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*Article*

# Mathematical Model of the Electronic Cam in Terms of Application in a Dosing Machine

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**Abstract:** The article analyses the use of servomotors in the control systems of industrial equipment, focusing on the alternative offered by position and speed synchronization in relation to classical mechanical mechanisms. A complete methodology is presented to determine the dynamic parameters of the adopted kinematic system using electronic motion profiles. The results obtained constitute a mathematical model of the execution chain and an analysis of the basic quantities for linear motion, supported by actual measurements of the drive parameters. The merit of the article is to show that the servomotors can significantly simplify the design of the device, make it more flexible in adaptation to different assortments, and allow integration with systems predicting the technical condition of the device. The analysis of the results revealed significant differences in the constant rotational speed of the servomotor, which do not align with previous findings. The results suggest that changing the angular working range of the assembly to the range (205°;270°) could significantly affect the generated linear acceleration, reducing the risk of stalling. The calculations and graphs conducted allowed for the accurate representation of the actual mechanical system, considering its dynamic characteristics. The key conclusion is that precise mathematical modelling is essential to ensure the stability and durability of engineering components.

**Keywords:** synchronization; real-time control system; electronic motion profile; electronic cam; cam coupling

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## 1. Introduction

The economic starting point of any production device is its performance and the quality of the final product. In order to meet the wide range of requirements of the current market, newly designed devices must meet stringent requirements for production stability, consumption of process media, environmental safety and operation, minimizing the ecological footprint [1–3].

An increasingly common criterion in the design of industrial machines is their versatility and simplification of design. This is to ensure quick adaptation of the device to the production of various formats of the output product and to simplify the process of maintaining the proper technical condition of the device. As a result, operating costs are significantly reduced, while reliability is increased [4,5]. Newly designed industrial equipment is very often characterized by proven design solutions, supported by the experience of construction engineers and new technologies offered by the dynamically developing drive market [2,6]. The use of servomotors avoids the kinematic and dynamic nonlinearities of conventional mechanisms. On this basis, it can be assumed that the process of selecting components can be much simpler [2].

Because of the above criteria, designers are increasingly reaching for servomotors, appreciating their high potential. Servomotors are used to synchronize the position or speed of the slave axes with the real or virtual master axes, control the torque, and implement the point-to-point (p-to-p) trajectory [7,8]. They enable the generation of dynamic rotary and reciprocating movements of components with a significant mass and moment of inertia, while maintaining a high precision in position control [1,9,10].

The use of servomotors significantly simplifies the construction of the device. The kinematic chain is limited to the servomotor, gearbox, and executive apparatus, i.e., the pull-lever system or the cam mechanism [11]. This eliminates the need for complex torque transmission, bearing, and lubrication systems. Simplifying the design by eliminating unnecessary mechanisms allows to reduce the risk of failure and maintenance costs, including the costs of purchasing parts, their storage, and providing qualified personnel for service and repair activities. Compared to mechanical mechanisms, the servomotor control method, including the electronic cam, provides unparalleled flexibility and the ability to compensate for errors in the part manufacturing process [1,5,9,11].

In recent years, an innovative approach to the design of industrial processes, including processing equipment, has become noticeable. In the first place, an abstract approach to the activity has been suggested, focusing on the main elements and the effects achieved. To implement multiple layouts, you need to create interactions between them, with respect to the parent system [6]. Basing the design of devices on servomotors perfectly reflects the presented idea of creating a device model. The presented methodology in obtaining a mathematical model gives a general view of the activity performed. And the synchronization property satisfies the reference criterion of the parent system [6,7].

The use of the position synchronization functionality ensures constant positioning of the equipment, which is especially important in precise processing processes [3,10,12]. These processes often apply to objects where the geometry is irregular and the surface is easily deformable [1,9]. The dynamic capabilities offered by servomotors have made them an unrivalled solution in relation to classic actuators, i.e., pneumatic and hydraulic actuators [4,6,12].

High dynamic properties, precise position control, and the ability to design motion trajectories are widely used in the construction of processing machines, i.e., pouring and capping. An interesting example is the control of liquid sloshing by optimizing motion profiles, as a function of speed efficiency of the processing device [13]. Sloshing is a vibration or swaying that is most often in the form of asymmetric sine waves, which differ in frequency and modulus. The phenomenon of sloshing, classified as interference, is often amplified by the resonant vibrations of the device. These disturbances are compensated for by generating torque or changing velocity to suppress the defined disturbance. The whole process is carried out by the mathematical modelling of the disturbance, which requires a fast computational process to suppress the resulting disturbance in time [14].

In the era of Industry 4.0, a particularly important direction of development in the construction of machines based on servomotors is the monitoring of the condition of executive mechanisms. This research has focused on predicting the end of the life of parts, diagnostics, and determining the source of faults. Detailed diagnostics of the technical condition of the executive units has become possible thanks to advanced microprocessor systems that are an integral part of servo amplifiers–motion controllers. Precise measurements of speed, torque, etc., allow for quick reference to the adopted mathematical models of newly designed devices and taking the appropriate remedial steps [5,6].

The Motion Curve (Function) Editor allows tabular and graphical configurations of the start and end points of the disintervals into which the full course is divided. This makes it possible to precisely program the trajectory of motion and its derivatives, such as speed, acceleration, and spurt, which allows for the optimization of the production process. The graphical representation of the position and its subsequent derivatives after time are calculated for the defined constant rotational speed or linear master axis. The separated intervals are described by a function of the form [7,8,15]:

- linear interpolation;
- interpolation with third-degree spline functions;
- interpolation with Bezier curves;

- interpolation of a 5th degree polynomial;
- interpolation of a 7th degree polynomial;
- interpolation of a quadratic function.

The number of available interpolations may vary depending on the hardware platform used to implement the electronic motion profile. On the basis of the motion function, editors calculate successive derivatives of the function, which describe quantities, such as the following:

- velocity;
- acceleration;
- spurt.

Motion curve editors dedicated to the hardware platform do not allow you to define the kinematic layout of the executive apparatus. A tabular or graphical representation of the electronic motion profile is limited to the defined gear ratio and the mechanical characteristics of the motor [4,15].

Hardware platforms providing drive solutions enable integration with most industrial computers or PLCs. Interface solutions and dedicated visualization applications, e.g., in HMI, enable editing motion profiles directly from the workstation, bypassing the connection through a dedicated hardware platform IDE. Thanks to such functionalities, qualified construction staff can easily adjust the operating parameters of processing equipment to constantly changing production requirements, thus optimizing the manufacturing process in terms of efficiency and precision [7,8,11].

By simplifying the editing of motion profiles and being able to transfer it partially to a completely different hardware platform, it is possible to edit recipes, which allows for immediate and simple preparation of the machine for the production of a completely different end product. Recipe change can include all parameters directly related to the movement profile, e.g., the position of the master axis, the position of the slave axis, the type of characteristic describing the discontinuity interval, the speed of crossing the discontinuity interval boundaries, and other critical parameters that are required to start the production of a new product. This enables a quick response to dynamically changing market demand, reducing downtime, including changeover time [7,11].

The combination of these advanced solutions allows for intuitive management and real-time monitoring of production, which translates into better quality control and increased production flexibility. The result is increased automation and optimization, which has become the domain of modern production centers, constantly striving to maximize their position on the market [1,9,10,12].

