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A Procedure for Estimating Drought Duration and Magnitude at the Uniform Cutoff Level of Streamflow: A Case of the Weekly Flows of Canadian Rivers

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Abstract: At times, hydrological drought is defined using Q90 or Q95 (90% or 95% flows equaling or exceeding) or even at higher levels, such as Q75 as the cutoff level regardless of their seasonal variation (i.e., truncation at the uniform flow level). In the past, the estimation of drought length and magnitude at the aforesaid uniform cutoff levels of flow has been a challenging issue. A procedure is presented to first estimate the drought magnitude (M), which then forms the basis for estimating the drought duration or length (L). The drought magnitude (M) and the length of the critical period (L_{cr}) are estimated using the concept of behavior analysis prevalent in the hydrologic design of reservoirs. This information is used for estimating the drought length ($L_{T\text{-}e}'$, the estimated value of drought length for the return period of T weeks) involving a Markov chain model on the standardized weekly flow sequences. A weighted average of L_{cr} and $L_{T\text{-}e}'$ (=0.60 L_{cr} + 0.40 $L_{T\text{-}e}'$) results in the estimate of drought length, which is compatible to the observed counterpart. The performance of the procedure to estimate drought length was found to be satisfactory up to the truncation level of Q75, whereas the estimation of drought magnitude was rated as good.

Keywords: cutoff level; behavior analysis; drought probabilities; drought duration; intensity and magnitude; Markov chain; weekly flows

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1. Introduction

The annual, monthly, and daily identification and modeling of hydrological droughts has been the subject of investigations for which significant research material is available in the literature. At times the hydrological droughts have been defined when the flow drops below a uniform predefined level such as Q95 (95% flows are equal to or exceeding this level of flow or the probability of drought, q = 5% or 0.05), Q90, Q80, or Q70 [1–4], or even the mean or median (=Q50) and 75% of the mean [5], generally in case of daily flow sequences. Lately, an attempt has also been made to quantify drought duration and deficit volume on a weekly basis by Sharma and Panu [6,7]. A weekly time scale is a good compromising resolution between a long period of a month and a short period of a day. Put another way, a weekly analysis of drought is simple and tractable in a statistical sense, whereas a daily analysis might catapult into a complex relationship, which at times tends to be intractable. Furthermore, a weekly analysis is vindicated by the outcome of low flow studies in which a minimum annual 7-day average flow is found to be a rational descriptor of drought conditions by the task committee for low flow conditions ((TCLFE) [8] in USA and Gustard et al. [9] in UK). One major advantage of a weekly analysis is that it can easily be scaled down to daily values in terms of drought duration (or length). For example, if a 50-year drought extends for 10 weeks based on the cutoff level of Q90, then a drought period spanning 70 (=10 × 7) days can be inferred.

In a majority of hydrological drought studies dwelling on a seasonal basis, the truncation level is variable in the sense that statistics of the respective month, season, or week are used to truncate the series. For instance, if we are dealing with monthly flows, then droughts are identified with respect to monthly means or medians [6,10,11] or some statistic of respective monthly flow such as Q70 of the respective month [12]. In a statistical sense, the variable flow level simplifies the analysis by which the flow series can be easily made stationary and the resultant series can be analyzed using the known concepts of stochastic theory. Such a stationarization can be achieved through week-by-week standardization of the weekly flows, with the resultant series termed as the standardized hydrological index, SHI series [6]. The SHI series can be regarded as analogous to a standardized precipitation index (SPI) series [13,14] that is used in the ambit of meteorological drought. In this kind of analysis, although the truncation level is uniform in terms of probability, the uniformity of the truncation in terms of the flow is not preserved. The term uniform probability implies that when the analysis is done in the standardized domain of flows (SHI sequences), then the truncation is done at the uniform value of SHI_x, with the uniform counterpart drought probability, q_x. In doing so, constancy of q_x is preserved, but the constancy of the cutoff flow is distorted.

A recent study [15] has used meteorological and hydrological (streamflow) anomaly indices to assess water needs during the hydrological drought on a monthly basis. The above study is based on the truncation level approach and the theory of runs. There has been an introduction of copula and entropy based approaches to characterize the drought duration and the deficit volumes [16,17], and these approaches are gaining prominence, though their efficacy is still to be evaluated in comparison to the traditional approaches advanced by Dracup et al.[10] and Yevjevich et al. [11].

In practice, the information on drought parameters (i.e., duration and magnitude) at the uniform flow level of truncation is of greater importance, which is being addressed in this paper. The parameters of the hydrologic drought, viz., T-week drought duration, L_T and magnitude, M_T (standardized deficit volume), are of practical utility at uniform truncation levels of flow for the design and the operation of drought alleviating facilities and structures. The standardized deficit volume is named here as drought magnitude M_T (=D_T/σ) in which D_T is the deficit volume in volumetric units and σ is a characteristic standard deviation of the flow sequence, thus M_T is a dimensionless entity and suffix T stands for the return period (week). In the traditional approach, the drought length is estimated first (say by the Markov Chain methodology or the extreme number theorem), which is transformed into drought magnitude through the linkage relationship (drought magnitude = drought intensity $\times$ drought length [10]). In the present analysis, firstly the drought magnitude is determined, which then forms the basis for estimating the drought length. The concepts of behavior analysis [18,19] used in the hydrologic design of reservoirs, simple and conditional probabilities, and the Markov chain using the SHI sequences are invoked to establish T-week drought parameters (duration and magnitude). The weekly flows have been subjected to drought analysis with a uniform truncation level of Q95 and Q90 up to Q75.

2. Background of the Model

To begin with, the weekly flow data for the Upper Humber River (#23, Table 1) were analyzed for the reservoir volume (V_R) as well as D_T (drought deficiency volume) at the truncation (demand level) of Q75. It should be noted that the Upper Humber River was chosen because of its size (2210 km^2) and its long uninterrupted length of flow record of 68 years during which the persistence in the weekly SHI sequences has been modestly high (ρ = 0.48), with the coefficient of variation (=1.07). Most importantly, the watershed is least provoked by abstractions, and flows can be deemed entirely in a natural state. For this river, the Q75 flow based on the weekly flow duration curve of the historical data spanning from 1953 to 2020 (68 years = 3536 weeks) was found to be equal to 24.16 m^3/s. In the behavior analysis [18], the weekly flow sequence was truncated at the uniform flow level =24.16 m^3/s (demand level) and several episodes of full reservoir conditions (assumed to be above the cutoff level) were tracked. Behavior analysis was chosen as it has been found to be a simple procedure to estimate reservoir capacity, and it can be applied to the data of any time interval. The procedure takes into account autocorrelation, seasonality, and other flow parameters imbued in the historical data [18]. Further calculations were done using the water balance equation (i.e., behavior analysis), $V_t = V_{t-1} + Qt - Q75$ (all losses including evaporation were neglected for the sake of simplicity in the drought analysis). In the above water balance equation, V_t is the reservoir volume at the end of week t, V_{t-1} is the volume at the end of the week t−1, Q_t is the river inflow during week t, and Q75 is the outflow needed to meet the demand during the week t. In the process of counting, $V_t < Q75$ indicates no fill condition (deficit conditions) and $V_t \geq Q75$ indicates the reservoir full condition (surpluses). Under the condition, $V_t > 0$, the reservoir will spill over the water from the reservoir say through a spillway. The calculations are then begun with $V_{t-1} = 0$, and all deficits are counted below zero as magnitudes with a negative sign. All the values of $V_t \geq 0$ are set to zero. The deficit spell begins with zero, drops down to the minimum level and then begins to recover and ends again with zero after full recovery. All the values during this dry spell are negative and the minimum negative value represents the maximum deficit volume (V_R) during this dry period, i.e., critical period, L_{cr}. By applying the above water balance equation and a counting scheme, the maximum value of V_R was found = 221.99 m^3/s-week (absolute value) with the L_{cr} equal to 21 weeks.

