Optical Bistability in an Optomechanical System with *N*-Type Atoms under Nonresonant Conditions

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Abstract: In this paper, the phenomenon of the optical bistability of a cavity field is theoretically investigated in an optomechanical system containing an *N*-type atomic ensemble. In this hybrid optomechanical system, the atoms are coupled with two controlling light fields besides coupling with the cavity field. Under the nonresonant condition, we analyze the influences of the coupling strength between cavity and atoms, Rabi frequencies of the controlling light field, the detuning between the controlling light field and atoms, and pump field power on the optical bistable behavior of mean intracavity photon number. The nonlinear distribution of the mean intracavity photon number has a potential application in field optical switches and optical bistable devices.

Keywords: cavity optomechanical; optical bistability; nonlinearity

1. Introduction

For most objects in the macro world, light pressure does not affect their orbit of motion. However, for nano- or micromechanical oscillators, its motion can be mediated by the electromagnetic radiation pressure force, which results in studies about cavity optomechanics. In the system of cavity optomechanics, one can use coupling of the cavity field and the mechanical oscillator to produce many interesting quantum phenomena, such as the squeezing of the output field [1–3] or quantum entanglement between a macroscopic mechanical oscillator and a cavity field [4]. In addition, in the cavity optomechanical system, researchers can obtain optomechanically induced transparency [5,6]. Based on the technology of optomechanically induced transparency, its potential applications in phononic engineering, signal sensing and quantum memory are reported in recent [7–10].

On the other hand, there is considerable interest in a hybrid cavity optomechanical system containing atomic ensemble, or quantum well, or a Kerr medium, or optical parametric amplifier. More interesting physical phenomena, such as steady-state entanglement, bistable behavior, and squeezing spectra of transmitted field, ground-state cooling of micromechanical oscillators, and so on, are investigated in references [11–16]. In this paper, the influences of atomic coherence and quantum coherence on optical bistability in a hybrid cavity optomechanical system containing *N*-type four-level atoms are studied. And we obtain optical bistability by changing physical parameters. By solving the steady-state solution of Heisenberg–Langevin equation, we can get the analytical expressions of mean photon number of cavity field depending on physical parameters. The influences of coupling strength between cavity and atom, Rabi frequency of incident laser field, and detuning between cavity field and atom level on mean intracavity photon number are analyzed. At the same time, the influence of power of the pump field on mean intracavity photon number is also investigated.

The distribution of mean intracavity photon number shows optical bistable behavior due to the nonlinearity of system can be generated.

2. Theoretical Model

In this paper, we consider a physical system in Figure 1, where N-type four-level atoms are in optomechanical cavity. The cavity is formed by two mirrors M_1 and M_2, and mirror M_1 with partial transmittance is a fixed one and mirror M_2 with full reflectivity is a moving one. The atomic ensemble consists of N-type four-level atoms and atomic structures of energy level are shown in Figure 1b. In the direction perpendicular to the cavity axis, the metastable state $|2\rangle$ is coupled with higher levels $|3\rangle$ and $|4\rangle$ by two controlling fields Ω_1 with frequency ω_{c1} and Ω_2 with frequency ω_{c2}, respectively. The cavity mode with eigenfrequency ω_0 is driven by a pump field with frequency ω_d and power P through the port mirror. In addition, the cavity mode interacts with atom through coupling between levels $|3\rangle$ and $|1\rangle$ in the direction of the cavity axis. The Hamiltonian of the total system is written as

$$
\begin{aligned}
H =\ & \hbar\omega_0 a^\dagger a + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 x^2 + \sum_{j=1}^{M} \hbar(\omega_1 \sigma_{11}^j + \omega_2 \sigma_{22}^j + \omega_3 \sigma_{33}^j + \omega_4 \sigma_{44}^j) \\
& + \sum_{j=1}^{M} \hbar\left(Ga\sigma_{31}^j + \Omega_1 e^{-i\omega_{c1}t}\sigma_{32}^j + \Omega_2 e^{-i\omega_{c2}t}\sigma_{42}^j + H.c.\right) \\
& - \chi_0 a^\dagger a x + i\hbar\epsilon_d\left(a^\dagger e^{-i\omega_d t} - a e^{i\omega_d t}\right),
\end{aligned}
\tag{1}
$$

