2.5 Estimation of the hydrodynamic parameters

In this work local velocity field, shear stresses, turbulent kinetic energy and eddy diffusivity are processed in accordance with the works of Devanathan (1991) and Degaleesan (1997). The three-dimensional velocity and turbulent parameters have been calculating by using the equations which discussed in follows:

2.5.1 Liquid velocity Field

The differencing in time of subsequent positions of the particle yield instantaneous Lagrangian velocities. Then, let the coordinates of the particle are x_1, y_1 and z_1 at position 1 and time 1, which is given sampling instant and let the coordinates for next sampling instant are x_2, y_2, z_2 which is (position 2 and time 2). Thus, the midpoint of (x_1, y_1, z_1) and (x_2, y_2, z_2) is calculated as x, y, z and corresponding for cylindrical coordinates is r, θ, z. Then the compartment to which (x, y, z) or (r, θ, z) belongs is calculated, by determining the compartment indices of the midpoint (i, j, k). The velocity calculated by time differencing of (x_1,y_1,z_1) and (x_2,y_2,z_2), as shown in the equations below, is assigned to this compartment with indices i, j and k . Instantaneous (f = 50 Hz) axial and radial velocities over the tracer particle displacement vector $\overline{\imath - 1,\ \imath}$, are calculated as:

$$u_{z,i-1/2} = f(z_i - z_{i-1}) \qquad 1$$

$$u_{r,i-1/2} = f(r_i - r) \qquad 2$$

$$u_{\theta,i-1/2} = f(\theta_i - \theta_{i-1})\frac{r_i + t_{i-1}}{2} \qquad 3$$

In such a manner the instantaneous velocities can be calculated for every sampling instant, as the particle moves around the column following the liquid. Interpretation of the results for the velocity measurements is done by ensemble averaging, which involves averaging the instantaneous velocities measured in a compartmentover the entire duration of the experiment, this ensemble averaging is equivalent to the phasic averaging applicable for modeling.

Authoritative time averaging requires radioactive particle tracking (RPT) experiments work for a long time to collect appropriate data to gain enough statistics to deeply explain in the phase flow field. for this experiments, 24 hr was enough to ensure that the isotopes tracer particle visit all the places inside the split photobioreactor, to emphasize that the time-averaged consequent and combination liquid velocity had reached the plateau.

The velocity is calculated as mean (time-averaged) by ensemble averaging of the instantaneous (Ns) velocity vector components (u_r, u_θ, u_z) that are assigned to a given (i, j, k) compartment.

$$\overline{u_{p(i,j,k)}} = \frac{1}{N_S}\sum_{n=1}^{N_S} u_{p(i,j,k),n} \qquad p = r, \theta, z \qquad 4$$

Fluctuating velocity is then the difference between the instantaneous and mean velocity.

$$u'_{p(i,j,k)} = u_{p(i,j,k)} - \overline{u_{p(i,j,k)}} \qquad 5$$

2.5.2 Shear stress and turbulence kinetic energy

Turbulence parameters are substantial in the modeling of the dynamics in multiphase flows system. In the cylindrical split column, the Reynolds stresses can characterize the interactions in turbulent eddies in a liquid phase. Then, the evaluation of the Reynolds stresses is possible by the RPT technique with another parameter. When the fluctuating velocities are calculated, the turbulence parameters (turbulent kinetic energy and Reynolds stresses) can be estimated. Then, the turbulent stress tensor can be defined for cylindrical coordinates as shown below in Eq. (6):

$$\tau = \rho_l \begin{pmatrix} \overline{u_r' u_r'} & \overline{u_r' u_\theta'} & \overline{u_r' u_z'} \\ \overline{u_\theta' u_r'} & \overline{u_\theta' u_\theta'} & \overline{u_\theta' u_z'} \\ \overline{u_z' u_r'} & \overline{u_z' u_\theta'} & \overline{u_z' u_z'} \end{pmatrix} \ldots\ldots\ldots 6$$

In Eq. (6), the nine unknown components minimized to six components due to the symmetry of the stress tensor, namely:

Shear stresses: $\rho_l \overline{u_z' u_r'}$, $\rho_l \overline{u_\theta' u_r'}$, $\rho_l \overline{u_\theta' u_r'}$ 7

Where, $\overline{u_z' u_r'} = \overline{u_z' u_r'}$, $\overline{u_\theta' u_z'} = \overline{u_\theta' u_z'}$, also, $\overline{u_\theta' u_r'} = \overline{u_r' u_\theta'}$

Normal stresses: $\rho_l \overline{u_r' u_r'}$, $\rho_l \overline{u_z' u_z'}$, $\rho_l \overline{u_\theta' u_\theta'}$ 8

The objective of this work is to realize and understand the mechanisms of the turbulence in a cylindrical split airlift column. As constant liquid density as we approved for culturing system see Laith et, al. 2018, then the negative signs and density are not considered here. The pq component of the Reynolds stress tensor is calculated as:

$$\tau_{pq} = \overline{u'_{p(\iota,\jmath,k)} u'_{q(\iota,\jmath,k)}} = \frac{1}{N_S} \sum_{n=1}^{N_S} u'_{p(i,j,k),n} \, u'_{q(i,j,k),n} \qquad p, q = r, \theta, z \qquad 9$$

The turbulent kinetic energy (TKE) per unit mass is defined as:

$$k = \frac{1}{2}[(\bar{u}_z')^2 + (\bar{u}_r')^2 + (\bar{u}_\theta')^2] \qquad 10$$

2.5.3 Eddy diffusivity

the substantial parameters for quantifying and modeling of transport and liquid mixing in cylinder split column are Turbulent eddy diffusivities (eddies that can vary in size cause the mixing). RPT measured Lagrangian autocorrelation directly can be obtained the eddy diffusivity.

