Mathematical and Computational Applications, Vol. 20, No. 1, pp. 62-75, 2015
http://dx.doi.org/10.19029/mca-2015-006

A METHOD OF HYBRID MULTIPLE ATTRIBUTES GROUP DECISION MAKING WITH RISK CONSIDERING DECISION-MAKERS' CONFIDENCE

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Abstract- With respect to different confidence on evaluation result from different decision-makers to the hybrid multi-attribute with risk, an approach to group decision-making based on prospect theory and projection theory is proposed. Firstly, a tuple is established to record the evaluation result and the hybrid decision information. Then the element of the tuple is changed into a single triangular fuzzy number by the transformation rule. Considering the confidence degree from single decision-maker to evaluation information, the group decision information is aggregated and the weights of attributes are calculated based on the intuitionistic fuzzy set theory. The improved projection method and prospect theory are proposed to rank the alternatives respectively. Finally, an application case is given to demonstrate the effectiveness and feasibility of the proposed approach.

Key Words- Confidence, Prospect theory, Projection method, Intuitionistic fuzzy set, Multi-attribute group decision-making

1. INTRODUCTION

Research on multiple attribute group decision making (MAGDM) provides a solution that decision-makers can select the best one in limited alternatives. As the real application environment becomes more complex and uncertain, the hybrid MAGDM with risk is advanced. It fits varied situation with changing attributes as well as their weights, and decision-makers having bounded rationality and risk preference.

Extensive research works have done in the field of MAGDM. Many previous works focused on the incompleteness of attribute value and their weights, or the diversity of their expression form. For example, T. Y. Chen developed an interactive method for handling MAGDM, when information about attribute weight is incompletely known and the attribute values are expressed as interval type-2 trapezoidal fuzzy numbers [1]. G. W. Wei established an optimization model based on the basic ideal of traditional grey relational analysis (GRA) method to get the attribute weights when they are incompletely known [2]. Z. S. Xu and J. Chen developed an interactive method which transformed fuzzy decision matrices into their expected decision matrices, when the attributes weights are partly known [3]. Several works paid attention to the preference, incomplete information and weights of decision makers. For example, J. H. Park and II Y. Park extended the TOPSIS method to solve MAGDM problems, when all the preference information provided by the decision-makers in interval-valued intuitionistic fuzzy environment [4]. B. Gülçin and Ç. Gizem took into account incomplete information of decision-makers and made a method to improve the effectiveness of the evaluation process [5]. Z. B. Wu and J. P. Xu provided the concepts of an individual consistency index and a group consensus index to aid the group consensus process while keeping an acceptable individual consistency for each decision with multiplicative preference relations [6]. Z. P. Chen and W. Yang derived the weights of decision makers by aggregating the individual intuitionistic fuzzy decision matrices into a collective intuitionistic fuzzy decision matrix [7]. I. Palomares proposed a graphical monitoring tool based on self-organizing maps, that provided a 2-D graphical interface whose information is related to expert preferences and their evolution during the process of group decision making [8].

Certainly, all kinds of operators, as the base of some approaches, are proposed in order to adapt the above different decision condition. Some classical operators include the arithmetic interval-valued intuitionistic fuzzy Choquet aggregation operator [9], the extended 2-tuple weighted geometric and the extended 2-tuple ordered weighted geometric operators [10], the linguistic generalized power aggregation operators [11], 2-Tuple linguistic hybrid arithmetic aggregation operators, hesitant fuzzy aggregation operators [13], some Hamacher aggregation operators[14], a series of intuitionistic uncertain linguistic aggregation operators[15,16,17,18], and so on.

As the decision making environment becomes more complex and more uncertain, the hybrid MAGDM with risk is advanced. It fits varied situation that the attributes and their weights change, or the decision-makers have risk preference. In recent years, many scholars, including H. K. Soung [19], C. J. Rao [20], F. E. Genc [21], Guiwu Wei [22], Peide Liu [23-25], W. Hsu [26], Jui-Sheng Chou [27], have researched this kind of problem and achieved fruitful results.

The above research has a basic assumption that the decision-makers are perfect rationality and trust their assessment completely. But it is impossible, especially in the complex and uncertain environment. For example, when making a project investment decision, the decision-makers will predict or evaluate the economic benefits. Whatever the assessment result about the rate of return on investment is linguistic variable (Good), or crisp data (10%), or the interval number (from 10% to 15%), the decision-makers can’t trust the result completely because there are many uncontrolled factors. But the decision-maker can predict to what extent to trust this result. So it is necessary to explore the confidence issue about the decision-makers on assessment.

The confidence has become a hot issue in many fields, such as psychology, neurophysiology, marketing, computer science, and so on. However, very few papers have paid adequate attention to confidence in decision science. The aim of this paper is to propose a method of hybrid MAGDM with risk based on the decision-makers' confidence. So the rest of this paper is organized as follows: next section briefly introduces some basic concepts or methods related to our work. In section 3, we detail our proposed method, which considers decision-makers' confidence and bounded rationality. In section 4, an example is used to illustrate our method, and a comparative study about whether we consider the decision-makers' confidence is illustrated. The final section concludes.