where the first term on the right side of Equation (1) represents the energy of the cavity mode. The second and third terms are the Hamiltonian of the mechanical oscillator. The fourth term represents the free energy of atom. Each term in the second line of Equation (1) stands for coupling of cavity with atom, interaction of atom with controlling field Ω_1 and controlling field Ω_2, respectively. In the third line of Equation (1), the first term describes the energy of the cavity mode interacting with the mechanical resonator, and the second term represents the Hamiltonian of the cavity mode driven by the pump field. Some physical parameters are shown here: ω_i ($i = 1, 2, 3, 4$) is eigenfrequency of level $|i\rangle$, σ_{ij} stands for atomic operators, G is the coupling strength between the cavity mode and atom and related to number of atoms, χ_0 is the coupling strength between the cavity mode with the mechanical resonator and $\chi_0 = \frac{\hbar\omega_0}{L}$ with L the cavity length in the absence of the intracavity field, and ϵ_d is amplitude of the pump field and is related to the field power P by $\epsilon_d = \sqrt{\frac{2\kappa P}{\hbar\omega_d}}$ with 2κ being the cavity decay rate.
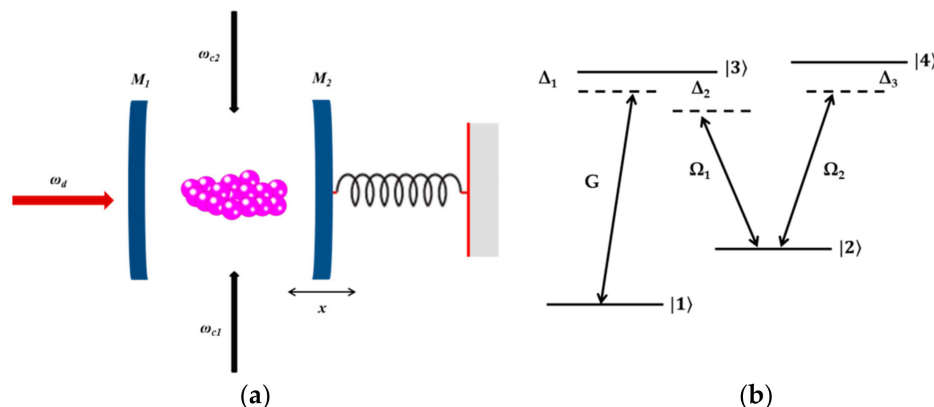
Figure 1. (**a**) Schematic of cavity optomechanical system containing an atomic ensemble. (**b**) Structures of N-type atomic level.

In this paper, we mainly discuss the influences of the coupling of the cavity mode interacting with atom and the existence of two controlling fields on the optical bistability. Under the condition of the weak excitation and atom number $M \gg 1$, we let $\sigma_{13} = \frac{1}{\sqrt{M}}\sum_{j=1}^{M}\sigma_{13}^j$, $\sigma_{12} = \frac{1}{\sqrt{M}}\sum_{j=1}^{M}\sigma_{12}^j$,

$\sigma_{14} = \frac{1}{\sqrt{M}} \sum_{j=1}^{M} \sigma_{14}^{j}$ stand for collective excitation operators [17–20]. In order not to lose generality, we take into account the decay of the cavity mode, the decay of the atomic ensemble and the thermal quantum noise. Using the Hamiltonian, we write the Heisenberg–Langevin equations of motion for the operators:

