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RELIABILITY ANALYSIS OF M/G/1 REPAIRABLE QUEUEING SYSTEM WITH MULTIPLE ADAPTIVE VACATIONS AND p-ENTERING DISCIPLINE

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Abstract- This paper considers the reliability of an M/G/1 queue with multiple adaptive vacations in which the arriving customers enter the system with probability p ($0<p\leq1$) during vacations. Through appropriate assumptions, the model is studied by the total probability decomposition technique and the tool of Laplace transform. Some reliability indices are studied. Moreover, we give some numerical examples to observe the effect of various parameters on the reliability indices.

Key Words- Reliability, Multiple adaptive vacations, Queue, p-entering discipline

1.INTRODUCTION

During the past three decades, queueing systems with server's vacations have been studied extensively and applied in many areas such as manufacturing systems, service and computer systems and communication network systems. Excellent surveys on the earlier works of vacation models have been reported by Doshi [1], Takagi [2], Tian and Zhang [3]. As to vacation queues, most works were concentrated on the study of models of multiple vacations and single vacation. However, multiple adaptive vacation policy is more general than most of the classical ones in the sense that the well known multiple vacation policy and single vacation policy become two extreme cases of this policy. It was first presented by Tian [4] that an $M/G/1$ queue with adaptive multistage vacation. In the past, several queueing models have been investigated and a considerable amount of work has been done on multiple adaptive vacations. Ma et al. [5] studied a general decrementing service $M/G/1$ queueing system with multiple adaptive vacations. Luo et al. [6] considered the queue length distribution of $M^X/G/1$ with adaptive multistage vacation. Sun et al. [7] discussed the queue $M^{\xi}/G/1$ with different arrival rates on multiple adaptive vacations and setup times. Simultaneously, some researches on the discrete time multiple adaptive vacation queues can be found in Sun [8], Zhang [9], Ma [10], Tang [11] etc. These literatures are mainly concentrated in the researches of the queue indices.

But as a matter of fact, many servers are deteriorative because of the aging effect and the accumulative wear. The problem of increasing reliability of servers becomes even more important and urgent in connection with the complex mechanization and automation of industrial processes. Yue and Zhao [12] considered the reliability analysis of some queueing models with reparable service station. Tang [13] investigated a single-server $M/G/1$ queueing system subject to breakdowns—some reliability and queueing problems. But they didn't study the case of server vacation. Yu et al. [14] investigated some reliability indices in $M^X/G(M/G)/1$ repairable queueing system with adaptive multistage delay vacation. Nevertheless, they only assumed that the arriving customers enter directly the system with probability 1. It is not fully consistent with the fact. Sometimes, the arriving customers maybe leave the system when they find the server being the vacation period. Liu and Tang [15] studied some reliability indices in $M/G/1$ repairable queueing system with p-entering discipline during server vacations, whereas they only considered the cases of vacations.

In this paper, we study the reliability of an $M/G/1$ queue with multiple adaptive vacations and p-entering discipline. The model is studied by the total probability decomposition technique and the tool of Laplace transform. Some reliability indices are studied. It is obvious that the models presented by Liu and Tang [15] are a special case of this model.

The rest of the paper is arranged as follows. In the next section, the model of the considered queueing system is described. In Section 3, we discuss the probability distribution of the system first failure time. In Section 4, we study the probability that the server fails at time t, that is, the unavailability of the server. In Section 5, we give some numerical examples to verify that the results we have obtained in this model are reasonable in special cases. Section 6 concludes this paper.