2. PRELIMINARIES

2.1. Prospect theory

The prospect theory (PT), found and defined by Kahneman and Tversky [28], thinks individuals' choices are uncertain at the risk-involved environment and proposes a novel behavior model in decision research. The prospect value in the PT can be calculated by the following equation:

$$V = \sum_{i=1}^{n} \pi\left(p_i\right) v\left(x_i\right) \tag{1}$$

where $v\left(x_i\right)$ is the S-shape value function, it describes how individuals determine the subjective values of outcomes. It can be represented as follows:

$$v\left(x_i\right) = \begin{cases} x_i^{a}, x_i \geq 0 \\ -\theta\left(-x_i\right)^{\beta}, x_i < 0 \end{cases} \tag{2}$$

$\pi(p_i)$ is the probability weighting function, which define the relations between probabilities and decision weights. The most common probability weighting function is defined by Wu [29], which is showed as following:

$$\pi\left(p_i\right) = \frac{p_i^r}{\left[p_i^r + \left(1 - p_i\right)^r\right]^{1/r}} \tag{3}$$

2.2. Fuzzy Set Theory

2.2.1 Triangular Fuzzy Number [30,31]

Definition 1: Let X be a universe set. A fuzzy subset A of X is defined with a membership function $\mu_A(x)$, which maps each element X in the subset A to a real number in the interval $[0,1]$. The functional value $\mu_A(x)$ indicates the grade of the membership of x in A. When $\mu_A(x)$ is large, its grade of membership of x in A is strong. A tuple $A=\left[a_1,a_2,a_3\right]$ ($a_1 \leq a_2 \leq a_3$) is called a triangular fuzzy number when its membership function is

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x < a_2 \\ 1, & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 < x \le a_3 \\ 0, & x < a_1 \quad or \quad x > a_3 \end{cases} \tag{4}$$

where a_1, a_2, a_3 are real numbers, and these constants reflect the fuzziness of the evaluation date. Any real number r can also be transformed triangular fuzzy number$[r, r, r]$.

Definition 2: Fuzzy decision-making matrix $A = (a_{ij})_{n\times m}$ is composed of triangular fuzzy number $a_{ij} = [a_{ij}^1, a_{ij}^2, a_{ij}^3]$. Normalize each element in the matrix $A = (a_{ij})_{n\times m}$ into a corresponding elements in matrix $R = (r_{ij})_{n\times m}$, $r_{ij} = [r_{ij}^1, r_{ij}^2, r_{ij}^3]$ can be got by using the following formulas:

$$r_{ij}^p = \frac{a_{ij}^p - \min_j(a_{ij}^1)}{\max_j(a_{ij}^3) - \min_j(a_{ij}^1)}, \; p = 1,2,3 \qquad \textit{for utility type } a_{ij} \tag{5}$$

$$r_{ij}^p = \frac{\max_j(a_{ij}^3) - a_{ij}^p}{\max_j(a_{ij}^3) - \min_j(a_{ij}^1)}, \; p = 1,2,3 \qquad \textit{for cost type } a_{ij} \tag{6}$$

Definition 3: $A=[a_1,a_2,a_3]$ and $B=[b_1,b_2,b_3]$ are two triangular fuzzy numbers, the distance between A and B is defined as follows:

$$d_\lambda(A,B) = \frac{[(a_1+a_2)-(b_1+b_2)](1-\lambda)+[(a_2+a_3)-(b_2+b_3)]\lambda}{2} \tag{7}$$

where $\lambda \in [0,1]$, λ indicates the attitude of the decision makers to the risk.

2.2.2 Intuitionistic Fuzzy Set [32,33]

Definition 4: Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. The intuitionistic fuzzy sets A in X is an object having the form: $A = \{\langle x_j, \mu_A(x_j), v_A(x_j) | x_j \in X \rangle\}$.
where $\mu_A : X \mapsto [0,1], x_j \in X \to \mu_A(x_j) \in [0,1]$,
and $v_A : X \mapsto [0,1], x_j \in X \to v_A(x_j) \in [0,1]$.

For each $\forall x_j \in X$, $0 \le \mu_A(x_j) + v_A(x_j) \le 1$, then $\mu_A(x_j)$ and $v_A(x_j)$ represent the membership degree and non-membership degree of the element $x_j \in X$ to the set $A \subseteq X$ respectively. And define $\pi_A(x_j) = 1 - \mu_A(x_j) - v_A(x_j)$ to be the intuitionistic index of the element x_j in the set A, which is the degree of indeterminacy membership of the element x_j in the set A. It is obvious that $0 \le \pi_A(x_j) \le 1$ for $\forall x_j \in X$.

2.3 The Projection Method

In the projection method, a vector represents an assessment, and it sorts them with the angles between every evaluation result and the ideal one.