Behind the whole range of advantages offered by servomotors and their implementation in industrial equipment, one should also be aware of their disadvantages and limitations. Servomotors are characterized by higher costs compared to other, classic drive solutions. The total cost of the drive unit is not only influenced by the cost of the motor, but also by advanced motion controllers, cables that are loaded with the maximum length, and the electrical installation that is designed and compatible with the latest EMC guidelines. These are the elements necessary for their proper functioning. Servomotors operating in harsh conditions, e.g., explosion hazard zones, marine environments, etc., are characterized by increased resistance to operating conditions and thus a much higher price [7].

The installation and configuration of complex servo drive units is more complex and complicated than that of simpler drives, such as inverters or stepper motors. This requires skilled knowledge backed by experience to fine-tune speed and position loop controllers based on position error over time. Achieving a satisfactory effect of the drive unit may lead to long production downtimes [7,8,11].

Behind a number of superlatives resulting from the use of electronic motion profiles, they also bring a number of limitations to the design of the device. The complexity of the software, which often differs significantly between hardware platforms, makes creating, editing, and visualizing complex and requires advanced technical knowledge. Without proper experience and knowledge, there is a significant risk of making mistakes that may result in damage to the device [8,11].

Often, motion profiles created at the beginning of the device's operation need to be optimized. This is a very time-consuming process, during which parameters related to positioning and generated torque are analysed, interpreted, and finally modified [7,8,11]. Such activities require advanced controllers that provide adequate computing power. This process significantly extends the time of implementation of new devices for operation, despite constantly developing methods of self-optimization of systems synchronized by electronic motion profiles [1,6,9].

The subject of this article is to determine the function of the motion of the executive apparatus and quantities, i.e., speed, acceleration, and spurt as a function of the position of the main axis. The seemingly basic parameters of movement allows for the precise dynamic analysis and selection of the right servomotor in terms of speed and generated torque. The results obtained create a precise mathematical model of the designed assembly, which is necessary for the implementation of advanced diagnostic systems and predicting the end of the life of parts.

## 2. Materials and Methods

### 2.1. Description of the Tested System

The pull-and-lever system, driven by a servomotor, was analysed in the context of replacing conventional mechanical systems. The position of the system was calculated on the basis of the electronic motion profile, the boundaries of which are shown in Table 1. The adopted division results from the need to synchronize with the other equipment installed in the production device. The intervals of discontinuities were described by a polynomial of the 5th degree and a linear function. The position synchronization took place in relation to the parent system, which was the virtual axis. The position of the virtual axis was in the range of $(0°; 360°)$, and the maximum rotational speed was 150 rpm.

**Table 1.** Motion profile table.

| No. | ΔX [Deg] | ΔY [Deg] |
|---|---|---|
| 0 | 20 | −5 |
| 1 | 60 | 0 |
| 2 | 120 | −60 |
| 3 | 20 | 5 |
| 4 | 60 | 0 |
| 5 | 10 | 5 |
| 6 | 60 | 50 |
| 7 | 10 | 5 |

Based on the data contained in Table 1, we generated an electronic traffic profile presented in Figure 1. A graphical representation of the item and the next row of derivatives using Cam Builder Software IndraWorks Engineering ML 14V22 P17 can be found in the appendix.

The software IndraWorks Engineering ML 14V22 P17 was from Bosch Rexroth drives, the IndraDrive series. It offers sophisticated technological functions for the comprehensive implementation of drive issues in industrial automation. Users can parameterize and tune drives efficiently and precisely, easily adapting their operation to the dynamically changing requirements of the application being designed.

This software allows you to configure the topology of the drive communication network, including operating and synchronization modes. A clear and intuitive user interface simplifies the process of configuration, monitoring, and diagnostics. Advanced diagnostics allow the real-time monitoring of operating parameters and direct access to a descriptive list of all available parameters, error codes, and warnings. This allows for a quick diagnostic process and equally quick restoration of the full functionality of the application in the event of a failure.

The functionality also includes designing communication with other hardware platforms, i.e., PLCs or industrial computers. Thanks to communication based on most of the available communication protocols, the integration process with existing industrial automation applications is smooth and uncomplicated. This makes the drives of the IndraDrive series easy to integrate into complex industrial systems.

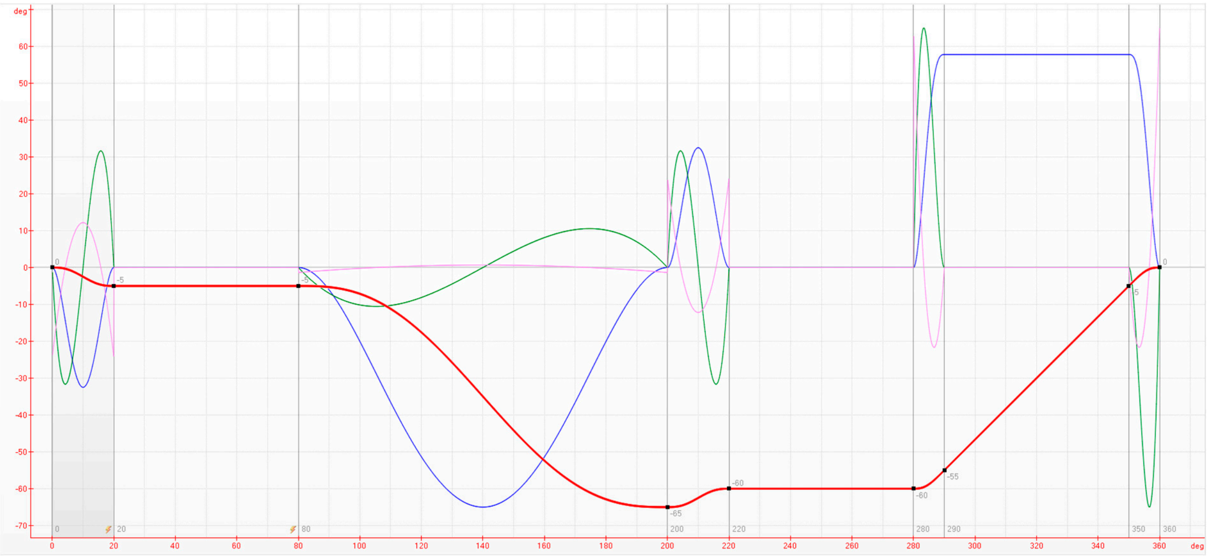

**Figure 1.** Electronic motion profile: red: synchronized axis position [deg], blue: synchronized axis speed [rpm], green: acceleration [rad/s$^2$], pink: synchronized axis spurt [rad/s$^3$].

The main processing material are pasted products, i.e., butter, milk fats with the addition of vegetable oils, margarines, cottage cheese, minced meat, lard, and yeast. The device in question has two working tracks that can carry out the packaging process in the range of 100–300 mL and with a capacity of up to 220 pieces/min [16].

The draw-lever system, analysed in this article and presented in Figure 2, is an integral part of the metering piston assemblies. The unit is made of two identical draw-lever systems, which are mirror-oriented to each other. The unit is driven by two separate servomotors that ensure synchronized movement and precise position control. Each system consists of the following:

- The L1 lever mounted on the gear shaft. The length of the L1 lever is crucial, as it is the distance between the piston rod axis and the axis of rotation of the gear shaft.
- The L2 link is the link between the lever and the piston rod of the metering piston. The role of the link is to transfer the rotary movement of the lever to the reciprocating movement of the piston rod.

