Supplementary Material: FSK/ASK Orthogonal Modulation System Based on Novel Noncoherent Detection and Electronic Dispersion Compensation for Short-Reach Optical Communications

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1. Theoretical Model for the DFB laser

The DFB laser is simulated by a traveling wave model (TWM) [17]:

$$\frac{dN(z,t)}{dt} = I(t)/(eV) - N(z,t)/\tau_c - v_g P_s(z,t) g(z,t)/[1+\varepsilon P_s(z,t)], \tag{S1a}$$

$$\left(\frac{1}{v_g}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)F(z,t) = \left\{-j\delta + \frac{1}{2}\left[\frac{\Gamma g(z,t)}{1+\varepsilon P_s(z,t)} - \alpha\right]\right\}F(z,t) + j\kappa R(z,t) + \tilde{s}^f(z,t), \tag{S1b}$$

$$\left(\frac{1}{v_g}\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right)R(z,t) = \left\{-j\delta + \frac{1}{2}\left[\frac{\Gamma g(z,t)}{1+\varepsilon P_s(z,t)} - \alpha\right]\right\}R(z,t) + j\kappa F(z,t) + \tilde{s}^r(z,t), \tag{S1c}$$

where $N(z,t)$ is the carrier density, $I(t)$ the injected current, e the electron charge, V the active region volume, τ_c the carrier lifetime, $v_g = c/n_g$ the group velocity, c the speed of light, n_g the group index, $P_s(z,t) = n_{eff}/(2hv_0)\sqrt{\varepsilon_0/\mu_0}\,\Gamma/(dwv_g)\cdot[|F(z,t)|^2 + |R(z,t)|^2]$ the photon density distribution, $n_{eff} = n_{eff}^0 - \lambda_0/(4\pi)\,\alpha_{LEF}\cdot\Gamma g(z,t)$ effective index, n_{eff}^0 effective index without injection, λ_0 the peak gain wavelength, α_{LEF} linewidth enhancement factor, Γ the optical confinement factor, $g(z,t) = a\ln[N(z,t)/N_0]$ the material optical gain, a the material gain coefficient, N_0 the transparent carrier density, h Planck's constant, v_0 the optical frequency corresponding to λ_0, ε_0 the permittivity of a vacuum, μ_0 the permeability of a vacuum, d thickness of active region, w width of active region, $F(z,t)$ the slowly varying envelopes of the forward propagating fields, $R(z,t)$ the slowly varying envelopes of the backward propagating fields, ε nonlinear gain suppression coefficient, j the imaginary unit, $\delta = \left[2\pi n_{eff}^0/\lambda_0 - 1/2\,\alpha_{LEF}\Gamma g(z,t) - \pi/\Lambda\right]$ the phase detuning factor from the Bragg wavelength, Λ Bragg grating period, α the optical modal loss, and κ grating coupling coefficient.

The magnitude of the spontaneous emission noise fields $\tilde{s}^f(z,t,\lambda_i)$ and $\tilde{s}^r(z,t,\lambda_i)$ are approximated as Gaussian random processes with a zero mean and satisfy the following autocorrelation function [17]:

$$\langle|\tilde{s}^{f,r}(z,t)||\tilde{S}^{f,r}(z',t')|\rangle = 2\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\Gamma\Upsilon g_{sp}hv_0}{n_{eff}}\delta(z-z')\delta(t-t'), \tag{S2}$$

where γ indicates the spontaneous coupling factor, g_{sp} is the spontaneous emission gain, and $\delta(\,)$ is Dirac's delta function. Again, the phase of the spontaneous emission noise fields is assumed to be uniformly distributed between 0–2π.

The finite bandwidth of the gain profile is modeled by an infinite impulse response (IIR) filter approach [40,41]:

$$|H(\omega)|^2 = \{(1-\eta)^2/[1+\eta^2-2\eta cos(\omega\Delta t)]\}, \tag{S3}$$

where η indicates the filter coefficient that controls the filter bandwidth and Δt is the time marching step in simulation.

2. Theoretical Model for the SOA

The numerical model that we have adopted to describe the SOA is given as [42–44]:

$$\frac{dN(z,t)}{dt} = \frac{I(t)}{eV} - \frac{N(z,t)}{\tau_c} - v_g \sum_{i=-M}^{M} P_s(z,t,\lambda_i) g(z,t,\lambda_i)/[1+\varepsilon P_{tot}(z,t)], \tag{S4a}$$

$$\begin{aligned}&\left(\frac{1}{v_g}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right) F(z,t,\lambda_i) \\ &= \left\{ j\left[\frac{1}{2}\alpha_{LEF}\Gamma g(z,t,\lambda_i)\right] + \frac{1}{2}\left[\frac{\Gamma g(z,t,\lambda_i)}{1+\varepsilon P_{tot}(z,t)} - \alpha\right]\right\} F(z,t,\lambda_i) + \tilde{s}^f(z,t,\lambda_i),\end{aligned} \tag{S4b}$$

$$\begin{aligned}&\left(\frac{1}{v_g}\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right) R(z,t,\lambda_i) \\ &= \left\{ j\left[\frac{1}{2}\alpha_{LEF}\Gamma g(z,t,\lambda_i)\right] + \frac{1}{2}\left[\frac{\Gamma g(z,t,\lambda_i)}{1+\varepsilon P_{tot}(z,t)} - \alpha\right]\right\} \mathrm{R}(z,t,\lambda_i) + \tilde{s}^r(z,t,\lambda_i),\end{aligned} \tag{S4c}$$

