

The arrangements (S1) and (S2) display the sensitivity equations obtained from the photoacclimated model utilized in this study.

\dot{x}_6	$=$	$\vartheta(1,1)x_6 + \vartheta(1,2)x_7 + \vartheta(1,3)x_8 + \vartheta(1,4)x_9 + \vartheta(1,5)x_{10} + \omega(1,1)$	$x_6(0)$	$=$	0
\dot{x}_7	$=$	$\vartheta(2,1)x_6 + \vartheta(2,2)x_7 + \vartheta(2,3)x_8 + \vartheta(2,4)x_9 + \vartheta(2,5)x_{10} + \omega(2,1)$	$x_7(0)$	$=$	0
\dot{x}_8	$=$	$\vartheta(3,1)x_6 + \vartheta(3,2)x_7 + \vartheta(3,3)x_8 + \vartheta(3,4)x_9 + \vartheta(3,5)x_{10} + \omega(3,1)$	$x_8(0)$	$=$	0
\dot{x}_9	$=$	$\vartheta(4,1)x_6 + \vartheta(4,2)x_7 + \vartheta(4,3)x_8 + \vartheta(4,4)x_9 + \vartheta(4,5)x_{10} + \omega(4,1)$	$x_9(0)$	$=$	0
\dot{x}_{10}	$=$	$\vartheta(5,1)x_6 + \vartheta(5,2)x_7 + \vartheta(5,3)x_8 + \vartheta(5,4)x_9 + \vartheta(5,5)x_{10} + \omega(5,1)$	$x_{10}(0)$	$=$	0
\dot{x}_{11}	$=$	$\vartheta(1,1)x_{11} + \vartheta(1,2)x_{12} + \vartheta(1,3)x_{13} + \vartheta(1,4)x_{14} + \vartheta(1,5)x_{15} + \omega(1,2)$	$x_{11}(0)$	$=$	0
\dot{x}_{12}	$=$	$\vartheta(2,1)x_{11} + \vartheta(2,2)x_{12} + \vartheta(2,3)x_{13} + \vartheta(2,4)x_{14} + \vartheta(2,5)x_{15} + \omega(2,2)$	$x_{12}(0)$	$=$	0
\dot{x}_{13}	$=$	$\vartheta(3,1)x_{11} + \vartheta(3,2)x_{12} + \vartheta(3,3)x_{13} + \vartheta(3,4)x_{14} + \vartheta(3,5)x_{15} + \omega(3,2)$	$x_{13}(0)$	$=$	0
\dot{x}_{14}	$=$	$\vartheta(4,1)x_{11} + \vartheta(4,2)x_{12} + \vartheta(4,3)x_{13} + \vartheta(4,4)x_{14} + \vartheta(4,5)x_{15} + \omega(4,2)$	$x_{14}(0)$	$=$	0
\dot{x}_{15}	$=$	$\vartheta(5,1)x_{11} + \vartheta(5,2)x_{12} + \vartheta(5,3)x_{13} + \vartheta(5,4)x_{14} + \vartheta(5,5)x_{15} + \omega(5,2)$	$x_{15}(0)$	$=$	0
\dot{x}_{16}	$=$	$\vartheta(1,1)x_{16} + \vartheta(1,2)x_{17} + \vartheta(1,3)x_{18} + \vartheta(1,4)x_{19} + \vartheta(1,5)x_{20} + \omega(1,3)$	$x_{16}(0)$	$=$	0
\dot{x}_{17}	$=$	$\vartheta(2,1)x_{16} + \vartheta(2,2)x_{17} + \vartheta(2,3)x_{18} + \vartheta(2,4)x_{19} + \vartheta(2,5)x_{20} + \omega(2,3)$	$x_{17}(0)$	$=$	0
\dot{x}_{18}	$=$	$\vartheta(3,1)x_{16} + \vartheta(3,2)x_{17} + \vartheta(3,3)x_{18} + \vartheta(3,4)x_{19} + \vartheta(3,5)x_{20} + \omega(3,3)$	$x_{18}(0)$	$=$	0
\dot{x}_{19}	$=$	$\vartheta(4,1)x_{16} + \vartheta(4,2)x_{17} + \vartheta(4,3)x_{18} + \vartheta(4,4)x_{19} + \vartheta(4,5)x_{20} + \omega(4,3)$	$x_{19}(0)$	$=$	0
\dot{x}_{20}	$=$	$\vartheta(5,1)x_{16} + \vartheta(5,2)x_{17} + \vartheta(5,3)x_{18} + \vartheta(5,4)x_{19} + \vartheta(5,5)x_{20} + \omega(5,3)$	$x_{20}(0)$	$=$	0