Definition 5: Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ be two vectors, then the projection $Q(\alpha, \gamma)$ can be defined as follows:

$$Q(\alpha,\gamma) = \left(\sum_{i=1}^{n} \alpha_i \gamma_i\right) \cdot \sqrt{\sum_{i=1}^{n} \alpha_i^2} \Bigg/ \left(\sqrt{\sum_{i=1}^{n} \alpha_i^2} \cdot \sqrt{\sum_{i=1}^{n} \gamma_i^2}\right) = \sum_{i=1}^{n} \alpha_i \gamma_i \Bigg/ \sqrt{\sum_{i=1}^{n} \gamma_i^2} \tag{8}$$

where the projection value $Q(\alpha, \gamma)$ is the projection of the vector α on the vector γ [34]. We can improve the projection method by considering the element importance in the vector. The weight $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is defined and the improved projection value is obtained as follows:

$$Q(\alpha,\gamma,\omega) = \frac{\left(\sum_{i=1}^{n} \omega_i \alpha_i \gamma_i\right) \cdot \sqrt{\sum_{i=1}^{n} (\omega_i \alpha_i)^2}}{\left(\sqrt{\sum_{i=1}^{n} (\omega_i \alpha_i)^2} \cdot \sqrt{\sum_{i=1}^{n} (\omega_i \gamma_i)^2}\right)} = \frac{\sum_{i=1}^{n} \omega_i \alpha_i \gamma}{\sqrt{\sum_{i=1}^{n} (\omega_i \gamma_i)^2}} \tag{9}$$

Obviously, the greater the value $Q(\alpha, \gamma, \omega)$, the closer the direction of the vector α to that of γ .

3. A DECISION-MAKING METHOD CONSIDERING CONFIDENCE

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes, and $DM = \{DM_1, DM_2, \ldots, DM_q\}$ be a set of decision-makers. Use a tuple $E_{ij}^k = (l_{ij}^k, u_{ij}^k, e_{ij}^k, r_{ij}^k, i_{ij}^k)$ to record that the decision-maker DM_k makes the evaluation for attribute C_j in the alternative A_i.

$l_{ij}^k, u_{ij}^k, e_{ij}^k$ mean the accepted minimum, expected optimum statement (maximum) and evaluation result respectively. And they can be expressed by crisp numbers, triangular fuzzy numbers and uncertain linguistic variables.

r_{ij}^k is a intuitionistic fuzzy variable of attribute weights, $r_{ij}^k = \left(\mu(r_{ij}^k), v(r_{ij}^k)\right)$, where $\mu(r_{ij}^k)$, $v(r_{ij}^k)$ are the degree of membership and the degree of non-membership of the weight of attribute C_j to the fuzzy concept "importance" respectively. $\mu(r_{ij}^k), v(r_{ij}^k) \in [0,1]$ and $\mu(r_{ij}^k) + v(r_{ij}^k) \le 1$ $\left(i=1,2,\ldots,m; j=1,2,\ldots,n; k=1,2,\ldots,q\right)$.

i_{ij}^k is a intuitionistic fuzzy variable for the confidence degree of evaluation result. $i_{ij}^k = \left(\mu(i_{ij}^k), v(i_{ij}^k)\right)$, where $\mu(i_{ij}^k)$ and $v(i_{ij}^k)$ are the degree of membership and the degree of non-membership of evaluation result to the fuzzy concept "confidence", respectively. $\mu(i_{ij}^k), v(i_{ij}^k) \in [0,1]$ and $\mu(i_{ij}^k) + v(i_{ij}^k) \le 1$, $\left(i=1,2,\ldots,m; j=1,2,\ldots,n; k=1,2,\ldots,q\right)$.

In the following, we will rank the alternative A_i, considering that the decision-makers are bounded rationality and they have incomplete confidence in their evaluation result. Our method involves the following steps:

Step 1. Form a decision-makers term and collect their assessment

The decision-makers term is composed of many experts in some field. They can make their evaluation based on their domain knowledge, experience and judgment. Use a tuple $E_{ij}^k = \left(l_{ij}^k, u_{ij}^k, e_{ij}^k, r_{ij}^k, i_{ij}^k\right)$ to record their evaluation information to facilitate the next steps.

Step 2. Transform the different data type into triangular fuzzy number

Uncertain linguistic term set $S = \{s_t \mid t = 0,1,\ldots,l\}$ (l is an even number) is composed of non-negative integers, where s_t represents a possible value for a linguistic variable. And the set is ordered: $s_\alpha \ge s_\beta$, if $\alpha \ge \beta$. So s_0 and s_l are the lower and upper limits, respectively. As the increase of t in the set S, the evaluation meaning is enhancing. The uncertain linguistic term set can be transformed into triangular fuzzy number by some defined laws. For example, when $l = 6$, the Tab.1 can be got.

Table 1. Triangular Fuzzy Number and Corresponding Linguistic Terms

Linguistic term	**Linguistic meaning**	**Abbreviation**	**Fuzzy scales**
S_0	Extreme bad	EB	(0,0,0.1)
S_1	Very bad	VB	(0,0.1,0.2)
S_2	Bad	B	(0.2,0.3,0.4)
S_3	Medium	M	(0.4,0.5,0.6)
S_4	Good	G	(0.6,0.7,0.8)
S_5	Very good	VG	(0.8,0.9,1)
S_6	Extreme good	EG	(0.9,1,1)

Step 3. Defuzzify, aggregate and normalize

$$\bar{l}_{ij} = \min_{k=1}^{q}(l_{ij}^k), i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; \tag{10}$$

$$\bar{u}_{ij} = \max_{k=1}^{q}(u_{ij}^k), i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; \tag{11}$$

where $\bar{l}_{ij}, \bar{u}_{ij}$ are the accepted minimum, expected optimum state (maximum) of group decision-makers to the attribute C_j in the alternative A_i.

