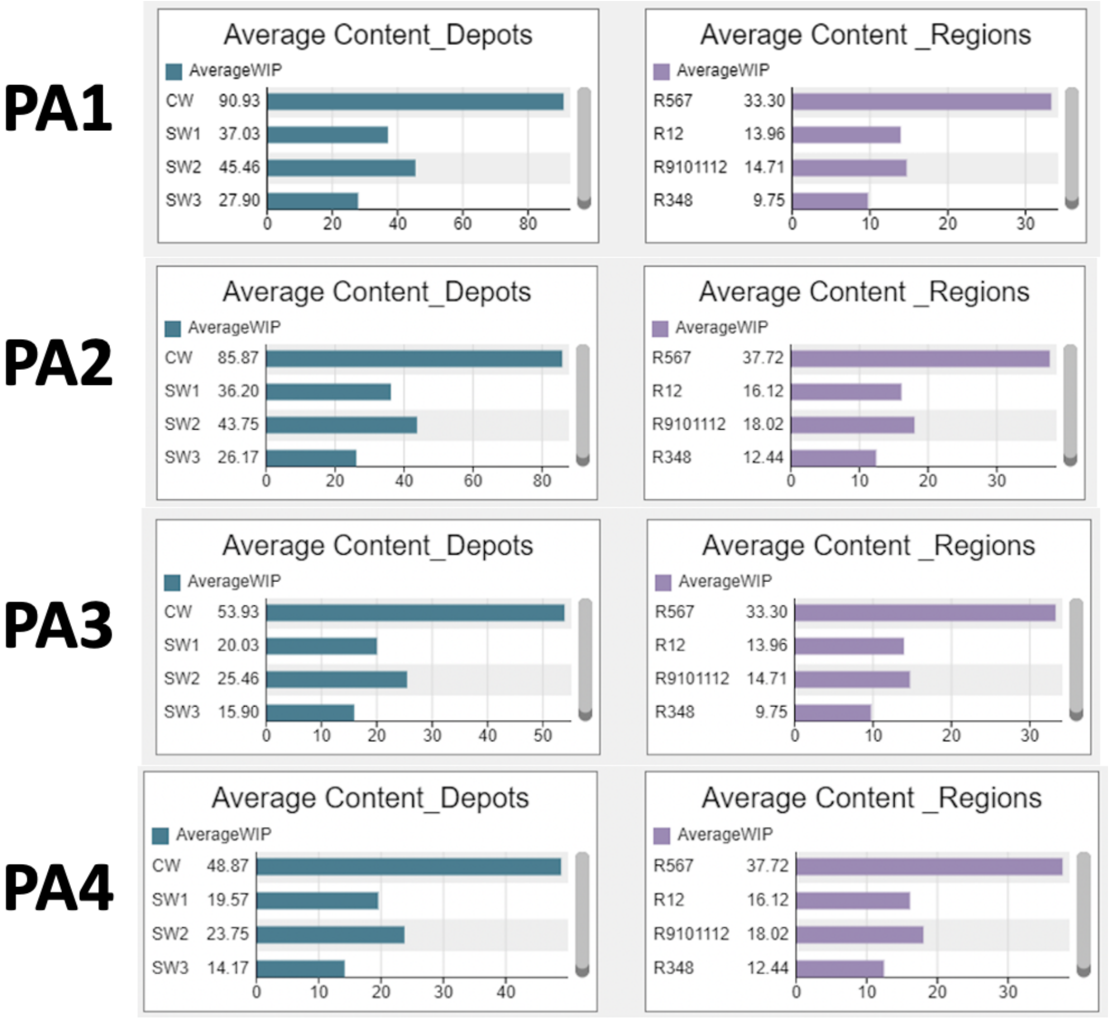

Figure 10. Average content for each alternative and for each node of the system.

Table 9. Ranking of alternatives according to Average Holding costs.

Alternatives	Average Holding Costs
PA2	3201.6
PA1	3166.8
PA4	2215.6
PA3	2169.2

Table 10. Ranking of alternatives according to Product Availability.