\dot{x}_{21}	$=$	$\vartheta(1,1)x_{21} + \vartheta(1,2)x_{22} + \vartheta(1,3)x_{23} + \vartheta(1,4)x_{24} + \vartheta(1,5)x_{25} + \omega(1,4)$	$x_{21}(0)$	$=$	0
\dot{x}_{22}	$=$	$\vartheta(2,1)x_{21} + \vartheta(2,2)x_{22} + \vartheta(2,3)x_{23} + \vartheta(2,4)x_{24} + \vartheta(2,5)x_{25} + \omega(2,4)$	$x_{22}(0)$	$=$	0
\dot{x}_{23}	$=$	$\vartheta(3,1)x_{21} + \vartheta(3,2)x_{22} + \vartheta(3,3)x_{23} + \vartheta(3,4)x_{24} + \vartheta(3,5)x_{25} + \omega(3,4)$	$x_{23}(0)$	$=$	0
\dot{x}_{24}	$=$	$\vartheta(4,1)x_{21} + \vartheta(4,2)x_{22} + \vartheta(4,3)x_{23} + \vartheta(4,4)x_{24} + \vartheta(4,5)x_{25} + \omega(4,4)$	$x_{24}(0)$	$=$	0
\dot{x}_{25}	$=$	$\vartheta(5,1)x_{21} + \vartheta(5,2)x_{22} + \vartheta(5,3)x_{23} + \vartheta(5,4)x_{24} + \vartheta(5,5)x_{25} + \omega(5,4)$	$x_{25}(0)$	$=$	0
\dot{x}_{26}	$=$	$\vartheta(1,1)x_{26} + \vartheta(1,2)x_{27} + \vartheta(1,3)x_{28} + \vartheta(1,4)x_{29} + \vartheta(1,5)x_{30} + \omega(1,5)$	$x_{26}(0)$	$=$	0
\dot{x}_{27}	$=$	$\vartheta(2,1)x_{26} + \vartheta(2,2)x_{27} + \vartheta(2,3)x_{28} + \vartheta(2,4)x_{29} + \vartheta(2,5)x_{30} + \omega(2,5)$	$x_{27}(0)$	$=$	0
\dot{x}_{28}	$=$	$\vartheta(3,1)x_{26} + \vartheta(3,2)x_{27} + \vartheta(3,3)x_{28} + \vartheta(3,4)x_{29} + \vartheta(3,5)x_{30} + \omega(3,5)$	$x_{28}(0)$	$=$	0
\dot{x}_{29}	$=$	$\vartheta(4,1)x_{26} + \vartheta(4,2)x_{27} + \vartheta(4,3)x_{28} + \vartheta(4,4)x_{29} + \vartheta(4,5)x_{30} + \omega(4,5)$	$x_{29}(0)$	$=$	0
\dot{x}_{30}	$=$	$\vartheta(5,1)x_{26} + \vartheta(5,2)x_{27} + \vartheta(5,3)x_{28} + \vartheta(5,4)x_{29} + \vartheta(5,5)x_{30} + \omega(5,5)$	$x_{30}(0)$	$=$	0
\dot{x}_{31}	$=$	$\vartheta(1,1)x_{31} + \vartheta(1,2)x_{32} + \vartheta(1,3)x_{33} + \vartheta(1,4)x_{34} + \vartheta(1,5)x_{35} + \omega(1,6)$	$x_{31}(0)$	$=$	0
\dot{x}_{32}	$=$	$\vartheta(2,1)x_{31} + \vartheta(2,2)x_{32} + \vartheta(2,3)x_{33} + \vartheta(2,4)x_{34} + \vartheta(2,5)x_{35} + \omega(2,6)$	$x_{32}(0)$	$=$	0
\dot{x}_{33}	$=$	$\vartheta(3,1)x_{31} + \vartheta(3,2)x_{32} + \vartheta(3,3)x_{33} + \vartheta(3,4)x_{34} + \vartheta(3,5)x_{35} + \omega(3,6)$	$x_{33}(0)$	$=$	0
\dot{x}_{34}	$=$	$\vartheta(4,1)x_{31} + \vartheta(4,2)x_{32} + \vartheta(4,3)x_{33} + \vartheta(4,4)x_{34} + \vartheta(4,5)x_{35} + \omega(4,6)$	$x_{34}(0)$	$=$	0
\dot{x}_{35}	$=$	$\vartheta(5,1)x_{31} + \vartheta(5,2)x_{32} + \vartheta(5,3)x_{33} + \vartheta(5,4)x_{34} + \vartheta(5,5)x_{35} + \omega(5,6)$	$x_{35}(0)$	$=$	0
\dot{x}_{36}	$=$	$\vartheta(1,1)x_{36} + \vartheta(1,2)x_{37} + \vartheta(1,3)x_{38} + \vartheta(1,4)x_{39} + \vartheta(1,5)x_{40}$			

