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Cost Efficiency Evaluation Based on a Data Envelopment Analysis Approach by Considering Undesirable Outputs on the Basis of the Semi-Disposability Assumption

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Abstract: It is considerably important to calculate the cost efficiency in data envelopment analysis for the efficiency evaluation of decision-making units. The present paper develops the classical cost efficiency model in which all the input prices are constant and certain for each decision-making unit, considering undesirable outputs under the semi-disposability assumption. The proposed models are interval and uncertain under the constant returns to scale and also variable returns to scale assumptions, for the easy solution of which, their lower and upper bounds are obtained on the basis of the theorem presented in the text. In order to simulate the proposed models and show their scientific capabilities, additionally, 56 electricity producing thermal power plants in Iran were studied in 2015. Results of the present study show that under both assumptions of constant returns to scale and variable returns to scale, the highest cost efficiency bounds belonged to the combined and steam cycle power plants. Moreover, the average of lower and upper cost efficiency bounds of the power plants under study were 34% and 35%, respectively, in 2015, under the constant returns to scale assumption, and 52% and 54%, respectively, under the variable returns to scale assumption.

Keywords: cost efficiency; undesirable outputs; semi-disposability assumption; data envelopment analysis

1. Introduction

Data envelopment analysis (DEA) is a method based on mathematical programming, and it was first presented by Chanrnes et al. [1]. This method is used to evaluate the relative efficiency of decision-making units (DMUs) doing the same functions, such as assessment and comparison of relative efficiency of organizational units such as the public departments of a ministry, schools, hospitals, department stores, bank branches, etc., which have homogeneous decision-making units. DEA is also widely used in ranking of DMUs. There are several ranking methods via DEA. For a survey of ranking methods, see Hadad and Hanani [2].

The most important information obtained for the data envelopment analysis models is the cost efficiency of DMUs. In fact, one of the most important aspects of the production analysis of organizations is the measurement of their cost and revenue efficiency [3]. In order to calculate the cost efficiency of DMU_o (the unit under evaluation), the cost efficiency model is indeed for seeking a unit that consumes the lowest cost for buying not greater than the inputs of the unit under evaluation so that they can produce outputs equal to the outputs of the unit under evaluation. The calculations of cost efficiency are used for cases in which prices are clearly known in each DMU (see Tone [4], Tone and Sahoo [5], Khorramabadi et al. [6], Ghiyasi [7], and Ashrafi and Kaleibar [8]), and even for cases in which the price information is incomplete and uncertain (see Thompson et al. [9], Schaffnit et al. [10], Kuosmanen and Post [11,12], Camanho and Dyson [13], Fang and Li [14,15], Fang and Hecheng [16], and Puri and Yadav [17]). These cases show that the DEA models can present a strong approximation of cost efficiency even when there is price uncertainty. Cost efficiency was first introduced by Farrell [3] and then developed by Färe et al. [18,19]. Where the input price information of each DMU is available, the cost efficiency evaluation can be conducted on the basis of Farrell's approach, and in other cases, in which the exact prices of inputs are not known in each DMU, but only the lower and upper bounds of such prices are available, it is necessary to propose an appropriate method for the calculation of cost efficiency. Research on the cost efficiency estimation with unknown and incomplete prices was first conducted by Thompson et al. [9] and Schaffnit et al. [10]. Results of their studies covered the calculation of an upper bound for cost efficiency using weight restriction techniques, in the form of the input cone assurance region (Thompson et al. [20]), and the cost efficiency lower bound was first proposed by Kuosmanen and Post [11]. Camanho and Dyson [13] developed some models for the estimation of cost efficiency bounds by using weight restrictions, considering the input price uncertainty.

The 2025 Perspective Document introduces Iran as a developed, active, and effective country in the world economy. Based on this document, one of the ways to progress and success in this country is to increase the competition power against other regional countries in terms of innovation, efficiency improvement, and internal production productivity so as to achieve economic development through technical, economic, and environmental considerations. Most firms and businesses including factories, hospitals, and trade centers are producing both desirable and undesirable outputs such as the emission of greenhouse gases, production wastes, poisoning sewers, harmful aerosols, and other undesirable effects such as noise pollution. Therefore, the modeling of such outputs has always received attention in the economic production theory. Such outputs, including the environmental factors, play an important role in the calculation of the environmental efficiencies of units. In the assessment of such units, the goal is to use a method that is compatible with the production theory so that it can decrease the undesirable outputs and increase the desirable outputs.

In a thermal power plant, for example, the generated electricity is the desirable output, and the generated NO_2 and CO_2 are the undesirable outputs. In order to evaluate the efficiencies of the units with undesirable outputs on the basis of DEA models, different approaches have been suggested, with one of them being ignoring the undesirable outputs, that is disregarding them in the production process. At any rate, ignoring the undesirable outputs means that they do not have any effects on the final evaluation, and therefore, this can give wrong or misleading results. In the new evaluations, therefore, the undesirable outputs are taken into considerations, and a new type of efficiency, called the environmental efficiency, is introduced since the environmental undesirable outputs can not be separated from their associated desirable outputs and a decrease of undesirable output leads to the decrease of its associated desirable output. Among researchers who have ignored undesirable outputs in their analysis, one can mention Pathomsiri et al. [21] and He et al. [22]. Another approach is to regard the undesirable outputs as the desirable inputs. The advantage of using this method is that different inputs can be given different weights and that the environmental efficiency can be calculated on the basis of the weights given to inputs [23]. The logic behind the modeling of undesirable outputs as the desirable inputs is that the lower values of undesirable outputs show a better performance of the model [24]. This method is called INP (It generates the same technology set as incorporating undesirable outputs U as inputs). Among the studies conducted on the basis of this method, one can mention the following: Pittman [25], Hailu and Veeman [26], Hailu [27], Korhonen and Luptacik [28], Sarkis and Cordeiro [24], and Yang and Pollitt [29]. Seiford and Zhu [30] maintained that if undesirable outputs are regarded as desirable inputs, the results obtained from the DEA method will not reflect the real production process, and on the other hand, one will seek to decrease the undesirable outputs and increase the desirable outputs, that is the simultaneous optimization of two opposite objects will be under consideration with these two types of output, although this is not important in the usual DEA methods.

Another approach to the efficiency evaluation of undesirable output units is based on the data transformation methods. In these methods, the undesirable outputs are transformed into desirable outputs by means of transformation functions, and therefore, the manner of undesirable output participation is based on transformation in these methods. One of the transformation methods is the additive inverse method, ADD (Additive inverse), in which the undesirable outputs U are transformed into desirable outputs with the $f(U) = -U$ value. This method was first proposed by Koopmans [31] and used and developed by Berg et al. [32]. Another type of transformation is based on $f(U) = -U + \beta$, called $TR\beta$, in which everything is dependent on β. Among those who have used this method, one can mention Ali and Seiford [33], Pastor [34], Scheel [35], Seiford and Zhu [30], Lin et al. [36], Zhou and Hu [37], and Liu et al. [38]. Another type of transformation is based on $f(U) = 1/U$, called multiplicative inverse, MLT (Multiplicative inverse), which was suggested by Golany and Roll [39]. Among those who used the transformation method, one can mention Lovell et al. [40] and Athanassopoulos and Thanassoulis [41].