2. MODEL DESCRIPTION

We consider an $M/G/1$ queueing system with multiple adaptive vacations and p-entering discipline on first come, first-serve (FCFS) basis in which the server will take a random maximum number, denoted by H, of vacations after emptying the system. Where H is generally distributed with probability distribution function (p.d.f.) $P(H=j)=h_j, j\geq 1$. Each vacation is an i.i.d. random variable, denoted by V. The average vacation time is finite and denoted by $\varepsilon(\varepsilon>0)$. The life length of the server has exponential distribution with distribution function $F_0(t)=1-e^{-\alpha t}(\alpha>0)$. When customers arrive and enter the system, the actual service time follows a general distribution function $G(t)$ with Laplace-stieltjes transform $g(s)$, and the average service time is finite and denoted by $\beta(\beta>0)$. At the beginning, the server is new, after the server fails, the repair time is denoted by Y with general distribution $Y(t)$ and Laplace-stieltjes transform $y(s)$, and $\gamma(\gamma>0)$ stands for the average repair time. When the arriving customers find the server being idle, they enter directly the system with probability 1 and the intervals denoted by $\{\tilde{\tau}_i, i\geq 1\}$ between customers are exponentially distributed with p.d.f. $F(t)=1-e^{-\lambda t}(\lambda>0)$. When customers find that the server is taking a vacation, they enter the system with probability $p(0<p<1)$ and the interarrival time has exponentially distributed with p.d.f. $\tilde{F}(t)=1-e^{-\lambda pt}(\lambda>0, 0<p<1)$ (see Tang and Mao [16]). Assume that all variable are all independent from each other. At the initial time $t = 0$, the number of customers presented in the system is $N(0) = j(j \geq 0)$ and the server does not take vacations when $N(0) = 0$.

3. THE PROBABILITY DISTRIBUTION OF SYSTEM FIRST FAILURE TIME

Firstly, we define the "generalized service time of the n th customer".
The "generalized service time $\tilde{\chi}_n (n \geq 1)$" of a customer (the n th arrival) is the time interval between the start and the end of the service of the n th arrived customer, which includes the repair time caused by system failure. The generalized service time $\tilde{\chi}_n (n \geq 1)$ follows general distributions with p.d.f. $\tilde{G}_n(t)$.

For $t \geq 0$, let $\tilde{G}_n^{[k]}(t) = P\{\tilde{\chi}_n \leq t$;the server has broken for k times during $\tilde{\chi}_n$ }, we have

$$\tilde{G}_n(t) = \sum_{k=0}^{\infty} \tilde{G}_n^{[k]}(t) = \sum_{k=0}^{\infty} \int_0^t Y^{(k)}(t-x) e^{-\alpha x} \frac{(\alpha x)^k}{k!} dG(x)$$

Because of the "generalized service time" of the n th customer is irrelevant to n, we denote the "generalized service time" by $\tilde{\chi}$. Treat $\tilde{\chi}$ as its counterpart in common queue model in which the server is always available, the system being discussing is equal to common $M/G/1$ queue system with $p-$ entering discipline and multiple adaptive vacations. The service times are independent identically distributed random variables with p.d.f $\tilde{G}(t)$. Its Laplace-stieltjes transform is given by

$$\tilde{g}(s) = \int_0^{\infty} e^{-st} d\tilde{G}(t) = g(s + \alpha - \alpha y(s)), \Re(s) \geq 0$$

It follows that the average "generalized service time" is written as

$$E[\tilde{\chi}] = (1 + \alpha\gamma)\beta$$

For mathematical clarity, we note that "generalized busy period" is from the instant when the system terminates idle period and begins to service customers until the system becomes idle again. Let $\tilde{b}$ represent the length of "generalized busy period" beginning with only one customer, similar to the discussing of the busy period of classic $M/G/1$ queue system, we have

Lemma 1. For $\Re(s) \geq 0$, $\tilde{b}(s)$ is the root with the absolute value of the equation $z = \tilde{g}(s + \lambda - \lambda z)$ within $|z| < 1$, and

$$\lim_{t\to\infty} \tilde{B}(t) = \lim_{s\to 0^+} \tilde{b}(s) = \begin{cases} 1, \tilde{\rho} \leq 1 \\ w < 1, \tilde{\rho} > 1 \end{cases}$$

$$E[\tilde{b}] = \begin{cases} \dfrac{\tilde{\rho}}{\lambda(1-\tilde{\rho})}, \tilde{\rho} < 1 \\ \infty, \tilde{\rho} \geq 1 \end{cases}$$

where $\tilde{\rho} = \lambda(1+\alpha\gamma)\beta$, which denotes the traffic intensity of system. w is the root of the equation $w = \tilde{g}[\lambda - \lambda w]$.