$$\begin{aligned}
\dot{x}_{56} &= \vartheta(1,1)x_{56} + \vartheta(1,2)x_{57} + \vartheta(1,3)x_{58} + \vartheta(1,4)x_{59} + \vartheta(1,5)x_{60} + \omega(1,11) & x_{56}(0) &= 0 \\
\dot{x}_{57} &= \vartheta(2,1)x_{56} + \vartheta(2,2)x_{57} + \vartheta(2,3)x_{58} + \vartheta(2,4)x_{59} + \vartheta(2,5)x_{60} + \omega(2,11) & x_{57}(0) &= 0 \\
\dot{x}_{58} &= \vartheta(3,1)x_{56} + \vartheta(3,2)x_{57} + \vartheta(3,3)x_{58} + \vartheta(3,4)x_{59} + \vartheta(3,5)x_{60} + \omega(3,11) & x_{58}(0) &= 0 \\
\dot{x}_{59} &= \vartheta(4,1)x_{56} + \vartheta(4,2)x_{57} + \vartheta(4,3)x_{58} + \vartheta(4,4)x_{59} + \vartheta(4,5)x_{60} + \omega(4,11) & x_{59}(0) &= 0 \\
\dot{x}_{60} &= \vartheta(5,1)x_{56} + \vartheta(5,2)x_{57} + \vartheta(5,3)x_{58} + \vartheta(5,4)x_{59} + \vartheta(5,5)x_{60} + \omega(5,11) & x_{60}(0) &= 0 \\
\dot{x}_{61} &= \vartheta(1,1)x_{61} + \vartheta(1,2)x_{62} + \vartheta(1,3)x_{63} + \vartheta(1,4)x_{64} + \vartheta(1,5)x_{65} + \omega(1,12) & x_{61}(0) &= 0 \\
\dot{x}_{62} &= \vartheta(2,1)x_{61} + \vartheta(2,2)x_{62} + \vartheta(2,3)x_{63} + \vartheta(2,4)x_{64} + \vartheta(2,5)x_{65} + \omega(2,12) & x_{62}(0) &= 0 \\
\dot{x}_{63} &= \vartheta(3,1)x_{61} + \vartheta(3,2)x_{62} + \vartheta(3,3)x_{63} + \vartheta(3,4)x_{64} + \vartheta(3,5)x_{65} + \omega(3,12) & x_{63}(0) &= 0 \\
\dot{x}_{64} &= \vartheta(4,1)x_{61} + \vartheta(4,2)x_{62} + \vartheta(4,3)x_{63} + \vartheta(4,4)x_{64} + \vartheta(4,5)x_{65} + \omega(4,12) & x_{64}(0) &= 0 \\
\dot{x}_{65} &= \vartheta(5,1)x_{61} + \vartheta(5,2)x_{62} + \vartheta(5,3)x_{63} + \vartheta(5,4)x_{64} + \vartheta(5,5)x_{65} + \omega(5,12) & x_{65}(0) &= 0 \\
\dot{x}_{66} &= \vartheta(1,1)x_{66} + \vartheta(1,2)x_{67} + \vartheta(1,3)x_{68} + \vartheta(1,4)x_{69} + \vartheta(1,5)x_{70} + \omega(1,13) & x_{66}(0) &= 0 \\
\dot{x}_{67} &= \vartheta(2,1)x_{66} + \vartheta(2,2)x_{67} + \vartheta(2,3)x_{68} + \vartheta(2,4)x_{69} + \vartheta(2,5)x_{70} + \omega(2,13) & x_{67}(0) &= 0 \\
\dot{x}_{68} &= \vartheta(3,1)x_{66} + \vartheta(3,2)x_{67} + \vartheta(3,3)x_{68} + \vartheta(3,4)x_{69} + \vartheta(3,5)x_{70} + \omega(3,13) & x_{68}(0) &= 0 \\
\dot{x}_{69} &= \vartheta(4,1)x_{66} + \vartheta(4,2)x_{67} + \vartheta(4,3)x_{68} + \vartheta(4,4)x_{69} + \vartheta(4,5)x_{70} + \omega(4,13) & x_{69}(0) &= 0 \\
\dot{x}_{70} &= \vartheta(5,1)x_{66} + \vartheta(5,2)x_{67} + \vartheta(5,3)x_{68} + \vartheta(5,4)x_{69} + \vartheta(5,5)x_{70} + \omega(5,13) & x_{70}(0) &= 0 \\
\dot{x}_{71} &= \vartheta(1,1)x_{71} + \vartheta(1,2)x_{72} + \vartheta(1,3)x_{73} + \vartheta(1,4)x_{74} + \vartheta(1,5)x_{75} + \omega(1,14) & x_{71}(0) &= 0 \\
\dot{x}_{72} &= \vartheta(2,1)x_{71} + \vartheta(2,2)x_{72} + \vartheta(2,3)x_{73} + \vartheta(2,4)x_{74} + \vartheta(2,5)x_{75} + \omega(2,14) & x_{72}(0) &= 0 \\
\dot{x}_{73} &= \vartheta(3,1)x_{71} + \vartheta(3,2)x_{72} + \vartheta(3,3)x_{73} + \vartheta(3,4)x_{74} + \vartheta(3,5)x_{75} + \omega(3,14) & x_{73}(0) &= 0 \\
\dot{x}_{74} &= \vartheta(4,1)x_{71} + \vartheta(4,2)x_{72} + \vartheta(4,3)x_{73} + \vartheta(4,4)x_{74} + \vartheta(4,5)x_{75} + \omega(4,14) & x_{74}(0) &= 0 \\
\dot{x}_{75} &= \vartheta(5,1)x_{71} + \vartheta(5,2)x_{72} + \vartheta(5,3)x_{73} + \vartheta(5,4)x_{74} + \vartheta(5,5)x_{75} + \omega(5,14) & x_{75}(0) &= 0 \\
\dot{x}_{76} &= \vartheta(1,1)x_{76} + \vartheta(1,2)x_{77} + \vartheta(1,3)x_{78} + \vartheta(1,4)x_{79} + \vartheta(1,5)x_{80} + \omega(1,15) & x_{76}(0) &= 0 \\
\dot{x}_{77} &= \vartheta(2,1)x_{76} + \vartheta(2,2)x_{77} + \vartheta(2,3)x_{78} + \vartheta(2,4)x_{79} + \vartheta(2,5)x_{80} + \omega(2,15) & x_{77}(0) &= 0 \\
\dot{x}_{78} &= \vartheta(3,1)x_{76} + \vartheta(3,2)x_{77} + \vartheta(3,3)x_{78} + \vartheta(3,4)x_{79} + \vartheta(3,5)x_{80} + \omega(3,15) & x_{78}(0) &= 0 \\
\dot{x}_{79} &= \vartheta(4,1)x_{76} + \vartheta(4,2)x_{77} + \vartheta(4,3)x_{78} + \vartheta(4,4)x_{79} + \vartheta(4,5)x_{80} + \omega(4,15) & x_{79}(0) &= 0 \\
\dot{x}_{80} &= \vartheta(5,1)x_{76} + \vartheta(5,2)x_{77} + \vartheta(5,3)x_{78} + \vartheta(5,4)x_{79} + \vartheta(5,5)x_{80} + \omega(5,15) & x_{80}(0) &= 0
\end{aligned} \tag{S2}$$