$$
\begin{aligned}
\dot{a} &= -\left(\kappa + i\omega_0 - \frac{i\chi_0 x}{\hbar}\right)a - iG\sqrt{M}\sigma_{13} + \epsilon_d e^{-i\omega_d t} + \sqrt{2\kappa}a_{in} \\
\dot{\sigma}_{13} &= -(\gamma_{13} + i\omega_3 - i\omega_1)\sigma_{13} - i\left(G\sqrt{M}a + \Omega_1 e^{-i\omega_{c1}t}\sigma_{12}\right) + \sqrt{2\gamma_{13}}f_{13}(t), \\
\dot{\sigma}_{12} &= -(\gamma_{12} + i\omega_2 - i\omega_1)\sigma_{12} - i\left(\Omega_1 e^{i\omega_{c1}t}\sigma_{13} + \Omega_2 e^{i\omega_{c2}t}\sigma_{14}\right) + \sqrt{2\gamma_{12}}f_{12}(t), \\
\dot{\sigma}_{14} &= -(\gamma_{14} + i\omega_4 - i\omega_1)\sigma_{14} - i\Omega_2 e^{-i\omega_{c2}t}\sigma_{12} + \sqrt{2\gamma_{14}}f_{14}(t), \\
\dot{x} &= \frac{p}{m}, \\
\dot{p} &= -\gamma_m p + \chi_0 a^\dagger a - m\omega_m^2 x + \xi(t),
\end{aligned}
\tag{2}
$$

where some decay rates are phenomenologically added: γ_m is the damping rate of the mechanical resonator, and γ_{13}, γ_{12}, γ_{14} denote the off-diagonal decay rates of atomic operators σ_{13}, σ_{12}, σ_{14}, respectively. a_{in} describes quantum vacuum fluctuation of the cavity field, $f_{13}(t)$, $f_{12}(t)$, $f_{14}(t)$ are the noise force acting on the atomic ensemble, and $\xi(t)$ represents the noise acting on the mechanical oscillator. Here, we only consider the average response characteristic of the system and the mean values of these noise operator are zero, so we can neglect the quantum fluctuations of the system. In addition, we assume all Rabi frequencies are real numbers.

In the rotating frame, we rewrite operators $a = \widetilde{a}e^{-i\omega_d t}$, $\sigma_{13} = \widetilde{\sigma}_{13}e^{-i\omega_d t}$, $\sigma_{12} = \widetilde{\sigma}_{12}e^{i(\omega_{c1}-\omega_d)t}$, $\sigma_{14} = \widetilde{\sigma}_{14}e^{-i(\omega_d-\omega_{c1}+\omega_{c2})t}$, and obtain the time evolutions for the expectation values of $\widetilde{a}, \widetilde{\sigma}_{13}, \widetilde{\sigma}_{12}, \widetilde{\sigma}_{14}, x, p$:

$$
\begin{aligned}
\left\langle \dot{\widetilde{a}} \right\rangle &= -\left(\kappa + i\Delta_0 - \frac{i\chi_0 x}{\hbar}\right)\langle\widetilde{a}\rangle - ig\langle\widetilde{\sigma}_{13}\rangle + \epsilon_d, \\
\left\langle \dot{\widetilde{\sigma}}_{13} \right\rangle &= -(\gamma_{13} + i\Delta_1)\langle\widetilde{\sigma}_{13}\rangle - i(g\langle\widetilde{a}\rangle + \Omega_1\langle\widetilde{\sigma}_{12}\rangle), \\
\left\langle \dot{\widetilde{\sigma}}_{12} \right\rangle &= -(\gamma_{12} + i(\Delta_1 - \Delta_2))\langle\widetilde{\sigma}_{12}\rangle - i(\Omega_1\langle\widetilde{\sigma}_{13}\rangle + \Omega_2\langle\widetilde{\sigma}_{14}\rangle), \\
\left\langle \dot{\widetilde{\sigma}}_{14} \right\rangle &= -(\gamma_{14} + i(\Delta_1 - \Delta_2 + \Delta_3))\langle\widetilde{\sigma}_{14}\rangle - i\Omega_2\langle\widetilde{\sigma}_{12}\rangle, \\
\langle\dot{x}\rangle &= \frac{\langle p\rangle}{m}, \\
\langle\dot{p}\rangle &= -\gamma_m\langle p\rangle + \chi_0\langle a^\dagger\rangle\langle a\rangle - m\omega_m^2\langle x\rangle,
\end{aligned}
\tag{3}
$$

where, $\Delta_0 = \omega_0 - \omega_d$ is the detuning between the pump field and the cavity mode, $\Delta_1 = \omega_{31} - \omega_d$ represents the detuning of the cavity mode with atom, and $\Delta_2 = \omega_{32} - \omega_{c1}$ and $\Delta_3 = \omega_{42} - \omega_{c2}$ are detunings of the two controlling fields, respectively. The coupling parameter $g = G\sqrt{M}$

