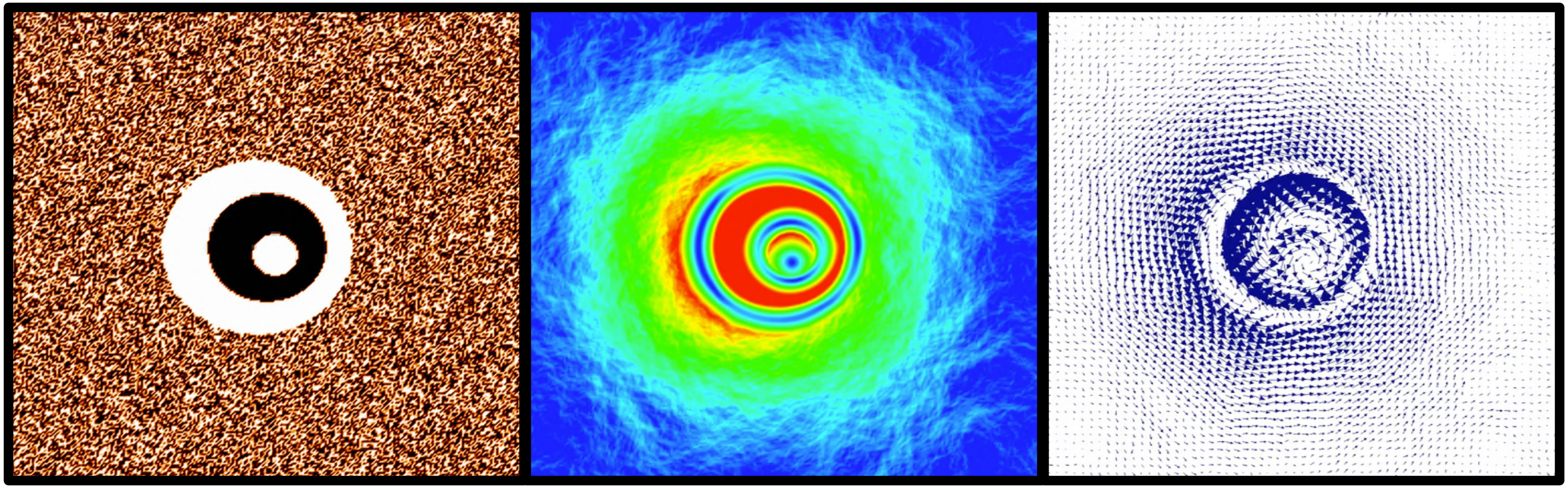
suite-CFD: *Open Source Fluid Dynamics Codes in MATLAB / Python*



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School of Science
The College of New Jersey

TCNJ THE COLLEGE OF NEW JERSEY

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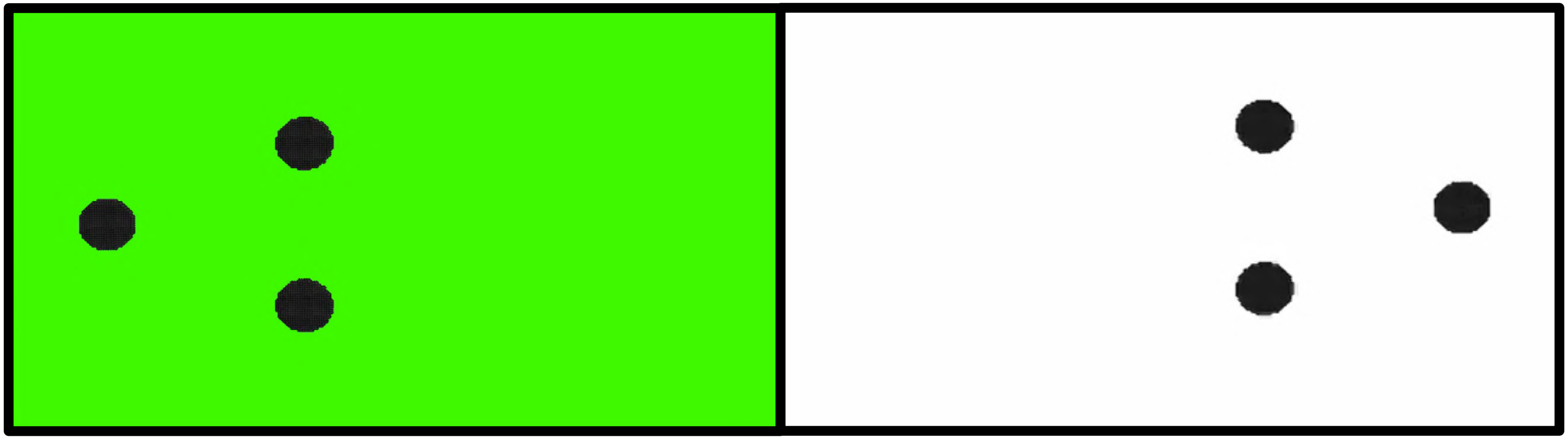

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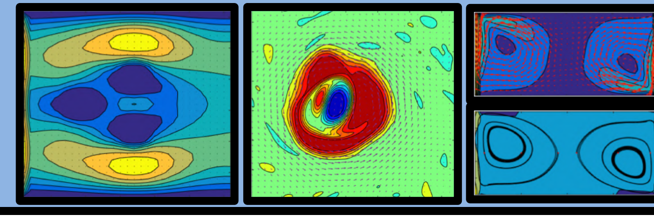
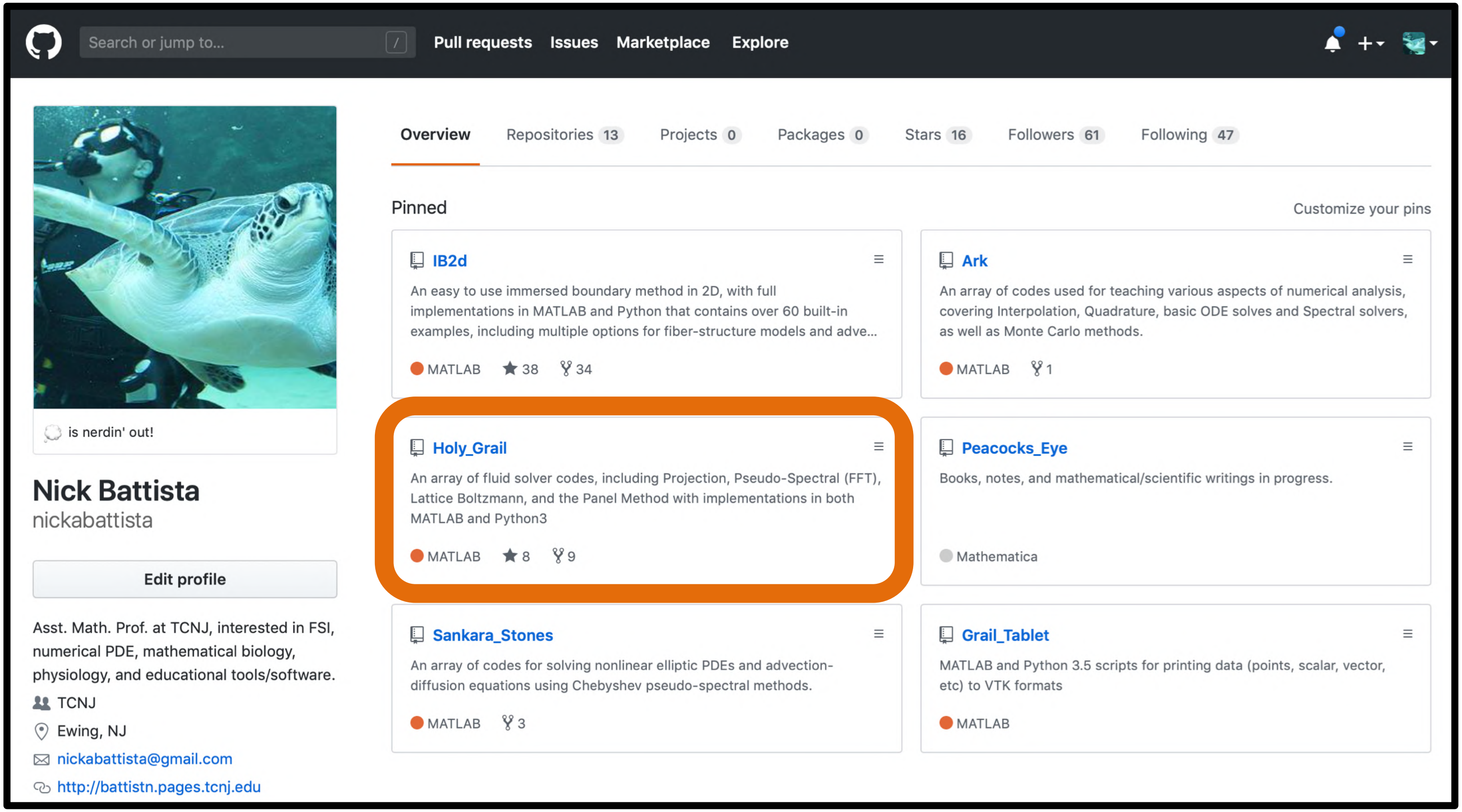
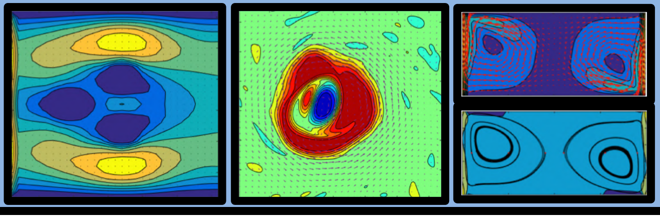
Numerical Fluid Solvers





Open Source Codes Available: github.com/nickabattista

Numerical Fluid Solvers



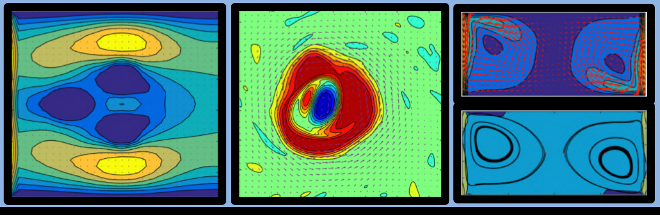
- **Fluid solvers for various applications**
  ("pick your preference")

- **Projection Methods**

- **Spectral Methods**

- **Lattice Boltzmann**

Numerical Fluid Solvers



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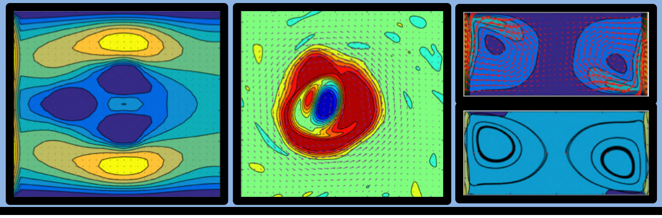
Numerical Fluid Solvers



- Fluid solvers for various applications
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- **Projection Methods**
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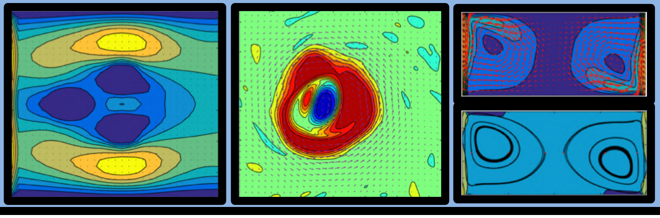
Code available:
github.com/nickabattista/Holy_Grail