Ranking	Alternatives
1	PA2 and PA4
2	PA1 and PA3

6. Conclusions

Simulation has proven to be relevant for multi-echelon inventory management. In this paper, we proposed a simulation-based approach for multi-echelon inventory system selection. The suggested approach starts with a characterization of the current supply chain network. The next step consists of determining the simulation study goals and validation parameters. After that, we identify the expected simulation benefits for the decision-maker. Then, we build the conceptual model for a multi-echelon inventory system as a fundamental phase before running the experimentation. Finally, we apply the simulation-based approach to select the best multi-echelon inventory system alternative using suitable simulation software that considers the supply chain specifications. The proposed simulation-based approach was applied to the case of the pharmaceutical products supply chain in the Moroccan public sector as well in Section 5. The main simulation objective was to compare different alternatives for multi-echelon inventory management through simulation and be able to advise on which one to proceed with. The results of the simulation provided insights into inventory levels for each node of the supply chain. Average inventories were depicted as well and used to calculate the average holding costs for each alternative. The levels of inventories across the supply chain network illustrated the product availability in the downstream stages as well. This helped in ranking different multi-echelon inventory system options according to both criteria: average holding costs and product availability. The results of the simulation demonstrated that adopting an installation stock (R, Q) policy at all levels of the network with an allocation of safety stocks in the most downstream stages is the best and most appropriate alternative for the pharmaceutical supply chain under study.

Appendix A. The Classical Economic Order Quantity Model

The most basic form of the classical economic order quantity model is based on the following assumptions:

- Demand is constant and continuous.
- Ordering and holding costs are constant over time.
- The batch quantity does not need to be an integer.
- The whole batch quantity is delivered at the same time.
- No shortages are allowed.

We will use the following notations:
H = holding cost per unit and time unit
S = ordering cost
D = demand per time unit
Q = batch quantity
C = costs per time unit

When shortages are not allowed and all demand is back-ordered and the case of no safety stock is needed, the inventory level will change over time and a batch is delivered exactly when the preceding batch is finished.

The relevant costs are the holding costs and ordering costs, which vary based on batch quantity Q. The cost is calculated as the following:

$$C = \frac{Q}{2}H + \frac{D}{Q}S \tag{A1}$$

Solving the Equation (A1) for Q we obtain the economic order quantity provided in the Formula (A2):

