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Communication

Theoretical Investigation of the Capacity of Space Division Multiplexing with Multimode Step-Index Air-Clad Silica Optical Fibers

Svetislav Savović [1,2], Alexandar Djordjevich [3], Isidora Savović [4], Branko Drljača [5], Ana Simović [2] and Rui Min [1,*]

1 Center for Cognition and Neuroergonomics, State Key Laboratory of Cognitive Neuroscience and Learning, Beijing Normal University at Zhuhai, Zhuhai 519087, China; savovic@kg.ac.rs
2 Faculty of Science, University of Kragujevac, R. Domanovića 12, 34000 Kragujevac, Serbia; asimovic@kg.ac.rs
3 Department of Mechanical Engineering, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong, China; mealex@cityu.edu.hk
4 Laboratory of Neurodegenerative Disease, School of Biomedical Sciences, LKS Faculty of Medicine, The University of Hong Kong, 21 Sassoon Road, Pokfulam, Hong Kong, China; u3008169@connect.hku.hk
5 Faculty of Sciences and Mathematics, University of Priština in Kosovska Mitrovica, L. Ribara 29, 38220 Kosovska Mitrovica, Serbia; branko.drljaca@pr.ac.rs
* Correspondence: rumi@doctor.upv.es

Abstract: We studied the effect of mode coupling on the space division multiplexing (SDM) capabilities of multimode step-index (SI) air-clad silica optical fibers by numerically solving the power flow equation. Mode coupling considerably reduces the length of these fibers at which space division multiplexing may be achieved with minimal crosstalk between neighboring optical channels, according to the findings. Up to 120 m and 30 m, respectively, the two and three spatially multiplexed channels in the investigated multimode step-index silica optical fibers can be used with low crosstalk. When building a space division multiplexing-based optical fiber transmission system, such characterization of optical fibers should be taken into account.

Keywords: air-clad silica optical fibers; microbends; mode coupling; space division multiplexing

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1. Introduction

Global network traffic has expanded dramatically in recent decades, owing primarily to the rapid expansion of the Internet [1]. Optical fiber systems now support the majority of this data flow. Multiplexing, fiber amplifiers, and high-efficiency spectral coding have all contributed to this capacity gain [1]. Optical data multiplexing is possible not just in wavelength, but also in space, time, polarization, and phase. Optical fiber transmission systems can benefit from the multiplexing technique [1,2]. SDM, which includes mode division multiplexing using few-mode fibers or multimode and/or core multiplexing utilizing multicore fibers, has gotten a lot of attention as a way to increase optical communication's multiplicative capacity [1,3–8]. SDM can operate at the same or separate wavelengths [9]. If the SDM channels inside the carrier fiber are assigned radially distributed optical signals at the same wavelength, the central channel is launched along the fiber axis in the form of a disk, whereas all other channels are in the form of concentric rings. Thus, one increases the capacity of the optical fiber link (Figure 1).

Silica optical fibers (SOFs) are suited to long-distance signal communication [10], while plastic optical fibers (POFs) with a large core are extensively used in short-distance (less than 100 m) networks [11]. Optical measurements, telecommunications, and sensor applications all use plastic-clad silica fibers (PCSFs) [12,13]. Currently, the only technology capable of reaching exceptionally high NAs is PCSFs and air-clad silica fibers. In commercially marketed PCSFs, NA is limited to ~0.46. An all-silica fiber is promising for SDM because of the material durability and sufficient silica purity for strong transmission performance in the large wavelength range of 300 nm to 2.4 μm [14–16]. In contrast, air-clad fibers can have an extraordinarily high numerical aperture ($NA_{meas} > 0.9$) [17]. Figure 2 is a cross-sectional sketch of an all-silica air-clad fiber with an annulus made up of a single ring of holes, produced at the University of Sydney [15]. The ring, or rings, of air holes, whose surface roughness has been measured to be 0.5 nm [18], are solely responsible for the NA of air-clad fibers. The thin bridges are typically between 100 and 400 nm thick, with the ring of holes forming a corrugated surface at the core. To decrease light leakage, the thickness was chosen.

