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Optimal Strategy of Unreliable Flexible Production System Using Information System

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Abstract: *Background*: Optimization approaches and a models can be applied for critical production systems that experience equipment failure, repair delays and product quality control in order to maximize the total profit. We can cite, as an example, flexible manufacturing systems. *Methods*: Our methodology involves developing a decision model integrated with an information system to coordinate various system operations, ensuring timely response to customer requests. The module of the information system is provided to optimally manage the production flow and parts ordering according to machine availability. The objective is to determine the optimal order thresholds of part batches that maximize the total profit. *Results*: Numerical results are provided to analyze the influence of system reliability and uncertainty on decision variables, offering insights into the system's performance and robustness. By using our method, the advancement of the flexible production systems is carried out by addressing key operational challenges and optimizing production processes for enhanced efficiency and profitability. *Conclusions*: To achieve this, an optimization algorithm is employed to identify optimal solutions that enhance profitability.

Keywords: production planning; carbon regulation; reconditioning; green logistics; optimization

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1. Introduction

In the current business landscape, the surge in the number of manufacturing companies has intensified competition significantly. Consequently, the imperative for survival and success in the global market necessitates the formulation of innovative strategies aimed at delivering exceptional customer service. Recognizing the pivotal role of high-quality customer service has become paramount in navigating the increasingly competitive environment shaped by the proliferation of manufacturing firms.

The Flexible Production System (FPS) excels in handling diverse parts, styles, and production quantities within a manufacturing shop floor. Its adaptability to changing market dynamics provides a significant edge, enabling efficient and rapid responses to customer requirements. As a pivotal manufacturing technology, FPS is widely embraced by companies seeking operational flexibility, effectively catering to various market segments. First, a comprehensive definition of flexible systems is required. These last ones are those that can adapt to changing conditions and requirements without requiring significant modifications to their core structure. They are characterized by modularity, allowing components to be independently altered or upgraded; scalability, enabling efficient handling of varying demands; and interoperability, ensuring seamless integration with other systems. Additionally, these dedicated systems exhibit resilience, maintaining performance and functionality despite disruptions or changes. This adaptability ensures that such systems remain efficient, effective, and relevant in dynamic environments.

The key components of FPS encompass workstations, material handling and storage systems, an integrated computer control system, and system control operators. The essence of manufacturing flexibility lies in the capacity to produce high-quality products at low costs while adhering to stringent delivery timelines. Embracing FPS allows companies not only to optimize their operations but also to meet diverse market demands with effectiveness and agility. As one of the pioneering works that delineates the concept of flexibility within FPS, we can cite the work of Browne et al. [1], in which their contribution involves categorizing flexibility into eight levels such as (1) machine flexibility, (2) product flexibility, (3) process flexibility, (4) production flexibility, (5) volume flexibility, and others. This classification aims to illustrate the comprehensive nature of flexibility within the overall FPS, providing insights into various facets of system adaptability. In fact, FPS is a remarkably adaptable production monitoring that comprises a seamlessly integrated, computer-controlled arrangement of numerically controlled machines, complemented by an automated material handling system [2].

The multifunctional machines within an FPS have the capability to execute a diverse range of operations concurrently, given the appropriate set of tools. Consequently, effective tool management becomes a crucial consideration in ensuring the smooth operation of such systems. By referring to the literature, several researchers have proposed sophisticated models associated with FPSthat can adapt to swiftly dynamic market trends and customer requirements. In their groundbreaking work on monitoring level, Abou Ali and Shouman [3] introduced a comprehensive simulation model that encompasses eight machines, storage buffer areas, a dedicated receiving zone, three robotic systems, and the integration of pallets. This innovative model serves as a significant advancement in understanding and optimizing manufacturing processes within a dynamic and automated framework. Moreover, Aldaihani and Savsar [4] have designed a sophisticated unreliable model designed to assess the efficiency of an FPS in the face of uncertain operational conditions. This includes factors such as random machining times, unpredictable loading/unloading intervals, and the variability introduced by random pallet transfer times. This pioneering model stands as a valuable contribution, shedding light on the system's performance in real-world scenarios marked by operational unreliability. Furthermore, Wahab et al. [5] introduced a versatile model designed to assess machine flexibility, taking into account uncertainties within the system. Based on advanced technologies, Hu et al. [6] discuss the utilization of advanced digital twin technologies in smart plants for accurate simulation and high-throughput data. To address the dynamic scheduling problem in FPS, the study employs deep reinforcement learning (DRL), specifically the deep Q-network (DQN). The scheduling problem is modeled as a Markov decision process (MDP) using a class of Petri nets called timed S3PR, considering manufacturing efficiency and deadlock avoidance. Continuing with smart manufacturing systems, Chakroun et al. [2] outline a study focused on characterizing and analyzing a smart FPS tailored for a company specializing in the production of brass accessories. The authors establish a simulation tool to create a numerical production platform for Industry 4.0, proficient in efficiently managing production and procurement through material requirement planning (MRP), a logistics warehouse, and a cyber-physical production system (CPPS). The study optimizes findings through a redesigned MRP 2 approach, incorporating load-capacity adjustment for a smart workshop and Industry 4.0 manufacturing planning. The integrated manufacturing system process significantly reduces assembly time for spherical bushels, enhancing control over production and assembly. Likewise, Bao et al. [7] propose the use of a genetic algorithm to address complex job shop scheduling problems in the flexible production shop floor. They focus on optimizing scheduling solutions, considering key performance indicators such as overdue jobs, total overdue time, job completion time, comprehensive load rate, and maximum load rate of machine tools. The study aims to improve processing efficiency and offers insights for the design and optimization of production scheduling algorithms based on genetic algorithms. Additionally, Habib et al. [8] present an integration of sustainability in flexible manufacturing systems and Industry 4.0 to address market demand fluctuations due to customization needs. Focusing on modular products like a 3D printer and an electric toothbrush, the research introduces a method applied to identify nine modules with specific functions and interface information.

