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HOMOTOPIC APPROXIMATE SOLUTIONS FOR THE GENERAL PERTURBED NONLINEAR SCHRÖDINGER EQUATION

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Abstract- In this work, a class of perturbed nonlinear Schrödinger equation is studied by using the homotopy perturbation method. Firstly, we obtain some Jacobi-like elliptic function solutions of the corresponding typical general undisturbed nonlinear Schrödinger equation through the mapping deformation method, and secondly, a homotopic mapping transform is constructed, then the approximate solution with arbitrary degree of accuracy for the perturbed equation is researched, it is pointed out that the series of approximate solution is convergent. Finally, the efficiency and accuracy of the approximate solution is also discussed by using the fixed point theorem.

Keywords- perturbed nonlinear Schrödinger equation; homotopic mapping; asymptotic method; approximate solution

1. INTRODUCTION

With the development of soliton theory in nonlinear science, searching for analytical exact solutions or approximate solutions of the nonlinear partial differential equations (NLPDEs) plays an important and significant role in the study of the dynamics of those nonlinear phenomena [1]. Many powerful methods have been used to handle these problems recently. For example, inverse scattering transformation [2], Hirota bilinear method [3], homogeneous balance method [4], Bäcklund transformation [5], Darboux transformation [6], projective Riccati equations method [7], the generalized Jacobi elliptic function expansion method [8] and so on [9]. But because of the complexity of NLPDEs, people can't find the exact solutions for many of them especially with disturbed term. Researchers had to develop some approximate methods for nonlinear theory, such as multiple-scale method [10], variational iteration method [11], indirect matching method [12] etc. The main essence of these methods is the study of nonlinear problems dealt with linear problems by using the approximate expansion.

The homotopy analysis method (HAM) was firstly proposed in 1992 by Liao [13], which yields a fast convergence for most of the selected problems. It also showed a high accuracy and a rapid convergence to solutions of the nonlinear partial evolution equations. After this, many types of nonlinear problems were solved with HAM by others, such as discrete KdV equation [14], a smoking habit model [15], and so on. As a special case of HAM, He proposed the homotopy perturbation method [HPM] [16]. Recently, based on the idea of HPM, Mo proposed the homotopic mapping method to handle some nonlinear problems with small perturbed term [17]. A great quantity of works about this subject have been researched by many authors, such as perturbed Kdv-Burgers equation [18], mid-latitude stationary wind field [19] etc.

In this paper, we extend the applications of HPM to solve a class of disturbed nonlinear Schrödinger equation in the nonlinear optics. And many useful results are researched.

2. MODEL AND HOMOTOPIC MAPPING

Consider the following generalized nonlinear Schrödinger equation with perturbed term

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\beta(z)\frac{\partial^2 u}{\partial t^2}+\delta(z)u|u|^2-i\alpha(z)u=\beta(z)f(u,z,t)\,. \tag{1}$$

If we let $t\to x, z\to t$, Eq.(1) turns to the following form

$$i\frac{\partial u}{\partial t}+\frac{1}{2}\beta(t)\frac{\partial^2 u}{\partial x^2}+\delta(t)u|u|^2-i\alpha(t)u=\beta(t)f(u,t,x)\,. \tag{2}$$

Where f is a perturbed term, which is a sufficiently smooth function in a corresponding domain. $\beta(t)$ and $\delta(t)$ are the slowly increasing dispersion coefficient and nonlinear coefficient respectively, $\alpha(t)$ represents the heat-insulating amplification or loss. The transmission of soliton in the real communication system of optical soliton is described by Eq.(2) with $f=0$ [20–25].

$$i\frac{\partial u}{\partial t}+\frac{1}{2}\beta(t)\frac{\partial^2 u}{\partial x^2}+\delta(t)u|u|^2-i\alpha(t)u=0\,. \tag{3}$$

In Ref.[20], Serkin and Belyaeva developed an effective mathematical algorithm to discover and investigate infinite numbers of novel soliton solutions for Eq. (3). In Ref.[21], Serkin and Hasegawa discussed the problem of soliton management described by Eq. (3). Many exact solutions of Eq.(3) have been obtained by the authors in Refs.[22-25].