The hyperbolic measurement approach is another approach to the undesirable outputs. In 1983, Pittman [42] published an article dealing with the undesirable outputs, in which he presented a model based on shadow prices. Inspired by Pittman's study, Färe et al. [43] tried to decrease the values of undesirable outputs. To this end, they had to divide the undesirable outputs by a number greater than one so that the solution could be found in the feasible space and be able to calculate the efficiency in the presence of undesirable outputs. In 1989, they published their findings in an article entitled "Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Nonparametric Approach". This article presented a different theory in which the desirable and undesirable outputs were combined into a single model. This approach allowed the desirable outputs to increase and the undesirable outputs to decrease at the same rate and proportion. On the other hand, an approach called the "weak disposability axiom" has been presented for dealing with the undesirable outputs. In recent years, many researchers have tried to model the undesirable outputs in the DEA framework. Färe et al. [43,44] were among the first who emphasized the modeling of undesirable outputs under the weak disposability axiom. Shephard [45,46] introduced the weak disposability axiom for the desirable and undesirable outputs. Based on this axiom, he presented the technology of treating the desirable and undesirable outputs. Färe and Grosskopf [47] explained that if the undesirable outputs were regarded as the desirable input, two problems would come to fore: first, the free disposability axiom between inputs and undesirable outputs implies that a finite amount of inputs can produce an infinite amount of undesirable outputs, which is contrary to physical laws. For example, using fixed values of energy, manpower, capital, and raw materials can produce infinite amounts of undesirable outputs, such as total suspended solids, and the like, which is physically impossible; secondly, the free disposability axiom does not clarify the relationship between the desirable and undesirable outputs. For example, it cannot clarify why pollutants can be decreased at the same rate and proportion of desirable outputs. This is known as the weak disposability axiom of desirable and undesirable outputs, introduced for the first time by Shephard, as was said above. In order to use the weak disposability axiom, the modeling presented by Färe and Grosskopf [47] applies only a single uniform abatement factor for every DMU, and the production possibility set is produced on the basis of this abatement factor. Kuosmanen [48] presented his non-parametric formulation of the weak disposability axiom, which preserves the linear property of the technology set. In his model, he considered a separate abatement factor for each DMU, on the basis of which he made the production possibility set. Among other studies based on the weak disposability axiom, one can refer to the following: Chung et al. [49], Zhou et al. [50], Podinovski and Kuosmanen [51], Kuosmanen and Kazemi Matin [52], Leleu [53], Fang [54], Lozano [55], and Yang et al. [56].

Another approach to the efficiency evaluation of units with undesirable outputs is the semi-disposability assumption. Recently, Chen et al. [57] developed the disposability concept for outputs, studying it in another perspective. They maintained that their method of evaluating the environment was more consistent with the actual production processes, and they called their new concept "semi-disposability". Based on this concept, through the use of advanced technologies and experienced management, and only by decreasing the desirable outputs by a small percentage, one can extensively decrease the undesirable outputs.

The electricity industry is a dynamic and effective industry, and it is closely related to the effective economic factors; therefore, it can play a significant role in terms of increased efficiency and productivity. In the electricity industry, production can be increased through two ways: increasing the production factors and better using the production factors by means of a better management of resources and new methods of combining them. Power plants (electricity production places) are the most important and the most capital intensive sections in the electricity industry. The ways of getting access to higher efficiency in power plants are the appropriate use of the workforce, the performance with the nominal capacity of power plants for the production of electricity by means of efficiency concepts, and paying attention to the management of production costs. Therefore, to gain all-out economic growth, and development, it is necessary to increase the efficiency and optimize the existing resources of every industry, and particularly, those of the electricity industry. To this end, the present study tries to evaluate the cost efficiencies of 56 thermal power plants of Iran in 2015 with a DEA approach, considering the undesirable outputs on the basis of the semi-disposability assumption.

The present study continues as follows: Section 2 presents the concept of cost efficiency. Section 3 deals with the extended strong disposability axiom, as well as the weak disposability axiom. Section 4 discusses the modeling of the production possibility set under the semi-disposability assumption. Section 5 presents the evaluation of cost efficiency, considering the undesirable outputs. Section 6 presents an experimental application for the practicing and assessment of the proposed model. Finally, Section 7 presents the conclusion and recommendations for future studies.

2. Cost Efficiency

Suppose that there is a set of K decision-making units $(DMU_k, k = 1, \ldots, K)$, as follows, with each using N inputs for the production of M outputs:

$$\begin{aligned} \mathbf{x}^k &= (x_1^k, \ldots, x_N^k)^T, \quad x_n^k \geq 0, \;\; \mathbf{x}^k \neq 0, \qquad n = 1, 2, \ldots, N, \\ \mathbf{v}^k &= (v_1^k, \ldots, v_M^k)^T, \quad v_m^k \geq 0, \;\; \mathbf{v}^k \neq 0, \qquad m = 1, 2, \ldots, M, \end{aligned} \tag{1}$$

The cost of buying raw materials (the cost vector) corresponding to the input vectors is denoted by $\mathbf{c} = (c_1, c_2, \ldots, c_N)^T$. Moreover, this cost vector is known and accessible and equal for each unit.

As was said in the Introduction, the cost efficiency model tries to find a unit that consumes the lowest cost for buying not greater than the inputs of the units under evaluation in order to produce the outputs equal to those of the unit under evaluation. Suppose that DMU_o is the unit under evaluation. The primal model of cost efficiency for the evaluation of DMU_o under the variable returns to scale (VRS) assumption is as follows [18]:

$$
\begin{aligned}
\min \quad & \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} z^k x_n^k \leq x_n, \qquad n = 1,2,\ldots,N, \\
& \sum_{k=1}^{K} z^k v_m^k \geq v_m^o, \qquad m = 1,2,\ldots,M, \\
& \sum_{k=1}^{K} z^k = 1, \\
& z^k \geq 0, \qquad k = 1,2,\ldots,K \\
& x_n \geq 0, \qquad n = 1,2,\ldots,N.
\end{aligned}
\tag{2}
$$

where $z^k (k = 1,2,\ldots,K)$ and $x_n (n = 1,2,\ldots,N)$ are, respectively, the intensity weights and the amount of inputs required for the production of a certain amount of outputs. As a matter of fact, Model (2) obtains the least cost for the production of any outputs, given the certain input cost vector $\mathbf{c}$. If $(\mathbf{x}^*, \mathbf{z}^*)$ is the optimal solution to Model (2), the cost efficiency of the o^{th} unit will be as follows:

$$
CE_{VRS}^o = \frac{\mathbf{c}^T \mathbf{x}^*}{\mathbf{c}^T \mathbf{x}^o} = \frac{\sum_{n=1}^{N} c_n x_n^*}{\sum_{n=1}^{N} c_n x_n^o} \tag{3}
$$

In fact, cost efficiency is defined as the minimum possible cost-to-existing cost ratio, with the value being always a number in the $[0, 1]$ interval.

Definition 1. *DMU_k is called cost efficient if and only if $CE_{VRS}^k = 1$.*

3. Weak Disposability Axiom and Extended Strong Disposability Axiom

As was said in the Introduction, some outputs in DEA might be undesirable outputs. For example, in an electricity producing power plant, the desirable output is the generated electricity, and the undesirable outputs are the generated SO_2 and CO_2 quantities. For this reason, modeling this type of output always receives the attention of researchers in the theory of economics and production. Färe et al. [43,44] were the first researchers to emphasize the modeling of undesirable outputs under the weak disposability axiom. Based on Shephard's viewpoint [45,46], Färe and Grosskopf presented the axiom as follows:

Consider K decision-making units $DMU_k (k = 1,\ldots,K)$. Suppose $\mathbf{x} = (x_1, x_2,\ldots,x_N) \in R_+^N$ is the input vector, $\mathbf{v} = (v_1, v_2,\ldots,v_M) \in R_+^M$ is the desirable output vector, and $\mathbf{w} = (w_1, w_2,\ldots,w_J) \in R_+^J$ is the vector of undesirable outputs. The weak disposability of outputs means that if the inputs are certain, a decrease in the undesirable outputs results in a decrease in desirable outputs in the same proportion. In other words, for the production possibility set T, we have:

$$
(\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T,\ 0 \leq \theta \leq 1 \Rightarrow (\mathbf{x}, \theta\mathbf{v}, \theta\mathbf{w}) \in T \tag{4}
$$

The weak disposability of outputs is also known as the "output congestion". In general, this axiom indicates that the decrease of some outputs needs a decrease corresponding to other outputs, or differently put, the decrease of an output is not possible without decreasing other outputs. For technologies producing both desirable and undesirable outputs, the weak disposability of outputs has been often imposed on the infrastructural technology in a way that any decrease of undesirable outputs needs a simultaneous decrease of desirable ones [58].