In the drought length and magnitude based analysis [7], the flow series of 3536 weeks is chopped at a uniform flow level of 24.16 m^3/s and several episodes of deficit (marked by 0) and surplus (marked by 1) conditions erupted. The episode with the largest length L_T and D_T (T = 3536) was identified, and these entities were estimated by a counting method. The L_T (largest drought length) and the D_T (largest deficit volume) were found = 17 weeks and 221.99 m^3/s-week, respectively. It can be seen that both V_R and D_T are equal but L_T (=17) is not equal to L_{cr} (=21). This behavior of equality of D_T and V_R was found in almost all the rivers at the chosen truncation levels. The above calculations were done using the historical observed data. So, the terms L_T and D_T are designated as $L_{T\text{-}o}$ and $D_{T\text{-}o}$ (subscript "o" stands for observed) from here onward. When L_T and D_T are estimated by a modeling scheme, they are designated by $L_{T\text{-}e}$ and $D_{T\text{-}e}$ (subscript "e" stands for estimated).

Table 1. Summary of statistical properties of weekly flows of the rivers under consideration.

Numeric Identifier of the River in Figure 1 with Name and Gauging Station Identity	Data Size (Years)	Area (km^2)	μ_o	cv_o	cv_{mx}	cv_{av}	cv_{gm}	ρ
[1] Fraser at Shelley, BC08KB001	70 (1951–2020)	32,400	817.34	0.90	0.82	0.36	0.28	0.75
[2] Athabasca River at Athabasca, AB07BE001	69 (1952–2020)	74,600	429.19	0.98	1.32	0.43	0.26	0.81
[3] *Bow River at Banff, AB05BB001*	110 (1911–2020)	2210	39.24	1.11	1.22	0.31	0.14	0.72
[4] *pipestone River at Karl lake, ON04DA001*	54 (1967–2020)	5960	59.05	1.04	1.56	0.63	0.41	0.89
[5] Neebing at Thunder Bay, ON02AB008	66 (1954–2019)	187	1.62	1.87	3.84	1.10	0.68	0.63
[6] Pic River near Marathon, ON02BB003	50 (1971–2020)	4270	50.10	1.24	2.16	0.71	0.48	0.74
[7] Pagwachaun at highway#11, ON04JD005	53 (1968–2020)	2020	53.08	1.45	2.77	0.79	0.48	0.74
[8] Nagamgami at highway#11, ON04JC002	70 (1951–2020)	2410	24.56	1.11	1.66	0.55	0.40	0.87
[9] Batchawana at Batchawana, ONBF001	50 ((1971–2020)	1190	22.38	1.38	2.75	0.74	0.52	0.62
[10] Goulis near Searchmont, ON02FB002	53 (1968–2020)	1160	18.37	1.32	2.69	0.75	0.55	0.69
[11] Whitson at Chemsford, ON02CF007	60 (1961–2020)	243	3.06	1.50	3.62	0.78	0.57	0.68
[12] North French near Mouth, ON04MF001	54 (1967–2020)	1190	95.72	1.29	2.44	0.71	0.46	0.72
[13] Labase River at North Bay, ON02DD013	54 (1975–2018)	70.4	0.91	1.49	3.24	0.96	0.79	0.44
[14] *Chippewa Creek at North Bay, ON02DD014*	54 (1975–2018)	37.3	0.62	1.11	2.10	0.81	0.67	0.43
[15] Commanda at Commanda, ON02DD015	46 (1975–2020)	106	1.76	1.22	2.31	0.77	0.66	0.58
[16] N. Magnetwan at Pickerel Lake, ON02EA010	52 (1969–2020)	149	2.86	1.26	2.54	0.82	0.70	0.51
[17] Becancour A Lyster, QC02PL001	46 (1923–1968)	1410	30.62	1.32	2.46	0.82	0.69	0.62
[18] Beaurivage A. Sainte Entiene, QC02PJ007	75 (1926–2000)	709	14.21	1.47	2.67	0.90	0.77	0.49
[19] *Lepreau River at Lepreau, NB01AQ001*	101 (1919–2019)	239	7.43	1.08	2.01	0.87	0.80	0.49
[20] Carruther at Saint Anthony, PE01CA003	59 (1962–2020)	46.8	0.97	1.33	2.89	0.82	0.62	0.48
[21] Bevearbank River at Kinsac, NS01DG003	88 (1922–2019)	97	3.04	1.09	1.53	0.90	0.84	0.43
[22] N. Margaree at Margaree valley, NS01FB001	90 (1929–2020)	368	17.01	0.96	1.60	0.68	0.59	0.46
[23] Upper Humber at Reidville, NF02YL001	68 ((1953–2020)	2210	80.29	1.07	1.57	0.66	0.60	0.48
[24] Torrent River at Bristol Pool, NF02YC001	61 (1960–2020)	624	24.90	1.07	2.21	0.62	0.53	0.57

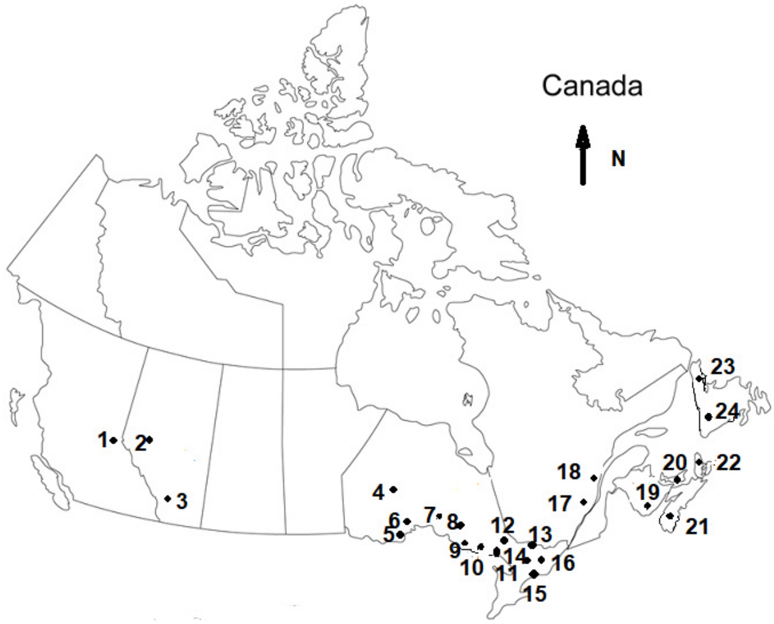

Figure 1. Map of Canada showing hydrometric gauging stations (source: Environment Canada).

Following the aforesaid calculations, the behavior of V_R, $D_{T\text{-}o}$, L_{cr}, and $L_{T\text{-}o}$ was investigated for all rivers [Table 1]. It was found that on a weekly basis, the value of $D_{T\text{-}o}$ using flow sequences was almost equal to V_R at the truncation levels from Q95 to Q75 for the Canadian rivers having ρ less than 0.90 [Tables 2–4]. That means the estimates of V_R can be obtained using behavior analysis and the estimate of V_R thus obtained can be standardized and designated as V_R'. The value of V_R' can be equated to the standardized $D_{T\text{-}o}$ (termed as drought magnitude $M_{T\text{-}o}$). Though the estimation of drought magnitude D_T can be easily accomplished by estimating V_R by behavior analysis, the estimation of L_T requires further investigation. The estimate of L_{cr} was generally found > $L_{T\text{-}o}$, so a need arose to determine other estimates of $L_{T\text{-}e}$ that could be combined with L_{cr}, with the resulting value equivalent to $L_{T\text{-}o}$. A method to estimate the $L_{T\text{-}e}$ to be equivalent to $L_{T\text{-}o}$ is the subject matter of the present paper.