The sensitivity equations are described by equations (S3) to (S102), which represent the elements $J\vartheta_{i,j}$ and $J\omega_{i,j}$ of the sensitivity equations shown in the arrangements (S1) and (S2).

$$\begin{aligned}
\vartheta(1,1) = & -D - \frac{I_0 L k_g \tilde{\mu} x \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} x \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{2 I_0^2 L k_g^2 \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{zc_0}{x_3} + 1 \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} - R
\end{aligned} \tag{S3}$$

$$\vartheta(2,1) = 0 \tag{S4}$$

$$\begin{aligned}
\vartheta(3,1) = & - \frac{I_0 K_{I^*} L a k_g \tilde{\mu} x^2 \gamma_m \left(-\frac{zc_0}{x_3} + 1 \right)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 z c_0 k_g \tilde{\mu} x}{x_3^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} x \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{2 I_0^2 K_{I^*} L a k_g^2 x \gamma_m}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} L a k_g x \gamma_m}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3^2 \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)}
\end{aligned} \tag{S5}$$

$$\begin{aligned}
\vartheta(4,1) = & \frac{I_0 K_{I^*} L a k_g \tilde{\mu} x^2 x_3 \gamma_m \left(-\frac{z c_0}{x_3} + 1 \right)}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} x \left(-\frac{z c_0}{x_3} + 1 \right) \left(-\frac{2 I_0^2 K_{I^*} L a k_g^2 x x_3 \gamma_m}{K_{il} (K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} - \frac{I_0 K_{I^*} L a k_g x x_3 \gamma_m}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} - \frac{K_{sI}^*}{K_{I^*} x_3 \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S6}$$

$$\vartheta(5,1) = 0 \tag{S7}$$

$$\vartheta(1,2) = -\frac{\rho_m y \left(1 - \frac{x_3}{z c_m} \right)}{K_y + y} \tag{S8}$$