For example, if both electricity (desirable output) and SO_2 (undesirable output) are produced by burning coal, then, based on the weak disposability axiom, by keeping the input constant, one has to reduce 10% of the electricity production in order to reduce 10% of the SO_2 emissions [59].

This can be explained in other ways. For example, if one intends to decrease pollutants, he/she can focus on some of the input vectors to filter the pollutants so that less input can be available for electricity production. As a result, desirable and undesirable outputs are decreased together [47].

Suppose that in a production activity, N inputs are used for producing M desirable outputs and J undesirable outputs, the $\mathbf{x} \in R_+^N$ vector has been consumed, $\mathbf{v} \in R_+^M$, $\mathbf{w} \in R_+^J$ show desirable and undesirable output vectors, respectively, and the set of observations is denoted by K. The production technology includes all the feasible $(\mathbf{x}, \mathbf{v}, \mathbf{w})$, which is characterized by $T = \{(\mathbf{x}, \mathbf{v}, \mathbf{w}) | \mathbf{x} \quad can \quad produce \quad (\mathbf{v}, \mathbf{w})\}$. As an equivalent, the set of outputs is characterized as $P(\mathbf{x}) = \{(\mathbf{v}, \mathbf{w}) \,|(\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T\}$. Färe and Grosskopf made the production possibility set on the basis of the following axioms:

- F_1 : Inclusion of each observation: the production possibility set includes every observation. In other words:

$$\forall k \in \{1, \ldots, K\} : \; (x^k, v^k, w^k) \in T \tag{5}$$

- F_2 : Free disposability for desirable inputs and outputs:

$$\forall(\mathbf{x}, \mathbf{v}, \mathbf{w}),\; \forall \mathbf{x}', \forall \mathbf{v}' : \big((\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T,\, 0 \le \mathbf{v}' \le \mathbf{v}, \mathbf{x}' \ge \mathbf{x} \Rightarrow (\mathbf{x}', \mathbf{v}', \mathbf{w}) \in T\big) \tag{6}$$

- F_3 : Weak disposability for desirable and undesirable outputs:

$$\forall(\mathbf{x}, \mathbf{v}, \mathbf{w}),\; \forall \theta : ((\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T,\, 0 \le \theta \le 1 \Rightarrow (\mathbf{x}, \theta \mathbf{v}, \theta \mathbf{w}) \in T) \tag{7}$$

- F_4 : convexity: technology T is convex.

Based on the axioms F_1, F_2, F_3, F_4, Färe et al. [43,47,60] presented the output sets under the constant returns to scale (CRS) and the variable returns to scale (VRS) assumptions with the weak disposability axiom as follows:

$$\begin{aligned} P_{CRS}^W(\mathbf{x}) = \{(\mathbf{v}, \mathbf{w}) \Big| & \sum_{k=1}^K z^k x_n^k \le x_n, && n = 1, 2, \ldots, N, \\ & \sum_{k=1}^K z^k v_m^k \ge v_m, && m = 1, 2, \ldots, M, \\ & \sum_{k=1}^K z^k w_j^k = w_j, && j = 1, 2, \ldots, J, \\ & z^k \ge 0, && k = 1, 2, \ldots, K\}. \end{aligned} \tag{8}$$

and:

$$P_{VRS}^{W}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \,\Big|\, \begin{array}{ll} \sum_{k=1}^{K} z^k x_n^k \leq x_n, & n = 1,2,\ldots,N, \\ \theta \sum_{k=1}^{K} z^k v_m^k \geq v_m, & m = 1,2,\ldots,M, \\ \theta \sum_{k=1}^{K} z^k w_j^k = w_j, & j = 1,2,\ldots,J, \\ \sum_{k=1}^{K} z^k = 1, & \\ z^k \geq 0, & k = 1,2,\ldots,K, \\ 0 \leq \theta \leq 1\}. & \end{array} \tag{9}$$

The θ parameter, corresponding to the weak disposability axiom, allows the uniform abatement of desirable and undesirable outputs.

One of the problems of the output set (9) is that it is nonlinear, which can be linearized by a simple method. Like Kuosmanen [48], one can linearize the nonlinear technology (9). To this end, the DMU_k weights, that is z^k, are partitioned into two parts:

$$z^k = \underbrace{\theta z^k}_{\lambda^k} + \underbrace{(1-\theta) z^k}_{\mu^k} \quad (k = 1,2,\ldots,K) \tag{10}$$

The first component, λ^k, shows the part of unit k's output that remains active (that is, $\lambda^k = \theta z^k$). The second component, that is μ^k, in turn, shows the part of unit k's output, which scales down the activity level (that is, $\mu^k = (1-\theta)\, z^k$). After applying the notation in the nonlinear technology (9), its linear equivalent can be rewritten as follows:

$$P_{VRS}^{W}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \,\Big|\, \begin{array}{ll} \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \leq x_n, & n = 1,2,\ldots,N, \\ \sum_{k=1}^{K} \lambda^k v_m^k \geq v_m, & m = 1,2,\ldots,M, \\ \sum_{k=1}^{K} \lambda^k w_j^k = w_j, & j = 1,2,\ldots,J, \\ \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1, & \\ \lambda^k, \mu^k \geq 0, & k = 1,2,\ldots,K, \}. \end{array} \tag{11}$$

As was said in Section 1, one of the approaches to undesirable outputs is to regard them as desirable inputs and impose the strong disposability axiom on them, which was called the "extended strong disposability axiom" in Liu et al. [61]. Interestingly enough, Hailu and Veeman [34] introduced this axiom as the "modified monotonicity condition" in the production possibility set structure. Liu et al. presented this axiom as follows:

Consider K decision-making units $(DMU_k, k = 1,\ldots,K)$. Suppose $\mathbf{x} = (x_1, x_2, \ldots, x_N) \in R_+^N$ is an input vector, $\mathbf{v} = (v_1, v_2, \ldots, v_M) \in R_+^M$ is the desirable output vector, and $\mathbf{w} = (w_1, w_2, \ldots, w_J) \in R_+^J$

is the undesirable output vector. The extended strong disposability axiom in the construction of production possibility set means that we consider the undesirable outputs as the desirable input, and then, we apply the standard strong disposability axiom to the inputs and outputs (desirable and undesirable). In other words, by showing the production possibility set with T, we have:

$$If\ (\mathbf{x},\mathbf{v},\mathbf{w}) \in T\ and\ \mathbf{v} \geq \mathbf{v}',\ \mathbf{w}' \geq \mathbf{w},\ \mathbf{x}' \geq \mathbf{x}\ then\ (\mathbf{x}',\mathbf{v}',\mathbf{w}') \in T \tag{12}$$

Based on F_1, the extended strong disposability, and F_4 axioms and given the studies of Yang and Pollitt [62] and Sueyoshi and Goto [63], the output sets have been notated under the CRS and VRS assumptions as follows:

$$\begin{aligned} P^S_{CRS}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \Big| & \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1,2,\ldots,N, \\ & \sum_{k=1}^{K} z^k v_m^k \geq v_m, && m = 1,2,\ldots,M, \\ & \sum_{k=1}^{K} z^k w_j^k \leq w_j, && j = 1,2,\ldots,J, \\ & z^k \geq 0, && k = 1,2,\ldots,K\}. \end{aligned} \tag{13}$$

and:

$$\begin{aligned} P^S_{VRS}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \Big| & \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1,2,\ldots,N, \\ & \sum_{k=1}^{K} z^k v_m^k \geq v_m, && m = 1,2,\ldots,M, \\ & \sum_{k=1}^{K} z^k w_j^k \leq w_j, && j = 1,2,\ldots,J, \\ & \sum_{k=1}^{K} z^k = 1, \\ & z^k \geq 0, && k = 1,2,\ldots,K\}. \end{aligned} \tag{14}$$

4. Modeling the Production Possibility Set with the Assumption of the Semi-Disposability

Semi-disposability for undesirable outputs shows that decision-making units can deal with undesirable outputs freely within the scope of current production technology, while decision-making units have to decrease desirable outputs at the same proportion to decrease undesirable outputs outside the scope.