For the maintenance level, Savsar [9] explores a comprehensive approach that integrates both simulation and analytical models. This methodology is employed to scrutinize the impact of corrective, preventive, and opportunistic maintenance policies on the overall productivity of a flexible manufacturing cell. The study delves into the intricate dynamics of these maintenance strategies, offering a nuanced understanding of their respective influences on the efficiency and performance of the manufacturing system. Based on Artificial Intelligence (IA) technologies, Chakroun and his colleagues introduce a predictive maintenance model leveraging Machine Learning (ML) within the context of a smart flexible shop floor, with a specific focus on health assessment for two assembly/packaging robots. Concerning optimal strategy of unreliable FPS, Pei et al. [10] present a preventive maintenance strategy for unreliable FPS aimed at improving efficiency and reducing costs. The strategy involves grouping elements for maintenance with optimized parameters, considering both reliability and cost factors. The study introduces a three-layer evaluation index system to accurately estimate FPS reliability, utilizing weights obtained through reliability importance modeling. Additionally, Xu et al. [11] address the challenge of optimizing the reliability allocation in FPS. The authors propose two dimension-reduction strategies: the Reliability-Weight Double-Threshold Qualification Strategy (RWTS) and the Bi-Level Optimization Strategy (BLOS). RWTS dynamically reduces the number of optimization variables by setting reliability thresholds and weight indexes for basic elements. BLOS transforms the overall reliability–allocation optimization problem into simpler allocations, improving convergence performance. Furthermore, Arasteh [12] delves into the intricate mathematical modeling of flexible production lines, where diverse part types are processed on machines with varying degrees of reliability. The key element in this modeling approach is the implementation of a priority rule, which guides the sequencing of tasks on these production lines. The complexity arises from the interplay of multiple factors, such as the varied nature of parts, the unreliability of the machines, and the strategic prioritization employed in the production process [13].

Based on this state of the art, this study presents a departure from the existing literature in several key aspects. Firstly, it delineates a focused exploration into the production dynamics of engraved and packaged glass pieces based on color specifications within flexible manufacturing systems (FMS). This narrow focus contrasts with the broader scope often observed in previous research within the domain of FMS optimization [14].

Secondly, the integration of an information system for real-time decision-making marks a notable departure from traditional theoretical modeling approaches prevalent in the literature [15–18]. By incorporating this technological component, the study emphasizes the practical imperative of agile responsiveness to evolving production demands, thus bridging the gap between theoretical frameworks and real-world implementation [19].

Thirdly, the explicit consideration of real-world production system characteristics, such as breakdowns and repair times, adds a layer of complexity and realism often lacking in theoretical models. This approach enriches the analysis by providing a more nuanced understanding of the challenges inherent in managing flexible manufacturing environments.

Finally, the study's primary focus on profit maximization sets it apart from previous works, which have typically explored a broader range of optimization objectives. This strategic emphasis underscores the study's relevance within the industrial context, where financial performance is a critical determinant of competitive success.

In this paper, we model and optimize a Flexible Manufacturing System allowing the production of engraved and packaged glass pieces according to their colors [20]. This involves developing a decision model based on an information system connected to the different operations of the flexible manufacturing system in order to respond to customer requests within a specific time frame. The originality of this work consists of taking into account the different concrete characteristics of a production system such as breakdowns, their repair times, and the quality control of its products. Therefore, comparing to the existing models in the literature that have addressed the FPS, our model takes into account the uncertain sources as in practice. In addition, to regulate the production flows and the buffers, we integrate a new module of information system that manages the operation according to the system events.

Now, let us shed light on queueing modeling; numerous researchers have presented intricate models in the realm of FPS that possess the capability to adapt to rapidly evolving market issues and customer needs. This specific area has garnered attention from various contributors in queueing modeling [15]. As in the prosed FPS in the under stochastic environment, the production flows and buffing are perturbed and may cause the profit to fall. In order to deal with these perturbations, we can act on parts supplies to optimize the system, with assuring the sufficient parts quantities for the production processes and at the same time avoiding the overflows in buffers, and this conducts to maximize the profit. The supplying of parts is commanded by defined thresholds to order new part batches. Thus, the objective of our work is to find the optimal thresholds that maximize the total profit. Subsequently, an optimization algorithm is devised to explore potential optimal solutions that maximize this profit. Additionally, numerical results are presented to demonstrate the impact of system reliability and uncertainty on decision variables, thereby underscoring the effectiveness of our model.

The paper presents a pioneering approach in the realm of flexible production systems, specifically tailored for the production of engraved and packaged glass pieces based on color distinctions. At its core, our work introduces the innovative Module of Information System and Logistic Operations Management (MISLOM), designed to efficiently oversee various system operations and manage parts ordering and production flow in accordance with batch requirements, machine states, and buffer levels. This novel method represents a significant scientific contribution by providing a comprehensive framework for modeling and optimizing flexible production systems to maximize total profit.

One of the key highlights of our research lies in the meticulous consideration of critical production system attributes, including equipment breakdowns, repair times, and product quality control measures. By integrating these factors into our decision model, coupled with an information system, we offer a holistic approach to coordinating system operations and ensuring timely responses to customer demands.

Furthermore, our study employs an optimization algorithm based on GA to explore potential optimal solutions that enhance overall profitability. The numerical results presented underscore the impact of system reliability and uncertainty on decision variables, thus validating the effectiveness of our model. To sum up, our work not only addresses the practical challenges faced in flexible production environments but also advances the theoretical understanding of optimizing such systems for maximum profitability.

What sets this work apart is its consideration of real-world production system characteristics, including equipment breakdowns, repair times, and product quality control. The goal is to manage system operations dynamically, leveraging the equipment status to meet product lot requirements and maximize overall profit. To achieve this goal, an optimization algorithm is designed to identify possible optimal solutions for profit maximization. Through numerical results, the study also assesses the impact of system reliability and uncertainty on decision variables, providing valuable insights into the effectiveness of the proposed model.

In summary, this study contributes to the literature by offering a nuanced exploration of FPS optimization within a specific production context, while also integrating real-time decision-making capabilities and emphasizing profit maximization. These distinctive features advance our understanding of effective management strategies in contemporary manufacturing environments, laying the groundwork for future research in this field.

The rest of this paper is organized as follows. In Section 2, FPS and the model explanations are provided. Section 3 presents the optimization methodology. In Section 4, numerical results are illustrated. Finally, conclusions and future work are given in Section 5.

2. Studied FPS and Model Explanations

In this section, the FPS is studied, and its operating is explained (see Figure 1).