$$Q^* = \sqrt{\frac{2SD}{H}} \tag{A2}$$

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References

1. Kouhizadeh, M.; Saberi, S.; Sarkis, J. Blockchain technology and the sustainable supply chain: Theoretically exploring adoption barriers. *Int. J. Prod. Econ.* **2021**, *231*, 107831. [CrossRef]
2. Dong, M. Process Modeling, Performance Analysis and Configuration Simulation in Integrated Supply Chain Network Design. Doctoral Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA, 2001
3. Yang, S.; Tan, C. Blockchain-based collaborative management of job shop supply chain. *Int. J. Simul. Model. IJSIMM* **2022**, *21*, 364–374. [CrossRef]
4. Chu, Y.; You, F.; Wassick, J.M.; Agarwal, A. Simulation-based optimization framework for multi-echelon inventory systems under uncertainty. *Comput. Chem. Eng.* **2015**, *73*, 1–16. [CrossRef]
5. Sbai, N.; Berrado, A. Simulation Models for Multi-echelon Inventory Management Problem: A Literature Review. In Proceedings of the 5th NA International Conference on Industrial Engineering and Operations Management, Detroit, MI, USA, 9–11 August 2020; p. 11.
6. Noordhoek, M.; Dullaert, W.; Lai, D.S.W.; de Leeuw, S. A simulation–optimization approach for a service-constrained multi-echelon distribution network. *Transp. Res. Part E Logist. Transp. Rev.* **2018**, *114*, 292–311. [CrossRef]
7. Attanayake, N.; Kashef, R.F.; Andrea, T. A simulation model for a continuous review inventory policy for healthcare systems. In Proceedings of the 2014 IEEE 27th Canadian Conference on Electrical and Computer Engineering (CCECE), Toronto, ON, Canada, 4–7 May 2014; pp. 1–6. ISSN: 0840-7789. [CrossRef]
8. Chu, Y.; You, F. Simulation-based method for optimizing multi-echelon inventory systems. In Proceedings of the 53rd IEEE Conference on Decision and Control, Los Angeles, CA, USA, 15–17 December 2014; pp. 1899–1904. ISSN: 0191-2216.
9. Zhou, R.; De Souza, R.; Ang, J.S.K. Improving supply chain performance through simulation. In Proceedings of the 2016 IEEE International Conference on Management of Innovation and Technology (ICMIT), Bangkok, Thailand, 19–22 September 2016; pp. 95–98.
10. Eddoug, K.; ElHaq, S.L.; Echcheikh, H. Performance evaluation of complex multi-echelon distribution supply chain. In Proceedings of the 2018 4th International Conference on Logistics Operations Management (GOL), Le Havre, France, 10–12 April 2018; pp. 1–10. [CrossRef]
11. Yan, B.; Liu, L. Simulation of multi-echelon supply chain inventory transshipment models at different levels. *SAGE J.* **2018**, *94*, 563–575. [CrossRef]
12. Abidi, M.; Lattouf, A.; Altarazi, S. A system dynamics simulation modeling for managing the inventory in multi-echelon multi-product pharmaceutical supply chain. In Proceedings of the Annual Simulation Symposium, Baltimore, MD, 15–18 April 2018; Society for Computer Simulation International: Baltimore, MD, USA, 2018; ANSS '18, pp. 1–10.
13. Xu, G.; Feng, J.; Chen, F.; Wang, H.; Wang, Z. Simulation-based optimization of control policy on multi-echelon inventory system for fresh agricultural products. *Int. J. Agric. Biol. Eng.* **2019**, *12*, 184–194. [CrossRef]
14. Zhang, Y.; Chai, Y.; Ma, L. Research on Multi-Echelon Inventory Optimization for Fresh Products in Supply Chains. *Sustainability* **2021**, *13*, 6309. [CrossRef]
15. Axsäter, S. *Inventory Control*; Springer: Berlin/Heidelberg, Germany, 2015; Volume 225.
16. Ech-Cheikh, H.; Elhaq, S.L.; Rachid, A.; Douraid, A. Simulating demand uncertainty and inventory control variability of multi-echelon distribution supply chain. In Proceedings of the 2014 International Conference on Logistics Operations Management, Rabat, Morocco, 5–7 June 2014; pp. 27–34. [CrossRef]
17. Axsäter, S.; Rosling, K. Notes: Installation vs. Echelon Stock Policies for Multilevel Inventory Control. *Manag. Sci.* **1993**, *39*, 1274–1280. [CrossRef]
18. Roundy, R. 98%-Effective Integer-Ratio Lot-Sizing for One-Warehouse Multi-Retailer Systems. *Manag. Sci.* **1985**, *31*, 1416–1430. [CrossRef]
19. Muckstadt, J.A.; Roundy, R.O. Chapter 2 Analysis of multistage production systems. In *Handbooks in Operations Research and Management Science*; Elsevier: Amsterdam, The Netherlands, 1993; Volume 4, pp. 59–131. . [CrossRef]
20. Clark, A.J.; Scarf, H. Optimal Policies for a Multi-Echelon Inventory Problem. *Manag. Sci.* **1960**, *6*, 475–490. . 10.1287/mnsc.6.4.475. [CrossRef]
21. Rosling, K. Optimal Inventory Policies for Assembly Systems Under Random Demands. *Oper. Res.* **1989**, *37*, 565–579. [CrossRef]
22. Sherbrooke, C.C. Metric: A Multi-Echelon Technique for Recoverable Item Control. *Oper. Res.* **1968**, *16*, 122–141. [CrossRef]
23. Sherbrooke, C.C. *Optimal Inventory Modeling of Systems: Multi-Echelon Techniques*; Springer Science & Business Media: Berlin, Germany, 2006; Volume 72.
24. Inderfurth, K. Safety stock optimization in multi-stage inventory systems. *Int. J. Prod. Econ.* **1991**, *24*, 103–113. [CrossRef]
25. Inderfurth, K.; Minner, S. Safety stocks in multi-stage inventory systems under different service measures. *Eur. J. Oper. Res.* **1998**, *106*, 57–73. [CrossRef]
26. Chakravarty, A.K.; Shtub, A. Simulated safety stock allocation in a two-echelon distribution system. *Int. J. Prod. Res.* **1986**, *24*, 1245–1253. [CrossRef]
27. Chopra, S.; Meindl, P. *Supply Chain Management: Strategy, Planning, and Operation, eBook, Global Edition*, 6th ed.; Pearson: Boston, MA, USA, 2015.