In order to obtain the approximate solution of Eq.(2),we make the transformation

$$u = A(t)\varphi(\xi)e^{i\eta}, \xi = k_1 x + c_1(t), \eta = k_2 x + c_2(t). \quad (4)$$

With the following consistency conditions

$$A(t) = ce^{\int_0^t \alpha(\tau)d\tau}, c_1(t) = -k_1 k_2 \int_0^t \beta(\tau)d\tau,$$

$$c_2(t) = \frac{1}{2}(a_2 k_1^2 - k_2^2)\int_0^t \beta(\tau)d\tau, \delta(t) = \frac{-a_4 k_1^2}{c^2}\beta(t)e^{-2\int_0^t \alpha(\tau)d\tau}. \quad (5)$$

Where k_1, k_2, a_2, a_4, c are arbitrary nonzero constants.

Substituting (4) into (2),we have

$$\varphi''_{\xi\xi} - a_2\varphi - 2a_4\varphi^3 = 2f(u,t,x)e^{-i\eta}/k_1^2. \quad (6)$$

If we let $f(u,t,x) = \frac{1}{2}k_1^2 f(\varphi)e^{i\eta}$, Eq.(6) becomes

$$\varphi''_{\xi\xi} - a_2\varphi - 2a_4\varphi^3 = f(\varphi). \quad (7)$$

When $f = 0$, we have

$$\varphi''_{\xi\xi} - a_2\varphi - 2a_4\varphi^3 = 0 . \quad (8)$$

By using the general mapping deformation method [9], we can obtain the following solutions of the corresponding undisturbed Eq. (3)

$$u_j = cF_j(k_1 x - k_1 k_2 \int_0^t \beta(\tau)d\tau)e^{\int_0^t \alpha(\tau)d\tau + i[k_2 x + \frac{1}{2}\int_0^t (a_{2_j} k_1^2 - k_2^2)\beta(\tau)d\tau]}.$$

With the consistency conditions

$$\delta(t) = \frac{-a_{4_j} k_1^2}{c^2}\beta(t)e^{-2\int_0^t \alpha(\tau)d\tau}, j = 0,1,2,3,\cdots.$$

Where F_j is an arbitrary solution of the equation $F_j'^2 = a_{0_j} + a_{2_j}F_j^2 + a_{4_j}F_j^4$, we can obtain the twenty-two classes of solutions F_j from Ref. [26], for example, if we let

$a_{0_0} = 1 - m^2, a_{2_0} = 2m^2 - 1, a_{4_0} = -m^2$, $F_0 = cn\xi$, we have

$$u_0 = ce^{\int_0^t \alpha(\tau)d\tau + i[k_2 x + \frac{1}{2}\int_0^t ((2m^2-1)k_1^2 - k_2^2)\beta(\tau)d\tau]} cn(k_1 x - k_1 k_2 \int_0^t \beta(\tau)d\tau).$$

Where $\delta(t) = \frac{m^2 k_1^2}{c^2}\beta(t)e^{-2\int_0^t \alpha(\tau)d\tau}$.

Remark 1: If we let $k_1 = c_1\sqrt{\frac{k_2}{2m^2-1}}, k_2 = c_2, ck = c_3\sqrt{\frac{-k_2m^2}{k_4(2m^2-1)}}$. We find that u_0 turns to the solution u_{31} in Ref.[24],and which was degenerated to the famous bright-soliton solutions u_1 in Ref.[25] when $m \to 1$.

Eq.(8) has the solution $\tilde{\varphi}_0 = cn[k_1x - k_1k_2\int_0^t \beta(\tau)d\tau]$.

