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Frequency Pulling and the Linewidth Enhancement Factor in Optically Injected Semiconductor Laser

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Abstract: The effect of the linewidth enhancement factor (LEF) on the frequency pulling behavior in optically injected lasers is theoretically investigated. The frequency pulling is found to be exponentially dependent on the LEF. This dependence is systematically revealed and explained.

Keywords: semiconductor lasers; optical injection; frequency pulling; linewidth enhancement factor

1. Introduction

The frequency pulling behavior in oscillators was discovered as early as 1946 [1], when Robert Adler derived an equation for an oscillator with an external injected signal to describe the pull-in behaviour. It took a while for this phenomenon to be reported in optically injected semiconductor lasers [2]. Thereafter, the existence of frequency pulling and frequency pushing when mapping the injection locking of an optically injected laser diode was reported [3,4]. A considerable region of frequency pulling was determined in the injection locking map toward the negative frequency detuning [5]. This frequency pulling was also observed in the low injection region, where the laser can be used as an amplifier of weak signals [6].

Another breakthrough was reported in 2004, when frequency pulling was used for synchronization of nonlinear oscillators [7]. A theoretical description, along with experimental observation, was also developed to explore the frequency pulling effect on pulse parameters in a mode-locked laser [8]. This frequency pulling was found to be largely dependent on the injection strength (i.e., the level of the injected signal power) [9]. Another study [10] of period-one oscillation in an optically injected semiconductor laser showed that there exists a competition between the red-shifting effect and the injection pulling effect inside the laser cavity. This competition determines the dependence of the free-running frequency on the injection level. The frequency pulling in that study was found to be confined to a small region around the free-running frequency. A pushed stable locking range was observed when two masters were injected simultaneously [11]. The direction of this pushing was found to be toward the negative detuning frequency with a low injection ratio.

It was expected that this phenomenon of frequency pulling would be useful in several applications. One of the most advantageous utilizations is the use of frequency pulling in the synchronization of nonlinear oscillators [7,12]. A recent study [13] showed that frequency pulling can be used to generate an optical frequency comb in a quantum cascade laser (QCL). Another recent study [14] provided a comprehensive (theoretical and experimental) study of injection locking and pulling in optoelectronic oscillators (OEO). That study found that frequency pulling depends on three parameters: the frequency detuning, the injection level, and the Q factor of the free-running OEO.

Class B lasers, including semiconductor lasers, have a very distinguished parameter called the linewidth enhancement factor (LEF), or α-factor. This factor describes the interaction between the photons and the carriers in the laser. This factor has been in the focus area of investigation since 1982 [15–17], with many methods measured [18–20]. Even

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though many studies have been carried out, the dependence of frequency pulling on the LEF has not yet been investigated to the best of my knowledge. This is very important in lasers used in telecommunication, where pulling can affect the quality of the signal and LEF can slightly change due to different factors. In this article, I investigate, from a theoretical perspective, the effect of the LEF on the frequency-pulling phenomenon in optically injected semiconductor lasers. Different acceptable values for the LEF were chosen, and each time I studied the frequency pulling. In this article, I explain the dynamics.

2. Method

The model used in this study was a simple model of optical injection looking described in detail in [21]. The rate equations for the slave laser (SL) electric field amplitude, phase, and carrier density can be stated, respectively, as follows:

$$\frac{d}{dt}E_o(t) = \frac{1}{2}G_N\Delta N(t)E_o(t) + \eta E\cos(\Delta t) \tag{1}$$

$$\frac{d}{dt}\varphi_o(t) = \frac{1}{2}\alpha G_N\Delta N(t) + \eta\frac{E}{E_o(t)}\sin(\Delta t) \tag{2}$$

$$\frac{d}{dt}N(t) = J - \frac{N(t)}{\tau_s} - G_N(N(t) - N_o)E_o^2(t) \tag{3}$$

where E_o (t) and E are the electric fields of the SL and the master laser (ML), respectively, G_N is the gain coefficient, $\Delta N(t)$ is the ratio of carriers in the cavity, or the so-called population inversion defined as $N - N_{th}$, where N is the carrier density and N_{th} is its value at the threshold, η is the coupling coefficient, $\Delta t_m = \Delta\omega_m t - \phi_o(t)$, where $\Delta\omega_m = \omega_m - \omega_o$ (the angular frequency detuning between the SL laser and the ML), ϕ_o (t) is the SL phase, α is the linewidth enhancement factor (LEF), and N_o is the carrier density at transparency. τ_s is the lifetime for spontaneous emission and non-radiative recombination. The injection level K can be expressed as the ratio of the injected field (E) of the ML to the free-running SL field (E_{os}), which is given by $E_{os} = \sqrt{\tau_p(J - N_{th}/\tau_s)}$, where τ_p is the photon lifetime. The results are presented in terms of the stability map of K versus Δf (where $\Delta f = \Delta\omega 2\pi$).