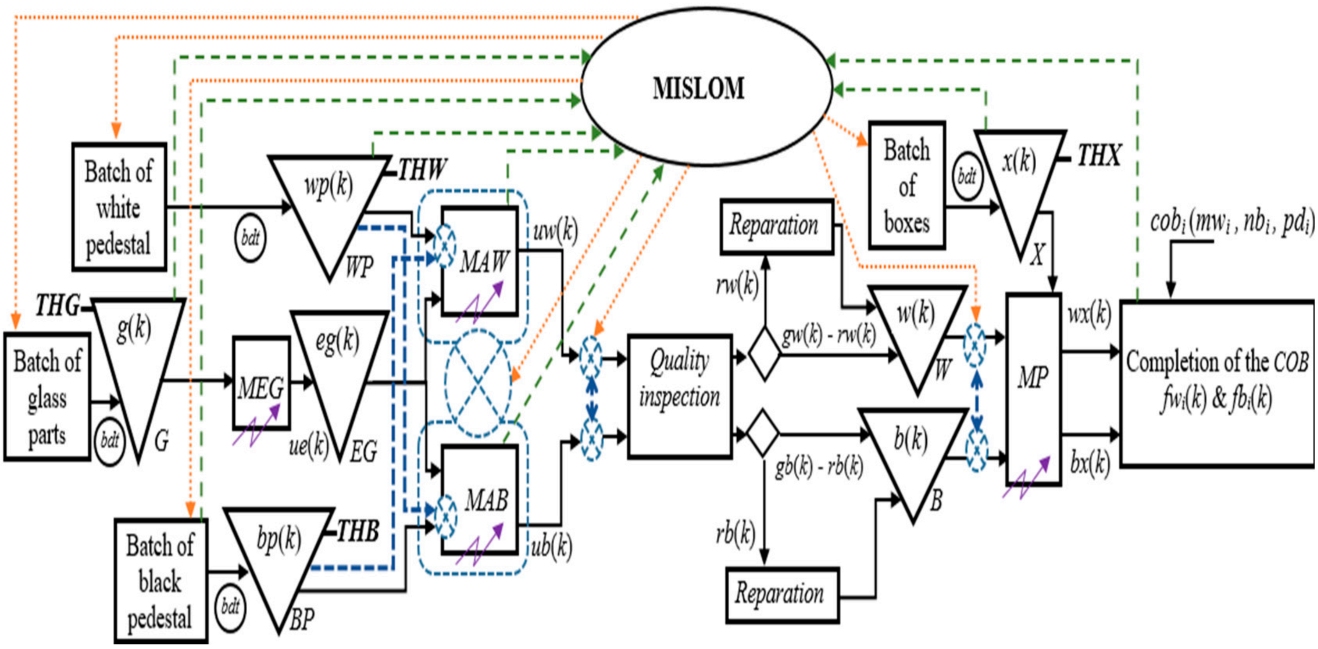

Figure 1. FPS with MISLOM.

The studied FPS (Figure 1) aims to produce a batch of gift or souvenir products. The product is a box that contains engraved glass placed on a plastic pedestal. The FPS produces two types of products: one with a white pedestal and the other with a black one. The FPS is composed of four machines (*MEG*, *MAW*, *MAB*, and *MP*) and seven buffers (*G*, *WP*, *BP*, *EG*, *W*, *B*, and *X*). The buffer *G* supplies the machine *MEG* that engraves the glass parts and fills the buffer *EG*. This last one supplies both machines *MAW* and *MAB*, which assemble the engraved glass parts with white and black pedestals, respectively. The buffer *WP* supplies the machine *MAW* with the white pedestals, and the buffer *BP* supplies the machine *MAB* with the black ones. The buffers *G*, *WP*, *BP*, and *X* are supplied by batches of parts that the producer has to order from a different supplier. As mentioned in the Introduction, we use discrete flow to model the system. Thus, it is assumed that the horizon study is disseized to *n* periods, and the variables are discrete and depend on the period index.

2.1. Notation and Model's Description

In this work, the optimization criterion is the total profit that counts sales revenues and different costs such as buffering and delays. Indeed, the total profit is defined as a measure to optimize our FPS, as its maximization implies the increasing of the sales and the reducing of the costs, and that induces to provide the optimal adjustment of the production flows in order to reduce the buffering and respect the planned deadline. Furthermore, as production flows in FPS are perturbed by the machines failures, it will be necessary to manage the supply of different parts and the buffering in order to avoid a shortage or an overflow of buffers. The supply of parts depends on the thresholds to order new batches (*THG*, *THW*, *THB*, and *THX*), which should be determined in a manner to avoid machine starvation or the overflow of parts. Thus, the objective of our work is to find the optimal thresholds *THG**, *THW**, *THB**, and *THX** that maximize the total profit *TP*.

The necessary variables and parameters of our model are presented in the following Table 1.

Table 1. Decision Variables.

Indexes, Parameters, Sets, and Decision Variables	
Indexes	
k	period index with $k = \{1, 2, \ldots, n\}$.
i	batch number
Parameters	
Z	study horizon
n	number of periods in Z
nbz	number of batch of finished products sold in Z
BE	batch size of glass parts
BWP	batch size of white pedestals
BX	batch size of empty boxes
BBP	batch size of black pedestals
Ueu	maximum production rate of the machine MEG
$MTBFG$	mean time between failures of the machine MEG
$MTTRG$	mean time to repair the machine MEG
$MTBFW$	mean time between failures of the machine MAW
$MTTRW$	mean time to repair the machine MAW
$MTBFB$	mean time between failures of the machine MAB
$MTTRB$	mean time to repair the machine MAB
$MTBFP$	mean time between failures of the machine MAP
$MTTRP$	mean time to repair the machine MAP
pf	price of one finished product (same price for both type of products)
cb	holding cost in buffer (same cost for all buffers)
cg	cost of one glass part
cwp	cost of one white pedestal
cbp	cost of one black pedestal
cbx	cost of one empty box
ce	cost for engraving one glass part
ca	assembling cost of one glass with one pedestal (same assembling cost for MAW and MAB)
cpk	packaging cost (same packaging cost for both type of products)
cr	repair cost (same repair cost for both type of products)
bdt	time to deliver one batch for the buffers
cp	penalty cost
TP	total profit function over Z
TP^*	optimal value of total profit
Sets	
mw_i	customer order of product with white pedestal in cob_i
nb_i	customer order of product with black pedestal in cob_i
$fw_i(k)$	actual number of product with white pedestal for cob_i
$fb_i(k)$	actual number of product with black pedestal for cob_i

Table 1. *Cont.*

Indexes, Parameters, Sets, and Decision Variables	
$g(k)$	buffer level of glass parts
$eg(k)$	buffer level of engraved glass parts
$wp(k)$	buffer level of white pedestals
$bp(k)$	buffer level of black pedestals
$x(k)$	buffer level of empty of boxes
$w(k)$	buffer level of assembled parts glass with white pedestal
$b(k)$	buffer level of assembled parts glass with black pedestal
$e(k)$	binary variable that represents the *BE* order
$q(k)$	binary variable that represents the *BWP* order
$y(k)$	binary variable that represents the *BBP* order
$v(k)$	binary variable that represents the *BX* order
$ue(k)$	production rate of the machine *MEG*
$uw(k)$	production rate of the machine *MAW*
$ub(k)$	production rate of the machine *MAB*
$wx(k)$	production rate of finished products with white pedestal in the machine *MP*
$bx(k)$	production rate of finished products with black pedestal in the machine *MP*
$rw(k)$	number of products with white pedestal that have an assembly defect
$rb(k)$	number of products with black pedestal that have an assembly defect
$rnd(k)$	random function that determines the percentage of products with an assembly defect
$\psi(k)$	state of the machine *MEG*
$\varphi(k)$	state of the machine *MAB*
$\phi(k)$	state of the machine *MAW*
$\eta(k)$	state of the machine *MP*
Decision variables	
THG	threshold to order new batch *BE* for the buffer *G*
THW	threshold to order new batch *BWP* for the buffer *WP*
THB	threshold to order new batch *BBP* for the buffer *BP*
THX	threshold to order new batch *BX* for the buffer *X*