28. Eruguz, A.S.; Sahin, E.; Jemai, Z.; Dallery, Y. A comprehensive survey of guaranteed-service models for multi-echelon inventory optimization. *Int. J. Prod. Econ.* **2016**, *172*, 110–125. [CrossRef]
29. Sbai, N.; Benabbou, L.; Berrado, A. Multi-echelon Inventory System Selection: Case of Distribution Systems. *Int. J. Supply Oper. Manag.* **2022**, *9*, 108–125. [CrossRef]
30. Sbai, N.; Berrado, A. A literature review on multi-echelon inventory management: The case of pharmaceutical supply chain. *MATEC Web Conf.* **2018**, *200*, 00013. [CrossRef]
31. Shah, N. Pharmaceutical supply chains: Key issues and strategies for optimisation. *Comput. Chem. Eng.* **2004**, *28*, 929–941. [CrossRef]
32. Grznar, P.; Gregor, M.; Gaso, M.; Gabajova, G.; Schickerle, M.; Burganova, N. Dynamic simulation tool for planning and optimisation of supply process. *Int. J. Simul. Model.* **2021**, *20*, 441–452. [CrossRef]
33. Robinson, S. *Simulation: The Practice of Model Development and Use*, 2nd ed.; Red Globe Press: Houndmills, UK; New York, NY, USA, 2014.
34. FlexSim Software Products, Inc. Flexsim Software. Available online: https://www.flexsim.com/ (accessed on 12 January 2023).
35. Narita, V.T.; Ikeziri, L.M.; Souza, F.B.D. Evaluation of dynamic buffer management for adjusting stock level: A simulation-based approach. *J. Ind. Prod. Eng.* **2021**, *38*, 452–465. [CrossRef]
36. Jiao, Y.l.; Xing, X.c.; Zhang, P.; Xu, L.c.; Liu, X.R. Multi-objective storage location allocation optimization and simulation analysis of automated warehouse based on multi-population genetic algorithm. *Concurr. Eng.* **2018**, *26*, 367–377. [CrossRef]
37. Wan, J.; Zhao, C. Simulation Research on Multi-Echelon Inventory System in Supply Chain Based on Arena. In Proceedings of the 2009 First International Conference on Information Science and Engineering, Nanjing, China, 26–28 December 2009; pp. 397–400. ISSN: 2160-1291. [CrossRef]
38. Zhou, W.Q.; Chen, L.; Ge, H.M. A multi-product multi-echelon inventory control model with joint replenishment strategy. *Appl. Math. Model.* **2013**, *37*, 2039–2050. [CrossRef]
39. Robinson, S. Conceptual modelling for simulation Part II: A framework for conceptual modelling. *J. Oper. Res. Soc.* **2008**, *59*, 291–304. [CrossRef]
40. Vieira, G.E.; Júnior, O.C. A conceptual model for the creation of supply chain simulation models. In Proceedings of the Proceedings of the 37th conference on Winter Simulation, Orlando, FL, USA, 4–7 December 2005; Winter Simulation Conference: Orlando, FL, USA, 2005; WSC '05, pp. 2619–2627.
41. Chopra, S.; Meindl, P. *Supply Chain Management: Strategy, Planning, and Operation*, 6th ed.; Pearson: Boston, MA, USA, 2014.
42. La Région | Portail National des Collectivités Territoriales. Available online: https://www.collectivites-territoriales.gov.ma/fr/accueil (accessed on 12 January 2023).