In order to obtain the solution of Eq.(2), We introduce the following homotopic mapping $H(\varphi, p)$: $f(R)\times I \to R$,

$$H(\varphi, p) = L\varphi - L\tilde{\varphi}_0 + p\left(L\tilde{\varphi}_0 - 2a_4\varphi^3 - f(\varphi)\right) \quad . \tag{9}$$

Where $R = (-\infty, +\infty), I = [0,1]$, $\tilde{\varphi}_0$ is an initial approximate solution to Eq.(8), and the linear operator L is expressed as

$$L(u) = \varphi''_{\xi\xi} - a_2\varphi \ . \tag{10}$$

Obviously, from mapping (9), $H(\varphi,1) = 0$ is the same as Eq.(7). Thus the solution of Eq.(7) is the same as the solution of $H(\varphi, p)$ as $p \to 1$.

3. APPROXIMATE SOLUTION

In order to obtain the solution of Eq.(7), set

$$\varphi = \sum_{i=0}^{\infty}\varphi_i(\xi)\,p^i = \varphi_0 + p\varphi_1 + p^2\varphi_2 + \cdots . \tag{11}$$

If we let $\varphi_0 = \tilde{\varphi}_0$, noticed the analytic properties of $f, \tilde{\varphi}_0$ and mapping (9), we can deduce that the series of (11) are uniform convergence when $p \in [0,1]$ [16]. Substituting expression (11) into $H(u, p) = 0$, expanding nonlinear terms into the power series in powers of p , we compare the coefficients of the same power of p on both sides of the equation, we have

$$p^0 : L\varphi_0 = L\tilde{\varphi}_0 \quad , \tag{12}$$

$$p^1 : L\varphi_1 = f(\varphi_0) \quad , \tag{13}$$

$$p^2 : L\varphi_2 = 6a_4\varphi_0^2\varphi_1 + f_\varphi(\varphi_0)\varphi_1 \quad , \tag{14}$$

$\vdots$

$$p^n : L\varphi_n = F(\varphi_0, \varphi_1, \cdots, \varphi_{n-1}) +$$

$$2a_4 \sum_{k_1=0}^{3}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_2}\cdots\sum_{k_{n-1}=0}^{k_{n-2}} C_3^{k_1}C_{k_1}^{k_2}C_{k_2}^{k_3}\cdots C_{k_{n-2}}^{k_{n-1}}\varphi_0^{3-k_1}\varphi_1^{k_1-k_2}\varphi_2^{k_2-k_3}\cdots\varphi_{n-2}^{k_{n-2}-k_{n-1}}\varphi_{n-1}^{k_{n-1}}.$$

$\vdots$

Where $3 \ge k_1 \ge k_2 \ge \cdots \ge k_{n-1} \ge 0 \in N$, $\sum_{j=1}^{n-1} k_j = n-1, n \in N^+$ and

$$F(\varphi_0, \varphi_1, \cdots, \varphi_{n-1}) = \frac{1}{(n-1)!}\frac{\partial^{(n-1)}}{\partial p^{n-1}} f(\varphi_0, \varphi_1, \cdots, \varphi_{n-1})\bigg|_{p=0}.$$

From (12) we have

$$\varphi_0(\xi) = \tilde{\varphi}_0(\xi) \quad . \tag{15}$$

If we select $\varphi_1|_{\xi=0} = 0$,and from (13) we have

$$\varphi_1 = \frac{1}{\sqrt{a_2}}\int_0^\xi f(\varphi_0)(e^{\sqrt{a_2}(\xi-\tau)} - e^{-\sqrt{a_2}(\xi-\tau)})d\tau\ , \quad a_2 \ne 0 \quad . \tag{16}$$

$f(\varphi_0) = f(\varphi_0(\tau))$.

If we select $\varphi_2|_{\xi=0} = 0$,and from (13) we have

$$\varphi_2 = \frac{1}{\sqrt{a_2}}\int_0^\xi [6a_4\varphi_0^2\varphi_1 + f_\varphi(\varphi_0)\varphi_1](e^{\sqrt{a_2}(\xi-\tau)} - e^{-\sqrt{a_2}(\xi-\tau)})d\tau \quad . \tag{17}$$

Where $a_2 \ne 0$, $\varphi_0 = \varphi_0(\tau), \varphi_1 = \varphi_1(\tau)$.