In the condition of the steady-state, we get the solution of the amplitude $\langle\widetilde{a}_s\rangle$ of the cavity field

$$
\langle\widetilde{a}_s\rangle = \frac{\epsilon_d}{\kappa + \frac{g^2}{X} + i\Delta},
\tag{4}
$$

where $\Delta = \Delta_0 - \frac{\chi_0^2 I_a}{\hbar m\omega_m^2}$ with $I_a = \left|\langle\widetilde{a}_s\rangle\right|^2$ is the mean intracavity photon number and $X = \gamma_{13} + i\Delta_1 + \frac{\Omega_1^2[\gamma_{14}+i(\Delta_1-\Delta_2+\Delta_3)]}{\Omega_2^2+[\gamma_{12}+i(\Delta_1-\Delta_2)][\gamma_{14}+i(\Delta_1-\Delta_2+\Delta_3)]}$.

Next, we investigate the dependence of the mean intracavity photon number on the physical parameters of the system. From Equation (4), we obtain the expression obeyed by the mean intracavity photon number

$$
I_a\left[\left(\kappa + g^2 F_2\right)^2 + \left(\Delta_0 - \frac{\chi_0^2 I_a}{\hbar m\omega_m^2} - g^2 F_1\right)^2\right] = |\epsilon_d|^2,
\tag{5}
$$

where, $F_1 = \frac{\Lambda_1}{\Lambda_1^2 + \Lambda_2^2}$, $F_2 = \frac{\Lambda_2}{\Lambda_1^2 + \Lambda_2^2}$, $\Lambda_1 = \Delta_1 - \frac{\Omega_1^2 J_1}{J_1^2 + J_2^2}$, $\Lambda_2 = \gamma_{13} + \frac{\Omega_1^2 J_2}{J_1^2 + J_2^2}$, $J_1 = \Delta_1 - \Delta_2 - \kappa_2(\Delta_1 - \Delta_2 + \Delta_3)$, $J_2 = \gamma_{12} + \kappa_2 \gamma_{14}$, $\kappa_2 = \frac{\Omega_2^2}{\gamma_{14}^2 + (\Delta_1 - \Delta_2 + \Delta_3)^2}$. In addition, Equation (5) is a function about the cube of the mean intracavity photon number I_a, so the mean intracavity photon number can show bistable behavior within the given parameters range. In order to simplify the discussion, we assume that the system has a uniform dissipation and is in the nonresonant condition where the concrete value of detuning is shown in next section, and we also assume $\gamma_{13} = \gamma_{12} = \gamma_{14} = \gamma$, $\Delta_1 = \gamma, \Delta_2 = \Delta_3 = 2\gamma$.