I numerically integrated the rate in Equations (1)–(3) using the Runge–Kutta method in a commercial computer program (Matlab) to investigate the behavior of the system. The theoretical power spectra were obtained by applying a fast Fourier transform (FFT) to a chosen time window of the SL electric field time series. The colored injection locking maps shown in the results were obtained by running the system and recording the location of the SL peak outside the locking region to determine frequency pulling. All the parameters used in simulation were based in real characterized parameters as stated in [21] and shown in Table 1. This model showed very good agreement with experiments in many previous studies [21–23].

Table 1. Parameters used in this study's simulation.

Parameter	**Symbol**	**Value**	**Unit**
Wavelength	λ	1556.6	nm
Differential gain	G_N	1.4×10^{-12}	$m^3 s^{-1}$
Carrier lifetime	τ_s	0.43	ns
Photon lifetime	τ_p	1.8	ps
Coupling rate	η	9×10^{10}	s^{-1}
Transparency carrier density	N_o	1.1×10^{24}	m^{-3}
Threshold carrier density	N_{th}	1.5×10^{24}	m^{-3}
Normalized injection current	I/I_{th}	2	

3. Results and Discussion

In order to study the effect of the LEF on frequency pulling behavior, I first ran the model to draw the stability map for different LEF values (0, 1, 2, 3, 4, and 5), as shown in

Figure 1. The color bars represent the frequency pulling in GHz, which is the reason why inside the locking bandwidth the map takes a unified color, as the SL is totally locked to the ML. The colors change at both edges of the locking bandwidth (i.e., when frequency pulling takes place). The locking bandwidth (the area where the SL is locked to the ML) becomes boarder as the LEF increases. This phenomenon (the broadening of the locking region as the LEF increases) has been discussed in detail elsewhere [24]. The stable locking, however, shrinks as the LEF increases [24]. In terms of frequency pulling, it can be seen that frequency pulling is higher (darker colors) at higher injection levels [25].

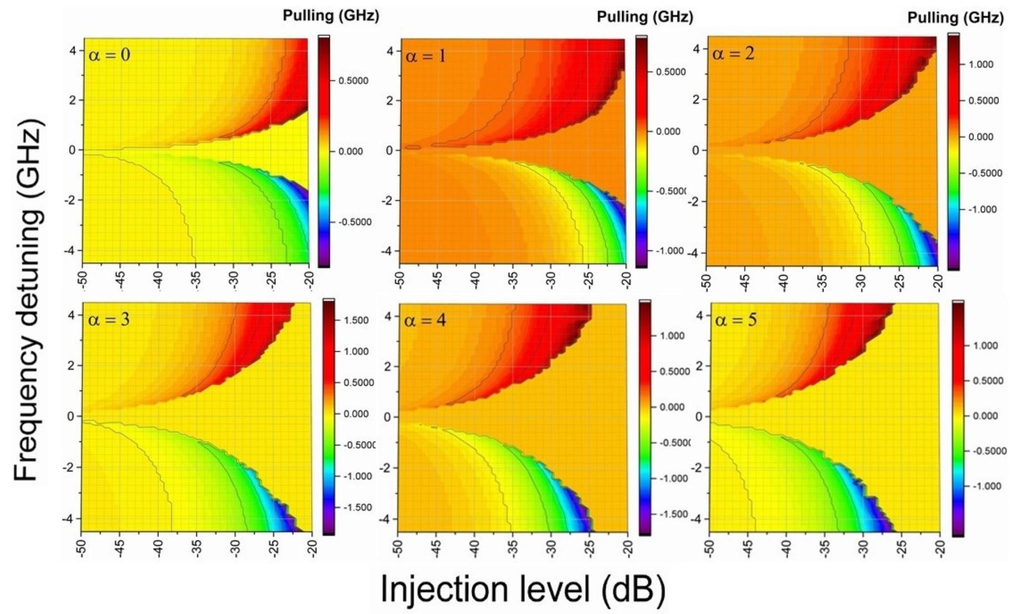

Figure 1. Injection locking map for six values of the LEF (0−5), showing the intensity of frequecy pulling at both edges of the locking region.