The dotted line indicates the maximum angular position of the lever, which translates into a linear stroke. At point B—which is the end of the piston rod—a hygienic piston was installed. Tight seating in the steel cylinder ensured proper tightness of the system in the production cycle and precise dosing of liquids (74.2 mm).

The 822 HS devices allowed product dispensing in the range of 100–300 mL [16]. The changing dose was implemented by a modifiable electronic motion profile, which allowed precise adjustment of the amount of material dispensed in real time. Such flexible adaptation to the changing conditions of the production process would not be possible with the use of mechanical mechanisms. Classic solutions are limited by fixed movement parameters and the lack of possibility of quick adaptation.

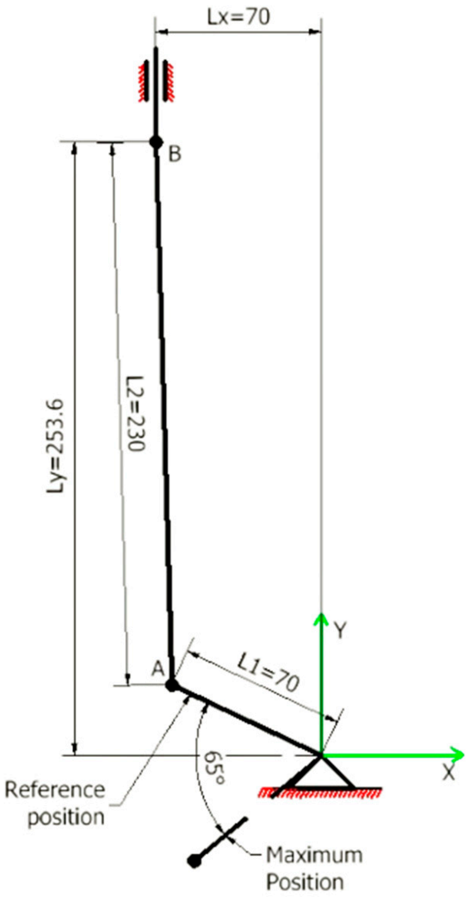

**Figure 2.** Execution chain kinematic system.

Changing the dose is often necessary from cycle to cycle of the device. Changes in product temperature, viscosity, density, and production line speed may result in fluctuations in the weight of the final product. The flexibility provided by the electronic cam allows for immediate and precise adaptation to these constantly changing conditions, which is crucial to maintaining high efficiency and quality of production.

The essence of this article is to determine the function of motion of the executive apparatus and to determine the key quantities, i.e., speed, acceleration, and spurt as a function of the position of the main axis. Seemingly basic motion parameters are necessary for further stages of design work, including the selection of the servomotor and gearbox, strength analysis, and the selection of bearing arrangements.

Based on the obtained mathematical model of the movement, it is possible to optimally select drive solutions, analyse various operating scenarios of the device, and accurately monitor the operating conditions. As shown in the research [1,5,17], comparing direct measurements from drives with the adopted mathematical model sets a completely new direction in the industry regarding the real-time diagnostics of actuator parts. As a result, we may acquire another level of efficiency and reliability with a device adapted to the realities of the modern manufacturing market.

Real-time monitoring of the operating status allows one to accurately track the technical condition of the equipment, identify deviations from accepted standards, and immediately respond to potential downtime resulting from equipment failure. This level of technological advancement not only increases efficiency, but also reduces maintenance and repair costs, increasing production uptime. As a result, companies can maximize their unrivalled position in the manufacturing market.

### Determination of the Motion Function of the Kinematic System

Equation of motion for point B depicted in Figure 2. It takes the following form:

$$\begin{gathered} y_B = [\cos(\alpha_1) * L_1] + [\cos(\mathrm{asin}(\tfrac{L_X - |\sin(\alpha_1) * L_1|}{L_2})) * L_2] - L_Y \\ x_B = 0 \end{gathered} \tag{1}$$

The working movement of the unit was divided into 8 compartments presented in Table 2. The division was due to the cooperation of the team in question with regard to the other equipment included in the device. Modifying the width of the compartment may lead to the desynchronization of the equipment or damage to it ($\Delta x$).

**Table 2.** Table of coefficients of the position function of point B.

| Interval | A | B | C | D | And | F |
|---|---|---|---|---|---|---|
| No. 0 | −0.00009375318850 | 0.000468766060870 | −0.006250222355561 | 0.000000207088533 | −0.000000388970875 | −0.000000048458332 |
| No. 1 | 0 | −5 | 0 | 0 | 0 | 0 |
| No. 2 | −0.000000014468037 | 0.000010127625759 | −0.002662118786617 | 0.324084025380577 | −18.5190869605351 | 397.975169692502 |
| No. 3 | 0.000009375310839 | −0.009844076379006 | 4.13138698156651 | −866.278725275157 | 90753.0097407992 | −3800191.05369414 |
| No. 4 | 0 | −60 | 0 | 0 | 0 | 0 |
| No. 5 | 0.000050000248613 | −0.071667031186851 | 41.0835468530585 | −11774.062460404 | 1686915.79185127 | −96662566.0039355 |
| No. 6 | 0.83336 | −296.67583 | 0 | 0 | 0 | 0 |
| No. 7 | 0.000050002046225 | −0.088336953382979 | 62.4192282221552 | −22050.9062159613 | 3894640.2872562 | −275126219.563166 |

The compartments of the electronic traffic profile are described by the following functions:

- Compartment No. 0: $y_0 = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$
- Compartment No. 1: $y_1 = Ax + B$
- Compartment No. 2: $y_2 = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$
- Compartment No. 3: $y_3 = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$
- Compartment No. 4: $y_4 = Ax + B$
- Compartment No. 5: $y_5 = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$
- Compartment No. 6: $y_6 = Ax + B$
- Compartment No. 7: $y_7 = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$

The values of the function coefficients were determined using the trend line—the functionality of the software Microsoft Excel 365 ver. 2407. The accuracy of coefficients to accurately represent the position curve shown in Figure 1 amounted to a minimum of 10 significant places.

### *2.2. Mathematical Analysis*

The axis position diagram shown in Figure 1 is presented in the following units:

$$\frac{pozycja\ osi\ slave\ [^\circ]}{pozycja\ osi\ master\ [^\circ]} \tag{2}$$

The unit shown in Equation (2) is sufficient only if the position of the actuator in question is presented. In order to continue the calculation, it is necessary to change the unit in which the time occurs $[s]$. This is due to the fact that the basic multiplicities characterising the system, i.e., speed, acceleration, and spurt, are calculated on the basis of the change in the antiderivative function per unit of time.