The customer orders one batch of products with a defined number for each type (nb_i and mw_i). The producer has to provide the ordered batch before the end of a planned period of time called the planned deadline (pd_i). Otherwise, a penalty is accrued according to the overtaking. As in a real file case, we take into account the delivery time (bdt) that one batch of parts takes between the producer order and its arriving to the buffer. That means that when the producer orders one batch of parts at the period k, it will be received in the buffer at the period $k + bdt$.

The quality of assembled parts is inspected and that will be deposited in the buffers *W* and *B*. When the quality of the assembling represents a defect, the assembled part will be sent to the reparation and then deposited in the buffers *W* or *B*. Indeed, the buffers *W* and *B* store the parts with white and black pedestals, respectively, and which will be packed in boxes by the machine *MP*. This last is supplied by the buffer *X* and builds the *COB* that will be completed when the numbers nb_i and mw_i are archived.

To consider a near real system, it assumed that the machines are unreliable and subject to breakdowns and repairs. Indeed, as in practice, in this work, we consider the machine failure time that is not negligible and represents a production time out and of course can delay the completion of the *COB* before the planned deadline. Each *COB* is represented by the vector cob_i (nb_i, mwi, pd_i) with the index i is the batch number. This batch vector is characterized by three parameters, nb_i and mw_i are the number of products with black and white pedestal, respectively, and represent the customer order. This means that nb_i and mw_i are defined by the customer and their sum represents the batch size.

The parameter pd_i is the planned deadline that is defined by the producer according to the ordered batch size. When the producer and the customer agree on the planned deadline, the production of the *COB* begins, and the producer is obliged to respect the planned deadline or to pay a penalty if it is overtaken. Thus, the proposed FPS aims to complete the ordered batch before the planned deadline in order to satisfy the customer and to avoid the delay penalty. As the FPS is characterized by stochastics events such as machine failures and repairs that may delay the batch, a module of information system and logistic operations management (MISLOM) is available to manage the parts ordering and production flow according to the required batch, machines states, and the buffers levels.

Figure 2 depicts a dynamic production system established within our laboratory (LGIPM) in France. This innovative system specializes in manufacturing engraved glass cubes of varying colors, each elegantly mounted on a plastic pedestal. Every meticulously crafted piece undergoes meticulous storage, assembly, and packaging processes before emerging as a refined finished product.

Figure 2. Real Flexible Production System (FPS).

One can use a white pedestal and the other with a black one (see Figure 3 below).

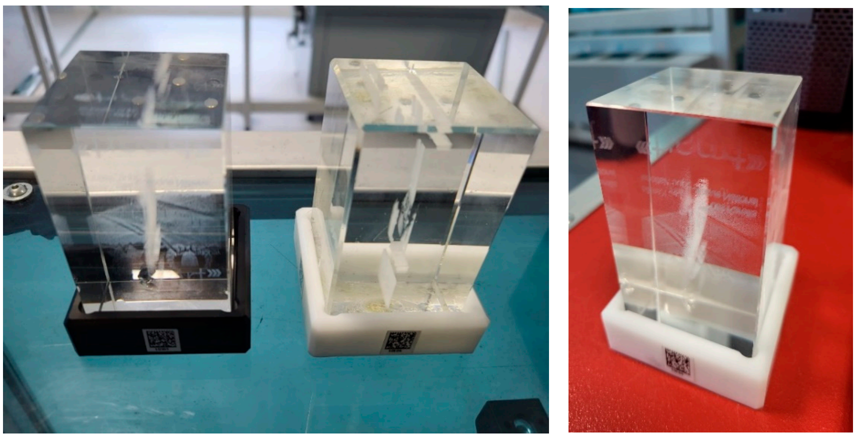

Figure 3. Engraved pieces with pedestals.

Once a piece is engraved and assembled with its chosen pedestal, it will be packaged (see Figure 4) and transported to the carousel station in order to store the product in the crates (see Figure 5).

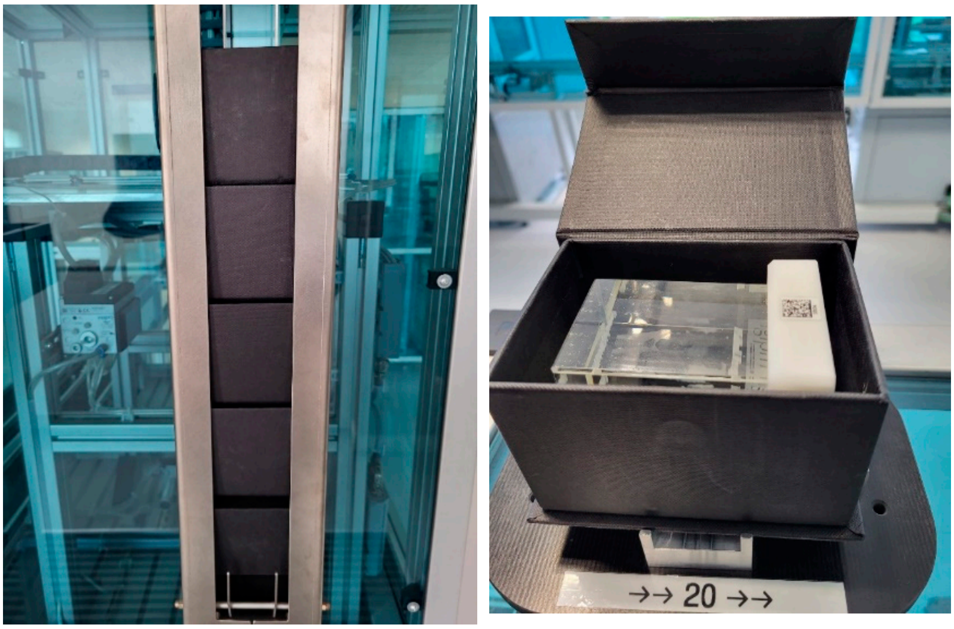

Figure 4. Packaged piece.