$$\vartheta(2,2) = -D + \frac{\rho_m x y \left(1 - \frac{x_3}{z c_m} \right)}{(K_y + y)^2} - \frac{\rho_m x \left(1 - \frac{x_3}{z c_m} \right)}{K_y + y} \tag{S9}$$

$$\vartheta(3,2) = \frac{\rho_m x y}{z c_m (K_y + y)} \tag{S10}$$

$$\vartheta(4,2) = 0 \tag{S11}$$

$$\vartheta(5,2) = 0 \tag{S12}$$

$$\begin{aligned}
\vartheta(1,3) = & \frac{I_0 L k_g \tilde{\mu} (-z c_0 + x_3) \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI}^{**} (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 k_g \tilde{\mu} (-z c_0 + x_3) \left(\frac{2 I_0^2 L k_g^2 \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S13}$$

$$\vartheta(2,3) = -\frac{\rho_m y \left(1 - \frac{x_3}{z c_m} \right)}{(K_y + y)^2} + \frac{\rho_m \left(1 - \frac{x_3}{z c_m} \right)}{K_y + x_2} \tag{S14}$$

$$\begin{aligned}
\vartheta(3,3) = & \frac{I_0 K_{I^*} Lak_g \tilde{\mu} x \gamma_m (-zc_0 + x_3)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 k_g \tilde{\mu} (-zc_0 + x_3) \left(\frac{2I_0^2 K_{I^*} Lak_g^2 x \gamma_m}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 y_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} Lak_g x \gamma_m}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3^2 \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 k_g \tilde{\mu}}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S15)
\end{aligned}$$

$$\begin{aligned}
\vartheta(4,3) = & \frac{I_0 K_{I^*} Lak_g \tilde{\mu} x \gamma_m (-zc_0 + x_3)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 y_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 y_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 k_g \tilde{\mu} (-zc_0 + x_3) \left(\frac{2I_0^2 K_{I^*} Lak_g^2 x \gamma_m}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 y_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} Lak_g x \gamma_m}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3^2 \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S16)
\end{aligned}$$

$$\vartheta(5,3) = 0 \quad (S17)$$

$$\begin{aligned}
\vartheta(1,4) = & - \frac{I_0^2 L k_g^2 \tilde{\mu} \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{K_{I^*} ax_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 L k_g \tilde{\mu} \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} - I^* \right) \left(\frac{K_{I^*} ax_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} - I^* \right) \left(\frac{2 I_0^2 L k_g^2 \left(\frac{K_{I^*} ax_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(\frac{K_{I^*} ax_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2} \quad (S18)
\end{aligned}$$

$$\vartheta(2,4) = 0 \quad (S19)$$

$$\begin{aligned}
\vartheta(3,4) = & - \frac{I_0^2 K_{I^*} L a k_g^2 \tilde{\mu} x \gamma_m \left(-\frac{zc_0}{x_3} + 1 \right)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 K_{I^*} L a k_g \tilde{\mu} x \gamma_m \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} - I^* \right)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 z c_0 k_g \tilde{\mu} \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} - I^* \right)}{x_3^2 \left(L \left(\frac{K_{I^*} axx_3 \gamma_{max}}{K_{I^*} + I^*} + bx + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} - I^* \right) \left(\frac{2 I_0^2 K_{I^*} L a k_g^2 x \gamma_m}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} L a k_g x \gamma_m}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)} \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_{max}} \right)} \quad (S20)
\end{aligned}$$

$$\begin{aligned}
\vartheta(4,4) = & \frac{I_0 K_{I^*} L a k_g \tilde{\mu} x x_3 \gamma_m \left(-\frac{z c_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right)}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{Q_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right) \left(-\frac{2 I_0^2 K_{I^*} L a k_g^2 x x_3 \gamma_m}{K_{il} (K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} - \frac{I_0 K_{I^*} L a k_g x x_3 \gamma_m}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right) \left(\frac{I_0 K_{I^*} L a k_g x x_3 \gamma_m}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} - 1 \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S21)
\end{aligned}$$

$$\vartheta(5,4) = 0 \quad (S22)$$

$$\begin{aligned}
\vartheta(1,5) = & - \frac{I_0 L k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right) \left(\frac{K_{I^*} a x_3 \gamma_m}{K_{I^*} + I^*} + b \right) (-\beta x_3 - q_g + 1)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right) \left(\frac{2 I_0^2 L k_g^2 \left(\frac{K_{I^*} a x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(\frac{K_{I^*} a x_3 \gamma_m}{K_{I^*} + I^*} + b \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} \right) (-\beta x_3 - q_g + 1)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2} \quad (S23)
\end{aligned}$$