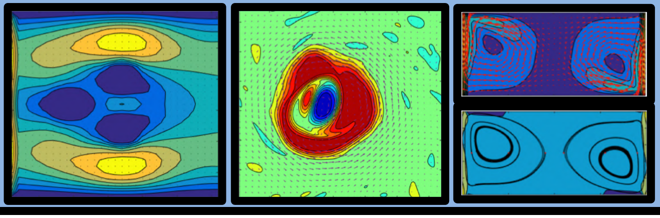
PROJECTION METHODS

Examples:

1. Cavity Flow in Rectangular Domain

2. Circular Flow in Square Domain

Overview of Numerical Scheme to follow



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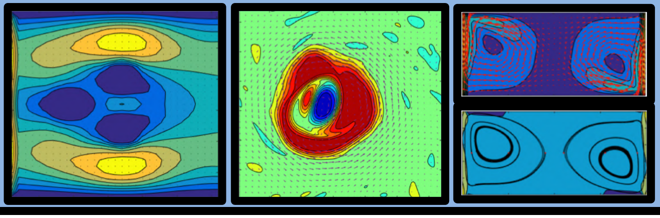
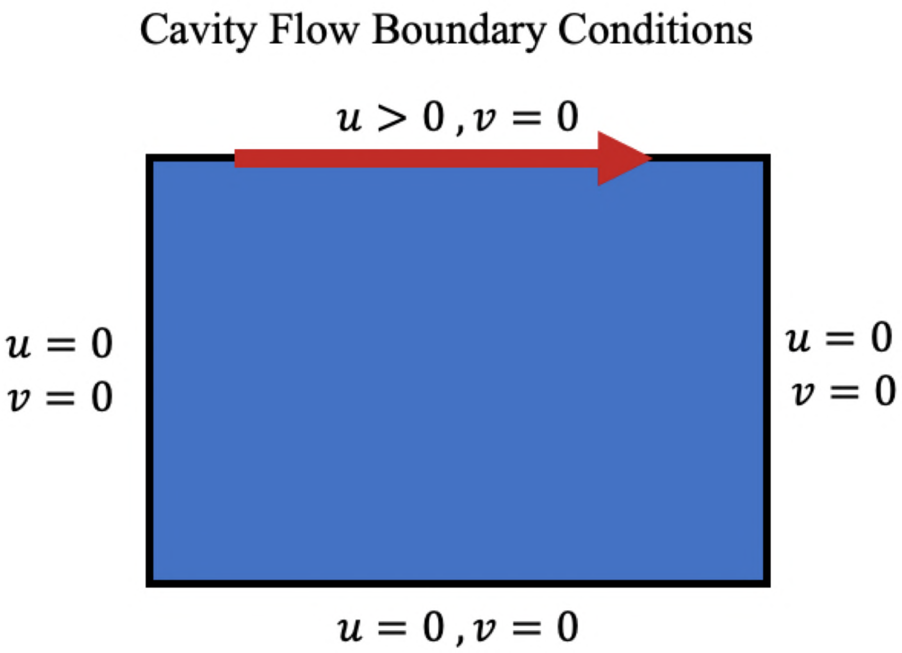
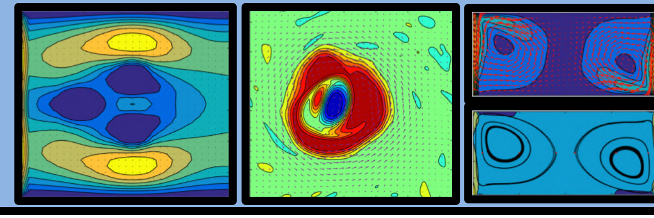
Projection Methods: Cavity Flow

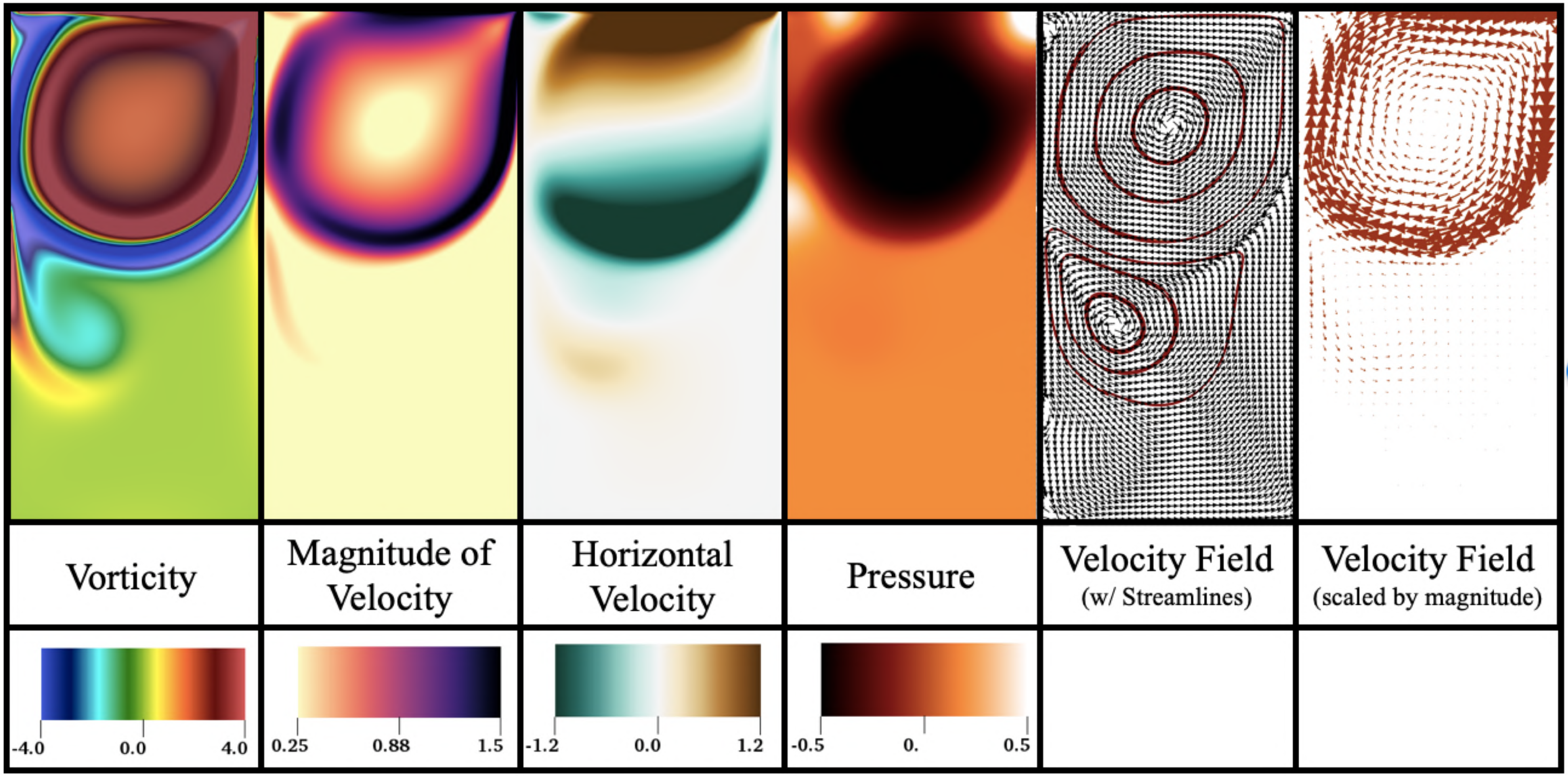




- Flow speed is ramped up across top of rectangular domain
- Observe qualitative differences due to differing Reynolds Numbers
- Measure horizontal velocity across middle of domain