Figure 5. Products stored in crates.

The equations that describe the system are presented in the next subsection.

2.2. *Mathematical Model*

a. Machine states

The machines of the FPS are either up or down. Thus, the states of the machines *MEG*, *MAW*, *MAB*, and *MP* are presented by the following equations:

$$\psi(k) = \begin{cases} 1 & if\ machine\ MEG\ \ is\ up \\ 0 & if\ machine\ MEG\ \ is\ down \end{cases} \tag{1}$$

$$\phi(k) = \begin{cases} 1 & if\ machine\ MAW\ \ is\ up \\ 0 & if\ machine\ MAW\ \ is\ down \end{cases} \tag{2}$$

$$\varphi(k) = \begin{cases} 1 & if\ machine\ MAB\ \ is\ up \\ 0 & if\ machine\ MAB\ \ is\ down \end{cases} \tag{3}$$

$$\eta(k) = \begin{cases} 1 & if\ machine\ MP\ \ is\ up \\ 0 & if\ machine\ MP\ \ is\ down \end{cases} \tag{4}$$

b. Buffers levels

The buffers levels are expressed by the following equations:

When the buffer level falls below the order threshold *THG*, one batch *BE* will be ordered. Considering the delivery time *bdt*, the buffer *G* receives the batch *BE* at period *k* when the level fell below *THG* at period $k - bdt$. In addition, it supplies the *MEG* by a quantity equal to the production quantity *ue*(*k*). Thus, the level of the buffer *G* at period *k* is given by the following equation:

$$\begin{gathered} g(k) = g(k-1) + BE{\cdot}e(k) - ue(k) \text{ with} \\ e(k) = \begin{cases} 1 & \text{whene } g(k-bdt) < THG \\ 0 & \text{otherwise} \end{cases} \end{gathered} \tag{5}$$

Similar to the equation of *g*(*t*), the buffer levels of *WP*, *BP*, and *X* are given by the following equations:

$$\begin{gathered} wp(k) = wp(k-1) + BWP{\cdot}q(k) - uw(k) \text{ with} \\ q(k) = \begin{cases} 1 & \text{whene } wp(k-bdt) < THW \\ 0 & \text{otherwise} \end{cases} \end{gathered} \tag{6}$$

$$\begin{gathered} wb(k) = wb(k-1) + BBP{\cdot}y(k) - ub(k) \text{ with} \\ y(k) = \begin{cases} 1 & \text{whene } wb(k-bdt) < THB \\ 0 & \text{otherwise} \end{cases} \end{gathered} \tag{7}$$

$$\begin{gathered} x(k) = x(k-1) + BX{\cdot}v(k) - u(k) \text{ with} \\ v(k) = \begin{cases} 1 & \text{whene } x(k-bdt) < THX \\ 0 & \text{otherwise} \end{cases} \end{gathered} \tag{8}$$

The buffer level *eg*(*k*) is given by the following equation:

$$eg(k) = eg(k-1) + eu(k) - uw(k) - ub(k) \tag{9}$$

The buffer level *w*(*k*) depends on the production rate of *MAW* and *MP* and that is given by the following equation:

$$w(k) = w(k-1) + uw(k) - wx(k) \tag{10}$$

Similar to the equation of *w*(*k*), the buffer level of *B* is given by the following equation:

$$b(k) = b(k-1) + ub(k) - bx(k) \tag{11}$$

c. Production rates

The production rate *eu*(*k*) depends on the *MEG* state and the level of the upstream buffer *G*. When the machine is up, the production rate takes its maximum (U_{eu}) if the buffer level $g(k)$ is higher or equal to U_{eu}. Otherwise, the production rate is equal to $g(k)$. Moreover, when the machine is down, the production rate is null.

$$ue(k) = \begin{cases} Ueu & if\ \ \psi(k) = 1\ \ and\ g(k) \geq Ueu \\ g(k) & if\ \ \psi(k) = 1\ \ and\ g(k) < Ueu \\ 0 & if\ \ \psi(k) = 0 \end{cases} \tag{12}$$

The production rate $uw(k)$ depends on the *MAW* state and both upstream buffers *EG* and *WP*. It is assumed that the machine *MAW* assembles one product in one period composed of one engraved glass from *EG* and one white pedestal from *WP*. Thus, if one of the buffers *EG* or *WP* is empty or the machine *MAW* is down, the production rate $uw(k)$ is null. Otherwise, $uw(k)$ equals 1 production unit.

$$uw(k) = \begin{cases} 1 & if\ \ \phi(k) = 1\ \ and\ eg(k) \geq 1\ \ and\ wp(k) \geq 1 \\ 0 & if\ \ \phi(k) = 0\ \ or\ eg(k) = 0\ \ or\ \ wp(k) = 0 \end{cases} \tag{13}$$

Similar to $uw(k)$, the production rate $ub(k)$ is given by the following equation:

$$ub(k) = \begin{cases} 1 & if\ \ \varphi(k) = 1\ \ and\ eg(k) \geq 1\ \ and\ bp(k) \geq 1 \\ 0 & if\ \ \varphi(k) = 0\ \ or\ eg(k) = 0\ \ or\ \ bp(k) = 0 \end{cases} \tag{14}$$

The machine *MP* produces two finished products at the same time, one with a white pedestal and the other with a black one. The machine *MP* packages both types in two flow rates $wx(k)$ and $bx(k)$ that are the number of finished products that built the *COB*. The production rate $wx(k)$ and $bx(k)$ depend on the buffer levels of *W*, *B*, *X* and the state of the machine *MP*.

$$ub(k) = \begin{cases} 1 & if\ \ \varphi(k) = 1\ \ and\ eg(k) \geq 1\ \ and\ bp(k) \geq 1 \\ 0 & if\ \ \varphi(k) = 0\ \ or\ eg(k) = 0\ \ or\ \ bp(k) = 0 \end{cases} \tag{15}$$

$$bx(k) = \begin{cases} 1 & if\ \ \eta(k) = 1\ \ and\ b(k) \geq 1\ \ and\ x(k) \geq 1 \\ 0 & if\ \ \eta(k) = 0\ \ or\ b(k) = 0\ \ or\ \ x(k) = 0 \end{cases} \tag{16}$$

d. MISLOM operating

The role of the MISLOM is to order automatically a batch of parts from the supplier when a buffer level falls below a fixed order threshold (see Equations (5)–(8)). Furthermore, the MISLOM has a more important role in that it regulates the production flow according to machines states and the achievement of the *COB*, indeed, for example, when the number of flinched products with a white pedestal reaches the ordered number (i.e., $fw_i(k) = mw_i$), but the number of flinched products with black pedestal is less than the ordered number (i.e., $fb_i(k) < nb_i$). In this case, and when the machine *MAW* is up, the MISLOM orders the *MAW* to assemble the glass parts with black pedestals using the buffer *WP*. In addition, it orders *MP* to package the flinched products with a black pedestal in place of the ones with a white pedestal. Thus, the flow of the products with a black pedestal increases and reaches quickly nb_i and then may the *COB* will be achieved before the planned deadline. This is vice versa when $fw_i(k) < mw_i$ and $fb_i(k) = nb_i$ (see decision flows in Figure 1). The MISLOM operating is illustrated by the following equations.

