## Supplementary Materials

### S1.    *Absorption model*

Figure S1 is a mutiple-tube mass balance model during the absorption of $CO_2$. Assume S is the cross section area of the tube, $\Delta z$ is the differential length, the u is the linear velocity of gas in the tube, the $-r_A$ is the reaction rate, the d is the diameter of the tube, and $C_A$ is the concentration of $CO_2$. According to mass balance at steady-state condition, the balance for multiple tube is showed as follow:
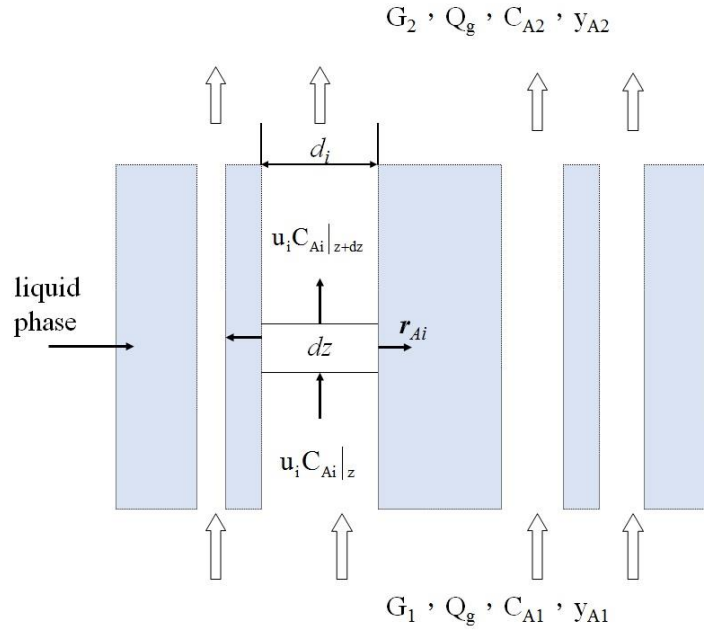


Figure S1. A mutiple-tube mass balance model

$$\sum_{i=1}^{N} S_i u_i (C_{Ai}|_z - C_{Ai}|_{z+\Delta z}) - \sum_{i=1}^{N} (-r_{Ai})(\pi d_i)\Delta z = 0 \qquad (s1)$$

Eq. (s1) is divided by $\Delta z$ and take a limit $\Delta z \rightarrow 0$, the equation becomes:

$$-\frac{\sum d(u_i S_i C_{Ai})}{dz} - \sum_{i=1}^{N} (-r_{Ai})(\pi d_i) = 0 \qquad (s2)$$

Because $G_{Ai}=u_i S_i C_{Ai}$, Eq. (s2) can be rewritten as follow:

$$-\frac{dG_A}{dz} - \sum_{i=1}^{N} (-r_{Ai})(\pi d_i) = 0 \qquad (s3)$$

where $G_A = \sum_{i=1}^{N} G_{Ai}$. Eq.(s3) multiplies by dz and takes an integration:

$$\int_{1}^{2} dG_A + \int_{1}^{2} \sum_{i=1}^{N} (-r_{Ai}) dA_i = 0 \qquad (s4)$$

or

$$G_{A2} - G_{A1} = \int_1^2 \sum_{i=1}^N (-r_{Ai}) dA_i \qquad (s5)$$

The average absorption rate can be defined as follow:

$$-\bar{r}_A = \frac{\sum \int_1^2 (-r_{Ai}) dA_i}{\sum \int_1^2 dA_i} = \frac{\sum \int_1^2 (-r_{Ai}) dA_i}{A} \qquad (s6)$$

Therefore, Eq.(s6) becomes:

$$G_{A2} - G_{A1} = \int_1^2 \sum_{i=1}^N (-r_{Ai}) dA_i = (-\bar{r}_A) A \qquad (s7)$$

If Eq. (s7) is the first stage in the column, all the other stage could be showed below:

First stage $\qquad G_{A2} - G_{A1} = (-\bar{r}_{A1}) A_1 \qquad (s8)$

Second stage $\qquad G_{A3} - G_{A2} = (-\bar{r}_{A2}) A_2 \qquad (s9)$

Third stage $\qquad G_{A4} - G_{A3} = (-\bar{r}_{A3}) A_3 \qquad (s10)$

.