It can also be seen that the locking bandwidths look slightly asymmetric around the zero-frequency detuning. This phenomenon is due to the small fluctuation in the carrier's density, which leads to another fluctuation in the refractive index, leading eventually to a small shift in the resonance frequency [21].

To specifically see the relation between the LEF and frequency pulling, I extracted data from Figure 1 to compare frequency detuning and frequency pulling for the chosen LEF values, as shown in Figure 2. Note that this figure is a cross-sectional type of the previous figures, taken at a constant injection level (−30 dB). The dashed line represents the locking bandwidth edges and the values of the LEF are indicated by different colors. The figure clearly illustrates that as the LEF increases, frequency pulling is enhanced. That is mainly because the LEF boosts the interaction between photons and carriers, resulting in a higher shift and increased pulling. The figure also shows that even when the LEF equals to zero, frequency pulling is still evident, notwithstanding its very low values.

Next, I show the different behaviors exhibited by the system at the points indicated by the arrows in Figure 2. Figure 3 illustrates the power spectra of the SL at these four points. In Figure 3a, the SL is pulled (0.2 GHz) by the ML (at 0.6 GHz) with zero LEF. The power of the resonance SL is slightly depleted, with one-sided four-wave mixing (FWM). The power of the SL is transferred to the ML signal during the pulling process, as described elsewhere [22]. This is a typical behavior of pulled ML. In Figure 3b, the SL is pulled (−1 GHz) by the ML (at −2.2 GHz), with the LEF equal to 4. The same behavior was observed again with enhanced FWM, as a result of the slightly high value of the LEF. In Figure 3c, even though the LEF is higher (5), the FWM is not enhanced as much as in the previous case. That is clearly because the frequency detuning in this case (3.7 GHz) is larger than in the previous one (−2.2 GHz). The same reason can be provided for the lower value of the frequency detuning in this case (around 0.8 GHz). However, in all three cases, the FWM peaks appear at the same spacing between the SL and the ML (i.e., the frequency detuning). Finally, Figure 3d shows a typical injection locking at −0.6 GHz and LEF = 1. The SL is nicely locked to the ML, with no side peaks at all.

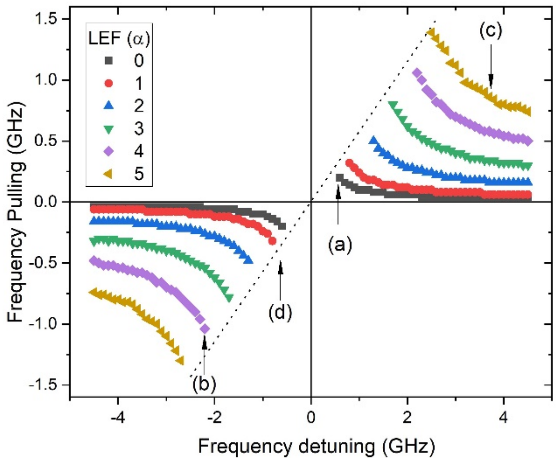

Figure 2. Frequency pulling vs. frequency detuning for different values of the LEF (0−5) at a constant injection level (−30 dB). The dashed line represents the locking bandwidth. The labels (**a**–**d**) represent the points at which the power spectra in Figure 3 are taken.