Firstly, we define $\Psi_i(t)$ as the p.d.f. of the first failure time of the server with i customers at $t=0$, and $\Psi_i(t)=p\{\tilde{X}\le t|N(0)=i\}$ with Laplace-stieltjes transform $\psi_i(s)$, $(i=0,1,2,\cdots)$, where $\tilde{X}$ denotes the first failure time length of the server. We have

Theorem 1. If $\Re(s)\ge 0$, the first failure time of server is given as follows

$$\psi_0(s)=\frac{\lambda\alpha}{(s+\lambda)(s+\alpha)}[1-\tilde{b}(s+\alpha)t(s)] \tag{1}$$

$$\psi_i(s)=\frac{\alpha}{s+\alpha}[1-\tilde{b}^i(s+\alpha)t(s)] \tag{2}$$

and the average first failure time is

$$\int_0^\infty td\psi_0(t)=\frac{1}{\alpha}+\frac{1}{\lambda}+\frac{\lambda p\varepsilon+H[v(\lambda p)]\{1-v(\lambda p)-\lambda p\varepsilon\}}{\lambda p\left\{1-v(\lambda p-\lambda p\tilde{b}(\alpha))+\sigma H[v(\lambda p)]\right\}}\tilde{b}(\alpha) \tag{3}$$

$$\int_0^\infty td\psi_i(t)=\frac{1}{\alpha}+\frac{\lambda p\varepsilon+H[v(\lambda p)]\{1-v(\lambda p)-\lambda p\varepsilon\}}{\lambda p\left\{1-v(\lambda p-\lambda p\tilde{b}(\alpha))+\sigma H[v(\lambda p)]\right\}}\tilde{b}^i(\alpha) \tag{4}$$

where

$\xi(s)=\{1-H[v(s+\lambda p)]\}[v(s)-v(s+\lambda p)]$, $\Lambda=s+\lambda p-\lambda p\tilde{b}(s+\alpha)$,

$\eta(s)=\{1-H[v(s+\lambda p)]\}[v(\Lambda)-v(s+\lambda p)]$, $\omega(s)=1-v(s+\lambda p)-\xi(s)$,

$\varpi(s)=1-v(s+\lambda p)-\eta(s)$, $\theta(s)=H[v(s+\lambda p)][1-v(s+\lambda p)]$,

$t(s)=\dfrac{s\omega(s)+\lambda p[\omega(s)-\theta(s)]}{s\varpi(s)+\lambda p[\varpi(s)-\tilde{b}(s+\alpha)\theta(s)]}$, $\sigma=v[\lambda p-\lambda p\tilde{b}(\alpha)]-v(\lambda p)-\tilde{b}(\alpha)[1-v(\lambda p)]$.

Proof. Let $s_k=\sum_{i=1}^{k}V_i$, $l_k=\sum_{i=1}^{k}\tilde{\tau}_i$ and $s_0=0$, $l_0=0$, then

$$\Psi_i(t)=P\{\tilde{X}\le t;X\le\tilde{b}^{[i]}\}+P\{\tilde{X}\le t;X>\tilde{b}^{[i]}\}$$

$$=P\{\tilde{X}\le t;X\le\tilde{b}^{[i]}\}+\sum_{j=1}^{\infty}h_jP\{s_j<\tilde{\tau}_1;\tilde{b}^{[i]}+\tilde{\tau}_1<\tilde{X}\le t;X>\tilde{b}^{[i]}\}$$

$$+\sum_{j=1}^{\infty}h_j\sum_{r=1}^{j}p\{s_{r-1}\le\tilde{\tau}_1<s_r;\tilde{\tau}_1+l_{k-1}\le s_r<\tilde{\tau}_1+l_k;\tilde{b}^{[i]}+s_r<\tilde{X}\le t;X>\tilde{b}^{[i]}\}$$