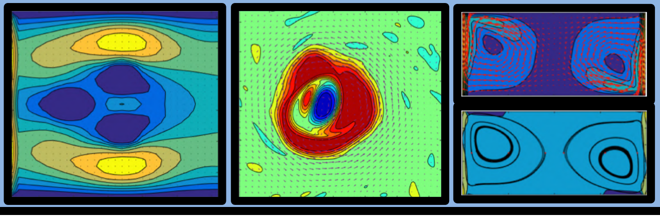
Projection Methods: Cavity Flow

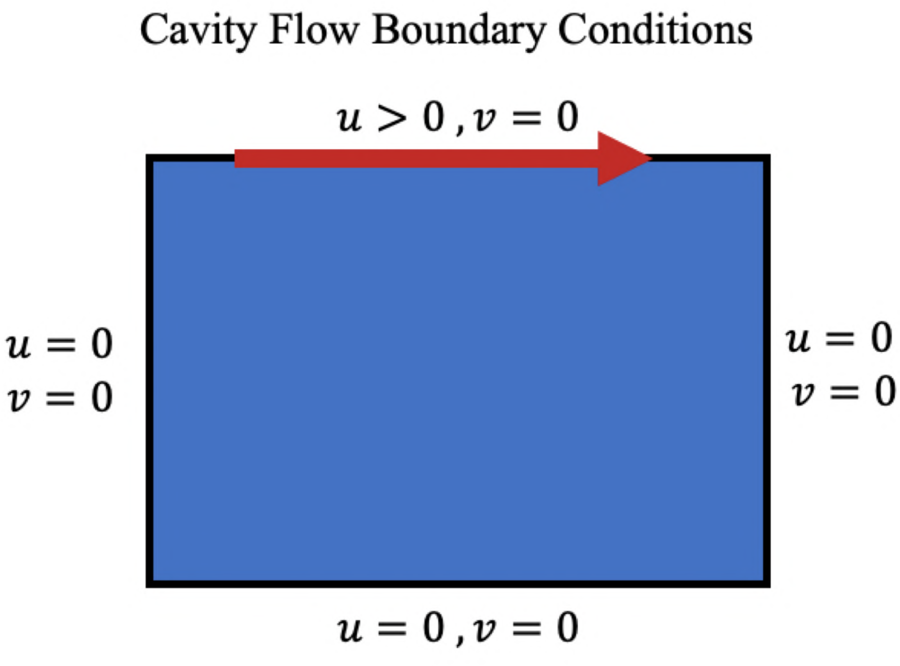


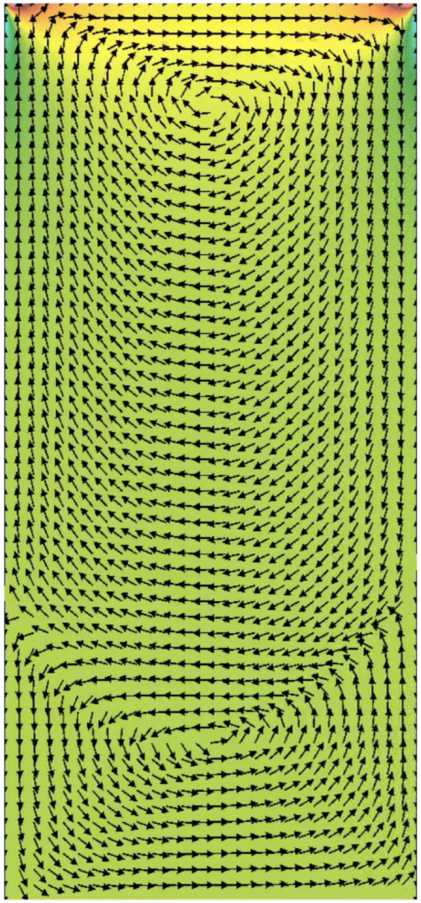


Simulation Data: Re=4000

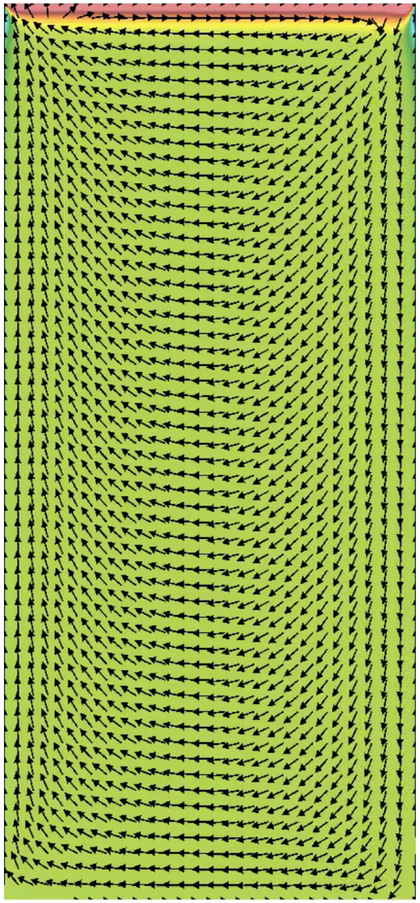
Projection Methods: Cavity Flow





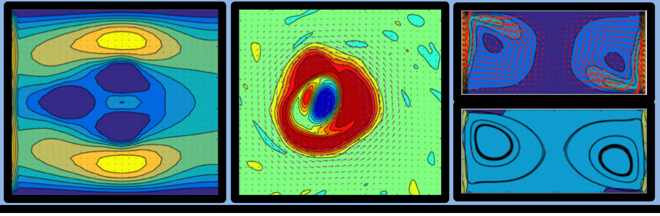


Re = 4

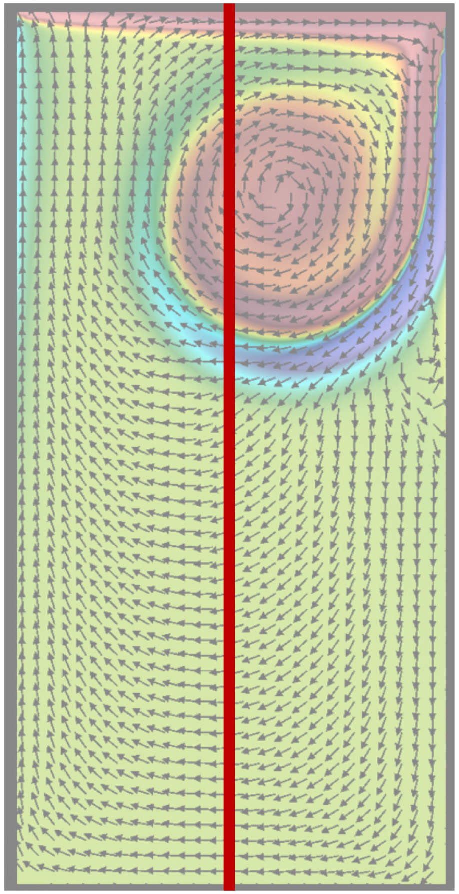


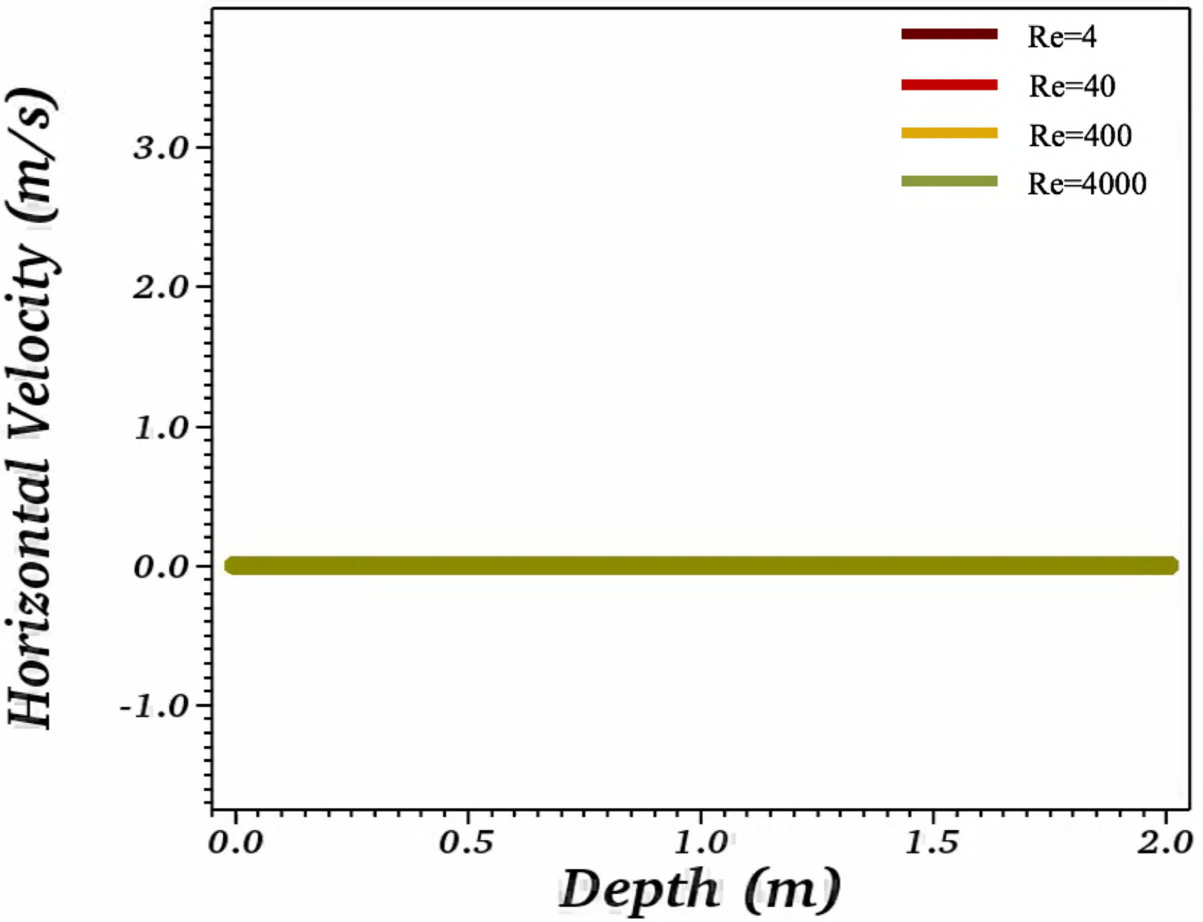
Re = 4000

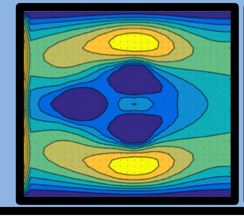
Projection Methods: Cavity Flow

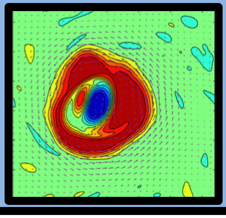


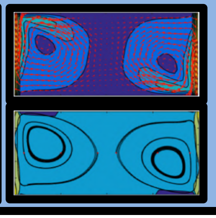
Measure **horizontal velocity** across *center of domain* for different Reynolds Numbers











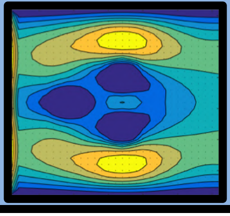
PROJECTION METHODS

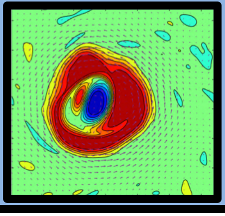
Examples:

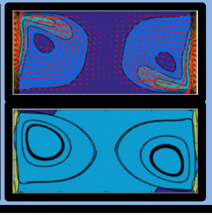
1. Cavity Flow in Rectangular Domain
2. **Circular Flow in Square Domain**