Based on this concept, through the use of advanced technologies and experienced management and only by decreasing the desirable outputs by a small percentage, one can extensively decrease the undesirable outputs. As an example, consider two power plants that consume coal as fuel: Let us call them "A" and "B". By means of advanced equipment and through competent managers, Power Plant A can decrease most of the harmful SO_2 emissions by just reducing a small percentage of electricity production. However, Power Plant B does not try to decrease the harmful SO_2 emissions. Here, the simultaneous change relationship between desirable and undesirable outputs presented by Färe and Grosskopf [47] in 2004 is not satisfied for Power Plant B since many harmful SO_2 emissions would be decreased if Power Plant B introduced advanced technologies and experienced management to neutralize the emissions. However, if Power Plant A tries to decrease the SO_2 emissions without advanced technologies and experienced management, it can do that only by reducing its electricity production.

In order to describe the semi-disposability for undesirable outputs, Chen et al. [57] defined the non-disposal degree concept as:

Definition 2. *For any undesirable output, a part cannot be decreased without cost, and its decrease leads to the simultaneous decrease of desirable outputs. The percentage of this part among the total undesirable output is known as the "non-disposal degree" of the undesirable output.*

As an example, Power Plant A can decrease the harmful SO_2 emissions by 90% by employing advanced technologies and competent management. However, 10% of the harmful SO_2 emissions cannot be decreased without decreasing the electricity production. Therefore, the non-disposal degree of harmful SO_2 emissions is 0.1 for Power Plant A.

Suppose that the production technology consists of all feasible (x, v, w) denoted by $\{(\mathbf{x}, \mathbf{v}, \mathbf{w}) | x \quad can \quad production \quad (\mathbf{v}, \mathbf{w})\}$. As an equivalent, the corresponding output set can be denoted by $P(\mathbf{x}) = \{(\mathbf{v}, \mathbf{w}) \,|(\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T\}$. Vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_J) \in R^J_+$ represents the non-disposal degree of undesirable outputs $\mathbf{w}$, and $(\mathbf{x}', \mathbf{v}', \mathbf{w}')$ represents an arbitrary production activity. Based on the concepts defined above, the output set under CRS with the semi-disposability assumption can be formulated as follows:

Case 1. If $\mathbf{w} > \alpha\mathbf{w}'$, then the undesirable outputs $\mathbf{w}$ have extended strong disposability. Therefore, under the premise of $\mathbf{w} > \alpha\mathbf{w}'$, if $(\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T$, $x \leq x'$, $v \geq v'$, $w \leq \mathbf{w}'$, then $(\mathbf{x}', \mathbf{v}', \mathbf{w}') \in T$. Therefore, in this case, given the output set obtained from Equation (13), the output set with the semi-disposability assumption can be notated as follows:

$$
\begin{aligned}
P_{CRS}^{SEMI-\ C1}(\mathbf{x}) = \{(\mathbf{v}, \mathbf{w}) \Big| & \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1, 2, \ldots, N, \\
& \sum_{k=1}^{K} z^k v_m^k \geq v_m, && m = 1, 2, \ldots, M, \\
& \sum_{k=1}^{K} z^k w_j^k \leq w_j, && j = 1, 2, \ldots, J, \\
& \sum_{k=1}^{K} z^k w_j^k > \alpha_j w_j, && j = 1, 2, \ldots, J, \\
& z^k \geq 0, && k = 1, 2, \ldots, K, \}.
\end{aligned}
\tag{15}
$$

Case 2. If $\mathbf{w} \leq \alpha\mathbf{w}'$, then the undesirable outputs $\mathbf{w}$ have the weak disposability. Therefore, under the premise of $\mathbf{w} \leq \alpha\mathbf{w}'$, if $(\mathbf{x}, \mathbf{v}, \mathbf{w}) \in T$ and $\mathbf{x} \leq \mathbf{x}'$, $\mathbf{v} \geq \mathbf{v}'$, $\mathbf{w} = \alpha\mathbf{w}'$, then $(\mathbf{x}', \mathbf{v}', \mathbf{w}') \in T$. Therefore, in this case, given the output set obtained from Equation (8), the output set with the semi-disposability assumption can be notated as follows:

$$P_{CRS}^{SEMI-\ C2}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \left| \begin{array}{ll} \sum_{k=1}^{K} z^k x_n^k \leq x_n, & n = 1,2,\ldots,N, \\ \sum_{k=1}^{K} z^k v_m^k \geq v_m, & m = 1,2,\ldots,M, \\ \sum_{k=1}^{K} z^k w_j^k = \alpha_j w_j, & j = 1,2,\ldots,J, \\ z^k \geq 0, & k = 1,2,\ldots,K,\}. \end{array} \right. \tag{16}$$

$\mathbf{w} = \alpha\mathbf{w}'$ can be divided into $\mathbf{w} \leq \alpha\mathbf{w}'$ and $\mathbf{w} \geq \alpha\mathbf{w}'$. Therefore, the output set, under CRS with the semi-disposability assumption, is the union of Case 1 and Case 2, and therefore, it can be notated as follows:

$$P_{CRS}^{SEMI}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \left| \begin{array}{ll} \sum_{k=1}^{K} z^k x_n^k \leq x_n, & n = 1,2,\ldots,N, \\ \sum_{k=1}^{K} z^k v_m^k \geq v_m, & m = 1,2,\ldots,M, \\ \sum_{k=1}^{K} z^k w_j^k \leq w_j, & j = 1,2,\ldots,J, \\ \sum_{k=1}^{K} z^k w_j^k \geq \alpha_j w_j, & j = 1,2,\ldots,J, \\ z^k \geq 0, & k = 1,2,\ldots,K\}. \end{array} \right. \tag{17}$$

As was said, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_J) \in R_+^J$ is a vector in a way that the value of each element α_j $(j = 1,2,\ldots,J)$ is between zero and one, which shows the non-disposal degree of undesirable outputs $\mathbf{w}$.

Interestingly enough, the non-disposal degree α_j $(j = 1,2,\ldots,J)$ is a constant, which can be determined in many ways, one of them being the Delphi method.

Moreover, Chen et al. [57] proposed the output set under VRS with the semi-disposability assumption. To this end, they assumed that $(\mathbf{x}', \mathbf{v}', \mathbf{w}')$ represented an arbitrary production activity and that the vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_J) \in R_+^J$ represented the non-disposal degree of the undesirable outputs $\mathbf{w}$. Based on what was said in the present section, the output set under VRS with the semi-disposability assumption can be formulated as follows:

Case 1. If $\mathbf{w} > \alpha\mathbf{w}'$, then undesirable outputs $\mathbf{w}$ have extended strong disposability. Therefore, in this case, given the output set obtained from Equation (14), the output set with the semi-disposability assumption can be formulated as follows:

$$P_{VRS}^{SEMI-\ C1}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \left| \begin{array}{ll} \sum_{k=1}^{K} z^k x_n^k \le x_n, & n = 1,2,\ldots,N, \\ \sum_{k=1}^{K} z^k v_m^k \ge v_m, & m = 1,2,\ldots,M, \\ \sum_{k=1}^{K} z^k w_j^k \le w_j, & j = 1,2,\ldots,J, \\ \sum_{k=1}^{K} z^k w_j^k > \alpha_j w_j, & j = 1,2,\ldots,J, \\ \sum_{k=1}^{K} z^k = 1, & \\ z^k \ge 0, & k = 1,2,\ldots,K, \}. \end{array} \right. \tag{18}$$

Case 2. If $\mathbf{w} \le \alpha \mathbf{w}'$, then the undesirable outputs $\mathbf{w}$ have weak disposability. Therefore, in this case, given the output set obtained from Equation (11), the output set with the semi-disposability assumption can be formulated as follows:

$$P_{VRS}^{SEMI-\ C2}(\mathbf{x}) = \{(\mathbf{v},\mathbf{w}) \left| \begin{array}{ll} \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \le x_n, & n = 1,2,\ldots,N, \\ \sum_{k=1}^{K} \lambda^k v_m^k \ge v_m, & m = 1,2,\ldots,M, \\ \sum_{k=1}^{K} \lambda^k w_j^k = \alpha_j w_j, & j = 1,2,\ldots,J, \\ \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1, & \\ \lambda^k, \mu^k \ge 0, & k = 1,2,\ldots,K, \}. \end{array} \right. \tag{19}$$

Therefore, the output set, under VRS with the semi-disposability assumption, is the union of Case 1 and Case 2, and therefore, it can be formulated as follows:

$$P_{VRS}^{SEMI}(\mathbf{x}) = P_{VRS}^{SEMI-\ C1}(\mathbf{x}) \cup P_{VRS}^{SEMI-\ C2}(\mathbf{x}) \tag{20}$$

5. Evaluation of Cost Efficiency, Considering the Undesirable Outputs

This section tries to present a new model for the measurement of cost efficiency, considering the undesirable outputs. The previous section presented the approach of Chen et al. [57] to the undesirable outputs and their production possibility technology. This section presents the measurement method for the cost efficiency in the presence of undesirable outputs based on their approach.