The Area Compensation Method [35] is used to defuzzy i_{ij}^k, which can be showed by the following Eq.(12):

$$F\left(p_{ij}\right) = \frac{1}{2}\int_0^1 \left(\inf_{x\in R} p_{ij}^{\alpha} + \sup_{x\in R} p_{ij}^{\alpha}\right) d\alpha \tag{12}$$

In the process of decision making, the degree of decision-makers' should be considered owing to all kinds of objective and subjective influences. So let $\bar{e}_{ij}$ be the aggregation assessment information based on the degree of decision-makers' confidence. When the weights of decision-makers are same, $\bar{e}_{ij}$ can be calculate by the Eq.(13):

$$\bar{e}_{ij} = \sum_{k=1}^{q} e_{ij}^k \cdot F\left(i_{ij}^k\right) \Big/ q \tag{13}$$

Normalize $\bar{l}_{ij}, \bar{u}_{ij}, \bar{e}_{ij}$ to obtain $\tilde{l}_{ij}, \tilde{u}_{ij}, \tilde{e}_{ij}$ by the Eq.(5) and Eq.(6), respectively.

Step 4. Obtain the negative ideal solution and the positive ideal solution

The negative ideal solution B and the positive ideal solution G can be obtained by the following two functions:

$$B = \left\{B_1, B_2, \ldots B_n\right\} = \left\{\min_{1\le i\le m}\left(\tilde{l}_{i1}\right), \min_{1\le i\le m}\left(\tilde{l}_{i2}\right), \ldots, \min_{1\le i\le m}\left(\tilde{l}_{in}\right)\right\} \tag{14}$$

$$G = \left\{G_1, G_2, \ldots G_n\right\} = \left\{\max_{1\le i\le m}\left(\tilde{u}_{i1}\right), \max_{1\le i\le m}\left(\tilde{u}_{i2}\right), \ldots, \max_{1\le i\le m}\left(\tilde{u}_{in}\right)\right\} \tag{15}$$

Step 5. Derive the value function

When the negative ideal solution B is selected as reference point, each alternative A_i is better than it, the decision makers gain and are risk-averse, so $\lambda < 0.5$. When the positive ideal solution G is selected as reference point, each alternative A_i is inferior to it, the decision makers are loss and risk-seeking, so $\lambda > 0.5$. Then the distance between alternative A_i to negative or positive ideal solution can be gained:

$$d_{\lambda<0.5}\left(A_i, B\right) = \left\{d(\tilde{e}_{i1}, B_1), d(\tilde{e}_{i2}, B_2), \dots, d(\tilde{e}_{in}, B_n)\right\}$$

$$d_{\lambda>0.5}\left(A_i, G\right) = \left\{d(\tilde{e}_{i1}, G_1), d(\tilde{e}_{i2}, G_2), \dots, d(\tilde{e}_{in}, G_n)\right\}$$

So the value function can be got by using following Eq.(16) and Eq.(17):

$$v_{ij}^{+}(d_{\lambda<0.5}(\tilde{e}_{ij}, B_j)) = (d_{\lambda<0.5}(\tilde{e}_{ij}, B_j))^{\alpha} \tag{16}$$

$$v_{ij}^{-}(d_{\lambda>0.5}(\tilde{e}_{ij}, G_j)) = -\theta(-d_{\lambda>0.5}(\tilde{e}_{ij}, G_j))^{\beta} \tag{17}$$

Step 6. Obtain the weight of attributes with decision-makers' confidence

When the confidence degree of decision-maker DM_k is i_{ij}^k, the weight of the attribute C_j in alternative A_i lies in the closed interval $[\mu(r_{ij}^k), 1-v(r_{ij}^k)]$, which is the range of weight. In the same way, the range of confidence from the decision-makers lies in a closed interval $[\mu(i_{ij}^k), 1-v(i_{ij}^k)]$. If $\sum_{j=1}^{n}\mu(i_{ij}^k) \le 1$ and $\sum_{j=1}^{n}[(1-v(i_{ij}^k)] \ge 1$, then there must be a $\omega_{ij}, \omega_{ij} \in [0,1] (j=1,2,\dots,n)$ satisfying $\sum_{j=1}^{n}\mu(i_{ij}^k) \le \sum_{j=1}^{n}\omega_{ij} \le \sum_{j=1}^{n}[1-v(i_{ij}^k)]$ and $\sum_{j=1}^{n}\omega_{ij} = 1$. Considering the decision-makers' confidence to the attributes weights, we can deduce the optimal weight $\omega^* = (\omega_1^*, \omega_2^*, \dots, \omega_n^*)$ as follows:

$$\max\left\{ Az_i = \frac{\sum_{k=1}^{q}\sum_{j=1}^{n}[1-v(i_{ij}^k) - \mu(i_{ij}^k)]\omega_{ij}}{q} \right\} \tag{18}$$

$$s.t\begin{cases} \omega_{ij}^l \le \omega_{ij} \le \omega_{ij}^u & \left(j=1,2,\dots,n\right) \\ \sum_{j=1}^{n}\omega_{ij} = 1 \end{cases}$$

where $\omega_{ij}^l = \min\left(\mu(r_{ij}^1), \mu(r_{ij}^2), \dots, \mu(r_{ij}^q)\right)$, $\omega_{ij}^u = \max\left(1\text{-}v(r_{ij}^1), 1\text{-}v(r_{ij}^2), \dots, 1\text{-}v(r_{ij}^q)\right)\left(j=1,2,\dots,n\right)$ and $i=1,2,\dots,n$ meets above Eq.(18).

Step 7. Deduce the improved projection value and relative proximity

According to the optimal weight vector ω_{ij}^* and Eq.(9), the improved projection values can be calculate as follows:

$$Q_i^-\left(\tilde{e}_{ij},B\right)=\frac{\sum_{j=1}^{n}\omega_{ij}^*\left[\tilde{e}_{ij}^1B_j^1+\tilde{e}_{ij}^2B_j^2+\tilde{e}_{ij}^3B_j^3\right]}{\sqrt{\sum_{j=1}^{n}\left(\omega_{ij}^*\right)^2\left[\left(B_j^1\right)^2+\left(B_j^2\right)^2+\left(B_j^3\right)^2\right]}} \tag{19}$$

$$Q_i^+\left(\tilde{e}_{ij},G\right)=\frac{\sum_{j=1}^{n}\omega_{ij}^*\left[\tilde{e}_{ij}^1G_j^1+\tilde{e}_{ij}^2G_j^2+\tilde{e}_{ij}^3G_j^3\right]}{\sqrt{\sum_{j=1}^{n}\left(\omega_{ij}^*\right)^2\left[\left(G_j^1\right)^2+\left(G_j^2\right)^2+\left(G_j^3\right)^2\right]}} \tag{20}$$

where $i=1,2,\ldots,m$ is the number of alternative. $Q_i^-\left(\tilde{e}_{ij},B\right)$ and $Q_i^+\left(\tilde{e}_{ij},\mathrm{G}\right)$ are project value from the group assessment $\tilde{e}_{ij}$ to the negative ideal solution B and the positive ideal solution G, respectively.

Let RD_i be the relative proximity, it can be calculated by the Eq.(21):

$$RD_i=\frac{Q_i^+\left(\tilde{e}_{ij},G\right)}{Q_i^+\left(\tilde{e}_{ij},G\right)+Q_i^-\left(\tilde{e}_{ij},B\right)} \tag{21}$$

Calculate and rank the relative proximity RD_i, the larger the relative proximity, the better the corresponding alternative is.

Step 8. Calculate the composite prospect value and rank

The composite prospect value V_{A_i} is obtained by the following Eq.(22):

$$V_{A_i}=\sum_{j=1}^{n}v_{ij}^+\pi^+(\omega_{ij}^*)+\sum_{j=1}^{n}v_{ij}^-\pi^-(\omega_{ij}^*) \tag{22}$$

Obviously the larger the composite prospect value, the better the alternative is.

4. AN ILLUSTRATIVE EXAMPLE

Let us suppose there is a company, which wants to invest a project from three possible alternatives $A_i\left(i=1,2,3\right)$. The decision-makers $DK_k\left(k=1,2,3\right)$ make decision according to three attributes $C_j\left(j=1,2,3\right)$, C_1,C_2 are respective social and ecological benefits, which can be expressed by seven linguistic terms (as showed in Tab.1). C_3 is economic benefit, which can be expressed by crisp numbers or triangular fuzzy numbers. Every decision-maker DK_k makes and records his assessment in the form of a tuple $E_{ij}^k = \left(l_{ij}^k, u_{ij}^k, e_{ij}^k, r_{ij}^k, i_{ij}^k\right)$. The group decision-makers will find the best alternative by our method. All evaluation information is showed as following Tab.2.