In the available cam editors, the main axis is assumed to convert units. This assumption also results in the constant time of a single team cycle. This relationship allowed us to determine the positions of the main axis as a function of time. For the calculations, it was assumed that the speed of the main axis was 150 [rpm]. Based on this, the total cycle time was determined as follows:

$$\omega = 2\pi * \frac{n[\text{rpm}]}{60[\text{s}]} = 2\pi * \frac{150}{60} = 15.70796323\left[\frac{\text{rad}}{\text{s}}\right] \tag{3}$$

$$t_C = \frac{2\pi}{\omega} = \frac{2\pi}{15.70796323} = 0.4[\text{s}] \tag{4}$$

In order to determine the next order of the time derivative, it is necessary to specify the time step *dt*. To achieve this, it is necessary to define the number of points into which we could divide the entire time period. When determining this step, we took into account the precision of calculations and the accuracy of our results. Accepting the appropriate number of points is important for the reliability and accuracy of the analysis of the dynamics of time phenomena. The following value was used for the calculation: $2^{10} = 1024$.

$$dt = \frac{t_C}{p-1} = \frac{0.4}{1024-1} = 3.91007 * 10^{-4}[\text{s}] = 0.000391007\ [\text{s}] \tag{5}$$

Using the following relation, the position of the master axis in a unit of time was determined, the graph of which is shown in Figure 3.

$$\alpha = \frac{\omega * t * 360^\circ}{2\pi}\ [^\circ] \tag{6}$$

where

$t$ [s] $\in (0; t_c)$—cycle time incremented by the value of $dt$.

By obtaining the positions of the main axis with high resolution as a function of time, it is possible to determine the *angular position of the synchronized axis*, the graph of which is shown in Figure 3. The calculations were made on the basis of the general form of the function, which was used to describe successive intervals of discontinuities, the table of coefficients presented in Table 2, and the widths of the intervals given in Table 1.

The determination of the angular position of the synchronized axis was calculated on the basis of the following relationships:

Compartment No. 0:

$$y_0 = A_0x^5 + B_0x^4 + C_0x^3 + D_0x^2 + E_0x + F \tag{7}$$

where $x \in (0^\circ; 20^\circ)$.

Compartment No. 1:

$$y_1 = A_1x + B_1 \tag{8}$$

where $x \in (20^\circ; 60^\circ)$.

Compartment No. 2:

$$y_2 = A_2x^5 + B_2x^4 + C_2x^3 + D_2x^2 + E_2x + F_2 \tag{9}$$

where $x \in (80^\circ; 200^\circ)$.

Compartment No. 3:

$$y_3 = A_3x^5 + B_3x^4 + C_3x^3 + D_3x^2 + E_3x + F_3 \tag{10}$$

where $x \in (200^\circ; 220^\circ)$.

Compartment No. 4:

$$y_4 = A_4x + B_4 \tag{11}$$

where $x \in (220^\circ; 280^\circ)$.

Compartment No. 5:

$$y_5 = A_5x^5 + B_5x^4 + C_5x^3 + D_5x^2 + E_5x + F_5 \tag{12}$$

where $x \in (280°; 290°)$.

Compartment No. 6:

$$y_6 = A_6 x + B_6 \tag{13}$$

where $x \in (290°; 350°)$.

Compartment No. 7:

$$y_7 = A_7 x^5 + B_7 x^4 + C_7 x^3 + D_7 x^2 + E_7 x + F_7 \tag{14}$$

where $x \in (350°; 360°)$.

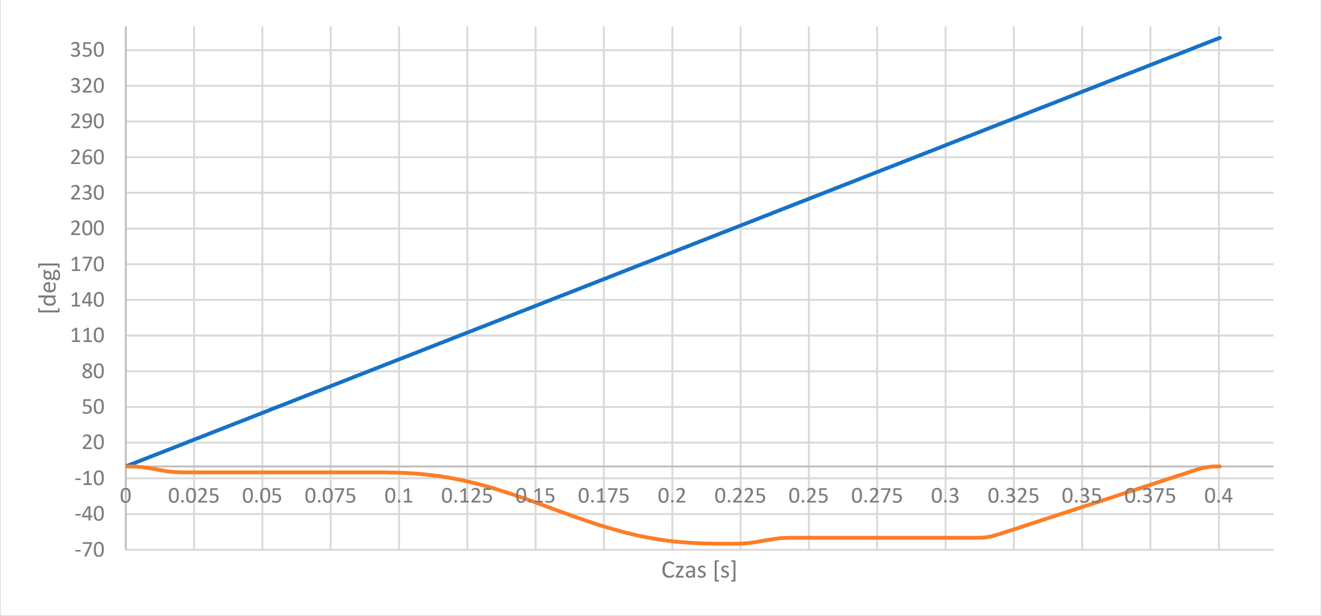

**Figure 3.** Chart of the position of the main and synchronized axis: blue: master axis position [deg], orange: synchronized axis position [deg].

The angular position, the position of the gear shaft, and its subsequent derivatives over time are only approximate conditions that prevail at the end of the execution chain. The inaccuracy resulted from the trigonometric functions describing the position of the final point of the executive system.

In order to describe the system in detail, a determination of the position of the kinematic system in mm is made. This is possible thanks to the determined angular position of the high-resolution synchronised axis substituted into (1). As a result of this arithmetic operation, the position of the synchronised axis at time t is obtained.

In order to determine the parameters, i.e., speed, acceleration, and spurt, the equation should be differentiated (1). Motion functions that describe the position of objects in two- or three-dimensional space are inherently complex, and their differentiation is time-consuming and complicated. To simplify the calculations, the following basic relation describing the antiderivative function was used:

$$f'(x) = \frac{dy}{dx} => f^{'}(x) = \frac{dy}{dt} \tag{15}$$

According to the presented relation, it is possible to determine subsequent derivatives, constituting the basic parameters of linear motion on the basis of successive, instantaneous values after time—the time step determined in Equation (5). Based on the above, the following relationships are recorded:

The equation for the velocity of point B.

$$v_{y_B} = \frac{dy_B}{dt}\left[\frac{\text{mm}}{\text{s}}\right] \tag{16}$$

The acceleration equation of point B.

$$a_{y_B} = \frac{dv_{y_B}}{dt}\left[\frac{\text{mm}}{\text{s}^2}\right] \tag{17}$$

The equation for the breakaway of point B.

$$j_{y_B} = \frac{da_{y_B}}{dt}\left[\frac{\text{mm}}{\text{s}^3}\right] \tag{18}$$

In order to thoroughly analyse and obtain a mathematical model of the system under consideration, a number of calculations were performed, the results of which are presented in the above diagrams. In Figures 4–7, position, speed, acceleration, and spurt as a function of time are presented.