.

.

Mth stage $\qquad G_{AM+1} - G_{AM} = (-\bar{r}_{AM}) A_M \qquad (s11)$

Take summation for all stages, the result is:

$$G_{AM+1} - G_{A1} = \sum_{j=1}^M (-\bar{r}_{Aj}) A_j = (-R_A) V_L \qquad (s12)$$

where $-\bar{r}_{Aj}$ means average rate for jth stage and $-R_A$ is:

$$-R_A = \frac{\sum_{j=1}^M (-\bar{r}_{Aj}) A_j}{V_L} \qquad (s13)$$

where $V_L$ is the liquid volume in the column; therefore, Eq.(s12) becomes:

$$-R_A = \frac{G_{AM+1} - G_{A1}}{V_L} \qquad (s14)$$

In Fig. 3, $G_{AM+1}$ and $G_{A1}$ are represented molar flow rate for inlet and outlet, respectively. In here, assume only two components, $CO_2(A)+N_2(B)$, in which B is the tie element. Due to this, the molar flow rate of B can be estimated below:

$$G_{BM+1} = G_{AM+1} \left[ \frac{1 - y_{AM+1}}{y_{AM+1}} \right]$$

(s15)

Therefore, the outlet concentration of A becomes:

$$G_{A1} = G_{BM+1} \left[ \frac{y_{A1}}{1 - y_{A1}} \right] = G_{AM+1} \left[ \frac{1 - y_{AM+1}}{y_{AM+1}} \right] \left[ \frac{y_{A1}}{1 - y_{A1}} \right]$$

(s16)

Substitute Eq.(s16) into Eq.(s14), the result is:

$$-R_A = \frac{G_{AM+1}}{V_L} \left( 1 - \left[ \frac{1 - y_{AM+1}}{y_{AM+1}} \right] \left[ \frac{y_{A1}}{1 - y_{A1}} \right] \right)$$

(s17)

This means that absorption rate can be calculated when the inlet and outlet concentrations of component A are available. On the other hand, overall mass-transfer coefficient can be derived by using mass balance and two-film model as shown in Fig. 3(b). Using differential mass balance in Eq.(s11), the mass balance at jth stage becomes:

$$Q_g dC_{Aj} = dG_{Aj} = (-\bar{r}_{Aj})dA_j = (-R_{Aj})dV_{Lj} = (K_G a)_j (C_{Aj} - H_j C_{LA})dV_{Lj}$$

(s18)

where $C_A$ is the concentration of $CO_2$ gas in the gas phase, $C_{LA}$ is the concentration of $CO_2$ gas in the liquid phase, and $H$ is the Henry's constant. In general, $C_A$ is much higher than $C_{LA}$ ($C_A \gg C_{LA}$). Therefore, Eq.(s18) becomes:

$$Q_g dC_{Aj} = (K_G a)_j C_{Aj} dV_{Lj}$$

(s19)

or

$$Q_g \frac{dC_{Aj}}{C_{Aj}} = (K_G a)_j dV_{Lj}$$

(s20)

Make a summation for Eq.(s20), it becomes:

$$Q_g \sum_{j=1}^{M} \frac{dC_{Aj}}{C_{Aj}} = \sum_{j=1}^{M} (K_G a)_j dV_{Lj}$$

(s21)

Take an integration in Eq.(s20) stage by stage,

No.1 $\qquad Q_g \int_1^2 \dfrac{dC_A}{C_A} = \int_1^2 (K_G a)_1 \, dV_{L1}$

No.2 $\qquad Q_g \int_2^3 \dfrac{dC_A}{C_A} = \int_2^3 (K_G a)_2 \, dV_{L2}$

No.3 $\qquad Q_g \int_3^4 \dfrac{dC_{A3}}{C_{A3}} = \int_3^4 (K_G a)_3 \, dV_{L3}$

.

.

.