From (4),(5),(11),(15),(16),(17) and mapping (9) we have the first and second order approximate Jacobi-like elliptic function solutions $u_{1\text{hom}}(x,t)$ and $u_{2\text{hom}}(x,t)$ of the generalized disturbed nonlinear Schrödinger equation (2) as follows:

$$\varphi_{1\text{hom}}(x,t) = cn[k_1x - k_1k_2\int_0^t \beta(\tau)d\tau] + \frac{1}{\sqrt{2m^2-1}}\int_0^\xi f(\varphi_0)(e^{\sqrt{2m^2-1}(\xi-\tau)} - e^{-\sqrt{2m^2-1}(\xi-\tau)})d\tau\ .$$

$$u_{1\hom}(x,t)= ce^{\int_0^t \alpha(\tau)d\tau+i[k_2x+\frac{1}{2}\int_0^t((2m^2-1)k_1^2-k_2^2)\beta(\tau)d\tau]}\varphi_{1\hom}(x,t).$$

$$\varphi_{2\hom}(x,t)=cn[k_1x-k_1k_2\int_0^t\beta(\tau)d\tau]+\frac{1}{\sqrt{2m^2-1}}\int_0^\xi f(\varphi_0)(e^{\sqrt{2m^2-1}(\xi-\tau)}-e^{-\sqrt{2m^2-1}(\xi-\tau)})d\tau$$

$$+\frac{1}{\sqrt{2m^2-1}}\int_0^\xi[-6m^2\varphi_0^2\varphi_1+f_\varphi(\varphi_0)\varphi_1](e^{\sqrt{2m^2-1}(\xi-\tau)}-e^{-\sqrt{2m^2-1}(\xi-\tau)})d\tau.$$

$$u_{2\hom}(x,t)= ce^{\int_0^t \alpha(\tau)d\tau+i[k_2x+\frac{1}{2}\int_0^t((2m^2-1)k_1^2-k_2^2)\beta(\tau)d\tau]}\varphi_{2\hom}(x,t) \quad .$$

With the same process, we can also obtain the N order approximate solution $\varphi_{n\hom}(x,t)$.

$$\varphi_{n\hom}(x,t)=cn[k_1x-k_1k_2\int_0^t\beta(\tau)d\tau]+\frac{1}{\sqrt{2m^2-1}}\int_0^\xi f(\varphi_0)(e^{\sqrt{a_2}(\xi-\tau)}-e^{-\sqrt{a_2}(\xi-\tau)})d\tau$$

$$+\frac{1}{\sqrt{2m^2-1}}\int_0^\xi[-6m^2\varphi_0^2\varphi_1+f_\varphi(\varphi_0)\varphi_1](e^{\sqrt{2m^2-1}(\xi-\tau)}-e^{-\sqrt{2m^2-1}(\xi-\tau)})d\tau$$

$$+\cdots+\frac{1}{\sqrt{2m^2-1}}\int_0^\xi(e^{\sqrt{2m^2-1}(\xi-\tau)}-e^{-\sqrt{2m^2-1}(\xi-\tau)})[F(\varphi_0,\varphi_1,\cdots,\varphi_{n-1})$$

$$-2m^2\sum_{k_1=0}^{3}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_2}\cdots\sum_{k_{n-1}=0}^{k_{n-2}}C_3^{k_1}C_{k_1}^{k_2}C_{k_2}^{k_3}\cdots C_{k_{n-2}}^{k_{n-1}}\varphi_0^{3-k_1}\varphi_1^{k_1-k_2}\varphi_2^{k_2-k_3}\cdots\varphi_{n-2}^{k_{n-2}-k_{n-1}}\varphi_{n-1}^{k_{n-1}}]d\tau.$$

$$u_{n\hom}(x,t)=ce^{\int_0^t \alpha(\tau)d\tau+i[k_2x+\frac{1}{2}\int_0^t((2m^2-1)k_1^2-k_2^2)\beta(\tau)d\tau]}\varphi_{n\hom}(x,t). \tag{18}$$