Table 2. The Evaluation Information from Group Decision-makers

		DM$_1$	**DM$_2$**	**DM$_3$**
A$_1$	C$_1$	M,EG,G,[0.25, 0.4],[0.9,0.05]	G,EG,VG,[0.35, 0.4],[0.8,0.1]	G,EG,VG,[0.3,0.45],[0.9,0.05]
	C$_2$	G, EG,G,[0.35,0.45],[0.85,0.05]	G, EG,VG,[0.30,0.5],[0.85,0.1]	M, EG,G,[0.3,0.5],[0.85,0.1]
	C$_3$	(4,6,8),(8,10,10),(6,8,10), [0.25,0.4],[0.9,0.05]	(5,5,5),(10,10,10),(7,8,9), [0.3,0.4],[0.9,0.1]	(6,6,6),(10,10,10),(6,8,9), [0.35,0.4],[0.9,0.05]
A$_2$	C$_1$	M,EG,G,[0.25 0.45],[0.9,0.05]	M,EG,VG,[0.25 0.4],[0.9,0.05]	B,EG,EG,[0.35 0.4],[0.8,0.05]
	C$_2$	B, EG,VG,[0.25,0.45],[0.85,0.1]	M, EG,EG,[0.35,0.35],[0.8,0.15]	M, EG,M,[0.35,0.45],[0.82,0.10]
	C$_3$	(4,5,6),(10,10,10),(7,7,7), [0.2,0.5],[0.86,0.05]	(4,4,4),(10,10,10),(8,9,10), [0.3,0.5],[0.75,0.2]	(4,5,6),(10,10,10),(7,8,9), [0.35,0.4],[0.75,0.1]
A$_3$	C$_1$	M,EG,G,[0.25 0.4],[0.75,0.12]	M,EG,VG,[0.3, 0.4],[0.85,0.1]	B,EG,VG,[0.3, 0.45],[0.82,0.1]
	C$_2$	B, EG,VG,[0.35,0.4],[0.9,0.1]	M, EG,VG,[0.3,0.4],[0.9,0.05]	M, EG,EG,[0.35,0.5],[0.75,0.15]
	C$_3$	(4,4,4),(8,9,10),(9,10,10), [0.25,0.5],[0.88,0.1]	(4,4,4),(9,10,10),(9,10,10), [0.3,0.45],[0.8,0.1]	(5,5,5),(10,10,10),(8,9,10), [0.2,0.5],[0.8,0.15]

The evaluation information can be aggregated and normalized by the Eq.(10)~(13), the result is showed by the Tab.3.

Table 3. The Aggregation and Normalization of Information

		l_{ij}	u_{ij}	$\sum DM$	r_{ij}	i_{ij}
A1	**C1**	(0.4,0.5,0.6),	(0.9,1,1),	(0.6583,0.7483,0.8383),	(0.25,0.40)	(0.80,0.05)
	C2	(0.4,0.5,0.6),	(0.9,1,1),	(0.5300,0.6183,0.7067),	(0.30,0.45)	(0.85,0.05)
	C3	(0,0,0),	(1,1,1),	(0.3000,0.6000,0.6400),	(0.25,0.40)	(0.90,0.05)
A2	**C1**	(0.2,0.3,0.4),	(0.9,1,1),	(0.5483,0.6392,0.7300),	(0.25,0.40)	(0.80,0.05)
	C2	(0.2,0.3,0.4),	(0.9,1,1),	(0.5955,0.6808,0.7387),	(0.25,0.35),	(0.80,0.05)
	C3	(0,0,0),	(1,1,1),	(0.3583,0.4473,0.5361),	(0.20,0.40)	(0.75,0.10)
A3	**C1**	(0.2,0.3,0.4),	(0.9,1,1),	(0.6257,0.7107,0.7957),	(0.25,0.40)	(0.75,0.10)
	C2	(0.2,0.3,0.4),	(0.9,1,1),	(0.7267,0.8142,0.8750),	(0.30,0.40)	(0.75,0.05)
	C3	(0,0,0),	(1,1,1),	(0.5700,0.7125,0.7583),	(0.20,0.45)	(0.80,0.10)

So the negative ideal solution B and the positive ideal solution G can be obtained by Eq.(14)~(15). $B = \{(0.2, 0.3, 0.4), (0.2, 0.3, 0.4), (0, 0, 0)\}$, $G = \{(0.9,1,1), (0.9,1,1), (1,1,1)\}$.

The distance between the alternative to the negative or positive ideal solution are calculated:

$$d_{\lambda=0.3}(A,B)=\begin{bmatrix}0.4503 & 0.3207 & 0.5010\\ 0.3410 & 0.3796 & 0.4295\\ 0.4137 & 0.5127 & 0.6695\end{bmatrix},\qquad d_{\lambda=0.8}(A,G)=\begin{bmatrix}-0.2147 & -0.3452 & -0.4140\\ -0.3236 & -0.2946 & -0.5261\\ -0.2538 & -0.1602 & -0.2834\end{bmatrix}$$

The positive prospect matrix V^+ and negative prospect matrix V^- of the alternatives can be obtained by Eq.(16)~(17), where $\alpha=\beta=0.88$ and $\theta=2.25$ came from the experimental data [36].

$$V^+=\begin{bmatrix}0.4955 & 0.3676 & 0.5443\\ 0.3880 & 0.4264 & 0.4753\\ 0.4599 & 0.5555 & 0.7025\end{bmatrix},\qquad V^-=\begin{bmatrix}-0.5810 & -0.8824 & -1.0355\\ -0.8337 & -0.7676 & -1.2786\\ -0.6732 & -0.4490 & -0.7418\end{bmatrix}$$

The weights ω_{ij}^* of attributes $C_j\,(j=1,2,3)$ in the alternative $A_i\,(i=1,2,3)$ can be calculated by Eq.(18), then $\omega_1^*=(0.25,0.45,0.30)$, $\omega_2^*=(0.45,0.35,0.20)$ and $\omega_3^*=(0.50,0.30,0.20)$.