Overview of Numerical Scheme to follow*

Projection Methods: Circular Flow

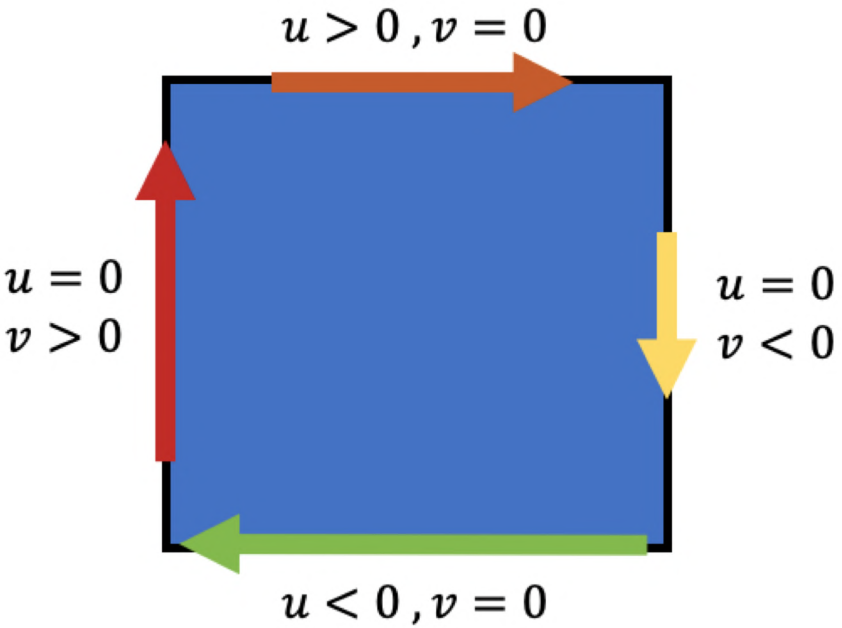




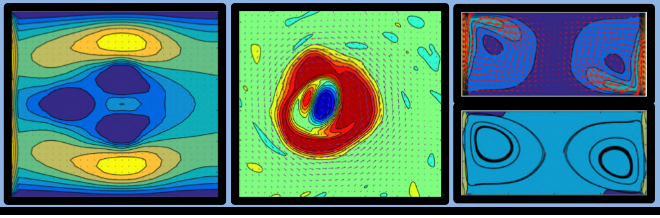


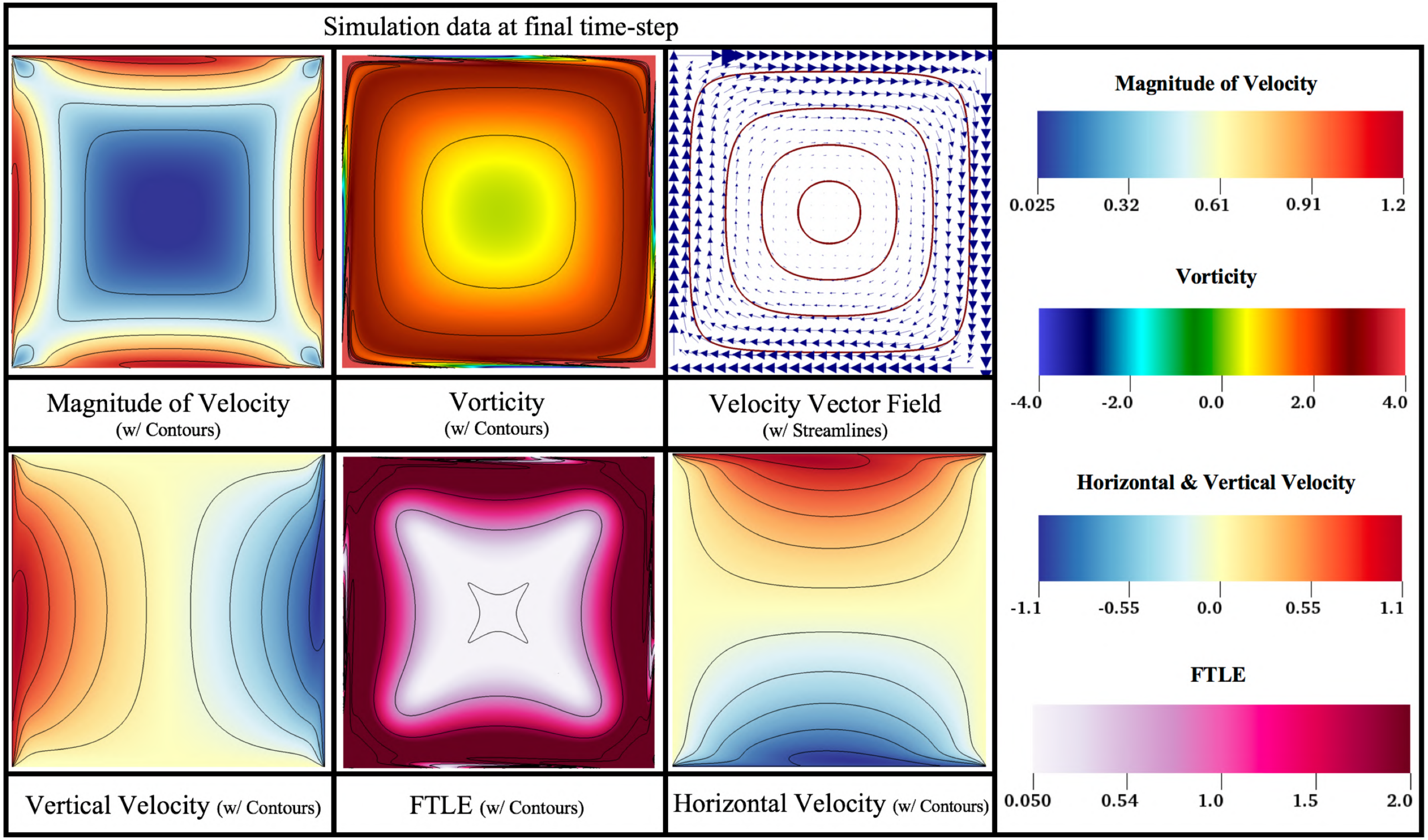
- Flow speed is ramped up across all sides of domain. Flow on each side is uniform in magnitude.
- Flow direction is ***clockwise*** around domain
- Observe qualitative differences due to differing Reynolds Numbers
- Measure horizontal velocity across middle of domain

Circular Flow Boundary Conditions



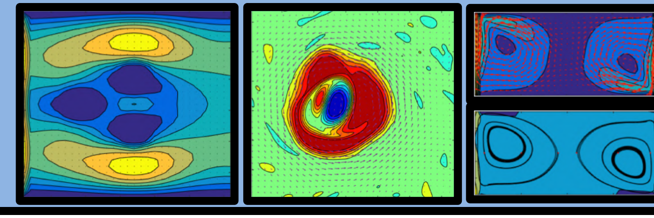
Projection Methods: Circular Flow





Re = 4000

Projection Methods: Circular Flow

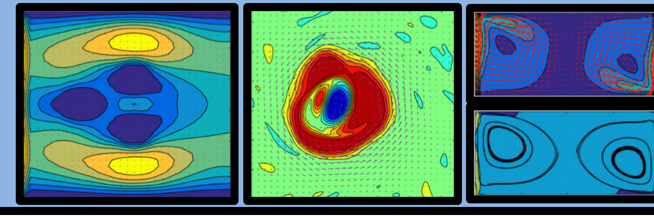


Magnitude of Velocity

Re = 400

Re = 1000

Re = 4000



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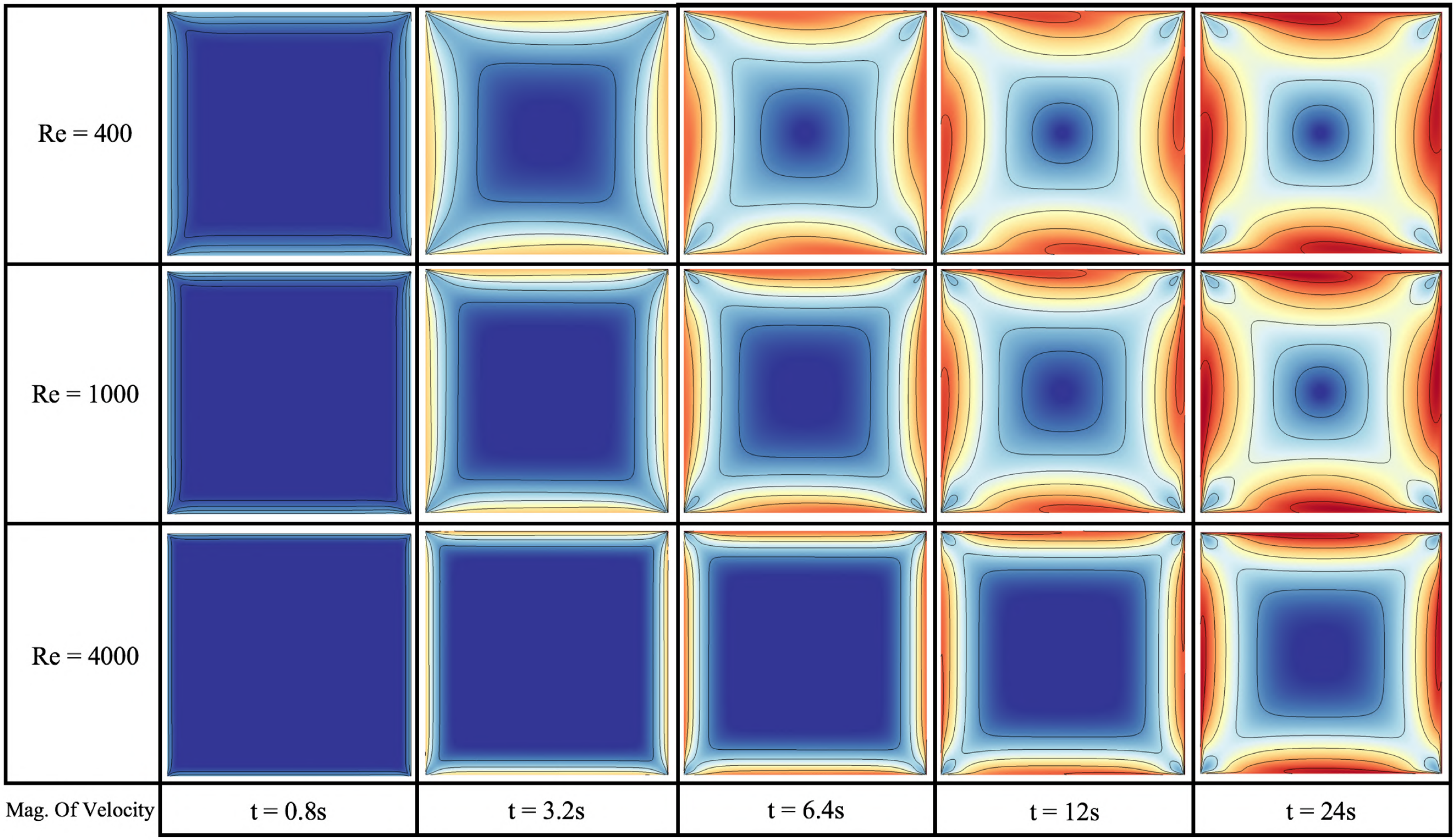
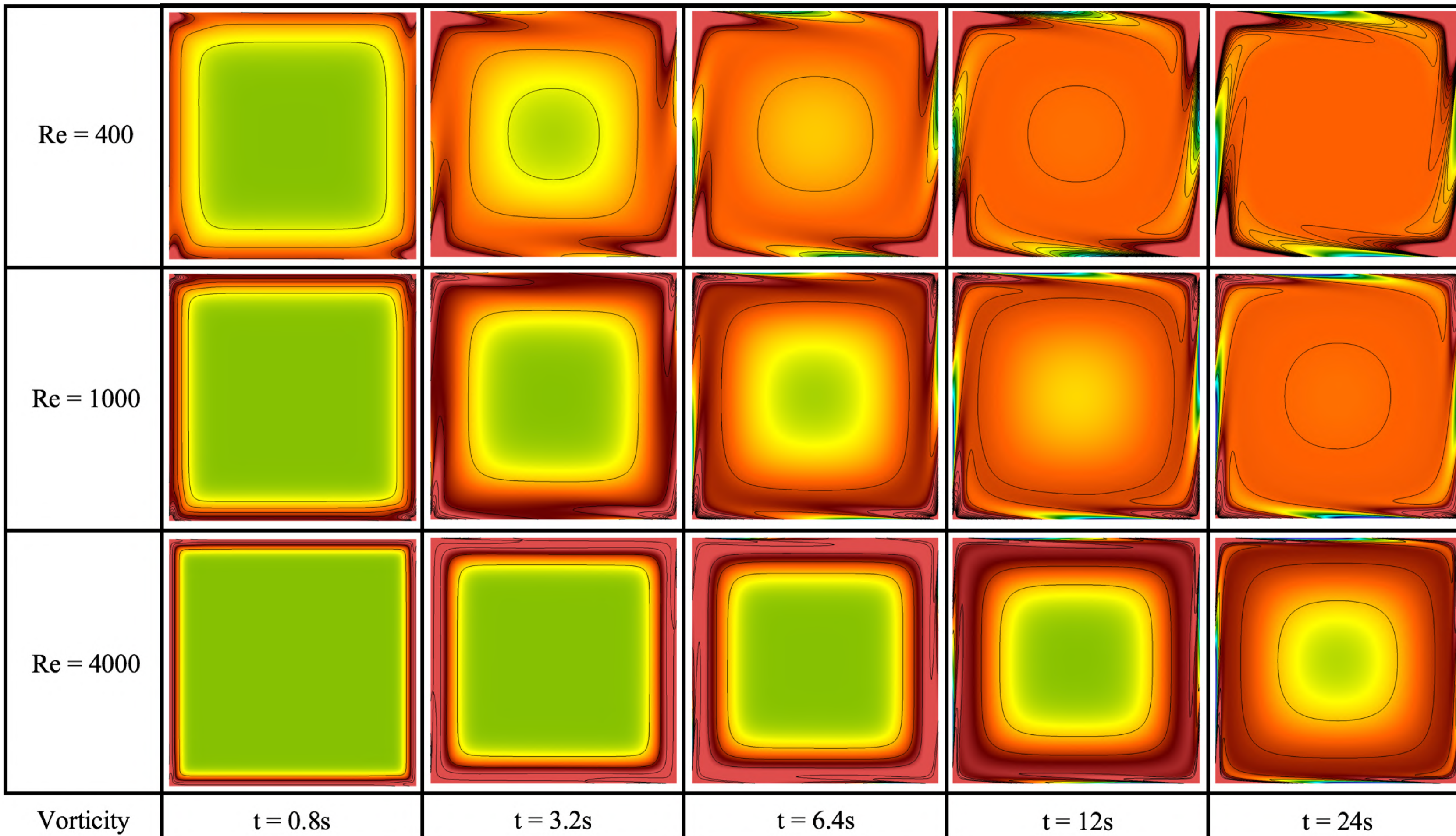
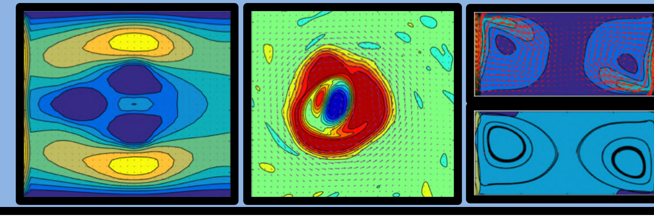