Where

$$G_n=F(\varphi_0,\varphi_1,\cdots,\varphi_{n-1})-$$
$$2m^2\sum_{k_1=0}^{3}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_2}\cdots\sum_{k_{n-1}=0}^{k_{n-2}}C_3^{k_1}C_{k_1}^{k_2}C_{k_2}^{k_3}\cdots C_{k_{n-2}}^{k_{n-1}}\varphi_0^{3-k_1}\varphi_1^{k_1-k_2}\varphi_2^{k_2-k_3}\cdots\varphi_{n-2}^{k_{n-2}-k_{n-1}}\varphi_{n-1}^{k_{n-1}},$$

$3\ge k_1\ge k_2\ge\cdots\ge k_{n-1}\ge 0\in N$, $\sum_{j=1}^{n-1}k_j=n-1, n\in N^+$ and

$$F(\varphi_0,\varphi_1,\cdots,\varphi_{n-1})=\frac{1}{(n-1)!}\frac{\partial^{(n-1)}}{\partial p^{n-1}}f(\varphi_0,\varphi_1,\cdots,\varphi_{n-1})\bigg|_{p=0}.$$

Remark 2: The N-order approximate solution $u_{n\hom}(x,t)$ is degenerated to the solitary wave approximate solution and trigonometric function approximate solution when the modulus $m \to 1$ or $m \to 0$.

If selecting different $\tilde{\varphi}_0$, we can obtain the other fifty-one types of approximate solutions of Eq.(2).

4. COMPARISION OF ACCURACY

In order to explain the accuracy of the expressions of the approximate solution represented by Eq.(18), we consider the small perturbation term $f = \frac{1}{2}\varepsilon k_1^2 e^{i\eta}\sin^n\varphi$,

$\varphi = \frac{1}{c}e^{-\int_0^t \alpha(\tau)d\tau - i(k_2 x + \frac{1}{2}(a_2 k_1^2 - k_2^2)\int_0^t \beta(\tau)d\tau)}u$, $0 < \varepsilon << 1$ in Eq.(2) .

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\beta(t)\frac{\partial^2 u}{\partial x^2} + \delta(t)u|u|^2 - i\alpha(t)u = \frac{1}{2}\varepsilon k_1^2\beta(t)e^{i\eta}\sin^n\varphi, \qquad n \in N^+ . \tag{19}$$

From the discussion of Section 3, we obtain the second order approximate Jacobi-like elliptic function solution of Eq.(19) as follows

$$\varphi_{2\hom}(x,t) = cn[k_1 x - k_1 k_2\int_0^t \beta(\tau)d\tau] + \frac{\varepsilon}{\sqrt{2m^2-1}}\int_0^\xi \sin^n(\varphi_0)(e^{\sqrt{2m^2-1}(\xi-\tau)} - e^{-\sqrt{2m^2-1}(\xi-\tau)})d\tau +$$

$$\frac{1}{\sqrt{2m^2-1}}\int_0^\xi [-6m^2\varphi_0^2\varphi_1 + \varepsilon n\sin^{n-1}(\varphi_0)\cos(\varphi_0)\varphi_1](e^{\sqrt{2m^2-1}(\xi-\tau)} - e^{-\sqrt{2m^2-1}(\xi-\tau)})d\tau$$

$$u_{2\hom}(x,t) = ce^{\int_0^t \alpha(\tau)d\tau + i[k_2 x + \frac{1}{2}\int_0^t((2m^2-1)k_1^2 - k_2^2)\beta(\tau)d\tau]}\varphi_{2\hom}(x,t) \quad . \tag{20}$$