The analysis of the diagrams allowed for a detailed understanding of the dynamic behaviour of the kinematic system under consideration, which ultimately translated into an efficient and precise design process. An important aspect of this analysis is the fact that all units are expressed in millimetres. This is important from the point of view of precise representation of the actual operation of the executive module.

It is worth noting that generic motion curve editors often present data in degrees, which can lead to inaccuracies and inaccuracies in the interpretation of results. In our case, using millimetres as the unit of measurement provides greater accuracy and a better representation of the real operating conditions of the system.

The above diagram is representative; the values of individual derivatives were scaled in order to reflect the phenomena occurring in the considered complex. Detailed values of the parameters, i.e., position, speed, acceleration, and spurt, are presented in the following graphs.

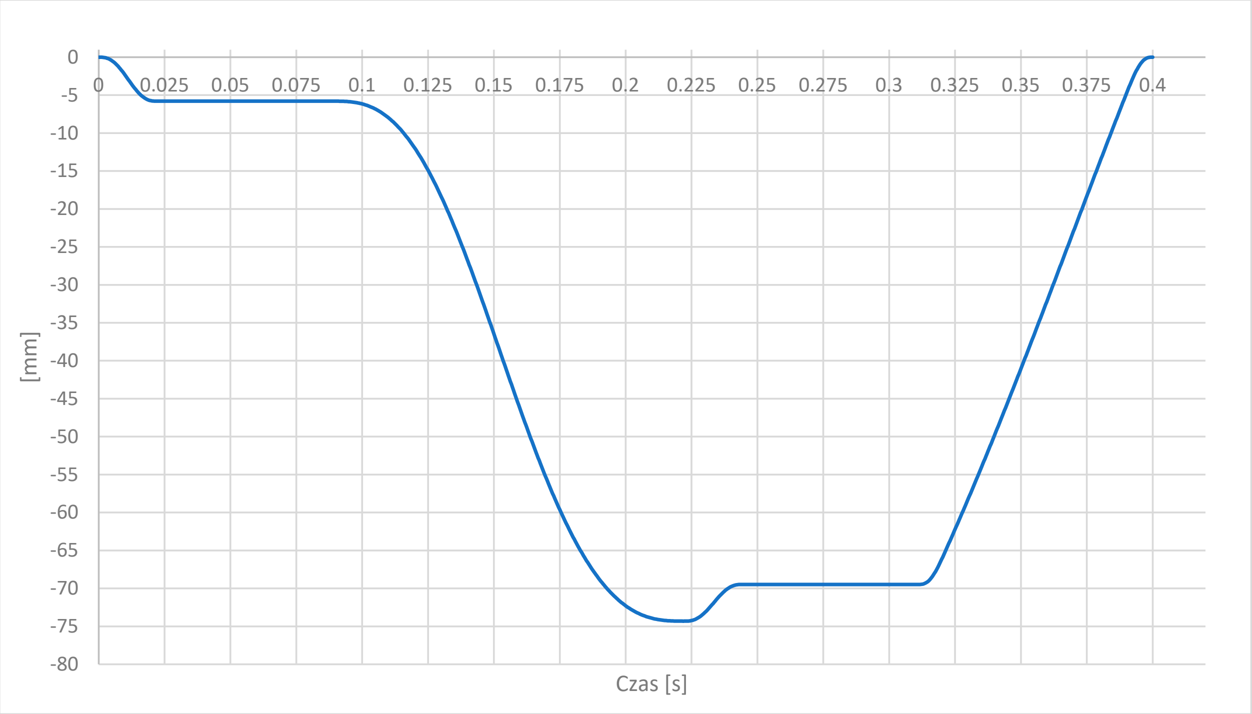

**Figure 4.** Position chart of point B.

In Figure 8, the complete kinematics of the considered executive system is presented. The purpose of this diagram is to demonstrate similarity to the diagram obtained in a dedicated electronic motion profile editor, designed for the drive Bosch Rexroth. Compare the charts in Figures 1 and 8 and it allows one to identify and better understand the differences between the calculations performed without taking into account the kinematic system and calculations with it included.