No.M $\qquad Q_g \int_{M-1}^M \dfrac{dC_A}{C_A} = \int_{M-1}^M (K_G a)_M \, dV_{LM}$

Addition of each stage, the result becomes:

$$Q_g \int_1^M \dfrac{dC_A}{C_A} = K_G a V_L \tag{s22}$$

where

$$K_G a = \dfrac{\int_1^2 (K_G a)_1 \, dV_{L1} + \int_2^3 (K_G a)_2 \, dV_{L2} + \ldots + \int_{M-1}^M (K_G a)_M \, dV_{LM}}{V_L} \tag{s23}$$

Make an integration for Eq.(s22), it becomes:

$$K_G a = \dfrac{Q_g}{V_L} \ln \dfrac{C_{AM}}{C_{A1}} \tag{s24}$$

Eq.(s24) can be used to evaluate the overall mass-transfer coefficient when $Q_g$, $V_L$, $C_{AM}$ and $C_{A1}$ are available.

Table S1. $CO_2$ loading data before and after regenerations.

| 1-s | $\alpha$ Before | T (℃) | $\alpha$ After |
|-----|-----------------|-------|----------------|
| $E_F$ | 0.8157 | 110 | 0.7523 |
| | | 120 | 0.7813 |
| | | 130 | 0.7932 |
| $R_A$ | 07050 | 110 | 0.6628 |
| | | 120 | 0.6724 |
| | | 130 | 0.6774 |
| $K_Ga$ | 0.6852 | 110 | 0.6423 |
| | | 120 | 0.6583 |
| | | 130 | 0.6656 |
| $\phi$ | 0.7381 | 110 | 0.6816 |
| | | 120 | 0.6956 |
| | | 130 | 0.7013 |
| **2-s** | **$\alpha$ Before** | **T (℃)** | **$\alpha$ After** |
| $E_F$ | 0.7941 | 110 | 0.7477 |
| | | 120 | 0.7537 |
| | | 130 | 0.7616 |
| $R_A$ | 0.6614 | 110 | 0.6309 |
| | | 120 | 0.6384 |
| | | 130 | 0.6416 |
| $K_Ga$ | 0.6667 | 110 | 0.6357 |
| | | 120 | 0.6385 |
| | | 130 | 0. 6432 |
| $\phi$ | 0.5595 | 110 | 0.4992 |
| | | 120 | 0.5002 |
| | | 130 | 0.5387 |

| 3-s | $\alpha$ Before | T (℃) | $\alpha$ After |
|---|---|---|---|
| $E_F$ | 0.7932 | 110 | 0.7374 |
| | | 120 | 0.7586 |
| | | 130 | 0.7639 |
| $R_A$ | 0.6183 | 110 | 0.5842 |
| | | 120 | 0.5895 |
| | | 130 | 0.5947 |
| $K_Ga$ | 0.6331 | 110 | 0.5795 |
| | | 120 | 0.6027 |
| | | 130 | 0.6103 |
| $\phi$ | 0.4667 | 110 | 0.4271 |
| | | 120 | 0.4325 |
| | | 130 | 0.4381 |

Table S2. Heat of regeneration data obtained according to $E_F$ optimum condition.

| $E_F$ | T (°C) | $q_{sen}$ (GJ/ton) | $q_{sol}$ (GJ/ton) | $q_{ads}$ (GJ/ton) | q (GJ/ton) |
|---|---|---|---|---|---|
| 1-s | 110 | 0.56 (0.148) | 2.00 (0.529) | 1.22 (0.323) | 3.78 |
| | 120 | 1.03 (0.155) | 4.41 (0.662) | 1.22 (0.183) | 6.66 |
| | 130 | 1.57 (0.177) | 6.10 (0.686) | 1.22 (0.137) | 8.89 |
| 2-s | 110 | 0.69 (0.193) | 1.66 (0.465) | 1.22 (0.342) | 3.57 |
| | 120 | 0.79 (0.200) | 3.32 (0.495) | 1.22 (0.305) | 5.33 |
| | 130 | 0.99 (0.154) | 4.22 (0.656) | 1.22 (0.190) | 6.43 |
| 3-s | 110 | 0.57 (0.147) | 2.02 (0.532) | 1.22 (0.321) | 3.80 |
| | 120 | 0.92 (0.167) | 3.33 (0.610) | 1.22 (0.223) | 5.46 |
| | 130 | 1.08 (0.181) | 3.66 (0.614) | 1.22 (0.205) | 5.96 |