$$\vartheta(2,5) = \frac{\alpha \rho_m x_2 \left(1 - \frac{x_3}{z c_m} \right)}{(K_y + x_2)^2} - \frac{\alpha \rho_m \left(1 - \frac{x_3}{z c_m} \right)}{K_y + x_2} \quad (S24)$$

$$\begin{aligned}
\vartheta(3,5) = & - \frac{I_0 K_{I^*} L a k_g \tilde{\mu} x \gamma_m \left(-\frac{z c_0}{x_3} + 1 \right) (-\beta x_3 - q_g + 1)}{(K_{I^*} + x_4) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 z c_0 k_g \tilde{\mu} (-\beta x_3 - q_g + 1)}{x_3^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 \beta k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right) (-\beta x_3 - q_g + 1) \left(\frac{2 I_0^2 K_{I^*} L a k_g^2 x \gamma_m}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} L a k_g x \gamma_m}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3^2 \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2} \\
& + \frac{a_{lf} \rho_m x_2}{z c_m (K_y + x_2)}
\end{aligned} \tag{S25}$$

$$\begin{aligned}
\vartheta(4,5) = & - \frac{I_0 K_{I^*} L a k_g \tilde{\mu} x x_3 \gamma_m \left(-\frac{z c_0}{x_3} + 1 \right) (-\beta x_3 - q_g + 1)}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right) (-\beta x_3 - q_g + 1) \left(-\frac{2 I_0^2 K_{I^*} L a k_g^2 x x_3 \gamma_m}{K_{il} (K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} - \frac{I_0 K_{I^*} L a k_g x x_3 \gamma_m}{(K_{I^*} + I^*)^2 \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} - \frac{K_{sl}^*}{K_{I^*} x_3 \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S26}$$

$$\begin{aligned}
\vartheta(5,5) = & - \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} + R
\end{aligned} \tag{S27}$$

$$\omega(1,1) = -x \tag{S28}$$

$$\omega(2,1) = Ds - x_2 \tag{S29}$$

$$\varpi(3,1) = 0 \quad (S30)$$

$$\varpi(4,1) = 0 \quad (S31)$$

$$\varpi(5,1) = 0 \quad (S32)$$

$$\begin{aligned} \varpi(1,2) = & \frac{I_0 k_g \tilde{\mu} x \left(-\frac{zc_0}{x_3} + 1 \right) \left(-\frac{2I_0 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} - \frac{k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx_1 + c \right) + k_g} \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2} \\ & + \frac{k_g \tilde{\mu} x \left(-\frac{zc_0}{x_3} + 1 \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \end{aligned} \quad (S33)$$

$$\varpi(2,2) = 0 \quad (S34)$$

$$\begin{aligned} \varpi(3,2) = & - \frac{I_0 k_g \tilde{\mu} (-zc_0 + x_3) \left(-\frac{2I_0 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} - \frac{k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} \right)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2} \\ & - \frac{k_g \tilde{\mu} (-zc_0 + x_3)}{\left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axx_3 \gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \end{aligned} \quad (S35)$$

$$\begin{aligned}
\omega(3,3) = & \frac{I_0 K_{I^*} Lak_g \tilde{\mu} x x_3 (-z c_0 + x_3)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 k_g \tilde{\mu} (-z c_0 + x_3) \left(\frac{2 I_0^2 K_{I^*} Lak_g^2 x x_3}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} Lak_g x x_3}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m^2} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2} \\
& - \frac{I_0 k_g \mu}{\left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + R - \frac{\rho_m x_2}{Q l (K_y + x_2)}
\end{aligned} \tag{S40}$$

$$\begin{aligned}
\omega(4,3) = & - \frac{I_0^2 K_{I^*} Lak_g^2 \tilde{\mu} x x_3 \left(-\frac{z c_0}{x_3} + 1 \right)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& - \frac{I_0 K_{I^*} Lak_g \tilde{\mu} x x_3 \left(-\frac{z c_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right) \left(\frac{2 I_0^2 K_{I^*} Lak_g^2 x x_3}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} Lak_g x x_3}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m^2} \right)}{\left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x x_3 \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S41}$$

$$\begin{aligned}
\omega(5,3) = & - \frac{I_0 K_{I^*} Lak_g \tilde{\mu} x z \left(-\frac{z c_0}{z} + 1 \right) (-\beta z - q_g + 1)}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{z} + 1 \right) (-\beta z - q_g + 1) \left(\frac{2 I_0^2 K_{I^*} Lak_g^2 x z}{K_{il} (K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 K_{I^*} Lak_g x z}{(K_{I^*} + I^*) \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m^2} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2}
\end{aligned} \tag{S42}$$

$$\omega(1,4) = - \frac{I_0 k_g \tilde{\mu} x (K_{I^*} + I^*) \left(-\frac{z c_0}{z} + 1 \right)}{K_{I^*} z \gamma_m \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2} \quad (S43)$$