$$=\int_0^t[1-\tilde{B}^{(i)}(x)]dF_0(x)+\sum_{j=1}^{\infty}h_j\int_0^t e^{-\alpha x}\int_0^{t-x}\Psi_1(t-x-y)V^{(j)}(y)d\tilde{F}(y)d\tilde{B}^{(i)}(x)$$

$$+\sum_{j=1}^{\infty}h_j\sum_{r=1}^{j}\sum_{k=1}^{\infty}\int_0^t e^{-\alpha x}\int_0^{t-x}e^{-\lambda py}\int_0^{t-x-y}\Psi_k(t-x-y-u)\frac{(\lambda pu)^k}{k!}e^{-\lambda pu}dV(u)dV^{(r-1)}(y)d\tilde{B}^{(i)}(x) \tag{5}$$

where $\tilde{b}^{[i]}$ presents the busy length beginning with i customers and $\tilde{B}^{(i)}(t)$ denotes i-fold convolution of $\tilde{B}(t)$. Similarly,

$$\Psi_0(t)=\int_0^t\Psi_1(t-x)dF(x) \tag{6}$$

Taking Laplace-stieltjes transform to (5) and (6) and using the following formula

$$\int_0^\infty t d\Psi_i(t) = -\frac{d\psi_i(s)}{ds}|_{s=0}$$

We can complete the proof.

4. THE PROBABILITY THAT THE SERVER FAILS AT TIME t

For $t \geq 0$, let

$\Gamma_i(t) = P\{$ The time t is during the server's "generalized busy period" $/N(0) = i$ $\}$, $\Gamma_i^*(s)$ represents the Laplace transform of $\Gamma_i(t)$ **,** and we get the following conclusion

Theorem 2. For $\Re(s) \geq 0$, the probability of system is busy during "generalized busy period" as follows

$$\Gamma_0^*(s) = \frac{\lambda}{s+\lambda}[1-\tilde{b}(s)\tilde{t}(s)] \tag{7}$$

$$\Gamma_i^*(s) = \frac{1}{s}[1-\tilde{b}^i(s)\tilde{t}(s)] \tag{8}$$

and

$$\lim_{t\to\infty}\Gamma_i(t) = \frac{\lambda p E[\tilde{b}]}{1+\lambda p E[\tilde{b}]} = \begin{cases} \dfrac{p\tilde{\rho}}{1-(1-p)\tilde{\rho}}, \tilde{\rho} < 1 \\ 1, \tilde{\rho} \geq 1 \end{cases}, \tag{9}$$

where

$\Delta = s + \lambda p - \lambda p \tilde{b}(s)$, $\tilde{\eta}(s) = \{1 - H[v(s+\lambda p)]\}[v(\Delta) - v(s+\lambda p)]$,

$\tilde{\varpi}(s) = 1 - v(s+\lambda p) - \tilde{\eta}(s)$, $\theta(s) = H[v(s+\lambda p)][1 - v(s+\lambda p)]$,

$$\tilde{t}(s) = \frac{s\omega(s) + \lambda p[\omega(s) - \theta(s)]}{s\tilde{\varpi}(s) + \lambda p[\tilde{\varpi}(s) - \tilde{b}(s)\theta(s)]}.$$

Proof. $\Gamma_0(t) = P\{\tilde{\tau}_1 < t \leq \tilde{\tau}_1 + \tilde{b}\} + P\{\tilde{\tau}_1 + \tilde{b} + \tilde{\tau}_2 < t;$ the time t is during the server's "generalized busy period"$\}$

$$= \int_0^t [1-\tilde{B}(t-x)]dF(x) + \sum_{j=1}^\infty h_j P\{s_j < \tilde{\tau}_2; \tilde{\tau}_1 + \tilde{b} + \tilde{\tau}_2 < t\}$$

$$+\sum_{j=1}^\infty h_j \sum_{r=1}^j \sum_{k=1}^\infty P\{s_{r-1} < \tilde{\tau}_2 \leq s_r; \tilde{\tau}_2 + l_{k-1} \leq s_r < \tilde{\tau}_2 + l_k; \tilde{\tau}_1 + \tilde{b} + s_r < t\} \tag{10}$$