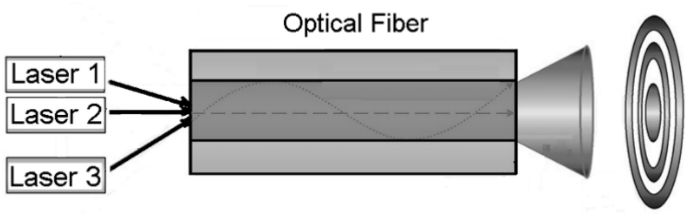

Figure 1. A schematic of a three-channel SDM system, consisting of a center disk and two concentric rings.

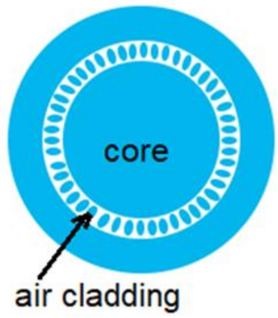

Figure 2. Sketch of cross section of air-clad silica fiber based on the design used in the experiments of [15].

Fiber imperfections and inhomogeneities introduced during the optical fiber fabrication process cause power transfer between adjacent modes [9,19,20]. Because SDM entails densely packed spatial channels in a fiber, mode coupling is critical since it allows crosstalk between channels. As a result, the expected beam parameters are altered. The optical power distribution of an optical fiber determines its far-field pattern, which is influenced by launch conditions, fiber parameters, and fiber length. Only short fibers will produce a highly defined ring radiation pattern when light is launched at a given angle $\theta_0 > 0$ with respect to the fiber axis. The boundaries of such a ring grow fuzzy at the end of longer fibers due to mode coupling. At "coupling length" L_c, where the highest-order mode ring-pattern evolves into a disk, an equilibrium mode distribution (EMD) is established.

Because information on SDM in multimode air-clad silica optical fibers is lacking in the literature, we investigated mode coupling in multimode SI air-clad silica optical fibers in this work by numerically solving the power flow equation. This fiber was previously investigated experimentally by Åslund et al. [15]. This allows one to determine the maximum fiber lengths for an SDM system that employs multimode SI air-clad silica optical fibers.

2. Power Flow Equation

Gloge's power flow equation is [19]:

$$\frac{\partial P(\theta,z)}{\partial z} = -\alpha(\theta)P(\theta,z) + \frac{D}{\theta}\frac{\partial}{\partial\theta}\left(\theta\frac{\partial P(\theta,z)}{\partial\theta}\right), \quad (1)$$

where $P(\theta,z)$ is the angular power distribution, θ is the propagation angle in respect to the core axis, z is the distance from the input end of the optical fiber, D is the constant coupling coefficient [19,21–23], and $\alpha(\theta)$ is the modal attenuation. Since $\alpha(\theta)$ need not be accounted for in solving Equation (1) for mode coupling [22,23], Equation (1) reduces to [15]:

$$\frac{\partial P(\theta,z)}{\partial z} = \frac{D}{\theta}\frac{\partial P(\theta,z)}{\partial \theta} + D\frac{\partial^2 P(\theta,z)}{\partial \theta^2}. \tag{2}$$

The explicit finite-difference approach [24] was used to derive a numerical solution of the power flow Equation (2) for a Gaussian launch-beam distribution of the form:

$$P(\theta,z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\theta-\theta_0)^2}{2\sigma^2}\right], \tag{3}$$

with $0 \le \theta \le \theta_c$, where θ_0 is the mean value of the launch angular distribution, σ is the standard deviation, and FWHM = $2\sigma\sqrt{2\ln 2} = 2.355\sigma$. We discretized Equation (2) using explicit finite difference method, so Equation (2) now reads [24]:

$$P_{i,j+1} = \left(\frac{\Delta z D}{\Delta\theta^2} - \frac{\Delta z D}{2\theta_{i,j}\Delta\theta}\right)P_{i-1,j} + \left(1 - \frac{2\Delta z D}{\Delta\theta^2}\right)P_{i,j} + \left(\frac{\Delta z D}{2\theta_{i,j}\Delta\theta} + \frac{\Delta z D}{\Delta\theta^2}\right)P_{i+1,j}, \tag{4}$$

where indexes i and j refer to the discretization step lengths $\Delta\theta$ and Δz for the angle θ and length z, respectively. This is a simple formula for $P_{i,j+1}$ at the $(i, j + 1)$th mesh point in terms of the known values along the jth distance row. The truncation error for the difference in Equation (4) is $O(\Delta z, \Delta\theta^2)$. The grid dimension in the θ direction is $N = \theta_c/\Delta\theta$ and the grid dimension in the z direction is $M = L/\Delta z$, where θ_c is the critical angle and L is the fiber length.