Vorticity

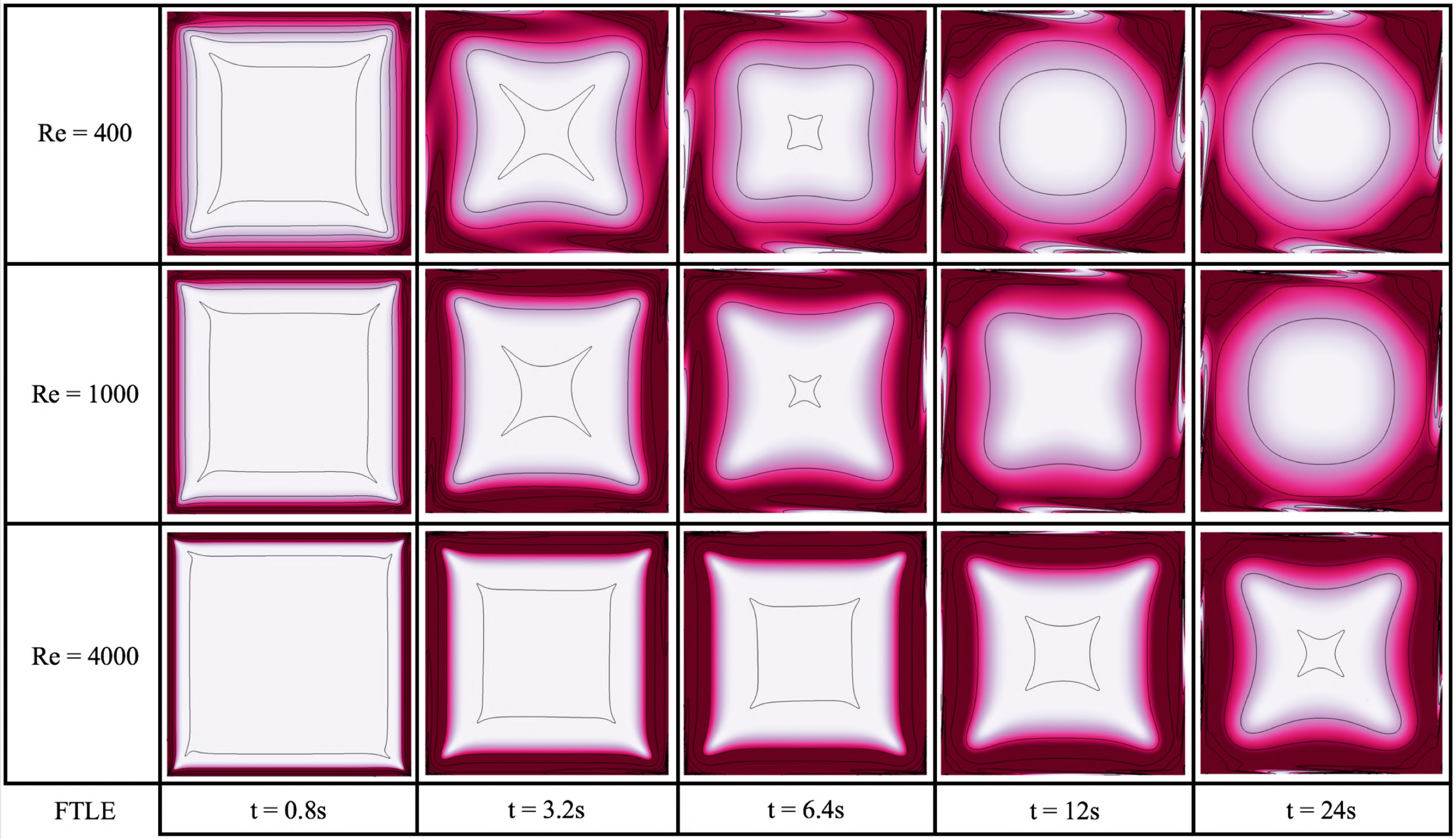


Projection Methods: Circular Flow



FTLE: finite-time Lyapunov exponents

(maximal ridges provide information regarding Lagrangian Coherent Structures)



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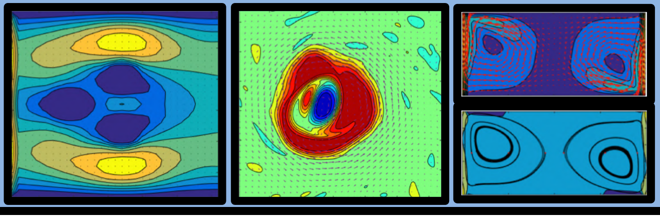
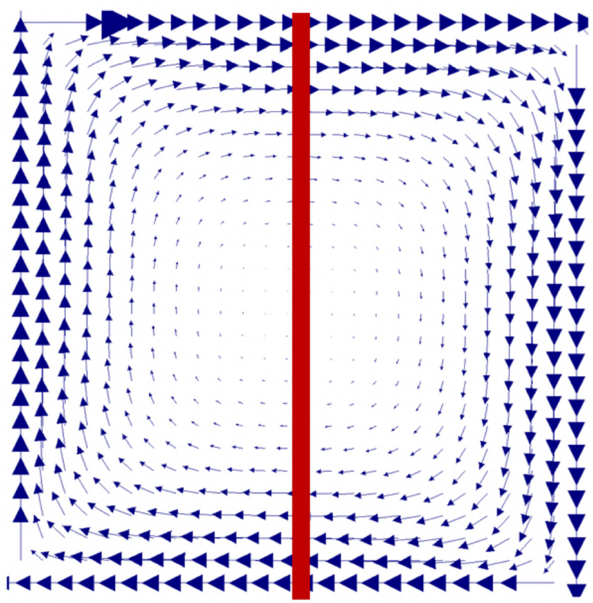
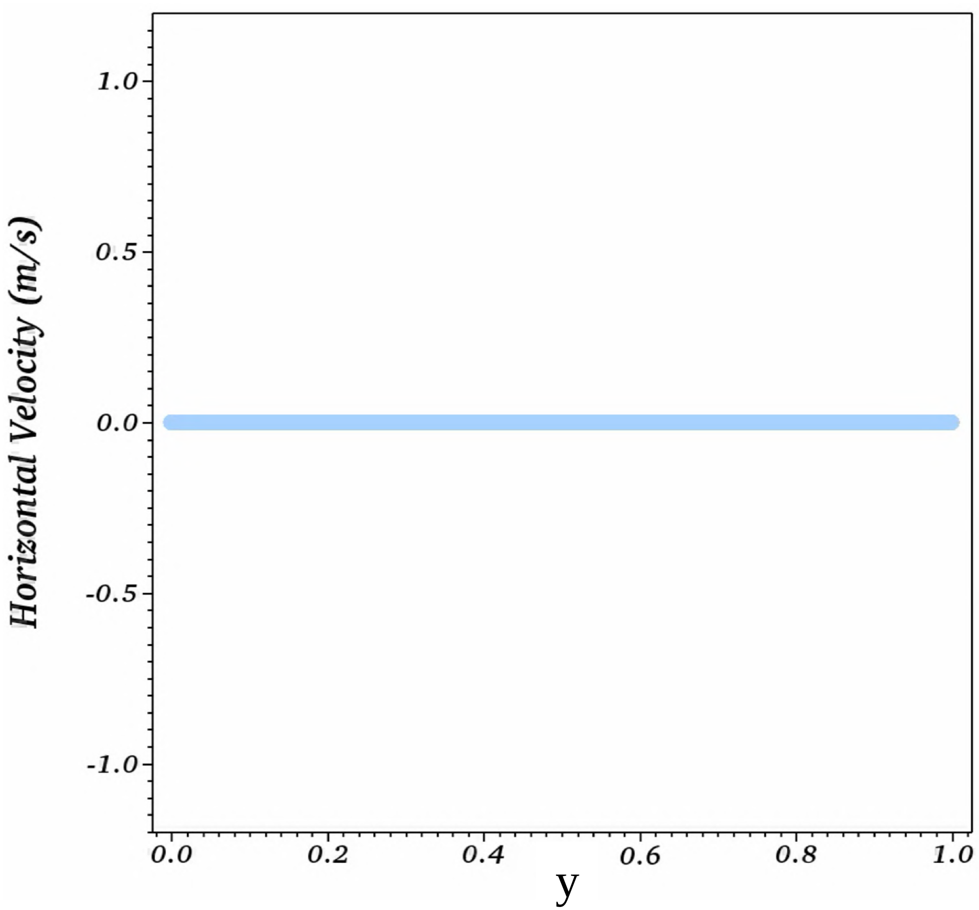
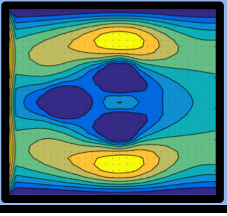
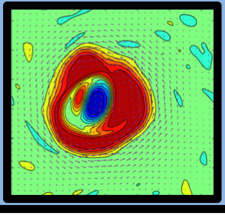
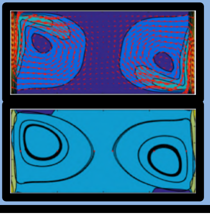






Projection Methods: Circular Flow

Diffusion Time Scales!

The viscous diffusion time can be thought of as the *time* it takes for *a fluid parcel to diffuse a particular distance on average*

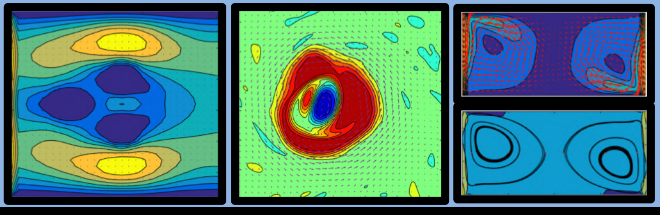
Mean squared distance

$$\tilde{t} \sim \frac{\tilde{l}}{\nu} \sim \frac{1}{\nu}$$

Viscous Diffusion Time

Kinematic Viscosity

Projection Methods: Circular Flow



Diffusion Time Scales!

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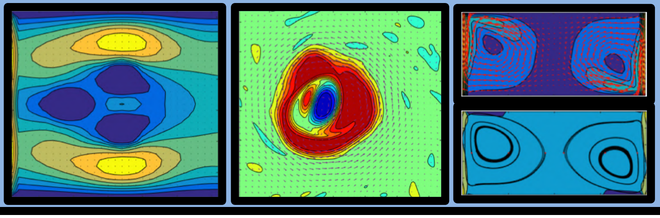
$$\tilde{t}_{Re=400} \sim \frac{1}{2.5/1000} = 400 \text{ s}$$

$$\tilde{t}_{Re=4000} \sim \frac{1}{0.25/1000} = 4000 \text{ s}$$

Dynamics evolve 10x slower towards middle of domain in the Re=4000 case!