$$\omega(2,4) = 0 \quad (S44)$$

$$\omega(3,4) = - \frac{I_0 k_g \tilde{\mu} (K_{I^*} + I^*) (-z c_0 + z)}{K_{I^*} z \gamma_m \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2} \quad (S45)$$

$$\omega(4,4) = - \frac{I_0 k_g \tilde{\mu} (K_{I^*} + I^*) \left(-\frac{z c_0}{z} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right)}{K_{I^*} z \gamma_m \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2} \quad (S46)$$

$$\omega(5,4) = - \frac{I_0 k_g \tilde{\mu} (K_{I^*} + I^*) \left(-\frac{z c_0}{z} + 1 \right) (-\beta z - q_g + 1)}{K_{I^*} z \gamma_m \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2} \quad (S47)$$

$$\omega(1,5) = - \frac{I_0 L k_g \tilde{\mu} x \left(-\frac{z c_0}{z} + 1 \right) \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)} \quad (S48)$$

$$+ \frac{I_0 k_g \tilde{\mu} x \left(-\frac{z c_0}{z} + 1 \right) \left(\frac{2 I_0^2 L k_g^2 \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} - \frac{K_{sl}^*}{K_{I^*} z \gamma_m} + \frac{K_{sl} (K_{I^*} + I^*)}{K_{I^*}^2 z \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl} (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2}$$

$$\omega(2,5) = 0 \quad (S49)$$

$$\begin{aligned}
\omega(3,5) = & \frac{I_0 L k_g \tilde{\mu}(-z c_0 + z) \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + x_4)}{K_{I^*} z \gamma_m} \right)} \\
& - \frac{I_0 k_g \tilde{\mu}(-z c_0 + z) \left(\frac{2 I_0^2 L k_g^2 \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} - \frac{K_{sl}^*}{K_{I^*} z \gamma_m} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*}^2 z \gamma_m} \right)}{2} \\
& - \frac{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)}{2}
\end{aligned} \tag{S50}$$

$$\begin{aligned}
\omega(4,5) = & - \frac{I_0^2 L k_g^2 \tilde{\mu} \left(-\frac{z c_0}{z} + 1 \right) \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)} \\
& - \frac{I_0 L k_g \tilde{\mu} \left(-\frac{z c_0}{z} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right) \left(-\frac{K_{I^*} a x z v}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_{max}}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{z c_0}{z} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} - I^* \right) \left(\frac{2 I_0^2 L k_g^2 \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(-\frac{K_{I^*} a x z \gamma_m}{(K_{I^*} + I^*)^2} + \frac{a x z \gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} - \frac{K_{sl}^*}{K_{I^*} z \gamma_m} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*}^2 z \gamma_m} \right)}{2} \\
& + \frac{\left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + x_4} + b x + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x z \gamma_m}{K_{I^*} + I^*} + b x + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)}{2}
\end{aligned} \tag{S51}$$

$$\begin{aligned}
\omega(5,5) = & - \frac{I_0 L k_g \tilde{\mu} \left(-\frac{zc_0}{z} + 1 \right) \left(-\frac{K_{I^*} axz\gamma_m}{(K_{I^*} + I^*)^2} + \frac{axz\gamma_m}{K_{I^*} + I^*} \right) (-\beta z - q_g + 1)}{\left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)} \\
& - \frac{I_0 L k_g \tilde{\mu} \left(-\frac{zc_0}{z} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} - I^* \right) \left(-\frac{K_{I^*} axz\gamma_m}{(K_{I^*} + I^*)^2} + \frac{axz\gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)} \\
& + \frac{I_0 k_g \tilde{\mu} \left(-\frac{zc_0}{z} + 1 \right) (-\beta z - q_g + 1) \left(\frac{2I_0^2 L k_g^2 \left(-\frac{K_{I^*} axz\gamma_m}{(K_{I^*} + I^*)^2} + \frac{axz\gamma_m}{K_{I^*} + I^*} \right)}{K_{il} \left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^3} + \frac{I_0 L k_g \left(-\frac{K_{I^*} axz\gamma_m}{(K_{I^*} + I^*)^2} + \frac{axz\gamma_m}{K_{I^*} + I^*} \right)}{\left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} - \frac{K_{sI^*}}{K_{I^*} z \gamma_m} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*}^2 z \gamma_m} \right)}{\left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} axz\gamma_m}{K_{I^*} + I^*} + bx + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + I^*)}{K_{I^*} z \gamma_m} \right)^2}
\end{aligned} \tag{S52}$$

$$\begin{aligned}
\omega(1,6) = & - \frac{I_0^3 k_g^3 \mu x_1 \left(-\frac{zc_0}{x_3} + 1 \right)}{K_{il}^2 \left(L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S53}$$