Table S3. Heat of regeneration data obtained according to $R_A$ optimum condition.

| $R_A$ | T (℃) | $q_{sen}$ (GJ/ton) | $q_{sol}$ (GJ/ton) | $q_{ads}$ (GJ/ton) | q (GJ/ton) |
|---|---|---|---|---|---|
| 1-s | 110 | 0.72 (0.134) | 2.92 (0.544) | 1.73 (0.322) | 5.37 |
| | 120 | 0.93 (0.135) | 4.23 (0.614) | 1.73 (0.251) | 6.89 |
| | 130 | 1.10 (0.140) | 4.98 (0.638) | 1.73 (0.222) | 7.81 |
| 2-s | 110 | 0.89 (0.159) | 2.99 (0.533) | 1.73 (0.308) | 5.61 |
| | 120 | 1.18 (0.190) | 3.29 (0.531) | 1.73 (0.279) | 6.20 |
| | 130 | 1.37 (0.199) | 3.81 (0.551) | 1.73 (0.250) | 6.91 |
| 3-s | 110 | 0.87 (0.163) | 2.78 (0.516) | 1.73 (0.321) | 5.39 |
| | 120 | 1.03 (0.182) | 2.90 (0.512) | 1.73 (0.306) | 5.66 |
| | 130 | 1.26 (0.181) | 3.97 (0.570) | 1.73 (0.249) | 6.96 |

Table S4. Heat of regeneration data obtained according to $K_Ga$ optimum condition.

| $K_Ga$ | T (℃) | $q_{sen}$ (GJ/ton) | $q_{sol}$ (GJ/ton) | $q_{ads}$ (GJ/ton) | q (GJ/ton) |
|---|---|---|---|---|---|
| 1-s | 110 | 0.56 (0.101) | 3.24 (0.586) | 1.73 (0.313) | 5.53 |
| | 120 | 0.87 (0.148) | 3.32 (0.560) | 1.73 (0.292) | 5.93 |
| | 130 | 1.20 (0.150) | 5.06 (0.633) | 1.73 (0.217) | 7.99 |
| 2-s | 110 | 0.60 (0.131) | 2.31 (0.497) | 1.73 (0.372) | 4.65 |
| | 120 | 0.67 (0.127) | 2.85 (0.543) | 1.73 (0.330) | 5.25 |
| | 130 | 0.80 (0.134) | 3.45 (0.577) | 1.73 (0.289) | 5.98 |
| 3-s | 110 | 0.32 (0.090) | 1.53 (0.427) | 1.73 (0.483) | 3.58 |
| | 120 | 0.56 (0.110) | 2.76 (0.547) | 1.73 (0.343) | 5.05 |
| | 130 | 0.75 (0.119) | 3.82 (0.606) | 1.73 (0.275) | 6.30 |

Table S5. Heat of regeneration data obtained according to $\phi$ optimum condition.

| $\phi$ | T (℃) | $q_{sen}$ (GJ/ton) | $q_{sol}$ (GJ/ton) | $q_{ads}$ (GJ/ton) | q (GJ/ton) |
|---|---|---|---|---|---|
| 1-s | 110 | 0.69 (0.130) | 3.24 (0.620) | 1.31 (0.250) | 5.23 |
| | 120 | 0.92 (0.151) | 3.88 (0.635) | 1.31 (0.214) | 6.11 |
| | 130 | 1.06 (0.125) | 6.13 (0.721) | 1.31 (0.154) | 8.50 |
| 2-s | 110 | 0.65 (0.135) | 2.86 (0.593) | 1.31 (0.272) | 4.82 |
| | 120 | 0.67 (0.121) | 3.48 (0.639) | 1.31 (0.240) | 5.45 |
| | 130 | 1.89 (0.212) | 5.73 (0.642) | 1.31 (0.219) | 8.93 |
| 3-s | 110 | 0.99 (0.201) | 2.61 (0.532) | 1.31 (0.267) | 4.91 |
| | 120 | 1.16 (0.190) | 3.60 (0.594) | 1.31 (0.216) | 6.06 |
| | 130 | 1.38 (0.197) | 4.32 (0.616) | 1.31 (0.187) | 7.01 |