The improved projection values, which are the projection of the alternatives $A_i\,(i=1,2,3)$ on the negative or positive ideal solution considering the weights of attributes, can be got by Eq.(19) and Eq.(20): $Q_1^+=1.8045$, $Q_2^+=1.7619$, $Q_3^+=2.0693$, $Q_1^-=1.5550$, $Q_2^-=1.5747$, $Q_3^-=1.7522$. Then relative proximity can be calculated by Eq.(21): $RD_1=0.5371$, $RD_3=0.5415$. Obviously $RD_3>RD_1>RD_2$, the best alternative is A_3.

The composite prospect value can be calculated by Eq.(22), which are $V_{A_1}=-0.4202$, $V_{A_2}=-0.5333$, $V_{A_3}=-0.0899$ respectively. Rank the alternatives to get $V_{A_3}>V_{A_1}>V_{A_2}$, the most desirable alternative is A_3.

The above two methods can gain the same conclusion. As a comparison, we still use the same two methods to analysis the above example, without considering the decision-makers' confidence. The results are shown in following table 4.

Table 4. The Result of Example and Comparison

Method	**The method of prospect theory**			**The projection method**		
Condition	**Non-confidence**	**confidence**	——	**Non-confidence**	**confidence**	——
——	$\overline{V_{A_i}}$	V_{A_i}	**Deviation** Δ	$\overline{RD_i}$	RD_i	**Deviation** Δ
A_1	-0.2133	-0.4202	0.2069	0.5363	0.5344	0.0019
A_2	0.1509	-0.5333	0.6842	0.5378	0.5281	0.0097
A_3	0.5184	-0.0899	0.6083	0.5569	0.5415	0.0154
Rank	A_3,A_2,A_1	A_3,A_1,A_2	——	A_3,A_2,A_1	A_3,A_1,A_2	——

From Table 4, we can know, when considering decision-makers' trust in the evaluation information, the result by the method of prospect theory and projection are both less than that of without confidence. Because when decision-makers don't consider their trust in the evaluation information, the evaluation information is complete certainty, and the evaluation results which they gain are the most perfect and ideal. But the confidence has weakened the evaluation result, which is gained under decision-makers' entirely rationality, and even it produces a relatively large deviation. In this case, the maximum deviation which can be generated by the method of prospect theory is 06842. Meanwhile the sorted results are not entirely consistent when we compare the sorted results based on confidence with that of without confidence.

5. CONCLUDING REMARKS

The traditional multi-attributes group decision methods fit the situation where decision-makers are perfect rationality. However, the decision-makers do not trust the evaluation information completely when they make decision in real life, which draws the concept of bounded rationality. In this paper, we focus on decision-makers' confidence in evaluation result and propose a method of hybrid multi-attribute group decision-making with risk. Firstly we transform the hybrid evaluation information into triangular fuzzy numbers, then defuzzy, aggregate and normalize them. Taking the single decision-maker's confidence into consideration, we calculate the weight of attributes based on intuitionistic fuzzy theory. We find that the weight value of same attribute may be different when the decision-makers have different confidence to evaluation result. This result accords with reality. The alternatives are ranked based on the improved projection method and prospect theory respectively. At last, a case demonstrates above two methods and proves the feasibility and effectiveness of our methods. All these have proven that considering the confidence of decision-makers is very necessary and reasonable in the process of decision making.

6. ACKNOWLEDGMENT

The author would like to express appreciation to the editor, the anonymous reviewers for their very helpful comments that improved the paper. This work is supported by National Science Foundation of China under Grant No.41271139, Science Research Foundation of Hunan Education Office under Grant No.1404444.

7. REFERENCES

1. T. Y. Chen, An interactive method for multiple criteria group decision analysis based on interval type-2 fuzzy sets and its application to medical decision making, *Fuzzy Optim Decis Making* **12,** 323–356, 2013.
2. G. W. Wei, GRA method for multiple attribute decision making with incomplete weight information in intuitionistic fuzzy setting, *Knowledge-Based Systems* **23(3),** 243–247, 2010.
3. Z. S. Xu and J. Chen, An interactive method for fuzzy multiple attribute group decision making, *Information Sciences* **177(1)**, 248–263, 2007.