3. Numerical Simulations and Discussion

In this section, we will discuss the influences of different physical parameters on the optical bistability. In our numerical analysis, our choices of physical parameters are based on reference [21], and their values are shown: $\lambda = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\kappa = 2\pi \times 215$ kHz, $\omega_m = 2\pi \times 947$ kHz, $\gamma_m = 2\pi \times 141$ Hz, and γ and Ω_0 are given by $\gamma = 2\pi \times 10^6$ Hz, $\Omega_0 = \pi \times 10^9$ Hz, respectively. Furthermore, we let quality factor of cavity mechanical system $Q = \frac{\omega_m}{\gamma_m} = 6700$. In Figure 2, we firstly investigate how the coupling strength of the cavity mode with atom affect the bistability when the other physical parameters are given. In Figure 2a, we find that the coupling strength of cavity-atom $g = 0$, the behavior of optical stability is the most obvious. Along with increasing the coupling strength, the phenomenon of optical bistability become gradually weak. From Equation (2), we know, when g is zero, atomic ensemble and cavity field are two separate systems. In this case, the cavity mode is only manipulated by mechanical resonator, and the system is in an optomechanical system. For the optomechanical system, optical bistability can be realized under the condition of steady state. When the coupling strength of cavity-mode increases gradually. The dependence of the mean intracavity photon number on the weak pump field decreases gradually. The system mainly shows the cavity field interacts with a two-level atomic ensemble, and its nonlinearly decreases little by little, so finally the phenomenon of optical bistability become weak. The dependence of the bistable region on the coupling strength of the cavity-atom is given by in Figure 2b, and we find when the g is greater than 33.33 MHz, the bistability will disappear completely. So, adding the coupling of cavity-atom can adjust the bistable behavior of the system according to the curves in Figure 2.
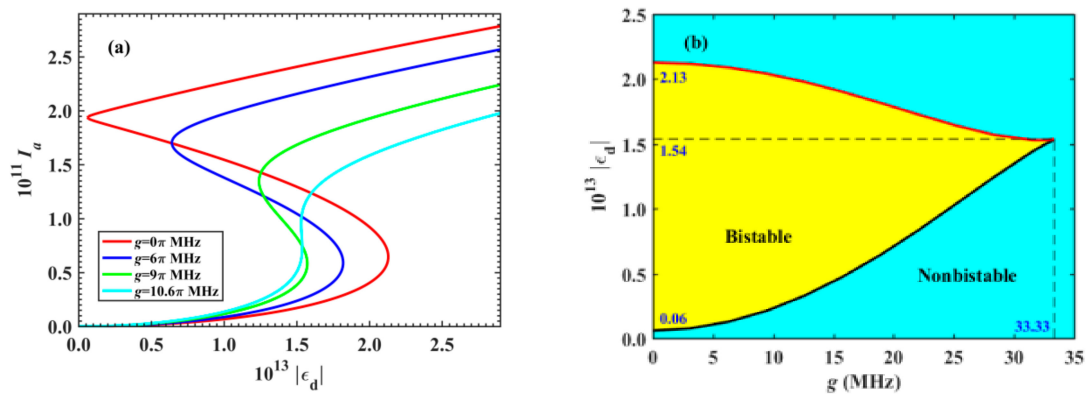
Figure 2. (**a**) Curves of the mean intracavity photon number versus the intensity of the pump field under the condition of different coupling of cavity-atom. (**b**) The relation of bistable region and the coupling of cavity-atom. Where $\gamma_{13} = \gamma_{12} = \gamma_{14} = \gamma$, $\Delta_1 = \gamma$, $\Delta_2 = \Delta_3 = 2\gamma$, $\Delta_0 = 20\gamma$, $\Omega_1 = \Omega_2 = 4.5\Omega_0$.

Next, we consider the effect of the controlling field Ω_1 on the bistable behavior of distribution of mean intracavity photon number. Here we give the values of some physical parameters: $\gamma_{13} = \gamma_{12} = \gamma_{14} = \gamma$, $\Delta_1 = \gamma$, $\Delta_2 = \Delta_3 = 2\gamma$, $\Delta_0 = 20\gamma$, $\Omega_2 = 4.5\Omega_0$, and $g = 6\pi \times 10^6$ Hz. We plot the mean intracavity photon number as a function of the intensity of the pump field when