3. Results and Discussion

In this study, we theoretically investigated the effect of mode coupling on SDM capabilities in a multimode SI air-clad silica optical fiber employed in a prior experiment by Åslund et al. [15]. The single-material air-clad fiber was made of low-grade natural silica and had material losses of less than 10 dB/km at 1550 nm. The fiber had NA = 0.54, a core diameter of d_{core} = 180 µm, and 59 bridges with a length of $l \sim 26$ µm to sustain it. The maximum thickness of the bridge was 340 nm. The constant coupling coefficient for the fiber was $D = 3.5 \times 10^{-5}$ rad^2/m at λ = 1550 nm [15,25], which we adopted in this work. The numerical solution to Equation (4) was obtained using discretization step lengths $\Delta\theta = 0.1°$ and $\Delta z = 0.001$ m.

Figure 3 illustrates the normalized output angular power distribution at two lengths of the multimode SI air-clad optical fiber obtained as a solution of Equation (2). The three launch beams in the Gaussian form with $(\mathrm{FWHM})_{z=0} = 3.9°$ and different input angles $\theta_0 = 0°$, 13°, and 26° represent three optical channels [15,25]. In the short SI air-clad optical fiber in Figure 3a, the mode coupling is minimal, and as a result there is no crosstalk between optical channels. Due to mode coupling, spatial power distributions broaden with increasing fiber length, so the three-channel SDM can be realized in this fiber up to a fiber length of z_{SDM} = 30 m (Figure 3b). In the case of two co-propagating optical channels, the two Gaussian launch distributions with input angles $\theta_0 = 0°$ and 26°, and $(\mathrm{FWHM})_{z=0} = 3.9°$, are investigated. One can see from Figure 4b that practical realization of two-channel SDM can be done up to a fiber length of z_{SDM} = 120 m. As can be seen, mode coupling severely restricts the length of multimode SI air-clad silica optical fibers that can be used for SDM (Table 1).

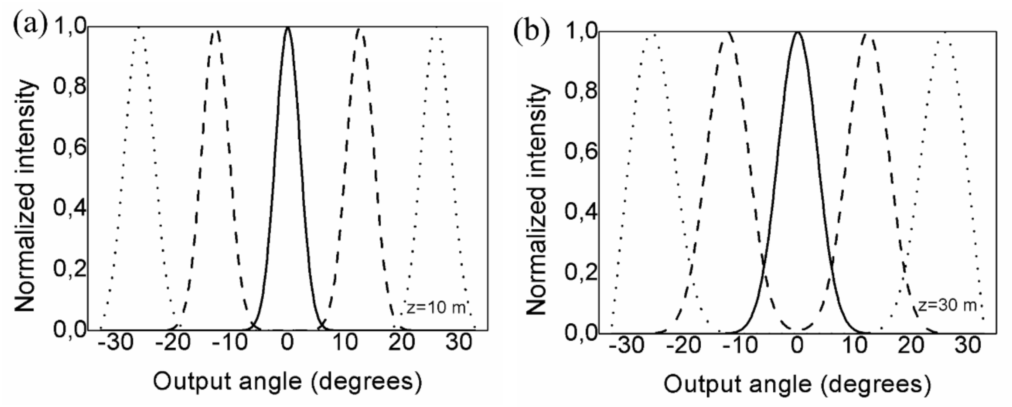

Figure 3. Normalized output intensity at different lengths of the multimode SI air-clad silica optical fiber obtained as solutions of Equation (2) for D = 3.5 × 10^{-5} rad^2/m, three launch beams in the Gaussian form, and input angles θ_0 = 0° (solid line), 13° (dashed line) and 26° (dotted line), with $(FWHM)_{z=0}$ = 3.9° for: (**a**) z = 10 m and (**b**) z = 30 m.