Estimation of Drought Length, $L_{T\text{-}e}$

It is mentioned in the foregoing section that L_{cr} is one estimate of $L_{T\text{-}e}$, which needs further work because of the discrepancy between $L_{T\text{-}o}$ and L_{cr}. To this end, recourse was taken for the estimation of $L_{T\text{-}e}$ by a Markov chain (MC) based relationship. To make use of the MC relationship, the weekly flow sequences must be stationary (i.e., stationarity up to the second order). The historical weekly flow sequences can be stationarized using the week-by-week standardization of flow sequences thus forming the SHI sequences. These SHI sequences form the platform for applying the MC based modeling methodology to derive an estimate of $L_{T\text{-}e}$ [6].

Since $M_{T\text{-}e}$ is a standardized value of the deficit volume, $D_{T\text{-}e}$, therefore $D_{T\text{-}e} = \sigma_{av} \times M_{T\text{-}e}$, where σ_{av} is the average of 52 values of the weekly standard deviations. It should be noted that σ_{av} was found to be the best estimator of the standard deviation for the above transformation (Sharma and Panu [20,21]). Further, the value of $M_{T\text{-}o}$ from the non-standardized weekly sequences is computed as $M_{T\text{-}o} = [V_R/\sigma_{av}] = V_R' = [D_{T\text{-}o}/\sigma_{av}]$, subject to the condition $V_R = D_{T\text{-}o}$, which is true for the range of uniform cutoff, Qx such as Q90, considered in the present analysis.

Table 2. Summary of calculations in the identification of the cutoff level for the estimation of drought length ($L_{T\text{-}e}$), Upper Humber River, Newfoundland, Canada.

River Identity	Qx, (q), V_R', Lcr	Cutoff SHIx = Z_0	q_1 *, q_q, q_p	$L_{T\text{-}e}'$, μ_d, $M_{T\text{-}e}'$	Decision	Model Φ, $M_{T\text{-}e}$	$L_{T\text{-}e}$
1	2	3	4	5	6	7	8
Upper Humber (#23, Table 1) T = 3536 σ_{av} = 53.10 ρ = 0.48	Q75 = 24.16, (q = 0.25), **4.18, 21** $L_{T\text{-}o}$ = **17**	(iii) = −1. 06 (iiia) = −0.86 (i) = −0.66	0.046, 0.389, 0.030 0.117, 0.528, 0.063 **0.242**, 0.575, 0.136	6, 0.51, 2.84 9, 0.55, 4.87 **11**, 0.60, 6.68	$M_{T\text{-}e} < V_R'$, next $M_{T\text{-}e} > V_R'$, q_1 is low, next cutoff (i) is fine	MC1 0.47, **4.20**	A = 16 b = 19 c = **17**
	Q80 = 20.47, (q = 0.20), **3.06, 20** $L_{T\text{-}o}$ = **16**	(iii) = −1.13 (iiia) = −0.91 (i) = −0.70	0.030, 0.358, 0.020 0.091, 0.497, 0.051 **0.210**, 0.552, 0.119	5, 0.50, 2.41 8, 0.54, 4.25 **10**, 0.59, 6.06	$M_{T\text{-}e} < V_R'$, next $M_{T\text{-}e} > V_R'$, q_1 is low, next cutoff (i) fine	MC1 0.63, **3.07**	a = 15 b = 18 c = **16**
	Q85 = 16.78 (q = 0.15), **2.02**, 17 $L_{T\text{-}o}$ = **15**	(i) = −0.74	**0.185**, 0.530, 0.107 At cutoff levels (iiia) and (iii) q = 0.071 and 0.02, very low	**10**, 0.58,5.54	Cutoff(i) fine, Level (iii) and (iiia) were rejected due to low q_1 values	MC1 0.82, **2.01**	a = 13 b = 15 c = **14**
	Q90 = 13.57 (q = 0.10), **1.27**, 16 $L_{T\text{-}o}$ = **14**	(i) = −0.78	**0.158**, 0.538, 0.087 At cutoff levels (iiia) and (iii) q = 0.056 and 0.014, very low.	**9**, 0.57, 5.42	Cutoff Level (i) is fine. Cutoff (iiia) and (iii) resulted in very low values of q_1, so were rejected	MC1 0.99, **1.27**	a = 13 b = 14 c = **13**
	Q95 * = 10.48 (q = 0.05), **0.70**, 15 $L_{T\text{-}o}$ = **9**	(i) = −0.81 (iiia) = −1.06	0.138, 0.518, 0.077 **0.043**, 0.379, 0.028	4, 0.51, 2.20 (*9*, *0.56*, *4.99*) **3**, 0.51, 1.27 (*5*, *0.51*, *2.75*)	Cutoff level (i) is too high—so next lower cutoff level (iiia) is fine	MC0 0.97, **0.70**	a = 9 b = 11 c = **10**

Asterisk (*) denotes that at Q95 level drought length resembles MC0 because the values of V_R' better correspond to the MC0 based values of $M_{T\text{-}e}'$. The values in italics show the relevant values with MC1 based lengths. However, from Q90 to Q75, MC1 yielded $M_{T\text{-}e}'$ values in better correspondence to V_R'. Note that all entities have no units except Qx, which is in m^3/s, and $L_{T\text{-}e}$ and $L_{T\text{-}e}'$ are in weeks.

Table 3. Summary of the results based on the calculations for estimated values, $L_{T\text{-}e}$ and $M_{T\text{-}e}$ at varying cutoff levels.

River Identity	V_R'	$M_{T\text{-}o}$	$L_{T\text{-}o}$	Qx *	q_1 *	q_q	q_p	Cutoff	$M_{T\text{-}e}'$	Φ	$M_{T\text{-}e}$	Lcr	$L_{T\text{-}e}'$	MC type	$L_{T\text{-}e}$ **		
															a	b	c
1	2			3				4				5			6		
Athabasca (#2, Table 1) T = 3588 σ_{av} = 183.97 ρ = 0.81	**0.93**	0.93	**15**	69.73	0.022	0.500	0.011	iiia	0.93	0.00	**0.93**	20	2	MC0	11	16	**13**
	1.99	1.90	**19**	79.77	0.025	0.538	0.012	iiia	3.00	0.50	**1.99**	24	17	MC1	15	20	**17**
	3.37	3.37	**21**	91.92	0.202	0.727	0.069	i	3.58	0.09	**3.36**	28	17	MC1	18	23	**20**
	4.56	4.56	**22**	102.32	0.211	0.728	0.073	i	9.58	0.67	**4.57**	28	17	MC1	22	25	**24**
	5.70	5.70	**23**	112.35	0.224	0.738	0.076	i	10.13	0.56	**5.69**	29	18	MC1	23	26	**24**
Goulis (#11, Table 1) T = 2756 σ_{av} = 13.84 ρ = 0.69	**1.50**	1.50	**31**	2.204	0.079	0.647	0.031	iiia	5.92	1.00	**1.53**	34	11	MC1	22	28	**25**
	4.10	4.10	**34**	3.288	0.095	0.654	0.037	iiia	6.39	0.48	**4.09**	37	12	MC1	24	31	**27**
	5.90	5.90	**35**	4.010	0.259	0.770	0.081	i	12.92	0.68	**5.86**	39	20	MC1	30	34	**32**
	7.36	7.36	**35**	4.587	0.275	0.782	0.083	i	13.75	0.59	**7.33**	39	22	MC1	31	35	**32**
	8.89	8.89	**35**	5.190	0.292	0.785	0.089	i	13.97	0.47	**8.86**	39	22	MC1	31	35	**32**
Bevearbank (#21, Table 1) T = 5096 σ_{av} = 2.74 ρ = 0.43	**0.24**	0.24	**12**	0.065	0.065	0.413	0.041	i	1.55	1.00	**0.58**	16	3	MC0	9	13	**11**
	0.80	0.80	**19**	0.174	0.145	0.515	0.082	ia	2.31	0.93	**0.79**	21	4	MC0	12	17	**14**
	1.79	1.79	**19**	0.324	0.168	0.518	0.097	ia	5.38	0.87	**1.77**	24	9	MC1	17	20	**18**
	3.20	3.20	**21**	0.511	0.203	0.541	0.117	ia	5.97	0.59	**3.22**	27	10	MC1	18	23	**20**
	4.63	4.63	**22**	0.696	0.255	0.566	0.148	ia	6.71	0.40	**4.59**	34	11	MC1	22	28	**25**

Asterisk (*) indicates Qx values at Q95, Q90, Q85, Q80, and Q75 levels for each river in column 1. The values of q (=q_x) are 0.05, 0.10, 0.15, 0.20, and 0.25, respectively, based on the Qx flow levels (flow duration curves). Asterisk (**) Indicates $L_{T\text{-}e}$ values (weeks) are computed for 3 options (a), (b), and (c) discussed in the text.