where $N(z,t)$ is the carrier density, $I(t)$ the injected current, e the electron charge, V the active region volume, τ_c the carrier lifetime, $v_g = c/n_g$ the group velocity, c the speed of light, n_g the group index, $P_s(z,t,\lambda_i) = n_{eff}/(2hv_i)\sqrt{\varepsilon_0/\mu_0}\,\Gamma/(dwv_g)\cdot[|F(z,t,\lambda_i)|^2 + |R(z,t,\lambda_i)|^2]$ the photon density distribution of the i^{th} wavelength channel, λ_i is the wavelength of the i^{th} $(i = 0, \mp 1, \mp 2, \ldots, \mp M)$ channel in the sliced spectrum, $n_{eff} = n_{eff}^0 - \lambda_0/(4\pi)\,\alpha_{LEF}\Gamma g(z,t,\lambda_i)$ effective index, n_{eff}^0 effective index without injection, λ_0 the peak gain wavelength, α_{LEF} linewidth enhancement factor, Γ the optical confinement factor, $g(z,t,\lambda_i) = a\ln[N(z,t)/N_0][1-0.5(\lambda_i - \lambda_0/\Delta\lambda_G)^2]$ the material optical gain, a the material gain coefficient, N_0 the transparent carrier density, $\Delta\lambda_G$ the gain profile width, h Planck's constant, v_i the optical frequency corresponding to λ_i, ε_0 the permittivity of a vacuum, μ_0 the permeability of a vacuum, d thickness of active region, w width of active region, $F(z,t,\lambda_i)$ the slowly varying envelopes of the forward propagating fields, $R(z,t,\lambda_i)$ the slowly varying envelopes of the backward propagating fields, ε nonlinear gain suppression coefficient, $P_{tot}(z,t) = \sum_{i=-M}^{M} P_s(z,t,\lambda_i)$ the total photon density distribution in all wavelength channels, j the imaginary unit, and α the optical modal loss.

The magnitude of the spontaneous emission noise fields $\tilde{s}^f(z,t)$ and $\tilde{s}^r(z,t)$ are treated in a similar way to equation (S2) in Section 1:

$$\langle |\tilde{s}^{f,r}(z,t)||\tilde{S}^{f,r}(z',t')|\rangle = 2\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\Gamma\Upsilon g_{sp} h v_0}{n_{eff}}\delta(z-z')\delta(t-t'), \tag{S5}$$

where γ is the spontaneous coupling factor, $R_{sp}(z,t,\lambda_i)$ the spontaneous emission rate, d_z the length of a subsection introduced by the spatial discretization of the active region along the wave propagation direction, and $\delta(\,)$ Dirac's delta function. The phase of the spontaneous emission noise fields is assumed to be uniformly distributed between 0–2π.

3. Theoretical Model for the Optical Fiber

The slow-varying envelope of a propagating optical pulse in the fiber can be described by the nonlinear Schrodinger equation (NSE) [20]:

$$\frac{\partial A(z,t)}{\partial z}+\frac{\alpha}{2}A(z,t)+\beta_1\frac{\partial A(z,t)}{\partial t}+\frac{j}{2}\beta_2\frac{\partial^2 A(z,t)}{\partial t^2}-\frac{1}{6}\beta_3\frac{\partial^3 A(z,t)}{\partial t^3} = j\gamma|A(z,t)|^2A(z,t), \tag{S6}$$

where A is the low-varying envelope of the optical field, α the fiber loss, β_1 the wave propagation constant, β_2 the second-order dispersion, β_3 the third-order dispersion, γ the fiber nonlinear parameter, and j the imaginary unit.

Transforming to a reference frame moving with the pulse and introducing the new coordinates $(T=t-\beta_1 z)$, the term β_1 can be eliminated in (S6) to yield:

$$\frac{\partial A(z,T)}{\partial z}+\frac{\alpha}{2}A(z,T)+\frac{j}{2}\beta_2\frac{A(z,T)}{\partial T^2}-\frac{\beta_3}{6}\frac{A(z,T)}{\partial T^3}=j\gamma|A^2(z,T)|A(z,T). \tag{S7}$$

According to the split-step method [22], the solution to (S7) is given by:

$$A(z+\Delta z,\omega)=\exp\,[j(1/2\,\beta_2\Delta z\omega^2-1/6\,\beta_3\Delta z\omega^3-1/2\,\alpha\Delta z)]F[A(z,T)], \tag{S8}$$

$$A(z+\Delta z,T)=exp\{j\Delta z\gamma|F^{-1}[A(z+\Delta z,\omega)]|\}\,F^{-1}[A(z+\Delta z,\omega)], \tag{S9}$$

where $F[\,]$ and $F^{-1}[\,]$ are the Fourier and inverse Fourier transforms, respectively. In our simulations, we have neglected dispersions higher than the second order as well as the fiber nonlinearity by setting $\gamma=0$, due to the relatively short transmission distance. As such, we can rewrite (S8) and (S9) as:

$$A(z+\Delta z,T)=F^{-1}\left\{exp\left[\frac{1}{2}(j\beta_2\Delta z\omega^2-\alpha\Delta z)\right]F[A(z,T)]\right\}. \tag{S10}$$