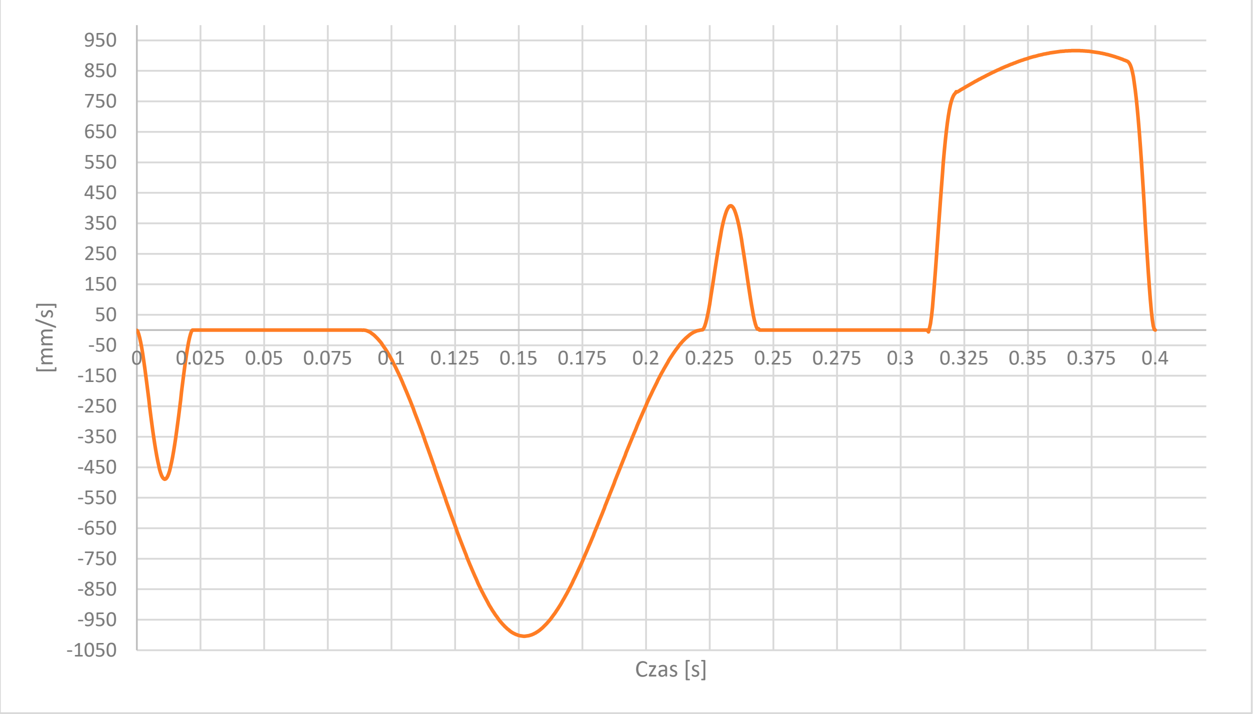

**Figure 5.** Velocity chart of point B.

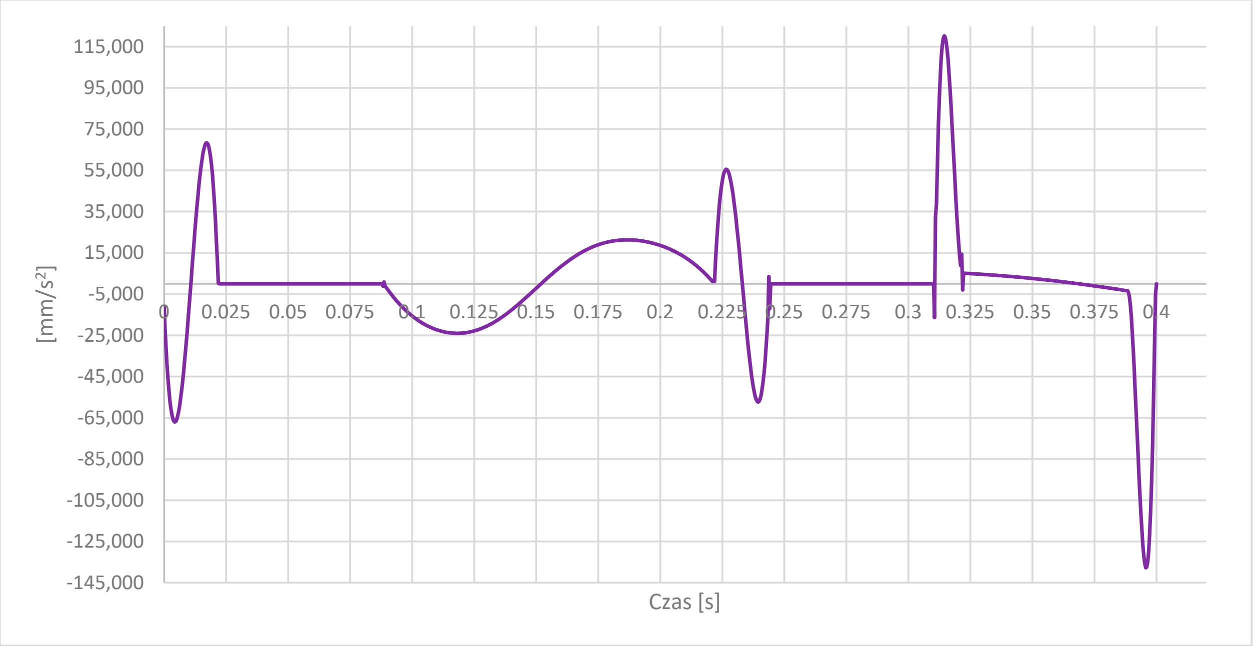

**Figure 6.** Acceleration chart of point B.

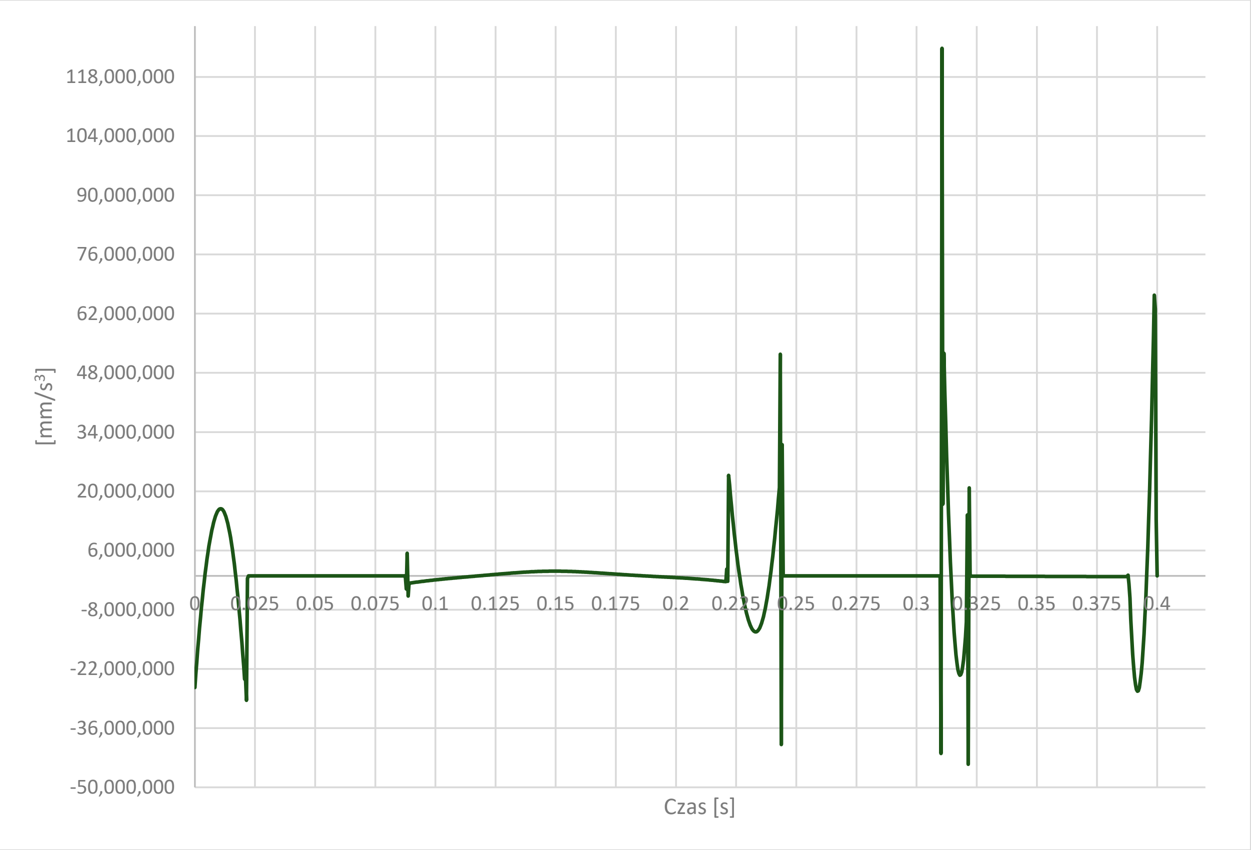

**Figure 7.** Jerk chart of point B.

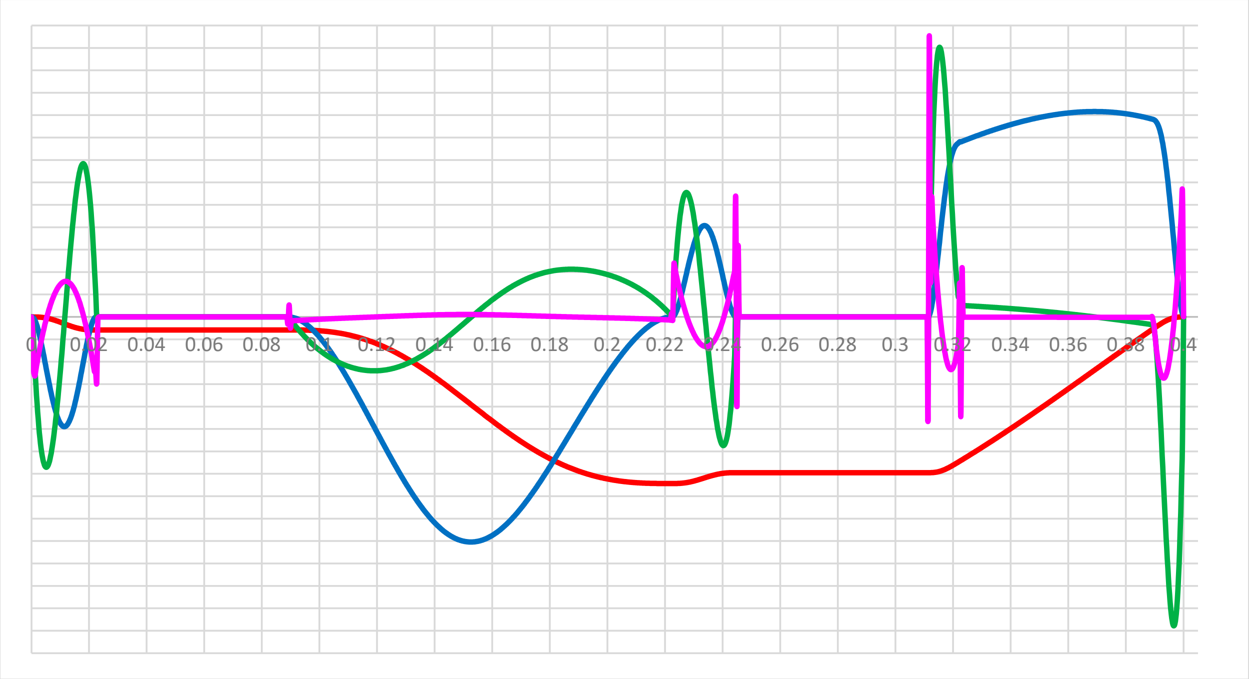

**Figure 8.** Actuator chart: red: synchronized axis position [mm], blue: synchronized axis speed [mm/s], green: acceleration [mm/s$^2$], pink: synchronized axis spurt [mm/s$^3$].

## 3. Results and Discussion

On the basis of the calculations performed, a mathematical model of the actuator system was obtained. The obtained number of results were an introduction to further design activities, i.e., the selection of construction materials, strength analysis, consideration of frictional wear, static, and dynamic load capacity of bearing arrangements, rod heads, the selection of drive and gears, etc. The obtained results were compared with the measurements of the drive parameters of the kinematic system in question.

The graphs show all units in millimetres (mm), which allows for a precise representation of the actual actuator. The adopted unit allowed for greater accuracy and better visualization of the system's operation, in contrast to general editors of electronic traffic profiles, which represent data in degrees, which may result in inaccuracies and inaccuracies in the interpretation of received messages. The calculations were limited to a single cycle of the assembly. This increased transparency and simplified the interpretation of the data and the identification of the key steps in the dispensing process.