Table 4. Summary of the results in the validation process in terms of estimated values, $L_{T\text{-}e}$ against $L_{T\text{-}o}$ at varying cutoff levels.

River Identity	Qx *	V_R	$M_{T\text{-}0}$	L_{cr}	$L_{T\text{-}0}$	Cutoff	Φ	$M_{T\text{-}e}$	Model Order	$L_{T\text{-}e}$ (a) †	(%) ∤	$L_{T\text{-}e}$ (b) ††	(%) ∤	$L_{T\text{-}e}$ (c) †††	(%) ∤
1			2					3		4	5	6	7	8	9
Lepreau River (#19, Table 1) T = 3588 σ_{av} = 183.97, ρ = 0.49	0.58	0.78	0.57	18	**12**	iii	0.78	0.78	MC0	10	−16.67	14	16.67	12	0.00
	1.03	1.78	1.78	24	**15**	iiia	0.55	1.78	MC1	15	0.00	19	26.67	17	13.33
	1.40	2.70	2.70	27	**19**	i	0.56	2.72	MC1	18	−5.26	22	15.79	19	0.00
	1.78	3.75	3.68	32	**19**	i	0.42	3.78	MC1	21	10.53	26	**36.84**	23	**21.05**
	2.19	5.17	5.17	34	**23**	i	0.27	5.20	MC1	22	−4.35	28	21.74	25	8.70
Pipestone River (#4,Table 1), T = 2808 σ_{av} = 37.31 ρ = 0.89	9.20	0.91	0.91	16	**13**	iiia	0.60	0.91	MC0	9	**−30.77**	13	0.00	11	**−15.38**
	10.84	1.52	1.52	17	**16**	iiia	0.02	1.52	MC1	17	**18.75**	17	18.75	17	18.75
	12.67	2.28	2.28	20	**18**	iiia	1.00	2.54	MC1	19	5.56	19	5.56	19	5.56
	14.46	3.08	3.08	22	**20**	iiia	0.93	3.04	MC1	20	0.00	21	5.00	20	0.00
	16.66	4.16	4.16	24	**22**	i	0.96	4.22	MC1	26	0.00	25	4.55	25	0.00
Chippewa River (#14, Table 1) T = 2288 σ_{av} = 0.48, ρ = 0.43	0.12	0.45	0.43	9	**7**	iiia	1	0.59	MC0	6	−14.29	7	0.00	7	0.00
	0.14	0.86	0.72	11	**10**	iiia	1	0.98	MC1	9	−10.00	10	0.00	9	−10.00
	0.16	1.23	1.23	16	**11**	iiia	0.95	1.24	MC1	12	9.09	14	27.27	13	18.18
	0.19	1.62	1.62	18	**13**	iiia	0.87	1.63	MC1	13	0.00	16	23.08	14	7.69
	0.21	2.27	2.27	20	**14**	iiia	0.74	2.27	MC1	15	7.14	17	21.43	16	14.29
Bow River (#3, Table 1) T = 5720 σ_{av} = 12.05 ρ = 0.72	7.11	1.77	1.77	20	**18**	i	0.99	1.77	MC1	17	−5.56	19	5.56	18	0.00
	7.67	2.67	2.67	22	**20**	i	0.86	2.69	MC1	18	−10.00	20	0.00	19	−5.00
	8.18	3.55	3.55	24	**21**	i	0.74	3.56	MC1	19	−9.52	22	4.76	20	−4.47
	8.63	4.34	4.34	26	**22**	i	0.63	4.37	MC1	20	−9.09	23	4.55	21	−4.55
	9.16	5.27	5.27	26	**24**	i	0.50	5.28	MC1	20	−10.67	23	**−4.17**	21	−12.56
Mean											−4.0%		12.0%		**3%**
Stan. error											11.4%		11.9%		**10.4%**

Asterisk (*) indicates Qx values at Q95, Q90, Q85, Q80, and Q75 levels in 5 rows for each river in column 1. (**a**) † indicates option-a, (**b**) †† indicates option-b, (**c**) ††† indicates option-c, and (%) ∤ indicates percent deviation.

Since V_R' = $M_{T\text{-}o}$, to find $L_{T\text{-}e}$, V_R' is set = $M_{T\text{-}e}$. In the process of equating V_R' to $M_{T\text{-}e}$, the following steps are pursued:

Step-1: Choose a suitable cutoff level corresponding to flow level, Qx (say Q90). The standardized version of the cutoff level (denoted as SHI_x) can be obtained in the following fashion.

Step-2: Let (i) = $SHI_o = (Qx - \mu_o)/\sigma_o$; (ii) = $SHI_{max} = (Qx - \mu_o)/\sigma_{max}$; (iii) = $SHI_{av} = (Qx - \mu_o)/\sigma_{av}$; or (iv) = $SHI_{gm} = (Qx - \mu_o)/\sigma_{gm}$, where σ_{max} is the maximum value, σ_{av} is the arithmetic mean, and σ_{gm} is the geometric mean of the 52 values of the weekly standard deviations. It has been mentioned earlier in the text that μ_o and σ_o are the overall mean and the standard deviations of the weekly historical (non-standardized) flow sequences. At times, cutoff levels [average of (i) and (ii) denoted as (ia)] or [average of (iii) and (i), denoted as (iiia)] can be combined to improve the results of the analysis.

To elucidate the point, there shall be six cutoff levels: (iv), (iii), (iiia), (i), (ia), and (ii). These levels, in general, in the ascending order are ((ii) being the highest), the most frequent levels being (iiia) and (i), whereas levels (ii) and (iv) are rare. Based on the chosen cutoff level SHI_x, the probabilities q_1, q_q, and q_p are computed by a counting method. At the Q95 level (drought probability q = 0.05), the q_1 is computed by the chosen cutoff level and it should be close to 0.05. Likewise, at the Q75 level, q_1 should be close to 0.25. So, the identification of an appropriate cutoff level is a trial and error procedure. It should be noted that q_1 is the drought probability computed by a counting method and q is the drought probability based on the flow duration curve, and they can be designated as q_x corresponding to cutoff flow Q_x.

Step-3: Once an appropriate cutoff level is chosen, obtain an estimate of $L_{T\text{-}e}{}'$ by using the MC1 [7] representation of drought lengths, expressed as follows.

$$L'_{T-e} = 1 - \frac{\log\left[F\,T(1-q_1)q_p \right]}{\log\left(q_q\right)} \tag{1}$$

where, F is the factor to account for the plotting position in the empirical estimation of the exceedance probability. In this analysis, the plotting position formula [22] developed for Canadian rivers has been used. The formula evaluates the exceedance probability (=0.75/(T + 0.25)), so F (=1.33 (1 + 0.25/T)) ≈ 1.33 as T is generally large. The term q_q stands for the conditional probability of the present period being drought given the previous period was also a drought and likewise q_p stands for the present period being drought given the previous period was wet. The probabilities q_q and q_p can be computed from the counting method. It should also be borne in mind that at low cutoff levels, such as Q95, $L_{T\text{-}e}{}'$ may be adequately represented by MC0 in which case $q_q = q_p = q_1$.