Suppose that there is a set consisting of K decision-making units $(DMU_k,\ \ k = 1,\ldots,K)$ as follows, with each using N inputs for the M desirable outputs and J undesirable outputs.

$$\begin{aligned}
&\mathbf{x}^k = (x_1^k, \ldots, x_N^k)^T, \quad x_n^k \geq 0, \quad \mathbf{x}^k \neq 0, \qquad n = 1,2,\ldots,N, \\
&\mathbf{v}^k = (v_1^k, \ldots, v_M^k)^T, \quad v_m^k \geq 0, \quad \mathbf{v}^k \neq 0, \qquad m = 1,2,\ldots,M, \\
&\mathbf{w}^k = (w_1^k, \ldots, w_J^k)^T, \quad w_j^k \geq 0, \quad \mathbf{w}^k \neq 0, \qquad j = 1,2,\ldots,J,
\end{aligned} \tag{21}$$

The cost of buying raw materials (the cost vector) corresponding to the input vectors is denoted by $\mathbf{c} = (c_1, c_2, \ldots, c_N)^T$. Moreover, this cost vector is known and accessible and equal for each unit.

Let DMU_o be the unit under evaluation. What follows is a cost efficiency model for the evaluation of DMU_o, considering the undesirable outputs on the basis of the output set under CRS with the semi-disposability assumption:

$$\begin{aligned}
\min \quad & KYG_{CRS} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} z^k x_n^k \leq x_n, \qquad n = 1,2,\ldots,N, \\
& \sum_{k=1}^{K} z^k v_m^k \geq v_m^o, \qquad m = 1,2,\ldots,M, \\
& \sum_{k=1}^{K} z^k w_j^k \leq w_j^o, \qquad j = 1,2,\ldots,J, \\
& \sum_{k=1}^{K} z^k w_j^k \geq \alpha_j^o w_j^o, \qquad j = 1,2,\ldots,J, \\
& z^k \geq 0, \qquad k = 1,2,\ldots,K.
\end{aligned} \tag{22}$$

where z^k $(k = 1,2,\ldots,K)$ and x_n $(n = 1,2,\ldots,N)$ are, respectively, the intensity weights and the amount of inputs required for the production of a certain amount of outputs (desirable and undesirable). As a matter of fact, Model (22) obtains the least cost for the production of any outputs (desirable and undesirable) under CRS with the semi-disposability assumption, given the known input cost $\mathbf{c}$.

As Chen et al. [57] pointed out, it is difficult to determine the crisp non-disposal degree to evaluate the environmental efficiency on the basis of the semi-disposability assumption in the practical decision-making processes. Therefore, the present study also assumes that α_j^o is uncertain and is within an interval such as $\left[\alpha_j^{Lo}, \alpha_j^{Uo}\right]$, where α_j^{Lo} and α_j^{Uo} denote the non-disposal degree lower bound and upper bound, respectively.

Theorem 1. *Let α_j^{Lo} and α_j^{Uo} be the lower and upper bounds, respectively, of the non-disposal degree of w_j^o. If the optimal solutions of Model (22) corresponding to α_j^{Lo} and α_j^{Uo} are KYG_{CRS}^{L*} and KYG_{CRS}^{U*}, respectively, then, $KYG_{CRS}^{L*} \leq KYG_{CRS}^{U*}$.*

Proof. We assume that DMU_o^L and DMU_o^U have the same inputs x_n^o $(n = 1,2,\ldots,N)$, desirable outputs v_m^o $(m = 1,2,\ldots,M)$, and undesirable outputs w_j^o $(j = 1,2,\ldots,J)$ with different non-disposal degrees α_j^{Lo} and α_j^{Uo}. By solving Model (22) for DMU_o^U, the present study obtains the optimal solution $(z_U^{1*}, \ldots, z_U^{K*}, KYG_{CRS}^{U*})$. $\alpha_j^{Uo} \geq \alpha_j^{Lo}$, and therefore, $\alpha_j^{Uo} w_j^o \geq \alpha_j^{Lo} w_j^o$ exists. It is clear that the optimal solution $(z_U^{1*}, \ldots, z_U^{K*}, KYG_{CRS}^{U*})$ for DMU_o^U is a feasible solution of DMU_o^L, rather than an optimal solution. However, it cannot be guaranteed that the optimal solution for DMU_o^L is the feasible solution for DMU_o^U. Given that the objective function is used to obtain a minimum value, we have $KYG_{CRS}^{L*} \leq KYG_{CRS}^{U*}$. □

Given Theorem 1, the cost efficiency lower and upper bounds for each DMU_o are obtained through the following models:

$$\begin{aligned}
\min \quad & KYG_{CRS}^{U} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1, 2, \ldots, N, \\
& \sum_{k=1}^{K} z^k v_m^k \geq v_m^o, && m = 1, 2, \ldots, M, \\
& \sum_{k=1}^{K} z^k w_j^k \leq w_j^o, && j = 1, 2, \ldots, J, \\
& \sum_{k=1}^{K} z^k w_j^k \geq \alpha_j^{Uo} w_j^o, && j = 1, 2, \ldots, J, \\
& z^k \geq 0, && k = 1, 2, \ldots, K.
\end{aligned} \tag{23}$$

and:

$$\begin{aligned}
\min \quad & KYG_{CRS}^{L} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1, 2, \ldots, N, \\
& \sum_{k=1}^{K} z^k v_m^k \geq v_m^o, && m = 1, 2, \ldots, M, \\
& \sum_{k=1}^{K} z^k w_j^k \leq w_j^o, && j = 1, 2, \ldots, J, \\
& \sum_{k=1}^{K} z^k w_j^k \geq \alpha_j^{Lo} w_j^o, && j = 1, 2, \ldots, J, \\
& z^k \geq 0, && k = 1, 2, \ldots, K.
\end{aligned} \tag{24}$$

If $(\mathbf{x}^*, \mathbf{z}^*)$ and $(\mathbf{x}^{**}, \mathbf{z}^{**})$ are the optimal solutions of, respectively Models (23) and (24), the upper and lower bounds of the cost efficiency of the o^{th} unit under CRS with the semi-disposability assumption are as follows:

$$CE_{CRS}^{Uo} = \frac{KYG_{CRS}^{U*}}{\mathbf{c}^T \mathbf{x}^o} = \frac{\mathbf{c}^T \mathbf{x}^*}{\mathbf{c}^T \mathbf{x}^o} = \frac{\sum_{n=1}^{N} c_n x_n^*}{\sum_{n=1}^{N} c_n x_n^o} \tag{25}$$

$$CE_{CRS}^{Lo} = \frac{KYG_{CRS}^{L*}}{\mathbf{c}^T \mathbf{x}^o} = \frac{\mathbf{c}^T \mathbf{x}^{**}}{\mathbf{c}^T \mathbf{x}^o} = \frac{\sum_{n=1}^{N} c_n x_n^{**}}{\sum_{n=1}^{N} c_n x_n^o} \tag{26}$$