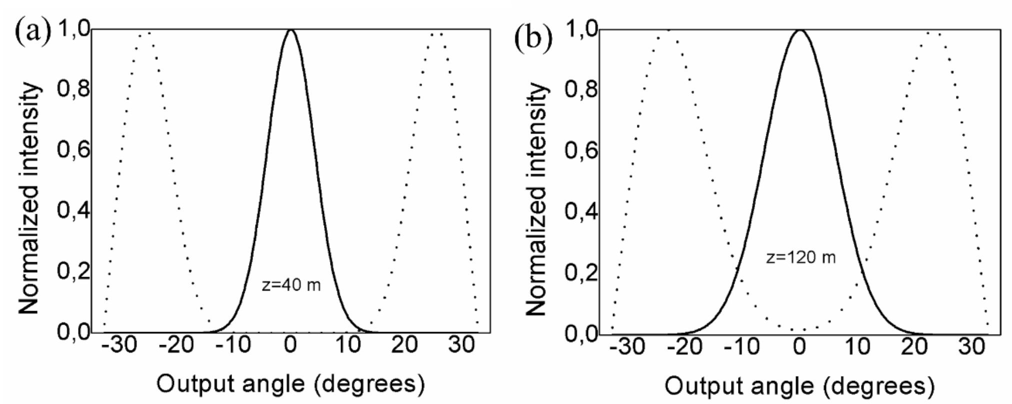

Figure 4. Normalized output intensity at different lengths of the multimode SI air-clad silica optical fiber obtained as solutions of Equation (2) for D = 3.5 × 10^{-5} rad^2/m, two launch beams in the Gaussian form, and input angles θ_0 = 0° (solid line) and 26° (dotted line), with $(FWHM)_{z=0}$ = 3.9° for: (**a**) z = 40 m and (**b**) z = 120 m.

Table 1. Length z_{SDM} for two and three spatially multiplexed channels with minimal crosstalk in SI air-clad silica optical fibers and PCSFs, with different numerical apertures NA and coupling coefficients D.

Fiber Type	NA	D (rad^2/m)	z_{SDM} (m) (2-Channel)	$_{SDM}$ (m) (3-Channel)
Air-clad silica fiber (this work)	0.54	3.5 × 10^{-5}	120	30
PCSF (1) [26]	0.4	1.28 × 10^{-5}	50	14
PCSF (2) [26]	0.37	4.5 × 10^{-5}	25	7

The coupling coefficient D of the multimode SI air-clad silica fiber investigated in this study is of the same order of magnitude as that of the previously investigated multimode SI PCSFs [26] (Table 1). SI PCSFs have a lesser capacity for SDM due to their lower NA of 0.37 to 0.4. For example, the capacity for three-channel SDM in the SI air-clad silica fiber investigated in this work, compared to PCSF (2) [26], is about four times greater, which is mainly due to the much higher NA of the SI air-clad silica fiber (these two fibers have a similar coupling coefficient D). On the other hand, despite the higher coupling coefficient D (which restricts the capacity for SDM) of the SI air-clad silica fiber compared to that of PCSF (1) [26], the capacity for three-channel SDM in the SI air-clad silica fiber compared to PCSF

(1) is about two times greater, due to the higher NA of the SI air-clad silica fiber. Therefore, in practical realization of SDM in PCSFs and air-clad silica fibers, the coupling coefficient is the dominant factor for lower NA PCSFs compared to high NA air-clad silica fibers. As a result, two- and three-channel SDM can be achieved in SI air-clad silica optical fibers with lengths longer than PCSFs (Table 1). Air-clad silica fibers are, hence, more suitable for SDM applications. It is worth noting that, in our previous works, we have shown that this kind of SDM can be employed in standard multimode SI plastic optical fibers at lengths of up to few meters [27–29] and multimode SI silica optical fibers at lengths of several hundreds of meters [30].

In order to investigate the influence of $(FWHM)_{z=0}$ of launch beam distribution on the capacity of SDM in the SI air-clad silica fiber, we solved Equation (1) for four different Gaussian beams with $(FWHM)_{z=0}$ = 0.5, 1, 2, and 3°. The numerical results are summarized in Table 2. One can see that, by increasing the width of the launch beam, the length of two- and three-channel SDM decreases. One can conclude that a narrower launch beam distribution is more desirable in practical realization of SDM in the investigated SI air-clad silica fiber.

Table 2. Length z_{SDM} for two and three spatially multiplexed channels with minimal crosstalk in SI air-clad silica optical fibers for different widths ($(FWHM)_{z=0}$) of the Gaussian launch beam distributions.

$(FWHM)_{z=0}$ (deg)	z_{SDM} (m) (2-Channel)	z_{SDM} (m) (3-Channel)
3.9	120	30
3	130	33
2	142	36
1	151	39
0.5	158	41

To conclude, an optical fiber with weaker mode coupling and higher NA should be employed in order to achieve a higher capacity of SDM. A further improvement of the capacity of SDM can be achieved by choosing a narrower launch beam.