Step-4: Obtain the initial estimate of $M_{T\text{-}e}$ by following the linear linkage relationship between drought magnitude (M) = drought intensity (I_d) × drought length (L) [10]. Thus, the $M_{T\text{-}e}$ can be expressed as

$$M_{T\text{-}e} = \mu_d \times L_c \tag{2}$$

in which L_c is a characteristic drought length obeying the following relationship (Sharma and Panu [20,21]).

$$L_c = \Phi\, L_m + (1-\Phi)\; L'_{T-e} \tag{3}$$

The value of the parameter Φ varies between 0 and 1, and its proper value can be estimated by a trial and error procedure, which will be explained in the following text.

The mean drought length (L_m) can be expressed as Sen [23]

$$L_m = \left(\frac{1}{1-q_q}\right) \tag{4}$$

It is tacitly assumed that μ_d, mean of the drought intensity, is approximated as independent of T, and it can be expressed by the truncated normal pdf of drought intensities as follows [23,24].

$$\mu_d = -\left[\frac{\exp\left(-0.5\, z_0^2\right)}{q_n\sqrt{2\pi}}\right] - z_0 \tag{5}$$

In Equation (5), q_n is the drought probability corresponding to z_0 (standard normal variate). The z_0 can be regarded as equivalent to the cutoff level (or truncation level), SHI_x. The q_n can be determined from the standard normal tables or the polynomial function [25]. In strict normal pdf conditions of SHI sequences, the value q_1 by the counting procedure is equal to q_n. Otherwise, minor deviation may occur. However, in the calculations, q_n based on the polynomial function should be used to ensure consistency.

Equation (2), involving μ_d, can be reduced to the following form.

$$M_{T-e} = \text{abs}\left\{-\left(\frac{\exp\left(-0.5\, z_0^2\right)}{q_n\sqrt{2\pi}}\right) - z_0\right\} L_c \tag{6}$$

If the parameter Φ = 0, then Equation (6) takes the following form.

$$M'_{T-e} = \text{abs}\left\{-\left(\frac{\exp\left(-0.5\, z_0^2\right)}{q_n\sqrt{2\pi}}\right) - z_0\right\} L'_{T-e} \tag{7}$$

After having chosen the value of the cutoff level (z_0 = SHI_x) and estimating the parameters (q_1, q_q, and q_p) by the counting method, the $L_{T\text{-}e}'$ and the $M_{T\text{-}e}'$ are computed. $M_{T\text{-}e}'$ (means Φ = 0) value so computed may be compared to V_R'. If $M_{T\text{-}e}' \geq V_R'$, then the model structure (MC1) is acceptable, and $M_{T\text{-}e}$ can be made equal to V_R' by adjusting the value of Φ within its range 0 and 1. If $M_{T\text{-}e}' < V_R'$, then matching should be done between $M_{T\text{-}e}'$ (based on $L_{T\text{-}e}'$ with MC0 structure of the model) and V_R'. Again, $M_{T\text{-}e}$ can be made = V_R' by adjusting the value of Φ within the range 0 and 1. This situation was found to occur with flow levels of Q95 and at Q90 (on a few occasions), as can be seen in Table 3 in the context of the Beaverbank River. The value of $L_{T\text{-}e}'$ so obtained should be combined with L_{cr} to arrive at an estimate of the $L_{T\text{-}e}$ as was discussed earlier in the text. The detailed calculations based on the modeling methodology are presented in the next section along with a flow diagram.

3. Data and Methods of Analysis

3.1. Data Acquisition

Data for the analysis constituted natural (i.e., unregulated) and uninterrupted flow records of 24 rivers across Canada (Figure 1), which are listed in Table 1. Twenty rivers were used for fitting the model whereas four rivers (written in italics) were used for validation of the model. Daily flow data for these 24 rivers were extracted from the Canadian Hydrologic Data Base, HYDAT, Environment Canada [26]. Selected rivers are representative of a wide range of drainage basins (37 to 32,400 km^2) and a long period of the historical database (1911 to 2020). Daily flows were transformed into weekly flows such that each of the first 51 weeks would be composed of 7 days, while week 52 would contain the remaining days. That is, the last week of the year would comprise 8 or 9 days (in case of a leap year).

3.2. Computation of Flow Statistics and Probabilities

The statistics such as μ_o (the overall mean), cv_o, cv_{mx}, cv_{av}, cv_{gm}, and ρ (lag-1 auto-correlation from the SHI sequences), were computed from historical weekly flow data, as shown in Table 1. Four variants of standard deviations were computed: σ_o, the overall standard deviation without standardizing the weekly flows; σ_{mx}, the maximum value among the 52 standard deviations; σ_{av} the arithmetic mean of 52 standard deviations; and σ_{gm} the geometric mean of these 52 standard deviations. The corresponding coefficients of variation were computed as cv_o (=σ_o/μ_o), cv_{mx} (=σ_{mx}/μ_o), and so on.

The observed values of drought duration ($L_{T\text{-}o}$) and drought deficiency volume ($D_{T\text{-}o}$) were computed using the historical weekly flow data. The computations are illustrated using three rivers, viz., the Athabasca (River #2), Goulis (River #10), and the Upper Humber (River #23) in Table 1. Likewise, behavior analysis as described briefly in Section 2 was used to estimate the V_R at the cutoff level of Q95, Q90, — Q75. The length of the critical period, L_{cr} was also computed from behavior analysis. In the majority of instances, $D_{T\text{-}o}$ was found equal to V_R. The entities $D_{T\text{-}o}$ and V_R were standardized using σ_{av} as a common denominator such that $M_{T\text{-}o}$ (=$D_{T\text{-}o}/\sigma_{av}$) and V_R' (=V_R/σ_{av}). The standard deviation σ_{av} was found to be the best estimator of M_T in the modeling mode as stated before [20,21], which is why it was used as a common denominator in the standardization process. For almost all the conditions of cutoff levels, V_R' was found equal to $M_{T\text{-}o}$ (Table 2, column 2). In other words, V_R' can be taken as an estimate of drought magnitude, which should compare well with the estimated value (denoted by $M_{T\text{-}e}$) by the modeling methodology.

The parameters q_1, q_p, and q_q were estimated by a counting method [6,7] by truncating the SHI series at the desired truncation (cutoff) level of SHI_x. Values above the truncation level were designated as surplus ′1′ and below the truncation level as deficit ′0′. Consider a number of scenarios involving occurrences of ′0′ as follows. The occurrences of 0 are, isolated single zeros (say n_1), and consecutive two 0′s, i.e., ′00′ (say n_2). Knowing that the total sample size is n, hence q_1 (=n_1/n), q_q (=n_2/n_1). Since there are only two numerals, namely 1 and 0, the number of 1′s = ($n - n_1$). The number of pairs ‘11′ occurring in succession (i.e., 1 preceded by 1) is counted (say n_2). Therefore, p_p (=$n_2/(n - n_1)$) and hence q_p (= (1 − p_p)). The probabilities q_1, q_p, and q_q were computed at each of the chosen cutoff levels (i.e., Q95, Q90, Q85, Q80, and Q75).