When $fb_i(k) < nb_i$ and $fw_i(k) = mw_i$, the expressions of $uw(k)$ and of $wx(k)$ are changed to the following equations:

$$uw(k) = \begin{cases} 1 & if\ \ \phi(k) = 1\ \ and\ eg(k) \geq 1\ \ and\ bp(k) \geq 1 \\ 0 & if\ \ \phi(k) = 0\ \ or\ eg(k) = 0\ \ or\ \ bp(k) = 0 \end{cases} \tag{17}$$

$$wx(k) = \begin{cases} 1 & if\ \ \eta(k) = 1\ \ and\ b(k) \geq 1\ \ and\ x(k) \geq 1 \\ 0 & if\ \ \eta(k) = 0\ \ or\ b(k) = 0\ \ or\ \ x(k) = 0 \end{cases} \tag{18}$$

When $fb_i(k) = nb_i$ and $fw_i(k) < mw_i$, the expressions of $ub(k)$ and of $bx(k)$ are changed by the following equations:

$$ub(k) = \begin{cases} 1 & if\ \ \varphi(k) = 1\ \ and\ eg(k) \geq 1\ \ and\ uw(k) \geq 1 \\ 0 & if\ \ \varphi(k) = 0\ \ or\ eg(k) = 0\ \ or\ \ uw(k) = 0 \end{cases} \quad (19)$$

$$bx(k) = \begin{cases} 1 & if\ \ \eta(k) = 1\ \ and\ w(k) \geq 1\ \ and\ x(k) \geq 1 \\ 0 & if\ \ \eta(k) = 0\ \ or\ w(k) = 0\ \ or\ \ x(k) = 0 \end{cases} \quad (20)$$

e. Delay penalty

When the planned deadline (pd_i) is not respected due to the delay in the *COB* completion, a penalty is incurred according to the overtaking. Thus, the delay penalty in a period k is expressed by the following equation:

$$dp(k) = \begin{cases} 1 & if\ \ bct_i(k) > pd_i \\ 0 & else \end{cases} \quad (21)$$

f. Repair actions

The number of products that have an assembly defect are determined with the random function $rnd(k)$.

For the products with white pedestal that have an assembly defect,

$$rw(k) = rnd(k) \times uw(k) \quad (22)$$

For the products with black pedestal that have an assembly defect,

$$rb(k) = rnd(k) \times ub(k) \quad (23)$$

g. Total profit

The total profit is the difference between the total sales revenues of batches and the total costs over the horizon Z.

$$\begin{aligned} TP = & \sum_{i=1}^{i=nbz} (mw_i + nb_i) \times pf \\ & - \sum_{k=1}^{k=n} [(g(k) + wp(k) + wb(k) + x(k) + eg(k) + w(k) + b(k)) \times cb \\ & + BE \times e(k) \times cg + BWP \times q(k) \times cwp + BBP \times y(k) \times cbp + BX \times v(k) \times cbx + ue(k) \times ce \\ & + (uw(k) + ub(k)) \times ca + (wx(k) + bx(k)) \times cpk + (rw(k) + rb(k)) \times cr + dp(k) \times cp] \end{aligned} \quad (24)$$

3. Optimization Methodology

The optimization of the mathematical model aims to find values of thresholds that maximize the total profit *TP*. While using only simulation is effective for determining total profits corresponding to all possible threshold values, it becomes computationally greedy and time-consuming. Therefore, an optimization method is employed to find an optimal or near-optimal solution within a reasonable computation time. In this paper, we focus on utilizing a metaheuristic method such as the Genetic Algorithm (GA). The GA is well established in the optimization literature for delivering satisfactory results in a short time frame. It facilitates the manipulation of a vector of decision variables, as in our case where we aim to determine the optimal values of four decision variables: *THG*, *THW*, *THB*, and *THX*. In this work, we have chosen the GA as on the one hand it is simple to manipulate and to develop, and on the other hand, it is efficient for finding solutions. In addition, comparing to exact methods that needs a hard development, the GA determines a good solution in a short time [19].

Consequently, we have devised an optimization methodology based on GA coupled with simulation of the objective function, represented by the total profit *TP* in our case.

Both the *TP* calculation and the GA were implemented using the C++ language with the free software Dev-C++ version 6.3. Within the GA, the decision variables are constrained within a vector, where each value is bounded between a minimum and a maximum value, set, respectively, to 1 and 100. The optimization methodology is illustrated in Figure 6 below for clarity.

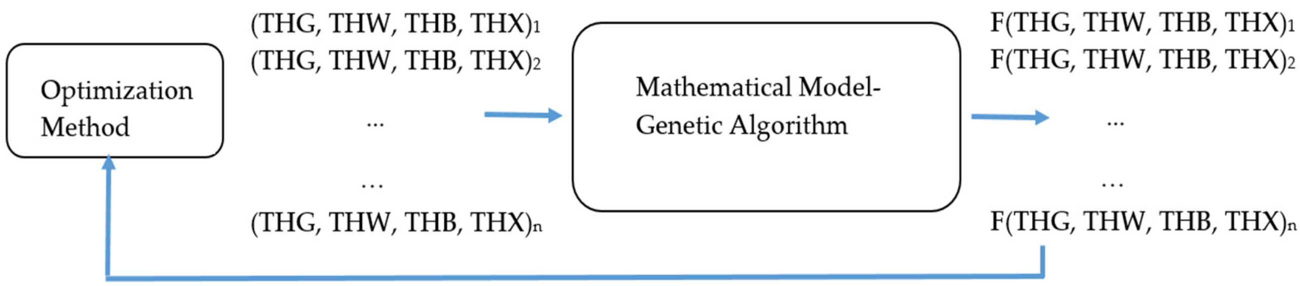

Figure 6. Optimization process of the decision variables (*THG**, *THW**, *THB**, *THX**).