$$\omega(2,6) = 0 \tag{S54}$$

$$\begin{aligned}
\omega(3,6) = & - \frac{I_0^3 k_g^3 \mu (-zc_0 + x_3)}{K_{il}^2 \left(L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S55}$$

$$\begin{aligned}
\omega(4,6) = & - \frac{I_0^3 k_g^3 \mu \left(-\frac{zc_0}{x_3} + 1 \right) \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g} - x_4 \right)}{K_{il}^2 \left(L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} ax_1 x_3 \gamma_m}{K_{I^*} + x_4} + bx_1 + c \right) + k_g} + \frac{K_{sI^*} (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)^2}
\end{aligned} \tag{S56}$$

$$\omega(5,6) = \frac{I_0^3 k_g^3 \mu \left(-\frac{z c_0}{x_3} + 1 \right) (-b_t x_3 - x_5 + 1)}{K_{il}^2 \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^3 \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sI}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)^2} \quad (S57)$$

$$\omega(1,7) = 0 \quad (S58)$$

$$\omega(2,7) = -\frac{x_1 x_2 \left(1 - \frac{x_3}{Ql} \right)}{K_y + x_2} \quad (S59)$$

$$\omega(3,7) = \frac{x_2 \left(1 - \frac{x_3}{Ql} \right)}{K_y + x_2} \quad (S60)$$

$$\omega(4,7) = 0 \quad (S61)$$

$$\omega(5,7) = -\frac{\alpha x_2 \left(1 - \frac{x_3}{Ql} \right)}{K_y + x_2} \quad (S62)$$

$$\omega(1,8) = 0 \quad (S63)$$

$$\omega(2,8) = \frac{\rho_m x_1 x_2 \left(1 - \frac{x_3}{Ql} \right)}{(K_y + x_2)^2} \quad (S64)$$

$$\omega(3,8) = -\frac{\rho_m x_2 \left(1 - \frac{x_3}{Ql} \right)}{(K_y + x_2)^2} \quad (S65)$$

$$\omega(4,8) = 0 \quad (S66)$$

$$\omega(5,8) = \frac{\alpha \rho_m x_2 \left(1 - \frac{x_3}{Ql} \right)}{(K_y + x_2)^2} \quad (S67)$$

$$\omega(1,9) = 0 \quad (S68)$$

$$\omega(2,9) = -\frac{\rho_m x_1 x_2 x_3}{Ql^2 (K_y + x_2)} \quad (S69)$$

$$\omega(3,9) = \frac{\rho_m x_2 x_3}{Ql^2 (K_y + x_2)} \quad (S70)$$

$$\omega(4,9) = 0 \quad (S71)$$

$$\omega(5,9) = -\frac{\alpha \rho_m x_2 x_3}{Ql^2 (K_y + x_2)} \quad (S72)$$

$$\omega(1,10) = -\frac{I_0 k_g \mu x_1}{x_3 \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right) \left(\frac{I_0 k_g}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sI}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S73)$$

$$\omega(2, 10) = 0 \quad (S74)$$

$$\omega(3, 10) = \frac{I_0 k_g \mu}{\left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S75)$$

$$\omega(4, 10) = - \frac{I_0 k_g \mu \left(\frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} - x_4 \right)}{x_3 \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S76)$$

$$\omega(5, 10) = - \frac{I_0 k_g \mu (-b_t x_3 - x_5 + 1)}{x_3 \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S77)$$

$$\omega(1, 11) = 0 \quad (S78)$$

$$\omega(2, 11) = 0 \quad (S79)$$

$$\omega(3, 11) = 0 \quad (S80)$$

$$\omega(4, 11) = 0 \quad (S81)$$

$$\omega(5, 11) = - \frac{\rho_m x_2 \left(1 - \frac{x_3}{Q_l} \right)}{K_y + x_2} \quad (S82)$$

$$\omega(1, 12) = 0 \quad (S83)$$

$$\omega(2, 12) = 0 \quad (S84)$$

$$\omega(3, 12) = 0 \quad (S85)$$

$$\omega(4, 12) = 0 \quad (S86)$$

$$\omega(5, 12) = - \frac{I_0 k_g \mu x_3 \left(-\frac{z c_0}{x_3} + 1 \right)}{\left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right) \left(\frac{I_0^2 k_g^2}{K_{il} \left(L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g \right)^2} + \frac{I_0 k_g}{L \left(\frac{K_{I^*} a x_1 x_3 \gamma_m}{K_{I^*} + x_4} + b x_1 + c \right) + k_g} + \frac{K_{sl}^* (K_{I^*} + x_4)}{K_{I^*} x_3 \gamma_m} \right)} \quad (S87)$$