3.3. Identification of Cutoff Level

Identification of the proper cutoff level (SHI_x) plays a crucial role. A value of SHI_x could take one of six forms, as discussed in Section 2. In the process of choosing the right value of SHI_x, one can begin from the level (iii) $SHI_x = SHI_{av}$ (= $(Qx - \mu_o)/\sigma_{av}$)) and compute the values of q_1, q_q, q_p, $L_{T\text{-}e}'$, and $M_{T\text{-}e}'$. At the chosen cutoff level, V_R' should be compared to $M_{T\text{-}e}'$ and the value of $M_{T\text{-}e}'$ should be $\geq V_R'$. Otherwise, the next cutoff level should be considered until the criterion $M_{T\text{-}e}' \geq V_R'$ has been satisfied. It should be borne in mind that $M_{T\text{-}e}'$ is a value of $M_{T\text{-}e}$ that is based on $\Phi = 0$. The value of Φ (=0) results in the highest value of $M_{T\text{-}e}$, whereas Φ (=1) yields the lowest value. Thus, $M_{T\text{-}e}'$ can be computed with MC0 or MC1 based $L_{T\text{-}e}'$, and matching can be done between V_R' and $M_{T\text{-}e}'$ to confirm the adequacy of the order of MC applicable at a particular Qx. This is particularly significant at Q95 and Q90 flow levels. All the above steps of calculations are depicted in the flow chart shown in Figure 2.

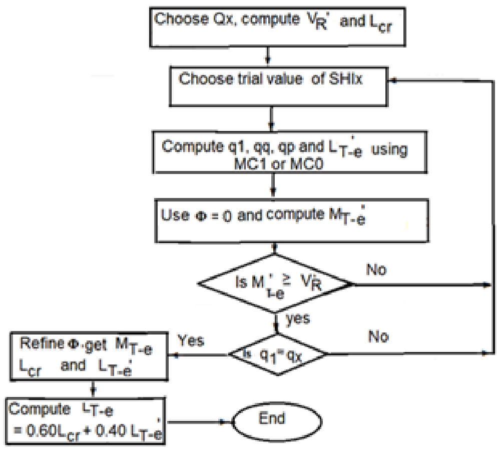

Figure 2. Flow chart of calculations to compute $M_{T\text{-}e}$ and $L_{T\text{-}e}$.

For elucidation, an example of the Upper Humber River (Table 2) is presented, at the Q75 level with cutoff level (iii) and MC1 based drought lengths, V_R' (=4.18) (behavior analysis), and $M_{T\text{-}e}'$ (=2.84). Thus, the criterion ($M_{T\text{-}e}' \geq V_R'$) was not satisfied, so prima-facie, cutoff level (iii) seemed to be inadequate. To meet the criterion, $M_{T\text{-}e}'$ should have a larger value, therefore, the next higher cutoff level (iiia) was tried. Using this cutoff level, the corresponding probability values (column 4), $M_{T\text{-}e}'$ and $L_{T\text{-}e}'$(column 5) were computed as shown in Table 2. For these computations, the criterion $M_{T\text{-}e}' \geq V_R'$ was satisfied, so cutoff level (iiia) seemed adequate. However, looking at the values of probabilities in column 4, it can be seen that q_1 (=0.117) is much lower than the expected value of q = 0.25 at Q75. So, there is further scope for adjustment in the cutoff level, therefore, the next level (i) = [$(Qx - \mu_o)/\sigma_o$] was attempted. Cutoff level (i) turned out to be appropriate in the sense that criterion $M_{T\text{-}e}' \geq V_R'$ was satisfied as well as q_1 (=0.242) resulted in being much closer to q = 0.25. Using cutoff level (i), the estimates of q_1, q_q, and q_p, (column 4) and $L_{T\text{-}e}'$ (column 5) were obtained. Likewise, by trial and error, Φ was found= 0.47 (column 7), which rendered $M_{T\text{-}e}$ (=4.20) closer to V_R' (=4.18). Thus, the calculations resulted in $L_{T\text{-}e}'$ = 11 weeks and L_{cr} = 21 weeks. These two values were combined to obtain (a) $L_{T\text{-}e}$ = (0.5L_{cr} + 0.50 $L_{T\text{-}e}'$) = 16; (b) = (0.75 L_{cr} + 0.25 $L_{T\text{-}e}'$) = 19; and (c) = (0.60 L_{cr} + 0.40 $L_{T\text{-}e}'$) = 17, which are shown in column 8. The value based on the weighted averaging option (c) compared with the observed value ($L_{T\text{-}o}$ = 17 in column 2) perfectly. Likewise, at Q80, Q85, and Q90 flow levels, similar calculations were repeated and cutoff level (i) was found adequate with MC1 for drought lengths. The relevant calculations are shown in Table 2.

Because, at the Q95 demand flow level, the values of $V_R{}'$ and L_{cr} are, respectively, 0.70 and 9, there arose a need to revisit cutoff level (i) with MC1 for the drought lengths. This combination resulted in $M_{T\text{-}e}{}'$(=4.99) >> $V_R{}'$ (=0.70), meaning that MC1 for length is an over-fit. Whereas for the MC0 condition $M_{T\text{-}e}{}'$ = 2.20, which compares relatively closer to $V_R{}'$ (=0.70). Although, with MC0 for the drought length, the criterion of drought magnitude was satisfied, yet the q_1 value was equal to 0.138 (column 4), which is much higher than the expected value of 0.05 (Q95). This called for a need to lower the cutoff level to (iiia). By doing so, q_1 reduced to 0.043 (column 4) with $M_{T\text{-}e}{}'$(=1.27), which is > $V_R{}'$ (=0.70). The calculations thus revealed that at the Q95 flow level, cutoff level (iiia) with MC0 for drought length is more appropriate. Using the revised value of q_1 and $L_{T\text{-}e}{}'$, the appropriate value of Φ was sought, which turned out to be 0.97 (column 7) and $M_{T\text{-}e}$ (=0.70) (column 7). Accordingly, the final three values of $L_{T\text{-}e}$ with three options of averaging are shown in column 8. At the Q95 cutoff level, option (a), i.e., the arithmetic averaging of $L_{T\text{-}e}{}'$ and L_{cr} matched $L_{T\text{-}o}$ perfectly (column 8).

Following the calculations for the Upper Humber River, all rivers (Table 1) were analyzed. The results based on the analysis of three typical rivers are summarized in Table 3. The three typical rivers from Table 1 are: the Athabasca River (#2 from Canadian Prairies), the Goulis River (#11 from northern Ontario), and the Beaverbank River (#21 from Atlantic Canada. These rivers represent a typical scenario in terms of the MC structure of drought lengths and cutoff levels. In a majority of cases, it was found that the optimal level of cutoff level yielded q_1 values close to q (=q_x) at Q75 or Q80 flow levels. Therefore, it is advisable to commence the analysis from Q75 downward to the Q95 level. At Q95 and Q90 flow levels, the values of q_1 are very sensitive to SHI_x and so q_1 may not correspond closely to q or q_x. Under such an ambiguous scenario, a SHI_x level that is one step lower than at Q75 and Q80 can be adopted (Table 3).

4. Results and Discussion

4.1. Estimation of $M_{T\text{-}e}$ and $L_{T\text{-}e}$: Fitting the Model Structure

The model structure as outlined in the aforesaid section was fitted on 20 rivers identified by bold numerals (Table 1). The remaining four rivers with italicized numerals and identity (#3 Bow River in Canadian prairies, #4 Pipestone River, western Ontario, #14 Chippewa River in northern Ontario, and #24 Lepreau River in Atlantic Canada) were used for model validation. The adequacy and the quality of fitting was adjudged by the Nash–Sutcliffe (NSE) efficiency and associated mean error (MER) criteria [20].