The optimization methodology relies on GA, which generates a new vector of decision variables for evaluation through the simulation program. Subsequently, the simulation program computes the corresponding *TP*, which is then fed back to the GA. Based on the *TP* value, the GA adjusts the decision variables to generate a new vector. This iterative process continues until the GA converges to the best possible solution or until a stopping criterion is met. This iterative loop ensures that the GA iteratively refines its solutions, ultimately converging towards an optimal or near-optimal solution.

After detailing the optimization methodology based on GA, it is evident that the study prioritizes efficiency in finding optimal or near-optimal solutions within a reasonable computation time. By iteratively refining decision variables through the GA-simulation loop, the approach aims to maximize *TP* while considering the system's constraints. The utilization of metaheuristic methods like GA highlights a commitment to exploring innovative techniques for tackling complex optimization problems. This optimization section sets the stage for further analysis and discussion regarding the performance and implications of the proposed methodology within the broader context of the study's objectives.

The proposed GA is based on one algorithm that performs an optimization problem with four discrete variables (*THG*, *THW*, *THB*, and *THX*). Each variable varies in the interval [1, 100]. In the first step, the optimization algorithm tests 500 sets of values (population) that are determined randomly. In the second step, the optimization algorithm explores the best solution obtained from the test and then repeats several iterations until finding the optimal solution. In each iteration, the optimization algorithm defines a new set of individuals for the decision variables and then uses the simulation program to evaluate the corresponding profit. According to the evaluation, the optimization algorithm performs crossover and mutation in order to determine the new set. When there is no improvement between two successive iterations, the GA optimization algorithm stops the iterations and then the optimal solution is found. To illustrate the process steps of the proposed GA, we have provided the schema below (Figure 7).

In the following section, we will present the numerical results obtained from applying the optimization methodology outlined earlier. These findings will shed light on the performance of the proposed GA in maximizing TP, while considering the system's constraints. By analyzing the numerical outcomes, we aim to evaluate the effectiveness and efficiency of our optimization methodology.

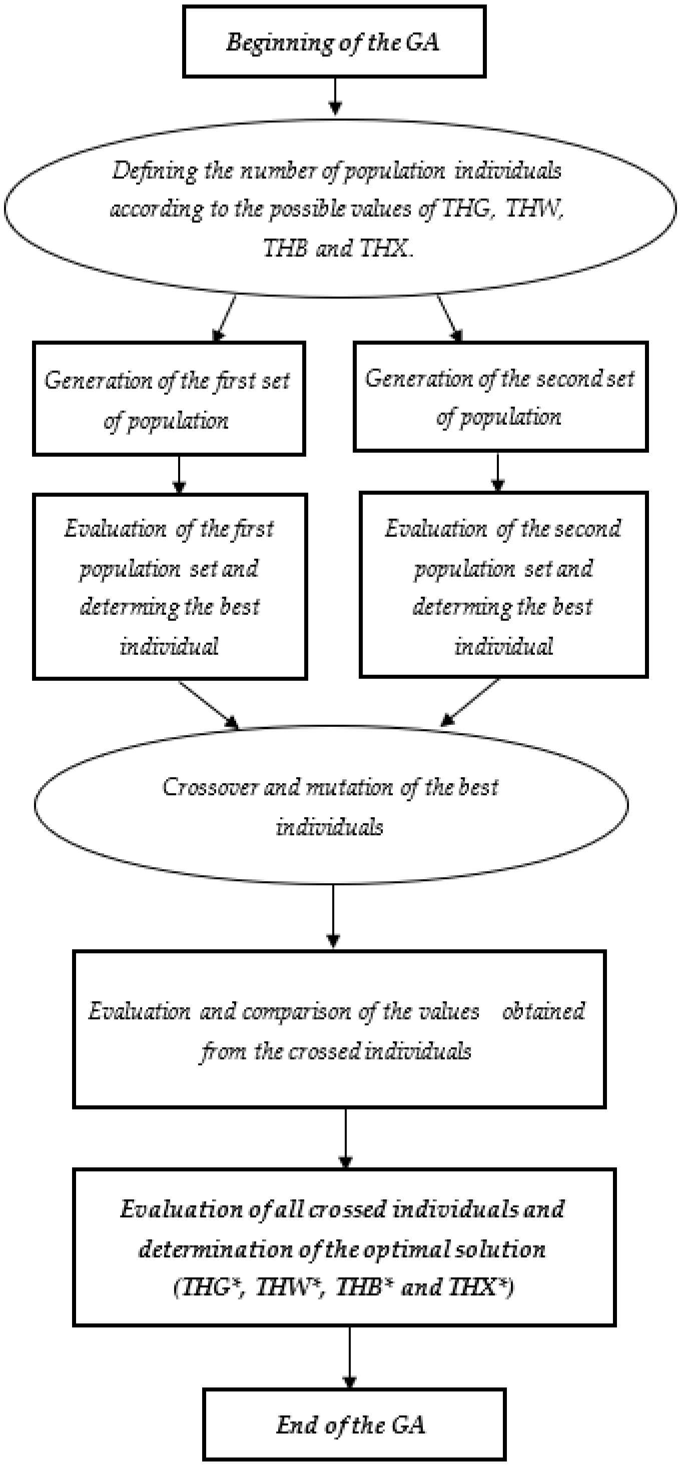

Figure 7. Schema of GA process.

4. Numerical Results

In this section, numerical results are presented to study the machines reliability and the delivery time on the optimal thresholds and the total profit. To evaluate the module MISLOM, numerical results are provided to compare the system with and without MISLOM.

In what follows, the data are used for the numerical results:

- $Z = 10^7$ periods;
- BE = 20;

- BWP = 10;
- BBP = 10;
- BX = 20;
- Ueu = 2;
- MTBFG = 16 periods;
- MTTRG = 3 periods;
- MTBFW = 16 periods;
- MTTRW = 3 periods;
- MTBFB = 16 periods;
- MTTRB = 3 periods;
- MTBFP = 16 periods;
- MTTRP = 3 periods;
- pf = 30 monetary units;
- cr = 2 monetary units;
- cb = 0.05 monetary units;
- cg = 2 monetary units;
- cwp = 1 monetary unit;
- cbp = 1 monetary unit;
- cbx = 0.05 monetary units;
- ce = 2 monetary units;
- ca = 0.07 monetary units;
- cpk = 0.08 monetary units;
- bdt = 3 periods;
- The value of the customer order either for product with a white or black pedestal is generated by the truncated normal distribution, where the average = 15 and the standard deviation = 12.

4.1. FPS with and without MISLOM

To study the benefit of the proposed module, we compare the results given by the optimization of the system with and without MISLOM. The results are presented in Table 2.

Table 2. Study of the FPS with and without MISLOM.