Using total probability decomposition technology, we have

$$\Gamma_0(t) = \int_0^t [1-\tilde{B}(t-x)]dF(x) + \sum_{j=1}^\infty h_j \int_0^t \int_0^{t-x} \Gamma_1(t-x-y) V^{(j)}(y) d\tilde{F}(y) d[F(x) * \tilde{B}(x)]$$

$$+\sum_{j=1}^\infty h_j \sum_{r=1}^j \sum_{k=1}^\infty \int_0^t \int_0^{t-x} e^{-\lambda p y} \int_0^{t-x-y} \Gamma_k(t-x-y-u) \frac{(\lambda p u)^k}{k!} e^{-\lambda p u} dV(u) dV^{(r-1)}(y) d[F(x) * \tilde{B}(x)] \tag{11}$$

For $i \geq 1$, $j \geq 1$, similarly, we can get

$$\Gamma_i(t)=1-\tilde{B}^{(i)}(t)+\sum_{j=1}^{\infty}h_j\int_0^t\int_0^{t-x}\Gamma_1(t-x-y)V^{(j)}(y)d\tilde{F}(y)d\tilde{B}^{(i)}(x)$$

$$+\sum_{j=1}^{\infty}h_j\sum_{r=1}^{j}\sum_{k=1}^{\infty}\int_0^t\int_0^{t-x}e^{-\lambda py}\int_0^{t-x-y}\Gamma_k(t-x-y-u)\frac{(\lambda pu)^k}{k!}e^{-\lambda pu}dV(u)dV^{(r-1)}(y)d\tilde{B}^{(i)}(x) \qquad (12)$$

Taking Laplace transform to (11) and (12), we obtain

$$\Gamma_i^*(s)=\frac{1-\tilde{b}^i(s)}{s}+\tilde{b}^{i-1}(s)\{\frac{s+\lambda}{\lambda}\Gamma_0^*(s)-[1-\tilde{b}(s)]\} \qquad (13)$$

Substituting (13) into the LST of (11) gives (7), and furthermore, we get (8) and (9).

Considering an unit reliability system, the life distribution of the server is $F(t)=1-e^{-\alpha t}\,(\alpha>0)$, after the system fails, the server will be repaired immediately, the repair time is subject to a general distribution, the average repair time is denoted by β . When the system has been repaired, the server begins to serve customers immediately, further assumption that the system is new at the instant $t=0$.

For $t\geq 0$, let

$\Phi(t)=P\{$The system fails at instant $t\}$, and $\varphi^*(s)$ be the Laplace transform of $\Phi(t)$, we have the following lemma

Lemma 2. If $\Re(s)\geq 0$, we have

$$\varphi^*(s)=\frac{\alpha[1-y(s)]}{s[s+\alpha-\alpha y(s)]} \qquad (14)$$

and

$$\lim_{t\to\infty}\Phi(t)=\lim_{s\to 0^+}s\varphi^*(s)=\frac{\alpha\beta}{1+\alpha\beta} \qquad (15)$$

For $t\geq 0$, let

$\Phi_i(t)=P\{$The system fails at instant $t\,/N(0)=i\}$, $\varphi_i^*(s)=\int_0^{\infty}e^{-st}\Phi_i(t)dt$, , $t\geq 0$.

We have

Theorem 3. For $\Re(s)\geq 0$ and $i\geq 0$, we can get

$$\varphi_i^*(s)=\frac{\alpha[1-y(s)]}{s[s+\alpha-\alpha y(s)]}\square[s\Gamma_i^*(s)] \qquad (16)$$

and the steady-state unavailability is

$$\lim_{t\to\infty}\Phi_i(t)=\begin{cases}\dfrac{p\tilde{\rho}\alpha\beta}{(1+\alpha\beta)[1-(1-p)\tilde{\rho}]},\tilde{\rho}<1\\ \dfrac{\alpha\beta}{1+\alpha\beta},\tilde{\rho}\geq 1\end{cases} \tag{17}$$