(Degaleesan, 1997) had been discussed the procedure in details to obtain the eddy diffusivities, thus calculating the eddy diffusivities is summarized in the governing equations which provided in this section. The location displacements of the isotope particle Yr, Yθ, Yz caused by the identical velocity fluctuation components were estimated according to the following equations:

$$Y_r(t) = \int_0^1 u_r'(t')dt' \qquad 11$$

$$Y_\theta(t) = \int_0^1 u_\theta'(t')dt' \qquad 12$$

$$Y_z(t) = \int_0^1 (u_z'(t') + \overline{\frac{du_z}{dr}}\Big|_{Y_z(t')} Y_z(t')).dt' \ldots\ldots \qquad 13$$

for axial direction displacement, Yz should also account for the significant time-averaged which is presence in gradient of the local axial velocity in the radial direction, The extra term in Eq. (13) disquiet the gradient in axial velocity in the radial direction due to flow which is anisotropic with radial no homogeneity in the split column. Eddy diffusivities have been defined by Degaleesan (1997) as the normal components of the eddy diffusivity tensor are then calculated as follows:

$$D_{rr}(t) = \int_0^1 \overline{u_z'(t').u_z'(t').}dt' = \frac{1}{2}\frac{d}{dt}\overline{Y_r^2(t)} \qquad 14$$

$$D_{\theta\theta}(t) = \int_0^1 \overline{u_\theta'(t').u_\theta'(t').}dt' = \frac{1}{2}\frac{d}{dt}\overline{Y_\theta^2(t)} \qquad 15$$

$$D_{zz}(t) = \int_0^1 \left[\overline{u_\theta'(t').u_\theta'(t') + \overline{\frac{du_z}{dr}}\Big|_{Y_z(t')}.(\int_0^1 \overline{u_z'(t').u_r'(t'').}dt'')}\right] = \frac{1}{2}\frac{d}{dt}\overline{Y_Z^2(t)} \qquad 16$$

The eddy diffusivities govern Eqs. (11)– (16), which all are related to the Lagrangian ensemble autocorrelation coefficient, which is given by:

$$R_{ij}(\tau) = \overline{u_i'u_j'(t+\tau)} \quad i,j = r,\theta,z \qquad 17$$

Where $i=j$ for the autocorrelation coefficient. For other components of the eddy diffusivity, D_{rz}, $D_{\theta r}$, and $D_{z\theta}$ can be calculated as follows:

$$D_{zr}(t) = \frac{1}{2}\left\{\int_0^1 \{\frac{du_z}{dr}\overline{(\int_0^1 u_r'(t)u_r'(\tau)\,d\tau)} + u_r'(t)u_z'(t')\}dt' + \int_0^1 \overline{u_r'(t').u_z'(t')}.dt'\right\} = \frac{1}{2}\frac{d}{dt}\overline{Y_z(t)Y_r(t)} \qquad 18$$

$$D_{r\theta}(t) = \int_0^1 \overline{u_r'(t').u_\theta'(\tau)}.d\tau = \frac{1}{2}\frac{d}{dt}\overline{Y_\theta(t)Y_r(t)} \qquad 19$$

$$D_{z\theta}(t) = \frac{1}{2}\left\{\int_0^1 \{\frac{du_z}{dr}\overline{(\int_0^1 u_\theta'(t)u_r'(\tau)\,d\tau)} + u_\theta'(t)u_z'(t')\}dt' + \int_0^1 \overline{u_\theta'(t').u_z'(t')}.dt'\right\} = \frac{1}{2}\frac{d}{dt}\overline{Y_z(t)Y_\theta(t)} \qquad 20$$

In RPT technique, the Lagrangian ensemble components velocities as a time series for tracer isotope particle which is used to determine the Lagrangian cross and auto-correlation coefficients. The eddy diffusivities concept can be explained as follows:

If the tracer particle enters at sometimes the compartment in the column, a counter is initiated and the tracer is tracked for a long time. Also, the tracking is started a new trajectory after a long time. To obtain enough statistics over a large number of the fictitious compartments of the split airlift column, this process is iterated for whole data. Therefore, when the experiment finished, there will be an ensemble of trajectories in the column for each single compartment. Depending on the ergodicity concept, this trajectories ensemble possibly showed as particles group's which were freed at that time and compartment and eventually spread outside the compartment (Degaleesan, 1997; Roy,2000).