By analysing the trajectory of motion depicted in Figures 4 and 9 (purple line), the most important and at the same time the most strenuous stage of movement is the dosing of the product accumulated in the space of the dosing cylinder (Figure 10). A product with a density of 900 $\frac{\text{kg}}{\text{m}^3}$, and a high kinematic viscosity offers a high resistance at the initial stage of dosing. This resistance is assumed to be related to the intermolecular cohesive forces of the product. The greater the bond strength, the greater the resistance, as evidenced by the rapid and momentary jump in torque generated by the servomotor.

It should be noted that the spurt diagram presented in Figure 7 reflects the shape of the measured torque generated by the drive shown in Figure 9. In the context of the spurt characteristics generated in the electronic motion profile editor, shown in Figure 1, it is clear that it did not reflect the operating conditions of the system at all. Such a discrepancy can lead to incorrect design assumptions and serious design errors.

The presented analyses show that motion profiles generated in the dedicated software can ignore important dynamic factors of the real system. In particular, the imprecise reproduction of the spurt may result in the improper selection of actuator components, which in turn affects the stability and reliability of the entire device.

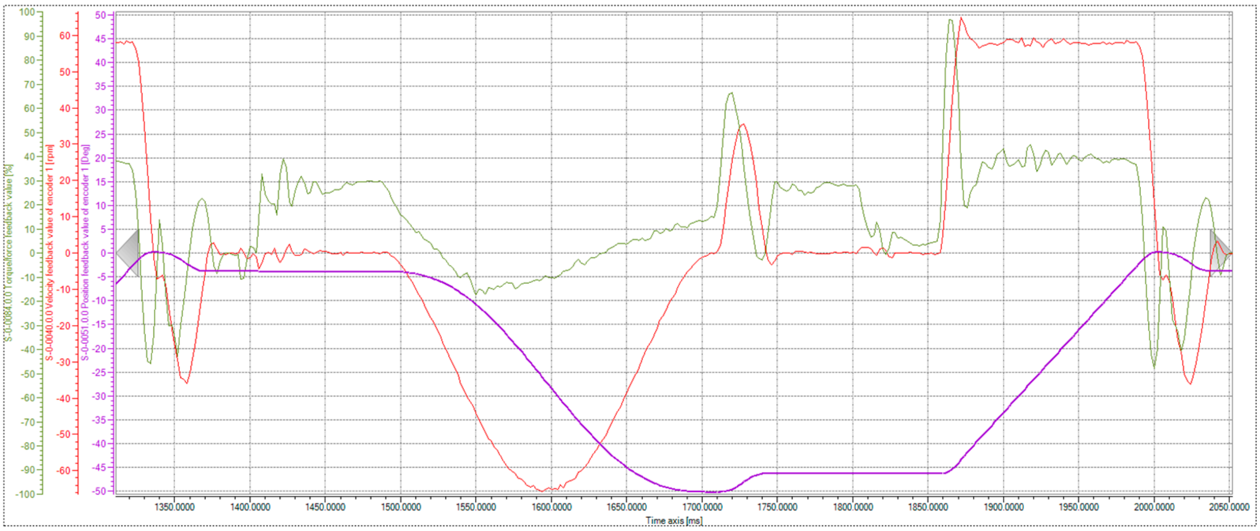

**Figure 9.** Measurement of actuator drive parameters: purple—position [deg], red—speed [rpm], green—torque [%] (Servomotor Max Torque Percentage).