Projection Methods: Circular Flow



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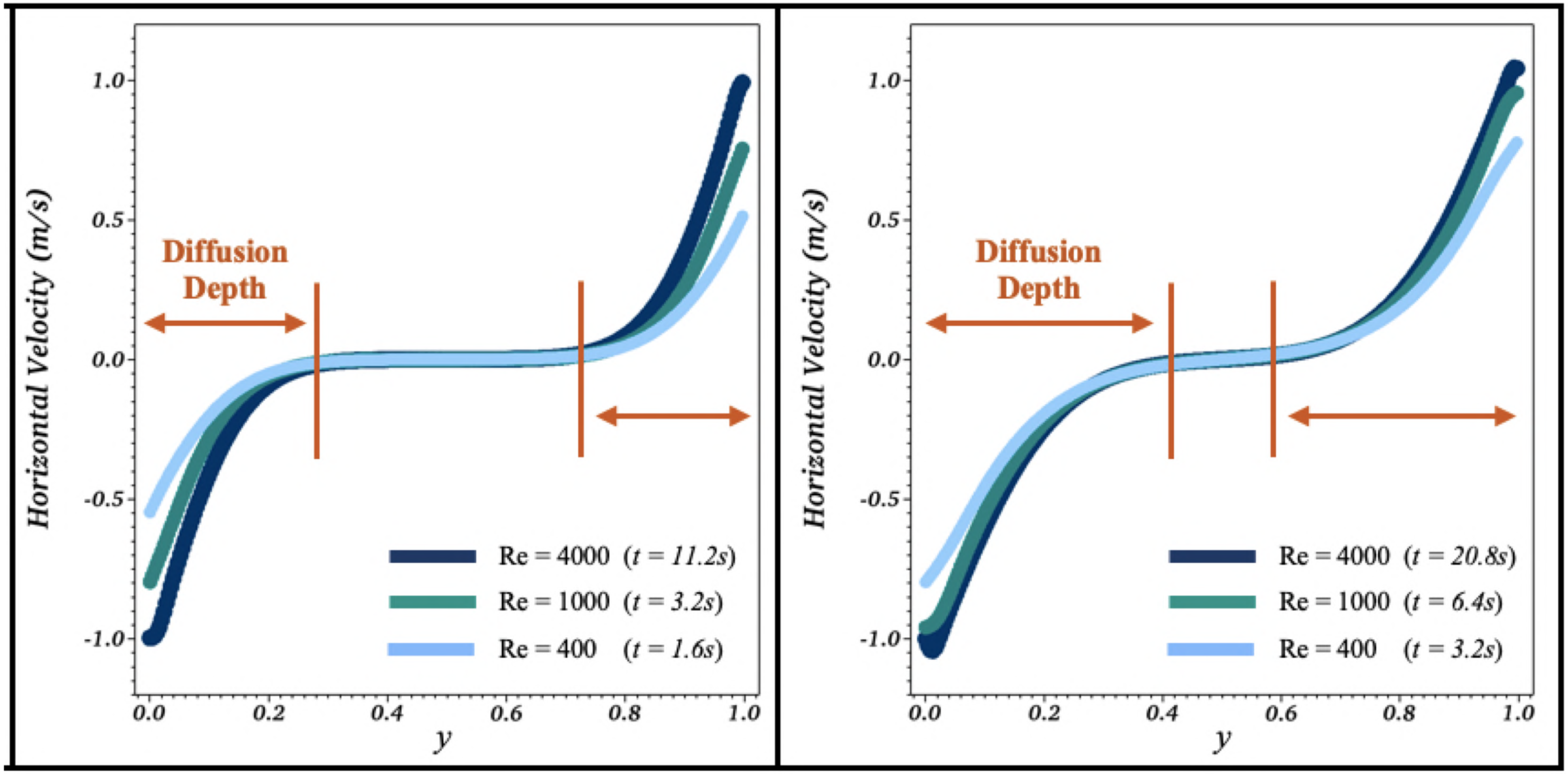
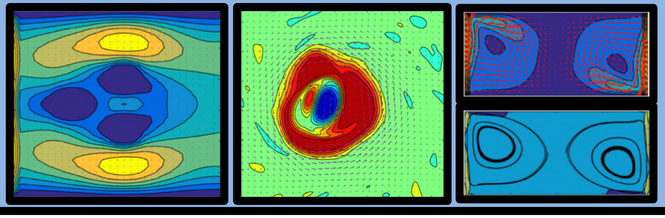


(look at the time in which these depths are achieved for each case)



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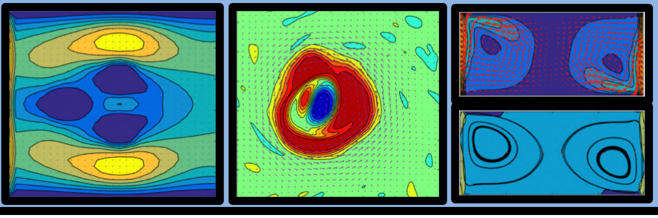
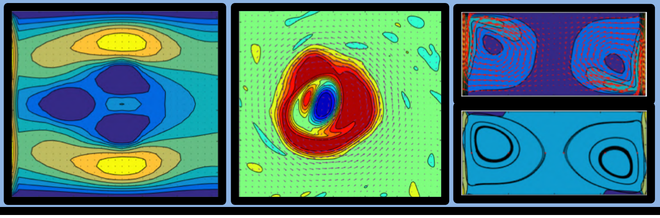


- **Use Operator Splitting + Helmholtz-Hodge Decomp.**

Projection Methods



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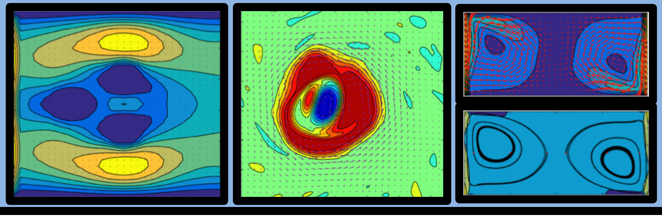
1. Compute auxiliary velocity -> IGNORE the pressure terms

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -\left(\mathbf{u}^n \cdot \nabla\right)\mathbf{u}^n + \nu \Delta \mathbf{u}^n$$

2. “Projection Step”

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho}\nabla p^{n+1}$$

Projection Methods



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NEXT STEP PRESSURE…
HOW TO GET IT?!

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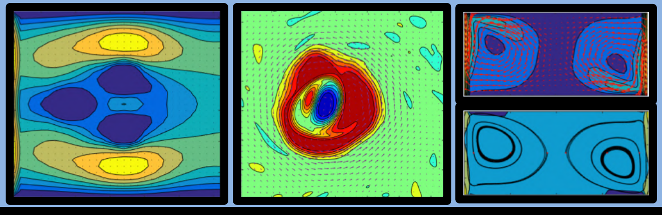
Projection Methods



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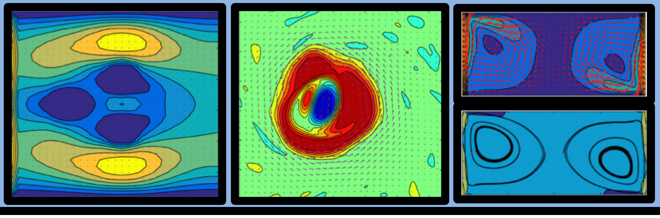
$$\mathbf{V} = \mathbf{V}_{sol} + \mathbf{V}_{irr} = \mathbf{V}_{sol} + \nabla\phi$$

$$\nabla \cdot \mathbf{V} = \Delta\phi$$

$$\mathbf{V}_{sol} = \mathbf{V} - \nabla\phi$$

$$\mathbf{u}_{incompressible} = \text{‘}\mathbf{u}\text{’} - \nabla\phi$$

Projection Methods



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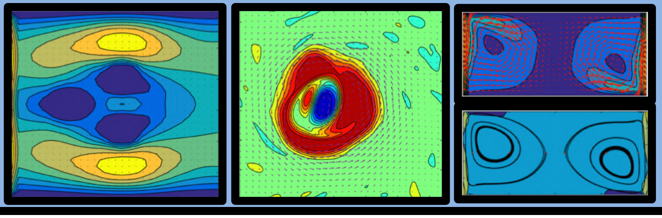

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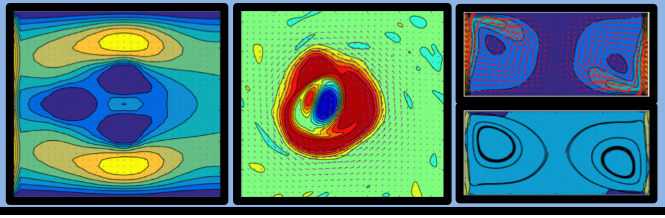
Projection Methods



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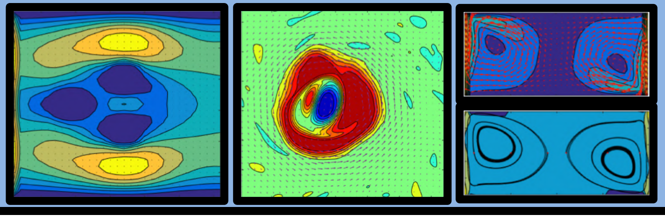
$$\nabla \cdot \left(\mathbf{u}^{n+1} - \mathbf{u}^*\right) = -\frac{\Delta t}{\rho} \nabla \cdot \nabla p^{n+1}$$

$$\nabla \cdot \mathbf{u}^{n+1} = 0$$

$$\frac{\Delta t}{\rho} \Delta p^{n+1} = \nabla \cdot \mathbf{u}^*$$

Projection Methods



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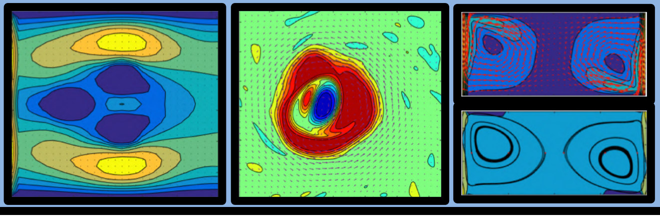

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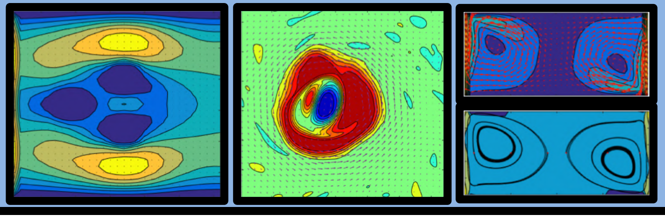

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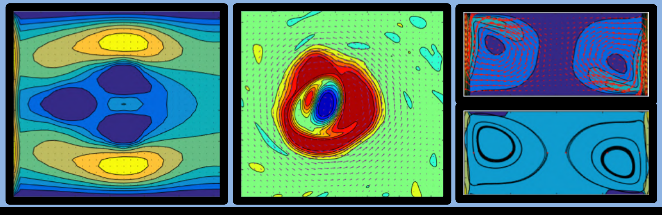
2a. Elliptic Solve for Pressure Field

$$\frac{\Delta t}{\rho} \Delta p^{n+1} = \nabla \cdot \mathbf{u}^*$$

2b. Projection Step

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho} \nabla p^{n+1}$$



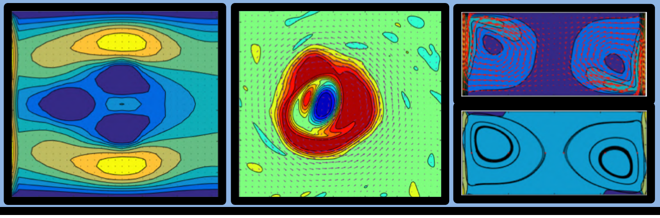
Spectral Method (FFT)