FPS	*THG**	*THW**	*THB**	*THX**	*TP**
Without MISLOM	8	7	6	6	1.63157×10^8
With MISLOM	16	9	10	11	2.4126×10^8

The table reveals that the optimal profit of the system with MISLOM is more important than the one without MISLOM. In addition, the optimal thresholds of the system with MISLOM are higher. As the role of MISLOM is to regulate the production flow according to machines states and the achievement of the *COB*, the system flows are optimized and the equipment are well exploited. As a result, the *COB* is usually completed on time, and the delays are avoided. Therefore, MISLOM manages the system in a way to accelerate the production process, and this allows us on the one hand to reduce the buffering costs and delay penalties and on other hand to obtain more and more ordered customer batches. Hence, the profit is improved with MISLOM. In addition, MISLOM has another role that it orders automatically a batch of parts when a buffer level falls below an order threshold and that avoids the case of a starved machine. As MISLOM accelerates the production process, the buffers should have considerable levels in order to supply machines, and this explains why the optimal thresholds of the system with MISLOM are higher. The system without MISLOM operates without information on equipment and the flows. Thus, flows are not well managed and that may cause delay in achieving the *COB*.

4.2. Impact of the Machines Reliability on the Decisions Variables

This subsection aims to study the impact of the machines reliability on the optimal thresholds and the total profit. Thus, for each machine, we vary the mean time between failures, and then the developed GA determines the corresponding optimal thresholds and total profit. The mean time to repair for each machine is fixed at the value 3 periods time (i.e., MTTRG = MTTRW = MTTRB = MTTRP = 3 periods time). Thus, in this case, the longer the mean time between failures, the more the machine reliability is important and that means that the machine breakdown time is less important compared to the total operating time. This study addresses, as in practice, how the machine breakdowns impact the system performance, and this serves as decision support when the machine reliability varies. The results are presented in Table 3.

Table 3. Study of the machines reliability.

MTBFG	*MTBFW*	*MTBFB*	*MTBFP*	*THG**	*THW**	*THB**	*THX**	*TP**
12	12	12	12	15	9	8	9	2.21607×10^8
16	16	16	16	16	9	10	11	2.4126×10^8
20	20	20	20	18	10	11	13	2.52446×10^8
30	30	30	30	23	12	12	16	2.72133×10^8

As observed, when the mean time to repair increases, the optimal thresholds and the total profit increase. When the mean time to repair increases, the machine availability increases too, and that increases the production quantity. Consequently, the more the mean time to repair increases, the more the *COB* is achieved quickly, and that increases the number of ordered customer batches, as when a batch is achieved, the producer receives a new order. Hence, the total profit increases. Of course, when the production process increases, the supplying of parts increases, and that explain that optimal thresholds increase with mean time to repair. The optimal threshold *THG** is more sensible in variation than the rest of the thresholds. As the machine *MEG* supplies both machines, *MAW* and *MAB*, the level of the buffer *G* should be sufficient to supply *MEG*, and then, the threshold *THG* is sensitive to the system behaviors such as the machine reliability.

4.3. Impact of bdt on the Decision Variables

This subsection studies the impact of the delivery time of one parts batch (*bdt*) on optimal thresholds and the total profit. Thus, *bdt* is varied, and the GA determines the corresponding optimal thresholds and total profit. Indeed, after ordering, the parts batch takes the period time to arrive to the buffers and that may cause a loss of stock in the system. Thus, we have the interest to study the impact of the delivery time on order thresholds.

The results are presented in Table 4.

Table 4. Study of the delivery time.

bdt	*THG**	*THW**	*THB**	*THX**	*TP**
1	11	5	4	7	2.43527×10^8
2	14	7	7	10	2.42294×10^8
3	16	9	10	11	2.41261×10^8
4	17	10	11	13	2.40212×10^8
5	19	12	12	15	2.36953×10^8

The results in the table reveal that when *bdt* increases, the optimal thresholds increase, but the optimal total profit decreases. When the delivery time is high, the system has to ensure the supply of the machines during the delivery time and which will increase the buffers levels. Thus, to keep high levels in the buffers, the ordering thresholds should be high, and this explains why the optimal thresholds increase with delivery time. In addition, when the optimal thresholds increase, the buffer levels become important and that increase the buffering costs, and then, the total profit decreases. Moreover, this study can be useful for managers to regulate the ordered thresholds according to the delivery time.

5. Conclusions

In this work, we presented a model and optimization approach for an FPS designed to produce engraved and packaged glass pieces depending on their colors. A new Module of Information System and Logistic Operations Management (MISLOM) is performed to manage the parts ordering and production flow according to the required batch, machines states, and the buffer levels. So, this work introduces a new mathematical model for a flexible production system which is specifically designed for producing engraved and packaged glass pieces based on their colors. The main objective is to maximize the total profit.

Notably, our work stands out for its comprehensive consideration of critical production system attributes, including equipment breakdowns, repair times, and product quality control. Our methodology involves developing a decision model integrated with an information system to coordinate various system operations, ensuring timely response to customer requests. The module of information system is provided to optimally manage the production flow and parts ordering according to machine availability.

Thus, this approach conjecture develops a decision model integrated with an information system to arrange various system operations, ensuring a timely response to customer requests. Then, to maximize the total profit, an optimization algorithm is used to identify optimal solutions that enhance profitability. Numerical results are provided to reveal and analyze the influence of system reliability and delivery time on ordering thresholds, offering insights into the system's performance and robustness. Via our methodology, the advancement of the FPS is carried out by addressing key operational challenges and optimizing production processes for better performance and greater productivity.

To sum up, our research provides practical insights for industry practitioners by validating the effectiveness of the MISLOM system. This validation equips decision-makers with evidence-based guidance for selecting optimization methods tailored to their operational needs. Additionally, our study contributes to theoretical advancements by identifying key strengths and limitations of different approaches, thereby enriching scholarly discourse in the field of optimization research. Overall, our research bridges the gap between theory and practice.

In our upcoming research, we plan to conduct a thorough comparative analysis to validate the effectiveness of our proposed method. This analysis will involve assessing the performance of our approach against alternative methods based on Artificial Intelligence (IA) commonly used in similar contexts. By leveraging benchmark datasets and diverse evaluation metrics, we aim to provide a comprehensive understanding of how our method stacks up against the competition in terms of accuracy, efficiency, and robustness. Through this comparative analysis, we seek to validate the practical utility of our approach and offer valuable insights for decision-makers in relevant domains.

For future works, data will be collected from MISLOM to develop further approaches based on AI. Moreover, predictive modes based on AI imply a profound change in process and activities to predict lot of requirements such as machines failures, customer ordering, and buffer levels.

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