Given Theorem 1, moreover, the cost efficiency upper bound of DMU_o under VRS with the semi-disposability assumption is presented as follows:

$$CE_{VRS}^{Uo} = \min\left\{ CE_{VRS}^{U*-C1} = \frac{KYG_{VRS}^{U*-\ C1}}{\mathbf{c}^T\mathbf{x}^o}\ ,\ CE_{VRS}^{U*-C2} = \frac{KYG_{VRS}^{U*-\ C2}}{\mathbf{c}^T\mathbf{x}^o} \right\} \tag{27}$$

where $KYG_{VRS}^{U*-\ C1}$ and $KYG_{VRS}^{U*-\ C2}$ denote the upper bounds of the optimal cost efficiency for DMU_o under VRS with the semi-disposability assumption on the basis of the output sets of $P_{VRS}^{SEMI-\ C1}(\mathbf{x})$ and $P_{VRS}^{SEMI-\ C2}(\mathbf{x})$, respectively, obtained by solving the following models:

$$\begin{aligned}
\min \quad & KYG_{VRS}^{U-\ C1} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} z^k x_n^k \le x_n, && n = 1,2,\ldots,N, \\
& \sum_{k=1}^{K} z^k v_m^k \ge v_m^o, && m = 1,2,\ldots,M, \\
& \sum_{k=1}^{K} z^k w_j^k \le w_j^o, && j = 1,2,\ldots,J, \\
& \sum_{k=1}^{K} z^k w_j^k > \alpha_j^{Uo} w_j^o, && j = 1,2,\ldots,J, \\
& \sum_{k=1}^{K} z^k = 1, && \\
& z^k \ge 0, && k = 1,2,\ldots,K.
\end{aligned} \tag{28}$$

and:

$$\begin{aligned}
\min \quad & KYG_{VRS}^{U-\ C2} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \le x_n, && n = 1,2,\ldots,N, \\
& \sum_{k=1}^{K} \lambda^k v_m^k \ge v_m^o, && m = 1,2,\ldots,M, \\
& \sum_{k=1}^{K} \lambda^k w_j^k = \alpha_j^{Uo} w_j^o, && j = 1,2,\ldots,J, \\
& \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1, && \\
& \lambda^k, \mu^k \ge 0, && k = 1,2,\ldots,K.
\end{aligned} \tag{29}$$

Given Theorem 1, furthermore, the cost efficiency lower bound of DMU_o under VRS with the semi-disposability assumption can be presented as follows:

$$CE_{VRS}^{Lo} = \min\left\{ CE_{VRS}^{L*-C1} = \frac{KYG_{VRS}^{L*-\ C1}}{\mathbf{c}^T\mathbf{x}^o}\ ,\ CE_{VRS}^{L*-C2} = \frac{KYG_{VRS}^{L*-\ C2}}{\mathbf{c}^T\mathbf{x}^o} \right\} \tag{30}$$

where $KYG_{VRS}^{L*-\ C1}$ and $KYG_{VRS}^{L*-\ C2}$ denote, respectively, the lower bounds of the optimal cost efficiency of DMU_o under VRS with the semi-disposability assumption on the basis of the output sets of $P_{VRS}^{SEMI-\ C1}(\mathbf{x})$ and $P_{VRS}^{SEMI-\ C2}(\mathbf{x})$, which are obtained by solving the following models:

$$\begin{aligned}
\min \quad & KYG_{VRS}^{L-C1} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} z^k x_n^k \le x_n, && n = 1, 2, \ldots, N, \\
& \sum_{k=1}^{K} z^k v_m^k \ge v_m^o, && m = 1, 2, \ldots, M, \\
& \sum_{k=1}^{K} z^k w_j^k \le w_j^o, && j = 1, 2, \ldots, J, \\
& \sum_{k=1}^{K} z^k w_j^k > \alpha_j^{Lo} w_j^o, && j = 1, 2, \ldots, J, \\
& \sum_{k=1}^{K} z^k = 1, \\
& z^k \ge 0, && k = 1, 2, \ldots, K.
\end{aligned} \tag{31}$$

and:

$$\begin{aligned}
\min \quad & KYG_{VRS}^{L-C2} = \sum_{n=1}^{N} c_n x_n \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \le x_n, && n = 1, 2, \ldots, N, \\
& \sum_{k=1}^{K} \lambda^k v_m^k \ge v_m^o, && m = 1, 2, \ldots, M, \\
& \sum_{k=1}^{K} \lambda^k w_j^k = \alpha_j^{Lo} w_j^o, && j = 1, 2, \ldots, J, \\
& \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1, \\
& \lambda^k, \mu^k \ge 0, && k = 1, 2, \ldots, K.
\end{aligned} \tag{32}$$

6. Case Study and the Analysis of Its Results

This section deals with the cost efficiency of electricity generation power plants in Iran. Since the electricity industry has an infrastructure role and is closely associated with the factors affecting economic development, it is a far-reaching industry. The electricity energy is a key factor for industrial, economic, and social development. The electricity industry today supplies the necessary energy for different services and industries (from small industries to giant nuclear power plants) in Iran, and for this reason, it is very important to study the cost efficiency of electricity producing power plants. Iran has rich resources of oil and gas (as the main fuel for thermal power plants), and therefore, it enjoys relatively powerful advantages in terms of energy supplies. Most power plants in Iran are thermal, and they produce nearly 90% of the required electricity. In general, there are various types of electricity power plants in Iran: atomic power plants, water power plants, thermal generators (steam, gas, combined cycle, diesel, combined steam and gas), solar factories, and other power plants.

For a developing country like Iran, the sources used for thermal power plants are strategically significant. These sources are non-renewable, and therefore, they should be optimally utilized. The environmental problems, moreover, should not be ignored when one deals with electricity producing power plants since various international agreements (such as the Kyoto International Protocol) emphasize the significance of environmental issues, and Iran has recently introduced many environmental programs such as those proposed in its 6th Development Plan; for these reasons, the present study focuses on the problems of thermal power plants excluding the diesel power plants, since they are only used on occasional conditions (such as supplying electricity to some remote military bases or some regions not connected to the nationwide network), and for this reason, the present study deals with other thermal power plants (such as steam, gas, and combined cycle). As these power plants have similar mechanisms and structures, they are regarded as similar units, and consequently, there is no limitation for using DEA for the evaluation of the cost efficiencies of these similar power plants.

The present study considers each power plant as a DMU, and the selection of input and output variables is one of the most important steps for the evaluation of the performance of power plants on the basis of the DEA approach. Inappropriate criteria might invalidate the evaluation results. The main criterion for the selection of DEA model inputs and outputs should be the data availability and the rate of influence they have on the performance of a power plant. The variables selected in this section were either derived from the studies on the performance evaluation of thermal power plants conducted in different countries by means of DEA or identified by experts and specialists. In fact, the main criteria for the selection of variables were the role(s) they played in the cost efficiency evaluation of a power plant, their uses in similar studies, and their availabilities. As a result, the input and output variables in the present study are as follows:

- A. Input variables
    1. Manpower: Manpower in the present study consists of the human forces working in each power plant in terms of persons. The present study distinguishes the specialized manpower from non-specialists, although there was no access, unfortunately, to separate statistics of manpower.
    Manpower price: average salaries in 2015.
    2. Nominal capacity: Instead of the capital index, the nominal capacity was used, in terms of megawatts, in each power plant as a substitute. There are problems in the power plants that impede capital computation.
    Price of nominal capacity: average nominal capacity in 56 power plants was used.
- B. Output variables
    1. Total rate of generated electricity: The desirable output obtained from each power plant is the total generated electricity in terms of megawatt/hour.
    2. Pollutant: Logically, this variable is considered to be an undesirable output. Unlike the desirable output, it should be decreased.