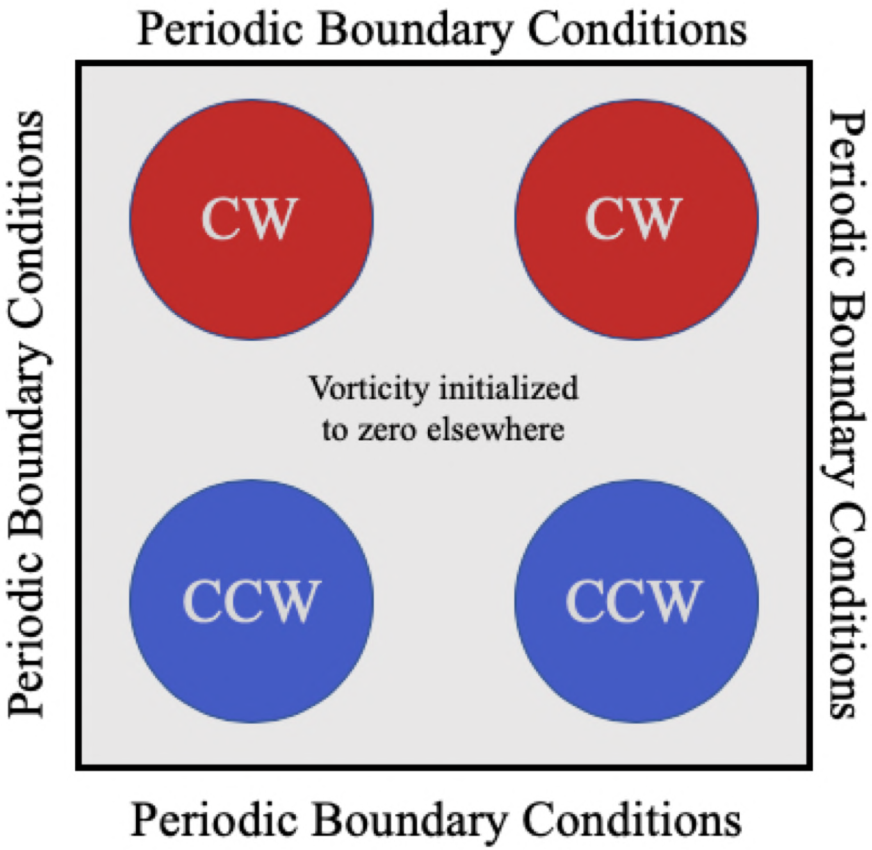
Examples:

1. **Interacting Regions of Vorticity**
2. Overlapping Regions of Vorticity
3. Evolution of vorticity from an initial vector velocity field

Overview of Numerical Scheme to follow

FFT: Interacting Regions





- Vorticity is initialized into each circular region uniformly, as either a positive or negative vorticity value
- CW = Positive initialization, CCW = Negative initialization
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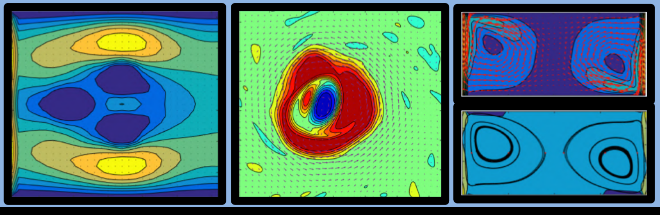
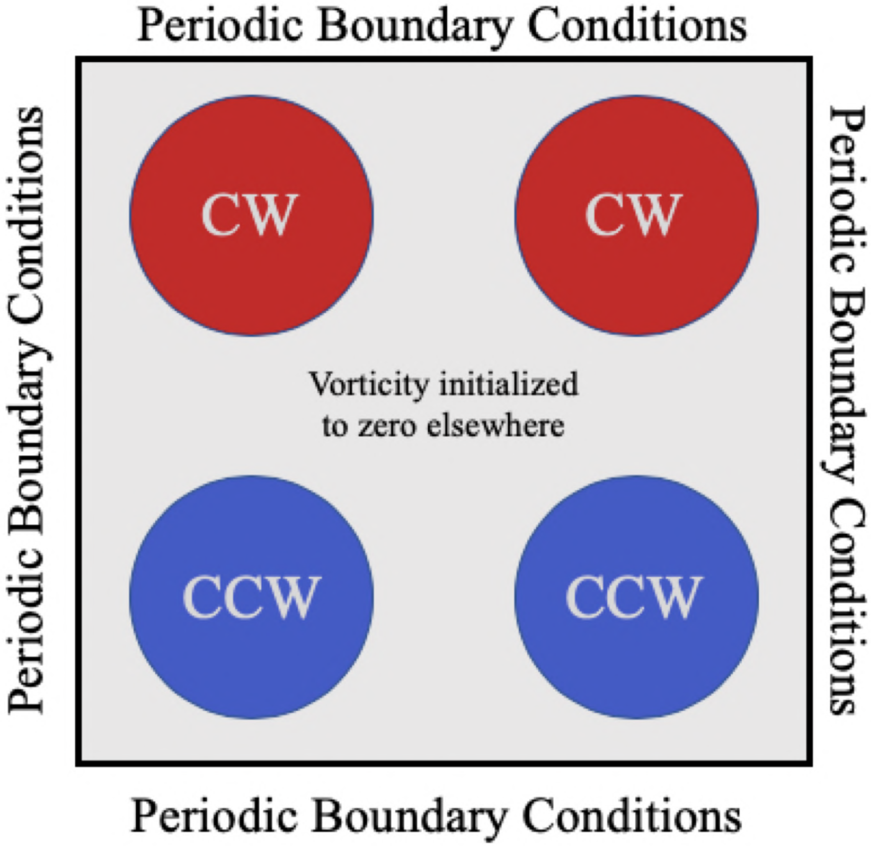
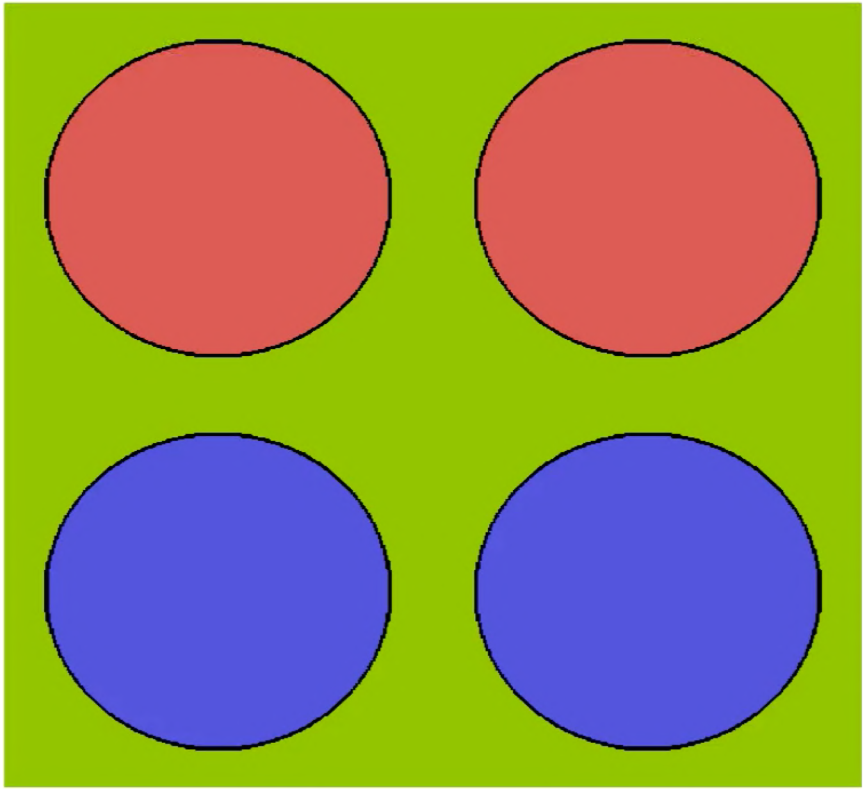
- Periodic boundary conditions on all sides of domain