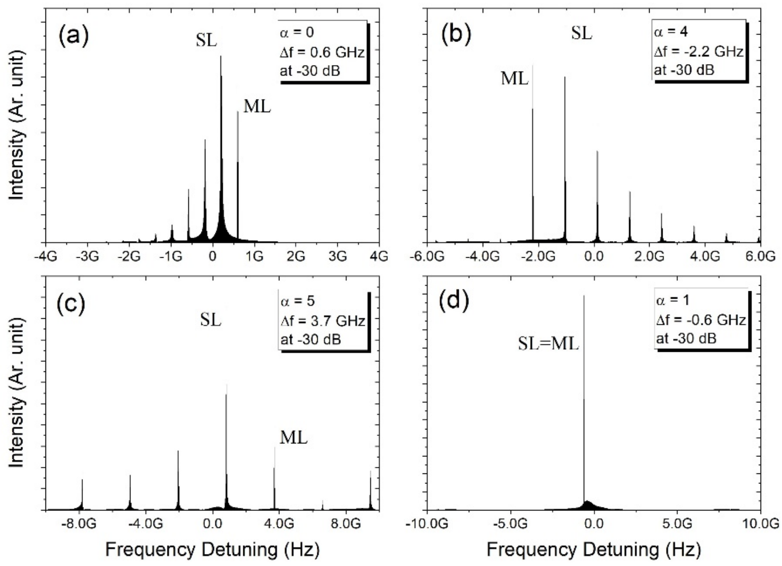

Figure 3. The power spectra of the laser output taken at the levels shown in Figure 2. (**a**) $\alpha = 0$ and $\Delta f = 0.6$ GHz, (**b**) $\alpha = 4$ and $\Delta f = -2.2$ GHz, (**c**) $\alpha = 5$ and $\Delta f = 3.7$ GHz and (**d**) $\alpha = 1$ and $\Delta f = -0.6$ GHz. Note that the injection level in all cases is constant (−30 dB).

To reveal the specific dependence of frequency pulling on the LEF, I drew them in comparison with each other at different frequency detuning values, as shown in Figure 4. It can be seen that frequency pulling increases as the LEF increases, due to the enhancement between the carriers and the photons. For any given frequency detuning (for instance 2.5 GHz, the black square), the pulling may jump from nearly zero to approximately 1.4 GHz, when the LEF increases from 1 to 5. This relationship is obviously exponential, revealing that frequency pulling is largely and sensitively dependent on the LEF. Moreover, this becomes exponentiality stronger with lower frequency detuning (a high interaction between the SL and the ML), while approaching linearity with very large frquency detuning. This feature can be utilized in many nonlinear optical applications.

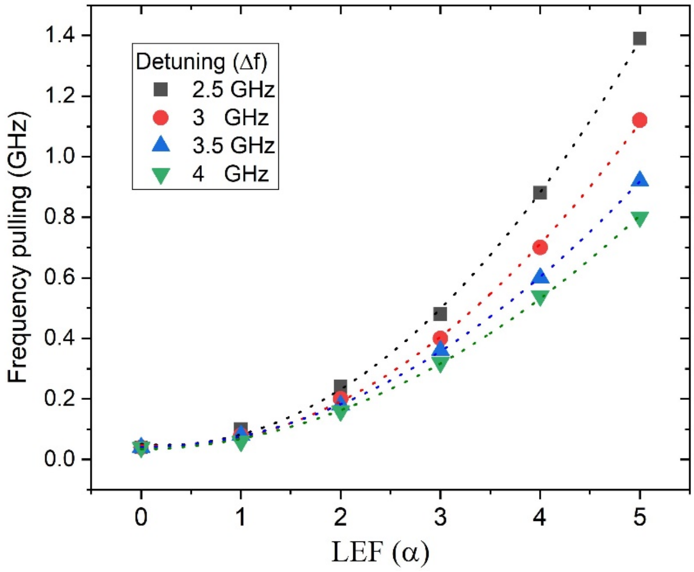

Figure 4. The dependence of frequency pulling on the LEF for different frequency detunings.

The figure also reveals that frequency pulling increases as the detuning decreases. That is mainly because as the injected signal comes closer to the free running laser, it starts stealing photons (i.e., gain) from the SL signal, leading to the observed frequency pulling. Even at zero LEF, frequency pulling seems to be present, but with no effect on the frequency detuning, as shown in the figure.

4. Conclusions

I theoretically investigated the dependence of the frequency-pulling phenomenon on the LEF in optically injected semiconductor lasers. The enhancement in the LEF was shown to boost frequency pulling exponentially. This dependence seemed to be stronger at lower frequency detuning, where the interaction of carriers was expected to be higher. This result is believed to contribute to further understanding of the nonlinear dynamics in optically injected semiconductor lasers and their unlimited applications, especially in terms of cryptographical communication and similar technologies. Further study should investigate the experimental validation of this theoretical prediction.

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