To better understand the proposed models, we assumed that the non-disposal degree of the pollutant variable was within the interval $[0.4, 0.9]$ for every unit (power plant) under evaluation. For example, $\alpha_j^L = 0/4$ (lower bound of non-disposal degree) and $\alpha_j^U = 0/9$ (upper bound of non-disposal degree) for every j $(j = 1, 2, \ldots, 56)$.

Figure 1 shows the structure of an electricity producing power plant as a DMU.

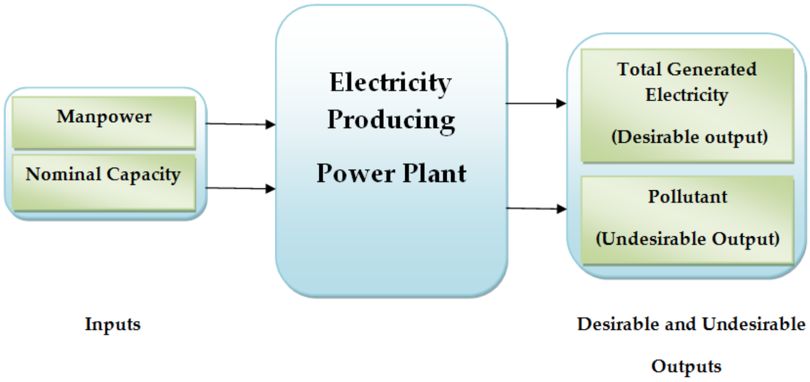

Figure 1. The structure of a power plant as a decision-making unit.

One of the main stages of research development is the data collection. The statistical universe in the present study consisted of 56 electricity producing power plants affiliated with the Ministry of Energy. The data and information collected for each variable (for each power plant) were provided to us in 2015 by the authorized Tavanir Company (the mother specialized company of electricity in Iran), and the descriptive statistics for this dataset are summarized in Table 1. Under the copyright brand regulations, the anonymity of electricity producing power plants was preserved. To analyze the data, the GAMS 24.5 software was used.

Table 1. Descriptive statistics for 56 electricity producing power plants.

Indicators	Minimum	Maximum	Mean	Standard Deviation
Manpower (No.)	42	2868	795.7857	605.469
Nominal capacity (MW)	53	765	252.8036	175.3114
Total rate of generated electricity (MWh)	61.452	15,169,522	3,877,387	3,699,915
Pollutant (ppm)	2622	2,936,547	832,661	693,511.8

Finally, the lower and upper bounds of cost efficiency of 56 power plants were evaluated under CRS and VRS with the semi-disposability assumption by means of Equations (25) and (26) (belonging to Models (23) and (24)) and Equations (27) and (30) (belonging to Models (28), (29), (31) and (32)), respectively, the results of which are shown in Table 2.

Based on Table 2, the mean lower and upper bounds of cost efficiency for the 56 power plants under the CRS assumption were 34% and 35%, respectively; moreover, under the VRS assumption, they were 52% and 54%, respectively. In other words, if the selected power plants decreased their use of inputs from 65% to 66%, on average, and from 46% to 48%, on average, without changing the amounts of electricity production and pollutants, they would be on the cost frontier under the CRS and VRS assumptions.

Figures 2 and 3 compare the cost efficiency lower and upper bounds of 56 thermal power plants, under CRS and VRS with the semi-disposability assumption.

Table 2. Results obtained from the evaluation of the lower and upper bounds of the cost efficiencies of the power plants selected in 2015 under constant returns to scale (CRS) and VRS with the semi-disposability assumption.

Power Plant	Under the CRS Assumption		Under the VRS Assumption	
	Lower Bound of Cost Efficiency	Upper Bound of Cost Efficiency	Lower Bound of Cost Efficiency	Upper Bound of Cost Efficiency
Combined cycle				
CC_1	0.9012	0.9012	1.0000	1.0000
CC_2	0.5190	0.5190	0.5317	0.5317
CC_3	0.3130	0.3130	0.3318	0.3318
CC_4	0.6243	0.6243	0.6485	0.6485
CC_5	0.5524	0.5524	0.6000	0.6095
CC_6	0.2532	0.2532	0.3481	0.3671
CC_7	1.0000	1.0000	1.0000	1.0000
CC_8	0.2878	0.2878	0.2914	0.2914
CC_9	0.0797	0.1753	0.2271	0.2590
CC_{10}	0.0682	0.1049	0.0982	0.1182
CC_{11}	0.1442	0.1768	0.2604	0.2930
CC_{12}	0.6486	0.6486	1.0000	1.0000
CC_{13}	1.0000	1.0000	1.0000	1.0000
CC_{14}	0.2340	0.2340	0.2372	0.2372
CC_{15}	0.2693	0.2693	0.6922	0.7051
CC_{16}	0.1294	0.2941	0.6469	0.6822
CC_{17}	0.0308	0.0616	0.4154	0.4154
CC_{18}	0.1819	0.1819	0.2500	0.2682
CC_{19}	0.4238	0.4238	0.4746	0.4915
CC_{20}	0.8751	0.8751	1.0000	1.0000
CC_{21}	0.9192	0.9192	1.0000	1.0000
CC_{22}	0.8059	0.8059	0.8692	0.8692
Steam				
S_1	0.6771	0.6771	0.7278	0.7278
S_2	0.4900	0.4900	0.5302	0.5302
S_3	0.3111	0.3111	0.8666	0.8666
S_4	0.2953	0.2953	0.3207	0.3715
S_5	0.3808	0.3808	0.3945	0.3945
S_6	0.1024	0.1085	0.3253	0.3373
S_7	0.1556	0.1667	0.5999	0.6110
S_8	0.1059	0.1323	0.2910	0.3069
S_9	0.2430	0.2430	0.2465	0.2465
S_{10}	0.1333	0.1333	0.2571	0.2571
S_{11}	0.5600	0.5600	1.0000	1.0000
S_{12}	0.1737	0.1737	1.0000	1.0000
S_{13}	0.0364	0.0546	0.4909	0.4909
S_{14}	0.5129	0.5129	0.6091	0.8462
S_{15}	0.1914	0.1914	0.3394	0.3579
S_{16}	0.0814	0.0814	0.1797	0.1831
Gas				
G_1	0.1803	0.1919	0.3197	0.3546
G_2	0.1749	0.1749	1.0000	1.0000
G_3	0.0121	0.0303	0.3273	0.3273
G_4	0.0157	0.0157	0.0706	0.0706
G_5	0.0224	0.0486	0.1028	0.1103
G_6	0.0105	0.0211	1.0000	1.0000
G_7	0.0077	0.0154	0.2077	0.2077
G_8	0.0181	0.0363	0.1399	0.1425
G_9	0.5542	0.5542	0.6987	0.7589
G_{10}	1.0000	1.0000	1.0000	1.0000
G_{11}	0.4069	0.4069	0.4276	0.4276
G_{12}	0.7294	0.7294	0.7411	0.7411
G_{13}	0.1942	0.1942	0.3235	0.3588
G_{14}	0.2001	0.2770	0.2385	0.2847
G_{15}	0.0247	0.0563	0.0769	0.0879
G_{16}	0.0294	0.0294	0.5196	0.5294
G_{17}	0.9092	0.9092	1.0000	1.0000
G_{18}	0.0857	0.1600	0.3143	0.3371
Mean	0.3444	0.3568	0.5287	0.5425

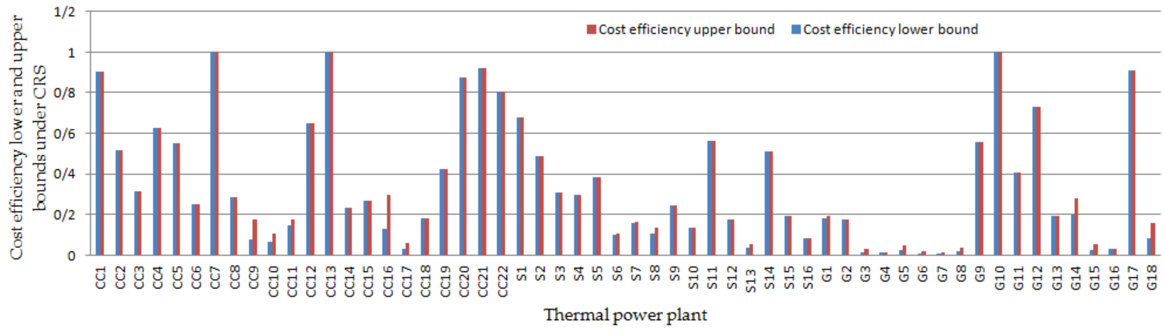

Figure 2. Comparisons of cost efficiency lower and upper bounds of 56 thermal power plants under *CRS* with the semi-disposability assumption.