Set $\varphi_{exa}(x,t) = \sum_{i=0}^{\infty}\varphi_i(x,t)$ to be an exact solution of Eq.(6), noticed that

$$L(\varphi_{exa} - \varphi_{2\hom})$$

$$= f(\varphi) + 2a_4\varphi_{exa}{}^3 - [2a_4\varphi_0{}^3 + f(\varphi_0) + 6a_4\varphi_0^2\varphi_1 + f_\varphi(\varphi_0)\varphi_1]$$

$$= \varepsilon\sin^n\left(\sum_{i=0}^{\infty}\varphi_i\right) + 2a_4(\sum_{i=0}^{\infty}\varphi_i)^3 - [2a_4\varphi_0{}^3 + \varepsilon\sin^n(\varphi_0) + 6a_4\varphi_0^2\varphi_1 + \varepsilon n\sin^{n-1}(\varphi_0)\cos(\varphi_0)\varphi_1]$$

$$= O(\varepsilon^2) \quad . \tag{21}$$

Where $0 < \varepsilon << 1$, selecting an arbitrary constant such that $\varphi_{exa}(0) = \varphi_{2\hom}(0)$, from the fixed point theorem [27], we have $\varphi_{exa}-\varphi_{2\hom}=O(\varepsilon^2)$, $0<\varepsilon<<1$ then

$$|u_{exa}-u_{2\hom}|=|A(t)e^{i\eta}[\varphi_{exa}-\varphi_{2\hom}]|$$

$$=\left|\frac{\varepsilon^2 An\sin^{n-1}(\varphi_0)\cos(\varphi_0)}{\sqrt{2m^2-1}}\int_0^{\xi}\sin^n(\varphi_0)(e^{\sqrt{a_2}(\xi-\tau)}-e^{-\sqrt{a_2}(\xi-\tau)})d\tau\right|=O(\varepsilon^2),\quad 0<\varepsilon<<1 .$$

Therefore, from the above result, we know that the approximate solution $u_{2\hom}$,obtained by asymptotic method, possesses better accuracy.

Setting $A(t)=1, k_1=k_2=1, \beta(t)=1, m\to 1, n=1, \xi\in[0,3]$ and $\varepsilon=0.01, 0.001$ for Eq.(20), we can get the comparison and simulation between $|u_{1\hom}(\xi)|$ and $|u_0(\xi)|$ in Table 1,Table 2, Fig.1 and Fig.2. From Figs. 1-2, it is easy to see that as $0<\varepsilon<<1$ is a small parameter, the solutions $|u_{1\hom}(\xi)|$ and $|u_0(\xi)|$ are very close to each other. This behaviour is coincident with that for the approximate solution of the weakly disturbed evolution equation (19).

Table 1. Comparison between $|u_{1\hom}(\xi)|$ and $|u_0(\xi)|$ when $\varepsilon=0.01$.

ξ	$\lvert u_{1\hom}(\xi)\rvert$	$\lvert u_0(\xi)\rvert$	Absolute error
0.1	0.9922415192	0.9950207490	0.0027792298
0.5	0.8854857025	0.8868188840	0.0013331815
1	0.6524340490	0.6480542737	0.0043797754
1.5	0.4397488886	0.4250960349	0.0146528537
2	0.2967042854	0.2658022288	0.0309020566
2.2	0.2586548862	0.2189185789	0.0397363073
3	0.1965393174	0.0993279274	0.0972113900

Table 2. Comparison between $|u_{1\mathrm{hom}}(\xi)|$ and $|u_0(\xi)|$ when $\varepsilon = 0.001$.

ξ	$\lvert u_{1\mathrm{hom}}(\xi)\rvert$	$\lvert u_0(\xi)\rvert$	Absolute error
0.1	0.9947428260	0.9950207490	0.0002779230
0.5	0.8866855658	0.8868188840	0.0001333182
1	0.6484922512	0.6480542737	0.0004379775
1.5	0.4265613203	0.4250960349	0.0014652854
2	0.2688924345	0.2658022288	0.0030902057
2.2	0.2228922096	0.2189185789	0.0039736307
3	0.1090490664	0.0993279274	0.0097211390