In general, for spatially multiplexed channels, it is difficult to precisely anticipate the amount of crosstalk that will prevent the system from operating by computing the normalized output power distribution for different fiber lengths. In practice, a transmission matrix should be employed for a more precise assessment of the SDM capacity of a specific fiber, taking into consideration the noise factors that are dependent on the receiver's individual implementation [27]. Our numerical results provide a good estimate of the optical fiber length, where SDM with three and two channels might be implemented with low crosstalk in the analyzed SI air-clad silica fibers.

4. Conclusions

The power flow equation was used to explore the effect of mode coupling on SDM in multimode SI air-clad silica optical fibers. We have shown that mode coupling limits the length at which the SDM can be realized in high-NA air-clad silica optical fiber. When compared to SI PCSFs with smaller NA, this constraint is less noticeable. For SDM, an optical fiber with a lower mode coupling and a higher NA is preferable. Furthermore, a narrower launch beam results in longer fiber lengths at which SDM can be realized. When building an optical fiber transmission system with space division multiplexing, such characterization of optical fibers should be taken into account.

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References

1. Richardson, D.; Fini, J.; Nelson, L. Space-division multiplexing in optical fibres. *Nat. Photonics* **2013**, *7*, 354–362. [CrossRef]
2. Savović, S.; Djordjevich, A.; Simović, A.; Drljača, B. Influence of mode coupling on angular division multiplexing in seven-core plastic optical fibers. *Laser Phys.* **2020**, *30*, 065103. [CrossRef]
3. Winzer, P.J. Optical networking beyond WDM. *IEEE Photonics J.* **2012**, *4*, 647–651. [CrossRef]
4. Li, G.; Bai, N.; Zhao, N.; Xia, C. Space-division multiplexing: The next frontier in optical communication. *Adv. Opt. Photonics* **2014**, *6*, 413–487. [CrossRef]
5. Brunet, C.; Ung, B.; Belanger, P.-A.; Messaddeq, Y.; LaRochelle, S.; Rusch, L.A. Vector mode analysis of ring-core fibers: Design tools for spatial division multiplexing. *J. Lightwave Technol.* **2014**, *32*, 4046–4057. [CrossRef]
6. Uden, R.G.H.; Huijskens, R.C.E.L.F.; Xia, C.; Li, G.; Schulzgen, A.; Waardt, H.; Koonen, A.; Okonkwo, C. Ultra-high-density spatial division multiplexing with a few-mode multicore fibre. *Nat. Photonics* **2014**, *8*, 865–870. [CrossRef]
7. Zhang, L.; Chen, J.; Agrell, E.; Lin, R.; Wosinska, L. Enabling Technologies for Optical Data Center Networks: Spatial Division Multiplexing. *J. Lightwave Technol.* **2020**, *38*, 18–30. [CrossRef]
8. Rommel, S.; Dodane, D.; Grivas, E.; Cimoli, B.; Bourderionnet, J.; Feugnet, G.; Morales, A.; Pikasis, E.; Roeloffzen, C.; Dijk, P.; et al. Towards a Scaleable 5G Fronthaul: Analog Radio-over-Fiber and Space Division Multiplexing. *J. Lightwave Technol.* **2020**, *38*, 5412–5422. [CrossRef]
9. Murshid, S.H.; Chakravarty, A.; Biswas, R. Attenuation and modal dispersion models for spatially multiplexed co-propagating helical optical channels in step index fibers. *Opt. Laser Technol.* **2011**, *43*, 430–436. [CrossRef]
10. Zhou, J.; Yang, C.; Sui, Q.; Wang, H.; Gao, S.; Feng, Y.; Liu, W.; Yan, Y.; Li, J.; Yu, C.; et al. Burst-Error-Propagation Suppression for Decision-Feedback Equalizer in Field-Trial Submarine Fiber-Optic Communications. *J. Lightwave Technol.* **2021**, *39*, 4601–4606. [CrossRef]
11. Apolo, J.; Ortega, B.; Almenar, V. Hybrid POF/VLC Links Based on a Single LED for Indoor Communications. *Photonics* **2021**, *8*, 254. [CrossRef]
12. Drljača, B.; Simović, A.; Djordjevich, A.; Savović, S. Wavelength dependence of equilibrium mode distribution and steady state distribution in W-type plastic-clad silica fibers. *Opt. Fiber. Technol.* **2020**, *54*, 102077. [CrossRef]
13. Savović, S.; Djordjevich, A. Mode coupling in multimode step-index plastic-clad silica fibers with corrugated surfaces. *Opt. Laser Technol.* **2017**, *97*, 400–404. [CrossRef]
14. Wang, W.; Wang, N.; Li, K.; Geng, Z.; Jia, H. A novel dual guided modes regions photonic crystal fiber with low crosstalk supporting 56 OAM modes and 4 LP modes. *Opt. Fiber. Technol.* **2020**, *57*, 102213. [CrossRef]
15. Åslund, M.L.; Canning, J. Air-clad fibres for astronomical instrumentation: Focal-ratio degradation. *Exp. Astron.* **2009**, *24*, 1–7. [CrossRef]
16. Corbett, J.C.W. A brief introduction to photonic crystal fibres for astronomical instrumentalists. *New Astron. Rev.* **2006**, *50*, 305–312. [CrossRef]
17. Wadsworth, W.J.; Percival, R.M.; Bouwmans, G.; Knight, J.C.; Birks, T.A.; Hedley, T.D.; Russel, P.S.J. Very high numerical aperture fibers. *IEEE Photonics Technol. Lett.* **2004**, *16*, 843–845. [CrossRef]
18. Canning, J.; Buckley, E.; Huntington, S.; Lyytikäinen, K. Using multi-micro channel capillaries for determination of the zeta potential of a microfluidic channel. *Electrochim. Acta* **2004**, *49*, 3581–3586. [CrossRef]
19. Gloge, D. Optical power flow in multimode fibers. *Bell Syst. Tech. J.* **1972**, *51*, 1767–1783. [CrossRef]
20. Garito, A.F.; Wang, J.; Gao, R. Effects of random perturbations in plastic optical fibers. *Science* **1998**, *281*, 962–967. [CrossRef]
21. Djordjevich, A.; Savović, S. Numerical solution of the power flow equation in step index plastic optical fibers. *J. Opt. Soc. Am. B* **2004**, *21*, 1437–1442. [CrossRef]
22. Gambling, W.A.; Payne, D.N.; Matsumura, H. Mode conversion coefficients in optical fibers. *Appl. Opt.* **1975**, *14*, 1538–1542. [CrossRef] [PubMed]