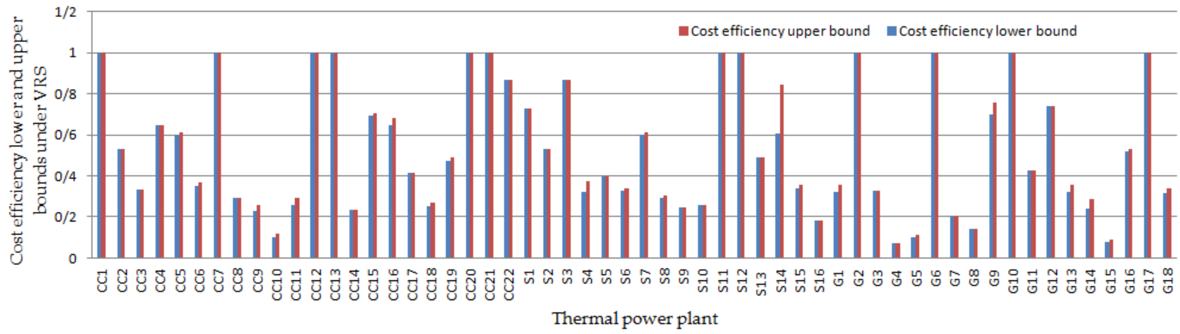

Figure 3. Comparisons of cost efficiency lower and upper bounds of 56 thermal power plants under *VRS* with the semi-disposability assumption.

Based on Table 2, for an easy explanation of the numbers and/or values in the table, the thermal power plants are divided into two classes: Class 1, the power plants that have the cost efficiency lower and upper bounds equal to one under CRS with the semi-disposability assumption; Class 2, the power plants that have the cost efficiency lower and upper bounds smaller than one under CRS with the semi-disposability assumption. The first class is the power plants having the cost efficiency lower and upper bounds equal to one under CRS with the semi-disposability assumption. These power plants are indeed the power plants that are theoretically referred to as "the cost efficient units" (the units that are on the cost efficient frontier). Based on Table 2, some power plants, for example CC_7, CC_{13}, and G_{10}, were among the power plants that had the lower and upper bounds of cost efficiency equal to one. The second class is the power plants that have cost efficiency lower and upper bounds smaller than one under CRS with the semi-disposability assumption, theoretically referred to as "cost inefficient power plants", and based on the results appearing in Table 2, they included the remaining power plants. Given the CRS assumption, these power plants were considered to be cost inefficient, for different reasons. Therefore, they were divided into two groups so that they could be studied more clearly. The first group was the power plants having the cost efficiency lower and upper bounds equal to one under VRS with the semi-disposability assumption. Based on Table 2, some power plants, for example, CC_1, CC_{12}, CC_{20}, CC_{21}, S_{11}, S_{12}, G_2, G_6, and G_{17}, were among this group of power plants. The second group was the power plants having the cost efficiency lower and upper bounds smaller than one under VRS with the semi-disposability assumption. Based on Table 2, some power plants, for example, CC_2, CC_3, CC_4, CC_5, CC_6, CC_8, CC_9, CC_{10}, CC_{11}, CC_{14}, CC_{15}, CC_{16}, CC_{17}, CC_{18}, CC_{19}, CC_{22}, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{13}, S_{14}, S_{15}, S_{16}, G_1, G_3, G_4, G_5, G_7, G_8, G_9, G_{11}, G_{12}, G_{13}, G_{14}, G_{15}, G_{16}, and G_{18}, were among this group of power plants.

Based on the results of the assessments of the cost efficiency lower and upper bounds of these 56 power plants, under CRS with the semi-disposability assumption, there were only three power plants on the cost frontier, and under VRS with the semi-disposability assumption, there were only 12 power plants on the cost frontier, having activities with the least costs. The reason for the inefficiency of some power plants is that they overused the inputs (that is, they used the inputs excessively) or else they used an improper combination of inputs. This means that the nominal capacity and the manpower capacity of these power plants were not proportionate to their production levels. One can predict that the power plants that were known to be cost inefficient could decrease their inputs and reach the frontier of cost efficiency by choosing an optimal combination of inputs without having to change their outputs.

Table 3 shows the mean cost efficiency lower and upper bounds in terms of type of power plant, under CRS and VRS with the semi-disposability assumption.

As Table 3 shows, under CRS and VRS with the semi-disposability assumption, the combined-cycle power plants had higher mean cost efficiency bounds than other power plants, followed by steam and gas power plants. There were two reasons for this. Firstly, the nominal capacity and manpower of these power plants were not proportional to their production levels. Secondly, most units of these power plants were off (offline) during low loading hours (and they joined the network only during the electricity consumption peak hours), and for this reason, parts of the power plant capacity were kept non-active for emergencies.

Table 3. Mean cost efficiency lower and upper bounds in terms of the type of power plant in 2015, under CRS and VRS with the semi-disposability assumption.

Power Plant	Mean Cost Efficiency Lower and Upper Bounds			
	Under the CRS Assumption		Under the VRS Assumption	
	Lower Bound of Cost Efficiency	Upper Bound of Cost Efficiency	Lower Bound of Cost Efficiency	Upper Bound of Cost Efficiency
Combined cycle	0.4664	0.4827	0.5873	0.5963
Steam	0.2781	0.2820	0.5111	0.5329
Gas	0.2541	0.2694	0.4726	0.4854

Moreover, Table 3 shows that the thermal power plants, including the gas power plants, were not highly efficient and that this was mainly caused by the environmental pollution. In order to promote the efficiencies of thermal power plants in Iran, it is necessary to deactivate many old power plants. Therefore, it is possible to increase the efficiencies of power plants by some parallel measures; this will contribute to local industries, and modern technologies will thrive in this country. Any type of planning including extension or contraction of power plants requires the current status of the efficiency and cost efficiency of power plants. The result of Table 3 strengthens the necessity of high attention by policy makers to gas and steam power plants.

7. Conclusions and Recommendations for Future

The present study developed the classical cost efficiency model in which all the input prices were fixed and known for each decision-making unit, considering the undesirable outputs under CRS and VRS with the semi-disposability assumption. The proposed models were interval and uncertain in both constant returns to scale and variable returns to scale assumptions, for the easy solution of which, the present study could obtain their lower and upper bounds by means of the theorem presented. To this end, in order to explain and analyze the proposed models, and given the importance of thermal power plants in terms of electricity production in the country, the present study evaluated the cost efficiencies of 56 electricity producing thermal power plants in 2015. Comparative results of the present study showed that under both constant returns to scale and variable returns to scale assumptions, the combined-cycle and steam power plants had the highest cost efficiency (lower and upper) bounds. Moreover, the mean lower and upper bounds of the cost efficiencies of the power plants under study in 2015 were 34% and 35%, respectively, under the constant returns to scale assumption, and 52% and 54%, respectively, under the variable returns to scale assumption. In other words, even if we intended to use optimally the existing capabilities with the current capacity without any development in the power plant capacity in the country, the generated electricity could be improved in the percentage interval of [65, 66] for the CRS assumption and in that of [46, 48] for the VRS assumption.

The following recommendations are suggested for future studies concerning the findings of the present study:

- Development of the proposed models considering the uncertain input prices.
- Generalization and extension of other cost efficiency models in the presence of undesirable outputs under the semi-disposability assumption.
- Generalization and expansion of the proposed models with imprecise input and output data such as the stochastic and interval data.

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