the controlling field Ω_1 has different values in Figure 3. Figure 3 shows the controlling field Ω_1 can obviously affect the bistable behavior. When the intensity of the controlling field becomes gradually strong, the threshold of bistability changes significantly, which means the nonlinearity of system is obviously enhanced. The main reason is that the controlling field Ω_1 make the atomic coherence of the system become stronger. We know the the controlling field Ω_1 couples with levels $|2\rangle$ and $|3\rangle$, and increasing its intensity will make states $|2\rangle$ and $|3\rangle$ go into dressed states which produce atomic coherence between states $|1\rangle$ and $|3\rangle$ when the cavity mode interacts with $|1\rangle$ and $|3\rangle$. This kind of atomic coherence may increase the nonlinearity of the system, so the behavior of bistability becomes obvious.
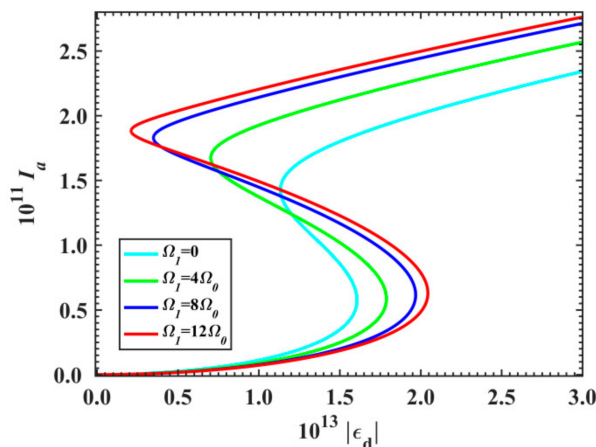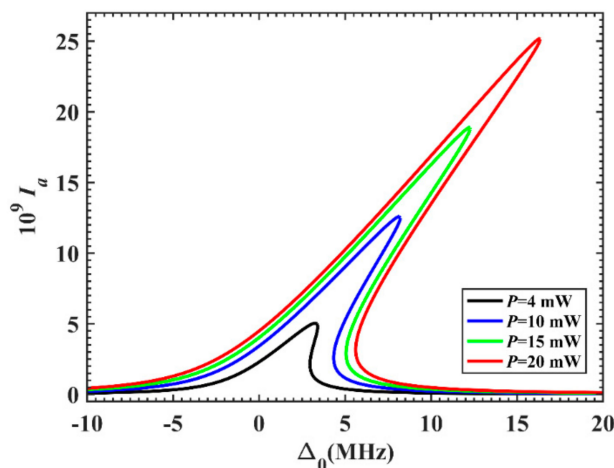

Figure 3. Curves of the mean intracavity photon number versus the intensity of the pump field under the condition with different the controlling field Ω_1, where $\gamma_{13} = \gamma_{12} = \gamma_{14} = \gamma$, $\Delta_1 = \gamma$, $\Delta_2 = \Delta_3 = 2\gamma$, $\Delta_0 = 20\gamma$, $\Omega_2 = 4.5\Omega_0$, $g = 6\pi \times 10^6$ Hz.

Application of the controlling fields can let atomic ensemble go into the states of atomic coherence, which affects the coupling of the cavity mode with the pump field. Now we analyze effects of the detuning between the cavity mode-pump field on the behavior of bistability (see Figure 4). Figure 4 shows we can obtain the optical bistability when changing the cavity-pump detuning. The bistable behavior does not take place when the cavity-pump detuning $\Delta_0 = 0$, that is the pump field are resonant with the cavity mode. With increasing the cavity-pump detuning, optical bistability occurs and becomes more and more significant. In this case where there exits atomic coherence, we can produce the optical bistability under the nonresonant condition. A nonresonant condition is easily realized in concrete physical experiments, so the introduction of atomic coherence makes nonlinearity of the system become stronger and optical bistability can be easily obtained.