Proof. 1) According to the model assumptions, the server will fail only happening during the generalized busy period. And at the beginning of the busy period, the server is always normal. So, the server fails only in some busy period at instant t, similar as Theorem2. For $i=0$,

$$\Phi_0(t)=\int_0^t Q_1(t-x)dF(x)+\sum_{j=1}^{\infty}h_j\int_0^t\int_0^{t-x}\Phi_1(t-x-y)V^{(j)}(y)d\tilde{F}(y)d[F(x)*\tilde{B}(x)]$$
$$+\sum_{j=1}^{\infty}h_j\sum_{r=1}^{j}\sum_{k=1}^{\infty}\int_0^t\int_0^{t-x}e^{-\lambda py}\int_0^{t-x-y}\Phi_k\left(t-x-y-u\right)\frac{(\lambda pu)^k}{k!}e^{-\lambda pu}dV(u)dV^{(r-1)}(y)d[F(x)*\tilde{B}(x)] \tag{18}$$

where $Q_k(t)=P\{\tilde{b}^{[k]}>t\geq 0;$ the server fails at the instant $t\}, k\geq 1$. For $i\geq 1$,

$$\Phi_i(t)=Q_i(t)+\sum_{j=1}^{\infty}h_j\int_0^t\int_0^{t-x}\Phi_1(t-x-y)V^{(j)}(y)d\tilde{F}(y)d\tilde{B}^{(i)}(x)$$
$$+\sum_{j=1}^{\infty}h_j\sum_{r=1}^{j}\sum_{k=1}^{\infty}\int_0^t\int_0^{t-x}e^{-\lambda py}\int_0^{t-x-y}\Phi_k(t-x-y-u)\frac{(\lambda pu)^k}{k!}e^{-\lambda pu}dV(u)dV^{(r-1)}(y)d\tilde{B}^{(i)}(x) \tag{19}$$

2) For $t\geq 0, i=1,2,3,\cdots$,

$\Phi(t)=P\{$The system fails at instant $t\}=P\{\tilde{b}^{[i]}>t\geq 0$, the server fails at instant $t\}$

$+P\{\tilde{b}^{[i]}<t$; the server fails at instant $t\}$

$=Q_i(t)+\int_0^t\Phi(t-x)d\tilde{B}^{(i)}(x), k\geq 1.$

Therefore

$$Q_i(t)=\Phi(t)-\Phi(t)*\tilde{B}^{(i)}(t) \tag{20}$$

Substituting (20) to (18) and (19), after taking Laplace transform, we will get

$$\varphi_i^*(s)=\varphi^*(s)[1-\tilde{b}^i(s)]+\tilde{b}^{i-1}(s)\{\frac{s+\lambda}{\lambda}\varphi_0^*(s)-\varphi^*(s)[1-\tilde{b}(s)]\} \tag{21}$$

Substituting (21) to the LST of (18), we get

$$\varphi_0^*(s)=\frac{\lambda\varphi^*(s)\left\{(s+\lambda p)\varpi(s)-(s+\lambda p)\tilde{b}(s)\omega(s)\right\}}{(s+\lambda)[(s+\lambda p)\varpi(s)-\lambda p\tilde{b}(s)\theta(s)]}, \tag{22}$$

Substituting (22) to (21), we have

$$\varphi_i^*(s)=\varphi^*(s)\{1-\tilde{b}^i(s)\frac{s\varpi(s)-\lambda p[\theta(s)-\varpi(s)]}{s\varpi(s)+\lambda p[\varpi(s)-\tilde{b}(s)\theta(s)]}\} \tag{23}$$

Compare (23) with (8), we can get $\varphi_i^*(s)=\varphi^*(s)[s\Gamma_i^*(s)]$, therefore, we can get (16) easily, furthermore, using the L'hospital, we can obtain (17).