23. Rousseau, M.; Jeunhomme, L. Numerical solution of the coupled-power equation in step index optical fibers. *IEEE Trans. Microwave Theory Tech.* **1977**, *25*, 577–585. [CrossRef]
24. Djordjevich, A.; Savović, S. Investigation of mode coupling in step index plastic optical fibers using the power flow equation. *IEEE Photonics Technol. Lett.* **2000**, *12*, 1489–1491. [CrossRef]
25. Savović, S.; Djordjevich, A.; Min, R. Investigation of mode coupling in step-index air-clad silica optical fibers. *Opt. Fiber Technol.* 2022, *submitted for publication*.
26. Savović, S.; Djordjevich, A. Mode coupling and its influence on space division multiplexing in step-index plastic-clad silica fibers. *Opt. Fiber Technol.* **2018**, *46*, 192–197. [CrossRef]
27. Tsekrekos, C.P.; Martinez, A.; Huijskens, F.M.; Koonen, A.M.J. Design considerations for a transparent mode group diversity multiplexing link. *IEEE Photonics Technol. Lett.* **2006**, *18*, 2359–2361. [CrossRef]
28. Savović, S.; Djordjevich, A.; Simović, A.; Drljača, B. Influence of mode coupling on three spatially multiplexed channels in multimode graded index plastic optical fibers. *Laser Phys.* **2020**, *30*, 115102. [CrossRef]
29. Savović, S.; Djordjevich, A.; Simović, A.; Drljača, B. Influence of mode coupling on three, four and five spatially multiplexed channels in multimode step-index plastic optical fibers. *Opt. Laser Technol.* **2018**, *106*, 18–21. [CrossRef]
30. Savović, S.; Djordjevich, A.; Simović, A.; Drljača, B. A transmission length limit for space division multiplexing in step-index silica optical fibers. *J. Mod. Opt.* **2019**, *66*, 1695–1700. [CrossRef]