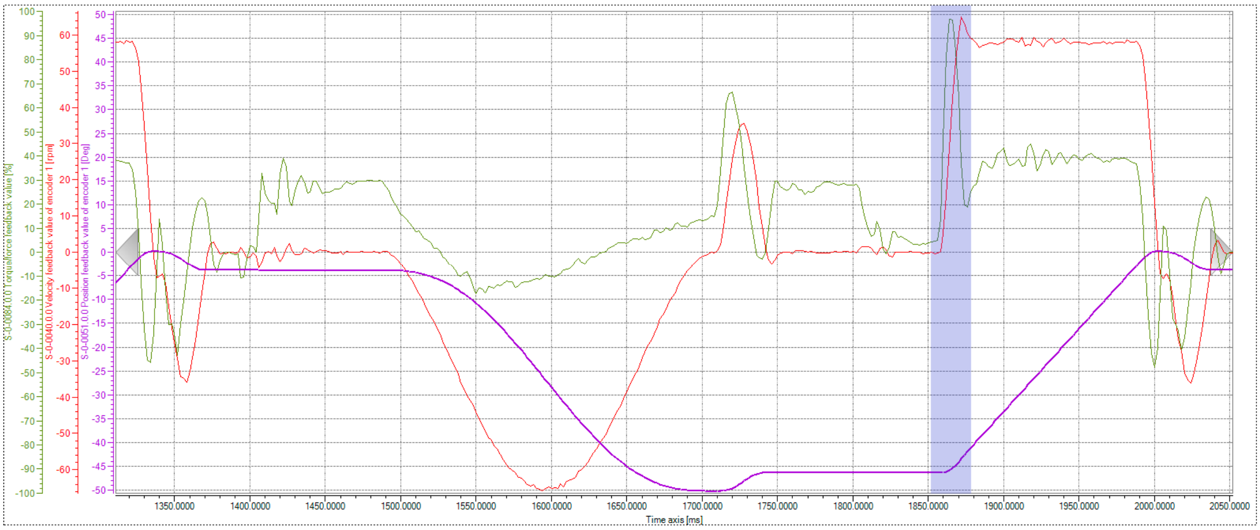

**Figure 10.** Start of product dispensing. The blue area indicates the start of the dosing cycle.

In practice, underestimating the spurt leads to excessive wear of bearing arrangements, rod heads, noise, and vibration of the equipment. The occurrence of a spurt resulting from rapid changes in acceleration causes the formation of large dynamic forces, which have a particularly negative impact on the executive system. The equation of the force acting on the system takes the following form:

$$F = ma = m\frac{dv}{dt} = \frac{dp}{dt} \tag{19}$$

From this equation, it is clear that the formation of large changes in acceleration has a direct effect on the executive system.

Continuing the analysis of the results obtained, another important difference is the constant rotational speed captured in the blue area in Figure 11 (red line), the course of which did not coincide with the results obtained in Figure 5. The constant rotational speed of the servomotor was the effect of the linear function described in interval No. 6 presented in Table 2. The obtained calculations showed that the course of the function within the range of No. 5 to No. 7 had a course similar to that of a parabola.

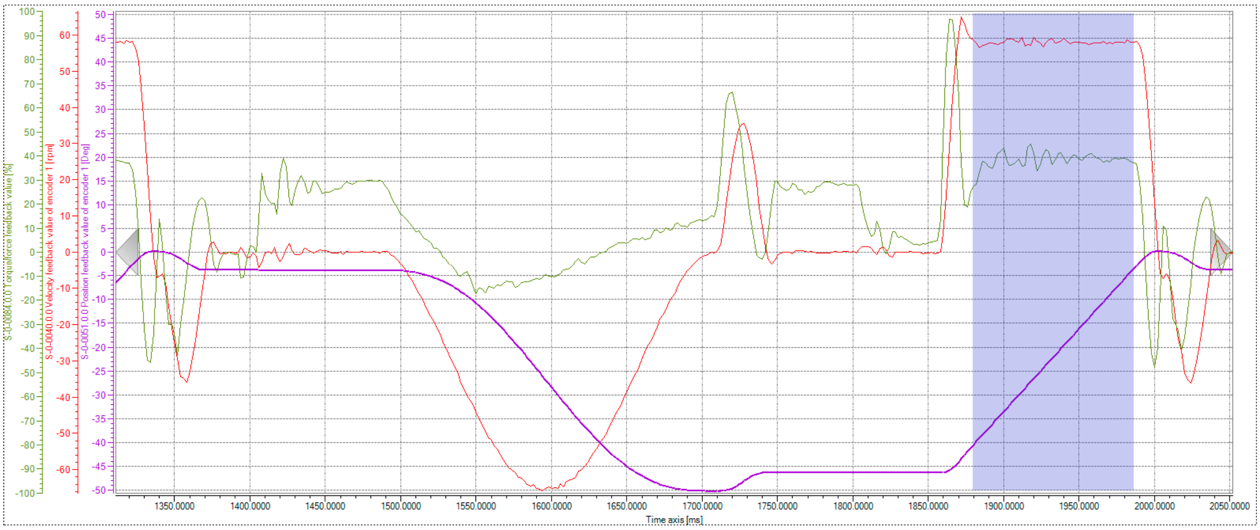

**Figure 11.** Constant speed section.

The difference shown is a consequence of the trigonometric functions with which the position of the executive member and its range of angular work were sown. The range of the angular work within the range of absolute measure $(220^\circ; 285^\circ)$ caused an increase in the linear velocity in the Y axis—the axis of work in the piston rod of the system, the maximum of which falls for 270°.

On the basis of the obtained mathematical model and the analysis performed, it is recommended to transfer the range of the angular work of the assembly to the interval. This change could significantly affect the generated linear acceleration, thus reducing the risk of the occurrence of the spurt phenomenon, the negative effects of which were described earlier $(205^\circ; 270^\circ)$.

This was due to the fact that the range of angular work directly affected the dynamic characteristics of the system. Moving the working range to the indicated angle range will ensure much smoother acceleration changes, which in the case of pasted products with high kinematic viscosity, will significantly reduce the wear of mechanical parts.

This paper presents an analysis of the results obtained from the mathematical model of the actuator system, which is a key step in the engineering design process. The calculations carried out and the generated diagrams made it possible to accurately map the real system, taking into account the basic parameters of linear motion. The analysis of the results focused on the problem of spurt and dynamic forces that particularly negatively affect the stability and durability of components.

The analysis showed that precise mathematical modelling is essential to accurately represent the actual behaviour of a mechanical system. Cam editors dedicated to drive solutions are only useful for general outlines of the motion trajectory, but do not take into account the key parameters resulting from the kinematic system. This can lead to serious design errors, which can disrupt the assumed reliability and durability of equipment that the modern process market expects.

## 4. Conclusions

Based on the above research and discussion it can be concluded that the primary scientific problem to be addressed is the optimization of kinematics and the efficiency of cable-lever systems used in industrial equipment. These systems offer an alternative to traditional cam mechanisms, potentially providing benefits in terms of precision and performance. Future research will focus on the following two key aspects:

- Kinematics Analysis and Modelling: There is a lack of sufficient research on the precise analysis of kinematics and the synchronization of movements in cable-lever systems. The goal is to develop advanced mathematical models and analytical methods that enable the precise modelling and optimization of these systems. The research will encompass both theoretical aspects of kinematics and practical approaches to movement synchronization, aiming to enhance the precision and efficiency of system performance.
- Durability and Reliability: It is crucial to investigate the durability and reliability of components used in cable-lever systems. This analysis will aim to assess the impact of these components on the stability and safety of the equipment, as well as to develop methods for improving their durability and reliability. Specifically, the research will focus on evaluating material wear, dynamic loads, and the effects of long-term operation on system performance and safety.

The planned research aims not only to advance the existing knowledge of cable-lever systems but also to provide practical solutions that can significantly impact the design and optimization of industrial equipment. The outcome of the doctoral dissertation is expected to include innovative approaches and recommendations that can be utilized in future engineering and research endeavours.

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