5. NUMERICAL RESULTS

We assume the service time, the interarrival time, the vacation time, the repair time and the life length of server may follow exponential distributions with the mean β, $1/\lambda$, ε, γ, $1/\alpha$,respectively. Moreover, the maximum number H of vacations of the server follows general geometric distributions with parameter p_h. Hence, we have

$$H(z)=\frac{p_h}{1-\bar{p}_h z}, E(H)=\frac{1}{p_h}$$

We fix $\varepsilon=3, \beta=1.2, \lambda=0.7$, $p_h=0.2, \alpha=0.01, \gamma=2$. In Fig. 1, we show the effect of p on the first failure time for $p=0.7$, 0.8 and 0.9. Which shows that the first failure time strictly decreases along with the increase of p. In Fig. 2, we show the effect of p_h on the first failure time for $p=0.7$, $\varepsilon=3$, $\beta=1.2$, $\lambda=0.7$, $\alpha=0.01$, $\gamma=2$ and different p_h. It shows that the first failure time strictly decreases along with the increases of p_h. In Fig.3, In Fig.4 we plot the curve of the effect of p on generalized busy period and the unavailability of the system, respectively. It shows that they strictly increase along with the increase of p.

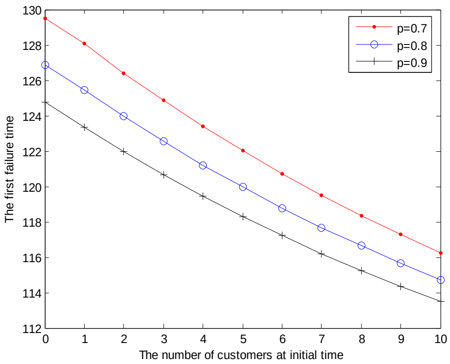

Fig.1. The effect of p on the first failure time

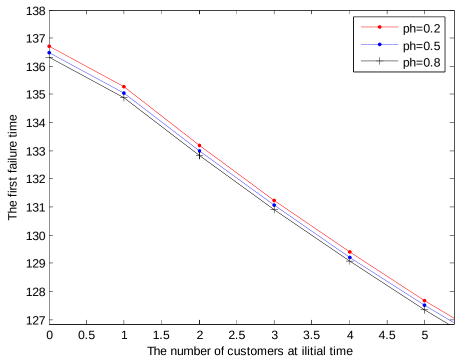

Fig.2. The effect of p_h on the first failure time

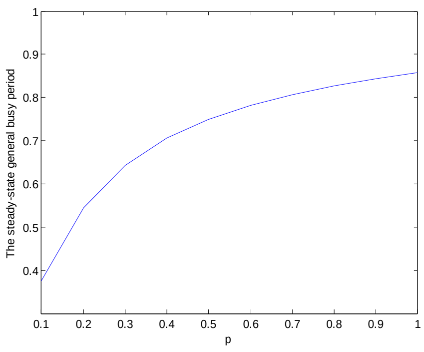

Fig.3. The effect of p on the generalized busy period

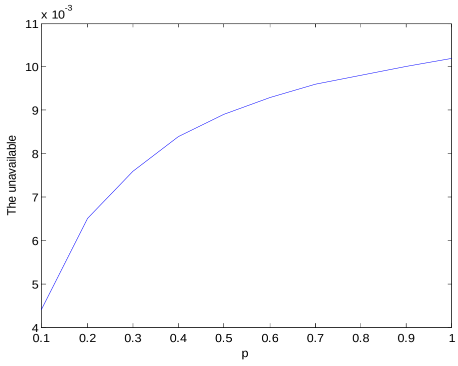

Fig.4. The effect of p on the unavailable

6. CONCLUSIONS

In this paper, we have carried out an analysis of the reliability of M/G/1 repairable queueing system with multiple adaptive vacations and p-entering discipline. Firstly, by using the total probability decomposition technique and the tool of Laplace transform, we have obtained the first failure time distribution and the average first failure time of the system. Then, utilizing the distribution of "generalized busy period" of the server, we have obtained the transient and steady state unavailability of the system. From the numerical results, we can see that appropriate controlling p and p_h will delay the first failure time and decrease the unavailable of system. This model has extensive applications in manufacturing systems, service and computer systems and communication network systems.

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