4. J. H. Park, II Y. Park, Y. C. Kwun and X. G. Tan, Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment, *Applied Mathematical Modeling* **35,** 2544–2556, 2011.
5. B. Gülçin and Ç. Gizem, A new incomplete preference relations based approach to quality function deployment, *Information Sciences* **206**, 30–40, 2012.
6. Z. B Wu and J.P. Xu, A consistency and consensus based decision support model for group decision making with multiplicative preference relation, *Decision Support System* **52**,757–767, 2012.
7. Z. P Chen and W. Yang, A new multiple attribute group decision making method in intuitionistic fuzzy setting, *Applied Mathematical Modeling* **35**, 4424–4437, 2011.
8. Iv´an Palomares, MENTOR: A graphical monitoring tool of preferences evolution in large-scale group decision making, *Knowledge-based System* **58**, 66–74, 2014.
9. F. G. Meng and J. Tang, Interval-valued intuitionistic fuzzy multi-attribute group decision making based on cross entropy measure and choquet integral, *International Journal of Intelligent Systems* **28(12)**, 1172–1195, 2013.
10. G. W. Wei, A method for multiple attribute group decision making based on the ET-WG and ET-OWG operators with 2-tuple Linguistic Information, *Expert Systems with Applications* **3**, 7895–1846, 2010.
11. L. G. Zhou and H. Y Chen, A generalization of the power aggregation operators for linguistic environment and its application in group decision making, *Knowledge-Based Systems* **26,** 216–224, 2012.
12. S. P. Wan, 2-Tuple linguistic hybrid arithmetic aggregation operators and application to multi-attribute group decision making, *Knowledge-Based Systems* **45**, 31–40, 2013.
13. M. M Xia, Z. S. Xu and N. Chen, Some hesitant fuzzy aggregation operators with their application in group decision making, *Group Decision and Negotiation* **22(2)**, 259–279, 2013.
14. P. D. Liu, Some Hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making, *IEEE Transactions on Fuzzy systems* **22(1)**, 83 – 97, 2014.
15. P. D. Liu, F. Jin, Methods for aggregating intuitionistic uncertain linguistic variables and their application to group decision making, *Information Sciences* **205(1)**, 58–71, 2012.
16. P. D. Liu, Z. M. Liu, X. Zhang, Some intuitionistic uncertain linguistic heronian mean operators and their application to group decision making, *Applied Mathematics and Computation* **230(1)**, 570–586, 2014.
17. P. D. Liu, Y. M. Wang, Multiple attribute group decision making methods based on intuitionistic linguistic power generalized aggregation operators, *Applied soft computing* **17**, 90–104, 2014.
18. P. D. Liu, Some generalized dependent aggregation operators with intuitionistic linguistic numbers and their application to group decision making, *Journal of Computer and System Sciences* **79 (1)**, 131–143, 2013.
19. H. K. Soung, S. A. Byeong, Interactive group decision making procedure under incomplete information, *European Journal of Operational Research* **116(3)**, 498–507, 1999.
20. C. J. Rao, X. P. Xiao and P Jin. Novel combinatorial algorithm for the problems of

fuzzy grey multi-attribute group decision making, *Journal of Systems Engineering and Electronics* **18(4)**, 774–780, 2007.
21. F. E. Genc, K. Mustafa and D. Akay, A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOP-SIS method, *Expert Systems with Applications* **36(8)**, 11363–11368, 2009.
22. G. W. Wei, Uncertain linguistic hybrid geometric mean operator and its application to group decision making under uncertain linguistic environment, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **17(2)**, 251–267, 2009.
23. P. D. Liu and X. Zhang, The study on multi-attribute decision-making with risk based on linguistic variable, *International Journal of Computational Intelligence Systems* **3(5),** 601–609, 2010.
24. P. D. Liu, J.T. Xia, P.P. Chi, A method of multi-attribute decision making under risk based on interval probability, *Journal of intelligent and fuzzy systems* **26 (6),** 3005–3011, 2014.
25. P. D. Liu, F. Jin, X. Zhang, Y. Su and M. H. Wang, Research on the multi-attribute decision-making under risk with interval probability based on prospect theory and the uncertain linguistic variables, *Knowledge-Based Systems* **24(4)**, 554–561, 2011.
26. W. Hsu, C. P. Tseng, W. L. Chiang and C. W. Chen, Risk and uncertainty analysis in the planning stages of a risk decision-making process, *Natural Hazards* **61**, 1355–1365, 2012.
27. J. S. Chou, C. S. Ongkowijoyo, Risk-based group decision making regarding renewable energy schemes using a stochastic graphical matrix model, *Automation in Construction*, **37**, 98–109, 2014.
28. D. Kahneman, A. Tversky, Prospect theory: an analysis of decision under risk, *Economica* **47**, 263-291, 1979.
29. G. Wu, R. Gonzalez, Curvature of the probability weighting function, *Management Science* **42**, 1676-1690, 1996.
30. L. A. Zadeh, Fuzzy set, *Information and Control* **8**, 338-353, 1965.
31. L.A. Zadeh, Towards a generalized theory of uncertainty (GTU)—an outline, *Information Sciences* **172**, 1-40, 2005.
32. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20**, 87-96, 1986.
33. K. T. Atanassov, Two theorems for intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **110**, 257-269, 2000.
34. Z.S. Xu, Q. L. Da, Projection method for uncertain multi-attribute decision making with preference information on alternatives, *International Journal of Information Technology & Decision Making* **57(3)**, 429-434, 2004.
35. P. Fortemps M. Roubens, Ranking and defuzzification methods based on area compensation, *Fuzzy Sets Systems* **82**, 319-330, 1996.
36. R. H. Thaler, Tversky A., Kahneman D., Alan Schwartz, The effect of myopia and loss aversion on risk taking: an experimental test, *The Quarterly Journal of Economics* **112(2)**, 647-661, 1997.