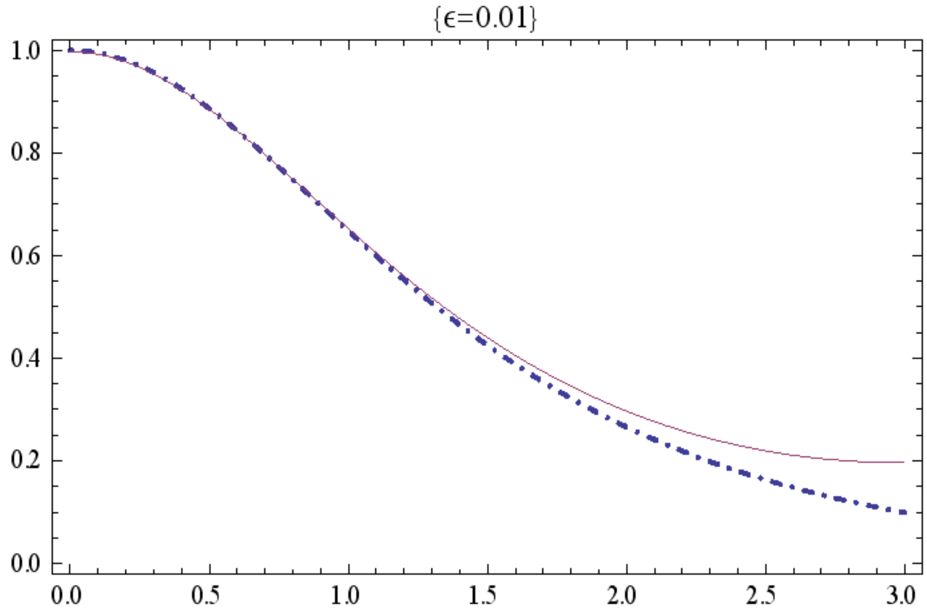

Figure 1. Comparison between the curves of solutions $|u_{1\mathrm{hom}}(\xi)|$ (solid line) and $|u_0(\xi)|$ (dashed line) with $\varepsilon = 0.01$.

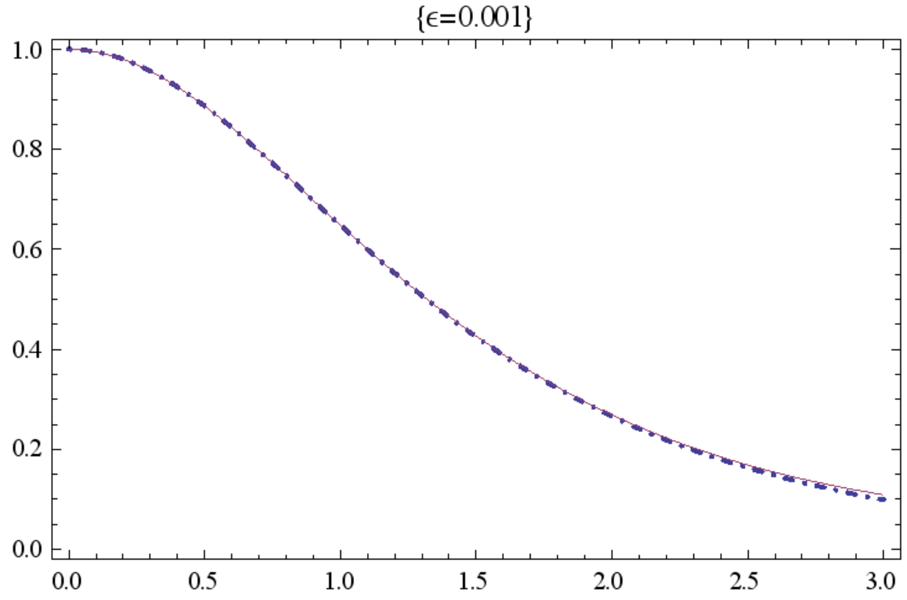

Figure 2. Comparison between the curves of solutions $|u_{1\text{hom}}(\xi)|$ (solid line) and $|u_0(\xi)|$ (dashed line) with $\varepsilon = 0.001$.

5. CONCLUSION

We researched a class of disturbed nonlinear Schrödinger equation with variable coefficients by using the homotopic mapping method, which is much more simple and efficient than some other asymptotic method such as perturbation method etc, the Jacobi-like function approximate solution with arbitrary degree of accuracy for the disturbed equation is researched, which shown that this method can be used to the soliton equation with complex variables, but it is still worth to research that whether or not the method can be used to the system with high dimension and high order.

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