The most important element discovered from the analysis was that in almost all cases $V_R{}' = M_{T\text{-}o}$ (column 2, Tables 3 and 4). This means one can safely use the equality $V_R{}' = M_{T\text{-}e}$ as a pivotal point to calculate the values of L_{cr} and $L_{T\text{-}e}{}'$ for a given condition of the uniform cutoff level, Qx. Thus, at a given Qx, $V_R{}'$ and L_{cr} remain unchanged with respect to any value of corresponding SHI_x [i.e., (i) for $(Qx - \mu_o)/\sigma_o$, (iii) for $(Qx - \mu_o)/\sigma_{av}$, etc.]. However, $M_{T\text{-}e}{}'$ does change with respect to the aforesaid SHI_x levels because of a change in the values of probabilities and thereby in $L_{T\text{-}e}{}'$ and μ_d (Equation (7)). Therefore, compute $V_R{}'$ (fixed value) from the behavior analysis and compare it with $M_{T\text{-}e}{}'$ computed from Equation (7). So, while moving the cutoff level SHI_x up and down, the corresponding probabilities, q_1, q_q, and q_p, are computed by the counting method and $M_{T\text{-}e}$ (with Φ, a value of which is found by the trial and error) is calculated. There are two estimates of $M_{T\text{-}e}$, namely, (1) MC0 based on $L_{T\text{-}e}{}'$ and μ_d and (2) MC1 based $L_{T\text{-}e}{}'$ and μ_d. The combination that yields the closest correspondence between $V_R{}'$ and $M_{T\text{-}e}$ is chosen.

For illustrative purposes, in the three selected rivers, the appropriate combination for various Qx levels is shown in Table 3. It can be noted that SHIx levels vary from river to river, though the frequent levels used are (iiia) and (i). Generally, it was observed that at the Q95 level, the appropriate MC order was found to be MC0 and from the Q90 to the Q75 levels, MC1 (with few exceptions at the Q90 level). Beyond the Q75 level, mostly MC1 did not seem to work, probably because in the higher range of Qx the L_T is better simulated by MC2, as demonstrated by Sharma and Panu [6].

The values of L_{cr} and $L_{T\text{-}e}'$ so computed are shown in column 5 (Table 3). It can be seen that L_{cr} and $L_{T\text{-}e}'$ display a large divergence between themselves and none of them matched to $L_{T\text{-}o}$, shown in bold numeral in column 2. So, the first thought that came to mind was to compute the average of L_{cr} and $L_{T\text{-}e}'$ [option (a)] as an estimator of $L_{T\text{-}e}$ and compare it with $L_{T\text{-}o}$. The $L_{T\text{-}e}$ values so obtained are shown in column 6, which shows some correspondence with $L_{T\text{-}o}$ values presented in column 2.

A plot (Figure 3A) between $L_{T\text{-}o}$ and $L_{T\text{-}e}$ based on option (a) was drawn on a 1:1 line, and statistics NSE and MER were evaluated. The plot resulted in NSE ≈ 84% but with a significant under-prediction (MER = −8.86%). To ameliorate the under-prediction, option (b), i.e., $L_{T\text{-}e}$ was computed as $L_{T\text{-}e}$ (=0.75 L_{cr} + 0.25 $L_{T\text{-}e}'$). This estimate of $L_{T\text{-}e}$ using option (b) proved less meaningful as NSE dropped to ≈ 80% accompanied by a significant over-prediction (MER ≈ 9%), as is shown in Figure 3B. Therefore, a more compromising estimate of $L_{T\text{-}e}$, option (c) was considered as $L_{T\text{-}e}$ (=0.60 L_{cr} + 0.40 $L_{T\text{-}e}'$). This new estimate of $L_{T\text{-}e}$ turned out better with an MER ≈ −1%, with an acceptable NSE ≈ 83% as is evinced by the plot in Figure 4. The values of $L_{T\text{-}e}$ so obtained are shown in column 9 in bold letters. In short, the weighted average of L_{CR} and $L_{T\text{-}e}'$ compared quite well with the $L_{T\text{-}o}$ in the process of model fitting.

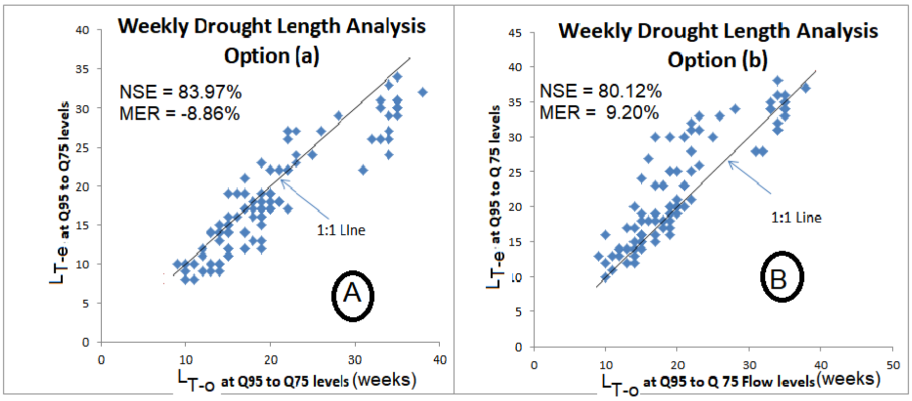

Figure 3. Comparison of estimated ($L_{T\text{-}e}$) and observed ($L_{T\text{-}o}$) drought lengths.

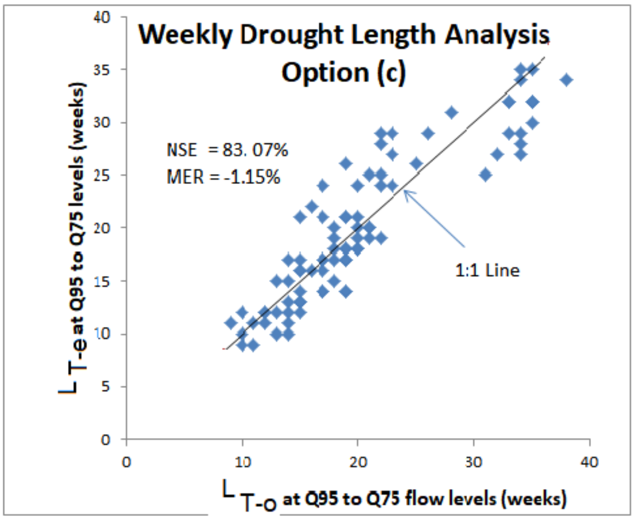

Figure 4. Comparison of estimated ($L_{T\text{-}e}$) and observed ($L_{T\text{-}o}$) drought lengths.

4.2. Validation of the Model Structure

To investigate the adequacy of the model, the $L_{T\text{-}e}$ values for the four rivers (Table 1, italicized identification) were estimated and compared with the observed ($L_{T\text{-}o}$) counterparts, and they are shown in Table 4. The validation affirmed that the modeling worked with the weighted averaging option (c) to be the best one. In this type of averaging, the relative difference between $L_{T\text{-}e}$ and $L_{T\text{-}o}$ was found to vary from −15 to 21%, with a mean deviation of 3% and a standard error of 10% (column 9, Table 4). The simple averaging tended to be next in line with mean deviation = −4% with the standard error of 11% (column 5, Table 4). The weighted averaging of option (b) was less satisfactory, as the mean deviation turned out to be excessively large = 12%, with the standard error of 12% (column 7, Table 4). Based on the foregoing discussion, one can easily conjure that the suggested procedure and model in this paper are robust for providing dependable performance in estimating drought magnitudes and lengths.

For weekly flow sequences at the uniform cutoff level, it may be worthwhile to compare the present methodology with that used by the authors [7]. In their paper, a traditional approach was used to first estimate the L_T, using weekly SHI sequences by truncating at a pulsating SHI_x, with two cutoff levels, viz. Q90 and Q95. The M_T was predicted by using the relationship ($M_T = \mu_d \times L_T$). The NSE and the MER for this kind of formulation turned out to be low (NSE = 72% for the L_T and 76% for M_T). In specific terms, there was acute under-prediction for the high end of M_T values. Using a different approach, the present paper proposes a predictive model with five cutoff levels (Q95 to Q75), and it has yielded predicted values of M_T (i.e., $M_{T\text{-}e}$) almost the same as the observed values (i.e., $M_{T\text{-}o}$). Likewise, the NSE for L_T turned out to be 83%, which falls within the acceptable range.