Finally, we discuss the dependence of the mean intracavity photon number on pump intensity. Figure 5 shows the mean intracavity photon number versus the cavity-pump field detuning Δ_0 for different pump field intensity. When the pump intensity is relatively weak ($P = 4$ mW), the cure of the mean intracavity photon number nearly approached Lorentzian distribution, which can be obtained in an optomechanical resonator containing a quantum well [12]. When the pump intensity changes from 4 mW to 20 mW, the mean intracavity photon number corresponds to three real roots for a given value of cavity-pump detuning in the region with cavity-pump detuning greater than 5 MHz, which means the cubic equation for the mean intracavity photon number (see Equation (5)) yields three real roots. In this case with high pump power, the signal of optical bistability can be produced.
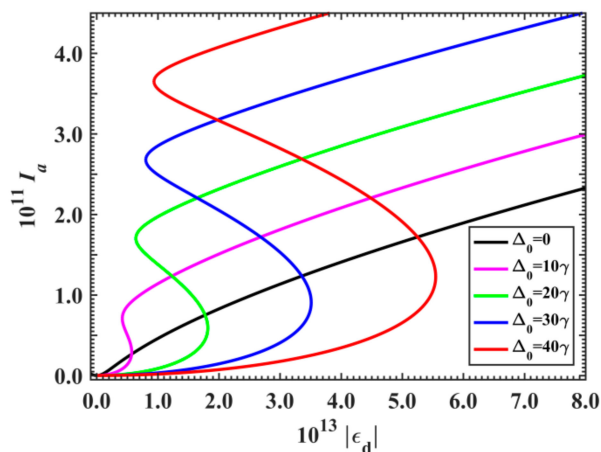
Figure 4. Curves of the mean intracavity photon number versus the intensity of the pump field under the condition with different detuning between the cavity mode-pump field, where $\gamma_{13} = \gamma_{12} = \gamma_{14} = \gamma$, $\Delta_1 = \gamma, \Delta_2 = \Delta_3 = 2\gamma, \Omega_1 = \Omega_2 = 4.5\Omega_0, g = 6\pi \times 10^6$ Hz.

Figure 5. Curves of the mean intracavity photon number versus cavity mode-pump field detuning under the condition with different pump field power, where $\gamma_{13} = \gamma_{12} = \gamma_{14} = \gamma, \Delta_1 = \gamma$, $\Delta_2 = \Delta_3 = 2\gamma, \Omega_1 = \Omega_2 = 4.5\Omega_0, g = 0$.

4. Conclusions

In conclusion, we have shown that the bistable behavior of the mean intracavity photon number in a hybrid optomechanical system containing *N*-type atomic ensemble can be generated due to the enhanced nonlinearity. Our model of *N*-type atomic ensemble is based on the research work of Li's group [22] where they obtain the optical bistability in *N*-type atomic ensemble under the coherent manipulation [23,24]. However, Li and his coworkers investigate the bistable behavior of a weak probe field [22]. We extend their research and place *N*-type atomic ensemble in cavity optomechanical system. We focus on the distribution of photon number of cavity field under the existence of two controlling fields. We have also taken note of the report about optical bistability in optomechanical system containing two-level atomic ensemble [25,26]. In a two-level atomic system, atom only couples with the cavity field and there is no atomic coherence in the system. After a multi-level atomic ensemble is introduced, besides the coupling between the atom and the cavity field, there are also interactions between the atom and the two controlling fields. Under the coupling of the controlling fields, atomic coherence will be appeared. Atomic coherence is also the source of electromagnetically induced transparency. In our model, atomic coherence originating from the interaction of atomic ensemble with the controlling fields makes the nonlinearity of system become strong, and the optical bistability of the mean intracavity photon number can easily be realized. Compared with two-level

atomic ensemble, our multi-level system provides more means to change the threshold of optical bistability. Our researches show that, under the nonresonant condition, we analyze the influences of the coupling strength between cavity and atoms, Rabi frequencies of the controlling light field, the detuning between the controlling light field and atoms, and pump field power on the optical bistable behavior of mean intracavity photon number. These research results indicate appropriate selections of parameters can change the threshold of bistability. At present, the cavity optomechanical technology has matured and electromagnetically induced transparency based on atomic coherence has been reported. By introducing a cold rubidium atom into the cavity optomechanical system, the optical bistability may be realized in the near future. In addition, the application of the controlling field, we can use atomic coherence to achieve optical bistability. This method of external intervention is an effective one for enhancing the nonlinearity of the system. The optical bistable property of the mean intracavity photon number has a potential application in field optical switches and optical bistable devices.

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Conflicts of Interest: The authors declare no conflict of interest.

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