In summary, the model fitting and validation with 24 rivers revealed that $M_{T\text{-}e}$ can be simulated adequately by taking it equal to V_R', which in turn allowed the determination of the length of the critical period (L_{cr}) via behavior analysis. In other words, at low uniform cutoff levels from Q95 up to Q75, a combination of L_{cr} with MC based $L_{T\text{-}e}'$ on SHI sequences can be used to estimate $L_{T\text{-}e}$.

4.3. An Illustrative Example for the Estimation of $M_{T\text{-}e}$ and $L_{T\text{-}e}$

Consider the Neebing River (near Thunder Bay), northern Ontario, Canada for which one needs to estimate the drought duration (weeks) and the magnitude (m^3) for a return period of 65 years (T = 3380 weeks = 52 × 65), with the following statistics: $\mu_o = 1.62$ m^3/s; $\sigma_o = 3.04$ m^3/s; $\sigma_{av} = 1.79$ m^3/s; and $\rho = 0.63$. A severe drought is defined when flow drops below the Q90 (= 0.089 m^3/s) level. One may investigate the additional requirement of water when the river has already receded below the Q90 level. The drought probabilities from the historical weekly data were computed as q = 0.10 (Q90), $q_p = 0.050$, and $q_q = 0.654$. The drought lengths were adequately simulated by MC0. These parameters can also be estimated using the equations involving ρ and a normal pdf of the SHI sequences [6].

A value of a 65-year drought duration $L_{T\text{-}e}'$ at the Q90 level can be computed to be approximately = 4 weeks (MC0), using Equation (1) and plugging in values of relevant parameters (T = 3380, q = 0.10, $q_p = 0.10$, $q_q = 0.10$ because of MC0). Based on the historical data, behavior analysis yields $L_{cr} = 36$ weeks. Therefore, $L_{T\text{-}e}$ (=0.60 × 36 + 0.40 × 4 =) is 23 weeks. $M_{T\text{-}e}$ was found to be = 0.58. When $M_{T\text{-}e}$ was converted to the total deficit volume (volumetric units) $D_{T\text{-}e} = M_{T\text{-}e} \times \sigma_{av} = 0.58 \times 1.79 \times (7 \times 24 \times 3600)$ $m^3 = 0.63 \times 10^6$ m^3 per week (note that the duration is in weeks and hence flow is to be converted on a weekly basis that brings in the multiplier 7 × 24 × 3600). Since drought is lasting for 23 weeks computed above, the total deficit volume at the Q90 level = $0.63 \times 10^6 \times 23$ $m^3 \approx 14.50 \times 10^6$ m^3. That means only 14.50×10^6 m^3 is available in the river, because the river is in the grip of a severe drought.

If the cutoff level for the drought-free condition is defined as Q75, then similar calculations (as done at Q90) will result in $L_{T\text{-}e} = 35$ weeks ($L_{cr} = 48$ and $L_{T\text{-}e}' = 16$) and $M_{T\text{-}e} = 2.16$. At Q75, the drought lengths fell in the regime of MC1 (with q = 0.25, $q_q = 0.70$, and $q_p = 0.12$). Therefore, the deficit volume would amount to $2.16 \times 1.79 \times 7 \times 24 \times 3600 \times 35 = 81.84 \times 10^6$ m^3. This volume of water met all water demands from the river, when the drought was not there.

To cater for needs during severe drought (Q90 level), we need the additional water to shore up to the drought-free level of Q75. Thus, the additional volume of water needed is 81.84 − 14.50 = 67.34 × 10^6 m^3, which must be arranged to restore the health of the river and to assuage other demands that the river has been coping with in the past. This could be met from external sources by diverting the water from reservoirs or by pumping from the lakes or similar water bodies. It can be appreciated that under the severe drought conditions (Q90), to restore the river flows to the modest drought free level (Q75), nearly 4.6 times more water is needed, i.e., (67.34/14.50). Likewise, to deal with extreme drought, at the Q95 level, the requirement would further shore up and would work out = (81.84 − 6.28) = 75.56×10^6 m^3, (i.e., 12 times = 75.56/6.28). At the Q95 level, calculations can be done with the MC0 regime of drought lengths with L_{cr} = 15, $L_{T\text{-}e}$ = 10, and $M_{T\text{-}e}$ = 0.58, and the available volume will work out to be 6.28 × 10^6 m^3.

4.4. A Comment on the Present State of Drought Magnitude Assessment and Cutoff Levels at the Uniform Flow Levels

The truncation level approach in assessing hydrological droughts originated in the 1960's [11], which used the mean or the median of the flows of the respective seasons (months) as the truncation levels. In doing so, the non-stationary series of drought variables is rendered stationary in the statistical sense. The analysis of stationary series is amenable to statistical analysis, using the theory of runs and Markovian processes. The stationary series, when standardized is tantamount to the standardized precipitation index series, SPI [13,14] used in the context of meteorological drought. The parameters of drought, viz. duration, magnitude and intensity, have been synthesized using the aforesaid variable cutoff levels (on a monthly or seasonal basis) and the results have been deemed satisfactory for drought mitigation purposes. One outcome of such analyses has catapulted in identifying interactions between meteorological and hydrological drought characteristics [27]. The other outcome has resulted in the estimation of drought deficit volume as an aid in planning and in evaluating the performance of the water storage systems [15–17,27,28].

A major challenge of drought assessment in the statistical sense is spurred by the desire for uniformity of the cutoff level throughout the flow regime, such as Q90, Q70, etc. In other words, if the demand level of water is relatively constant irrespective of seasons, then the analysis using the stochastic theory applicable to stationary time series turns out to be tedious. The current approach for handling such a problem on daily flow sequences is confined to the frequency distribution fitting of durations and magnitudes empirically abstracted from the historical data by truncating at the desired level [1–4]. In such an analysis, the linkage between the parameters of frequency functions of duration and magnitude, and that of the drought variable (such as streamflow series) is still to be defined. Ideally, the endeavor should be directed to the assessment of drought parameters based on the readily available statistics (μ, cv, and ρ) of the drought variable at the uniform cutoff level. This paper has demonstrated some success in meeting the above goal in which drought parameters are being predicted using the estimates of probabilities derived from the data. Most importantly, the estimates of reservoir volume and the length of the critical period (L_{cr}) have been used as key elements in the analysis.

5. Conclusions

Drought parameters, viz. $M_{T\text{-}e}$ and $L_{T\text{-}e}$ on a weekly basis at the uniform truncation level, such as from Q95 to Q75 flow levels, can be estimated from the analysis of weekly flow sequences using behavior analysis used in the estimation of reservoir volumes. The behavior analysis provided an estimate of reservoir volume, V_R' ($\approx M_{T\text{-}o}$) and that of L_{cr}. The Markov chain models (of order 1: MC1) and (of zero order: MC0) performed satisfactorily in simulating $L_{T\text{-}e}'$ by using the estimates of simple and conditional drought probabilities from weekly SHI sequences. The most important element in estimating the probabilities was the appropriate identification of the cutoff level (SHI_x). The frequently occurring cutoff levels were found to be $[(Qx - \mu_o)/\sigma_{av}]$, $[(Qx - \mu_o)/\sigma_o]$ and the average of these two entities. Using the hypothesis that drought magnitude = drought intensity × drought length, the scaling parameter Φ for the characteristic drought length was evaluated to fine-tune the estimates of $M_{T\text{-}e}$ to be equal to $V_R{}'$. The $L_{T\text{-}e}{}'$ from the MC analysis when combined with L_{cr} from the behavior analysis (i.e., $L_{T\text{-}e} = 0.60L_{cr} + 0.40L_{T\text{-}e}{}'$) yielded the drought length $L_{T\text{-}e}$, which was found compatible with the observed drought length, $L_{T\text{-}o}$.

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