FFT: Interacting Regions

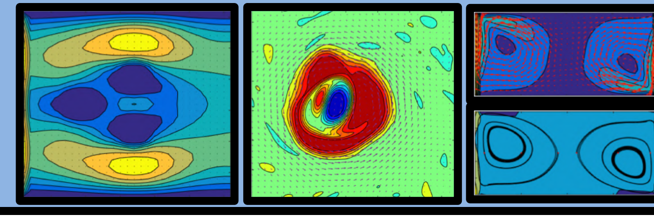


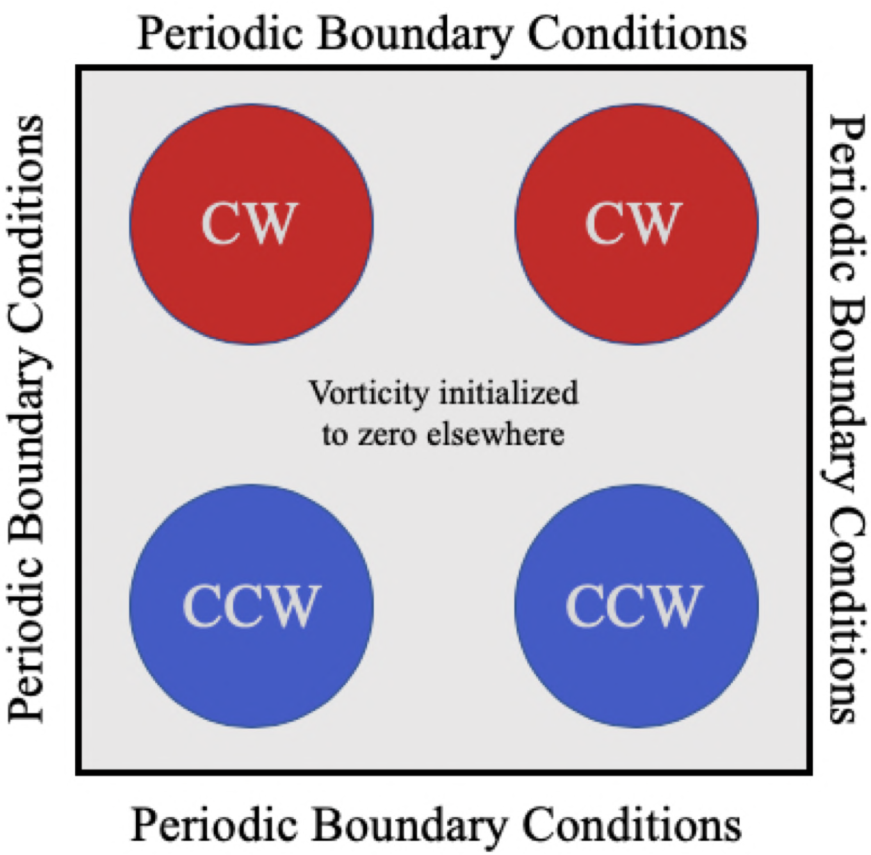


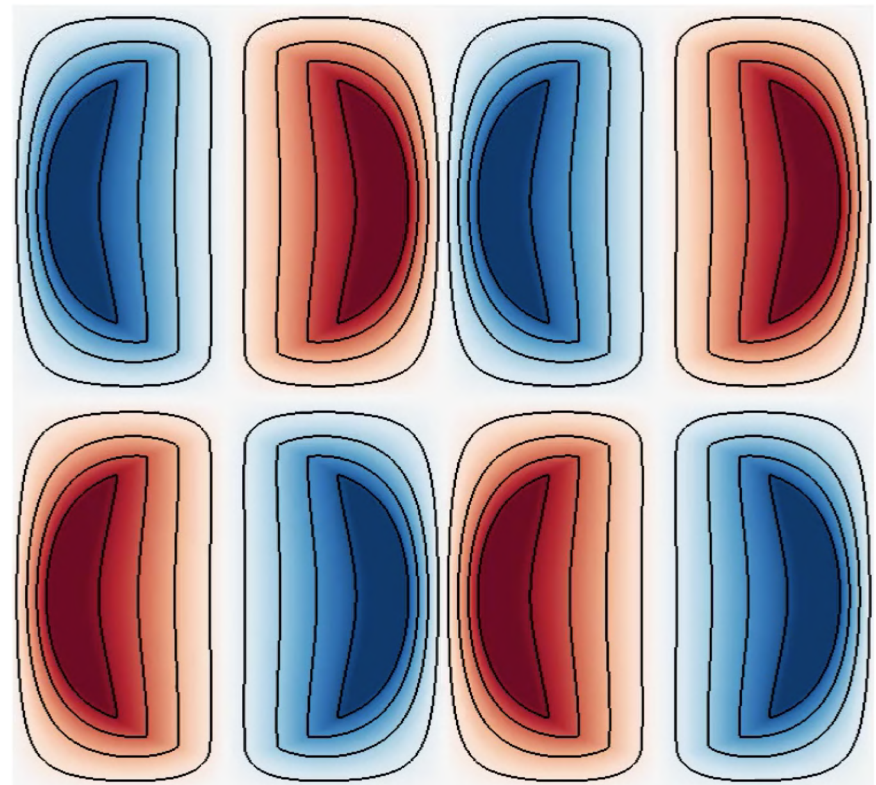


Visualization of Vorticity w/ Vorticity Contours

FFT: Interacting Regions

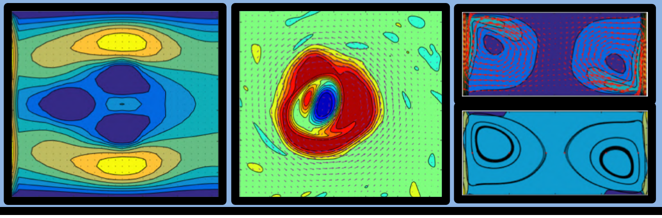


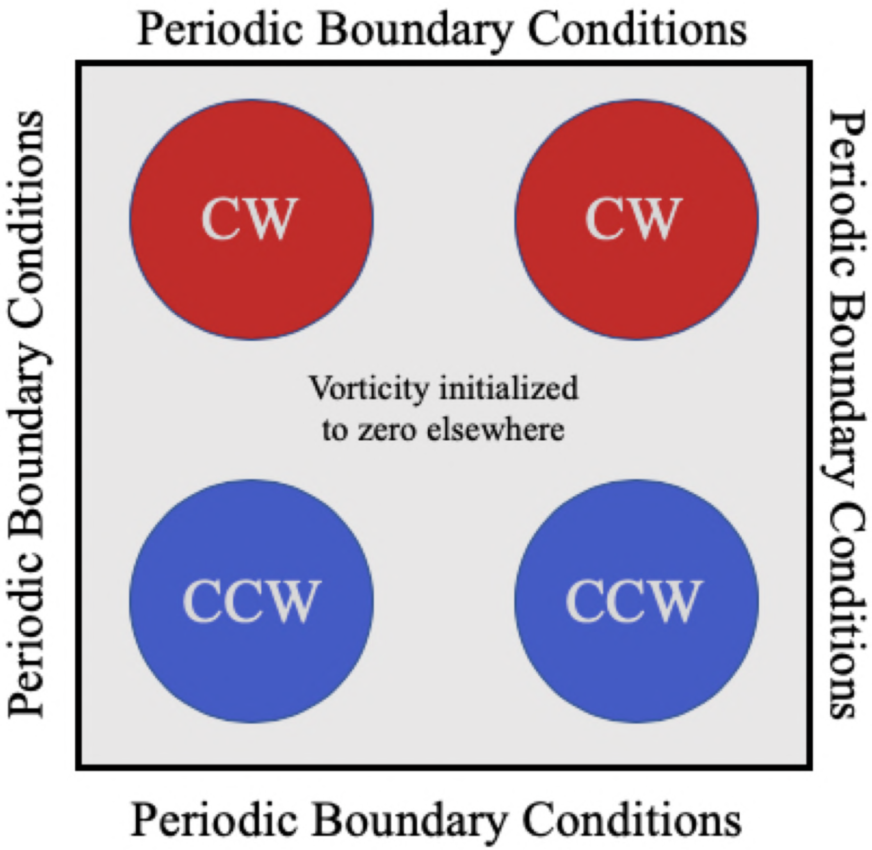




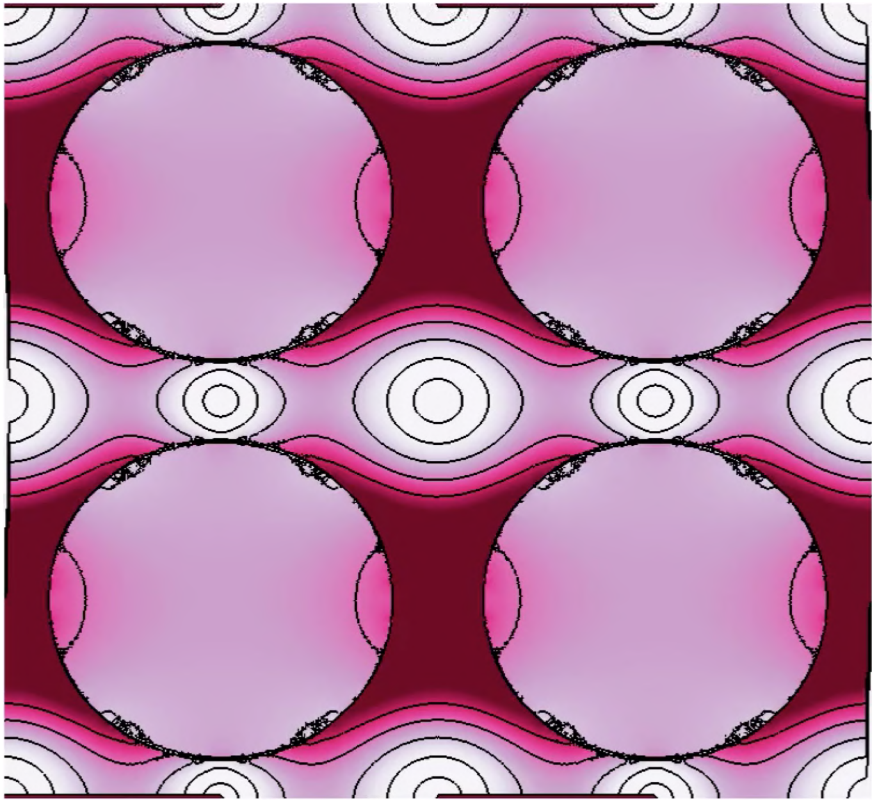
Visualization of Vertical Velocity w/ its Contours

FFT: Interacting Regions



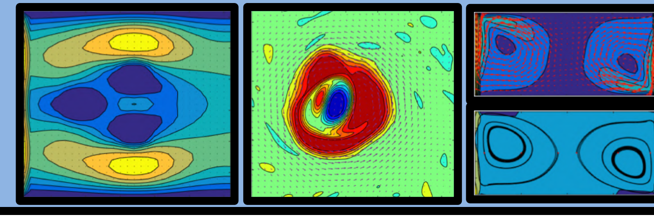


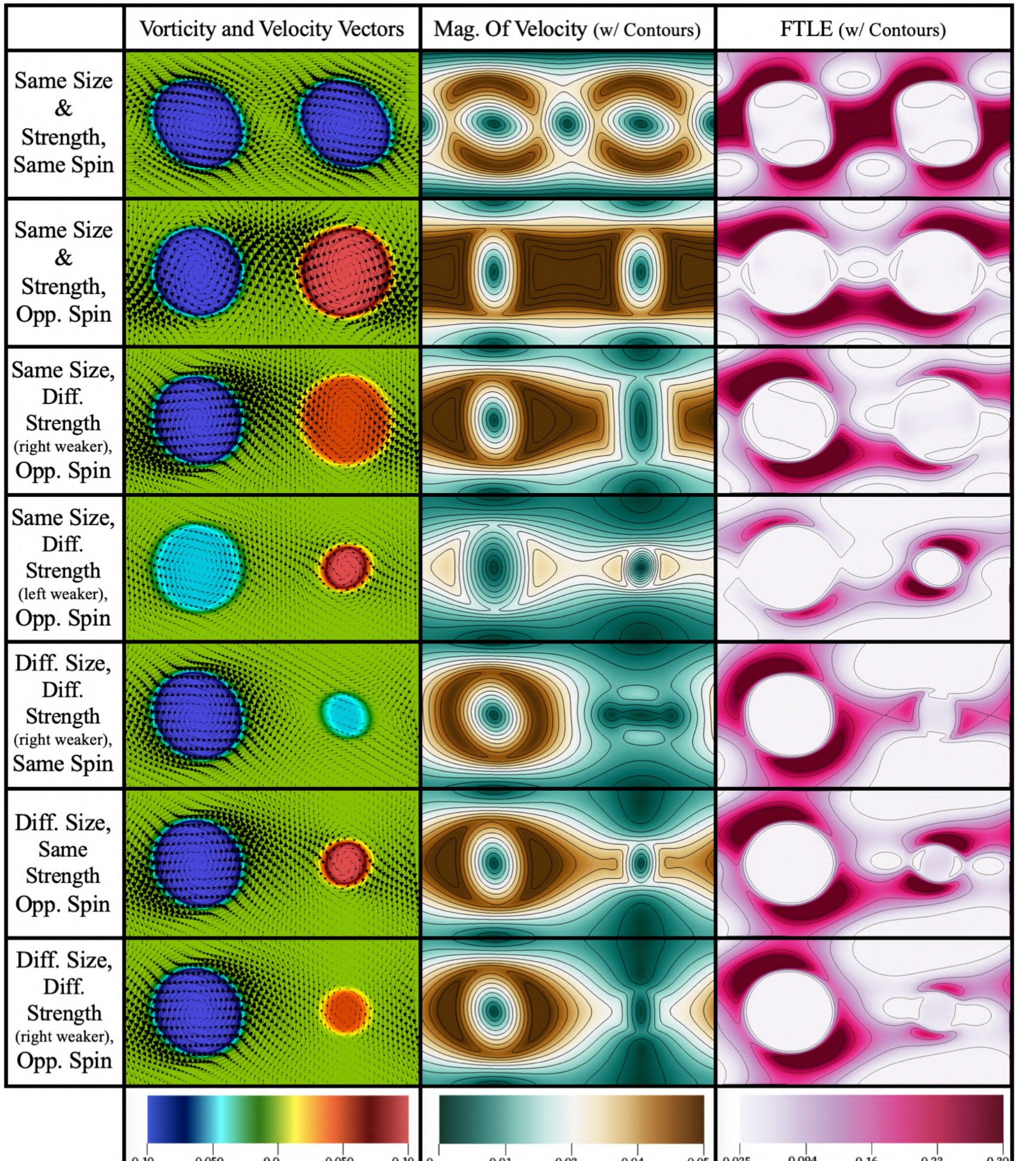
High FTLE values can be thought to be regions of high fluid mixing



Visualization of FTLE w/ its Contours

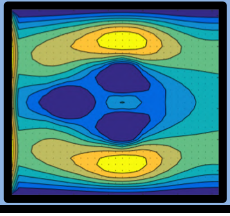
FFT: Interacting Regions

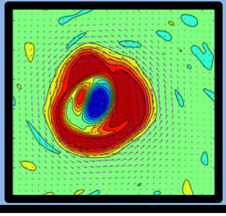


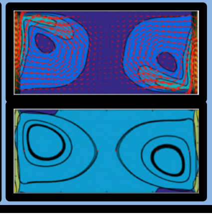


Instead of 4 interacting regions, place 2 Side-By-Side and simulate all possible cases of:

1. Region size
2. Region's vorticity magnitude
3. Region's vorticity initialization value (both positive, both negative, mixed)







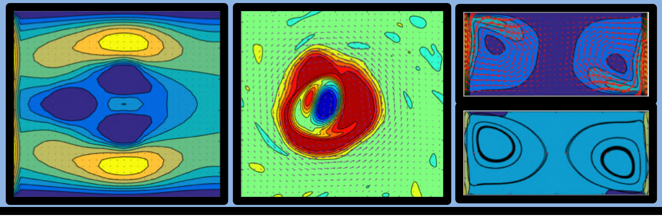
Spectral Method (FFT)

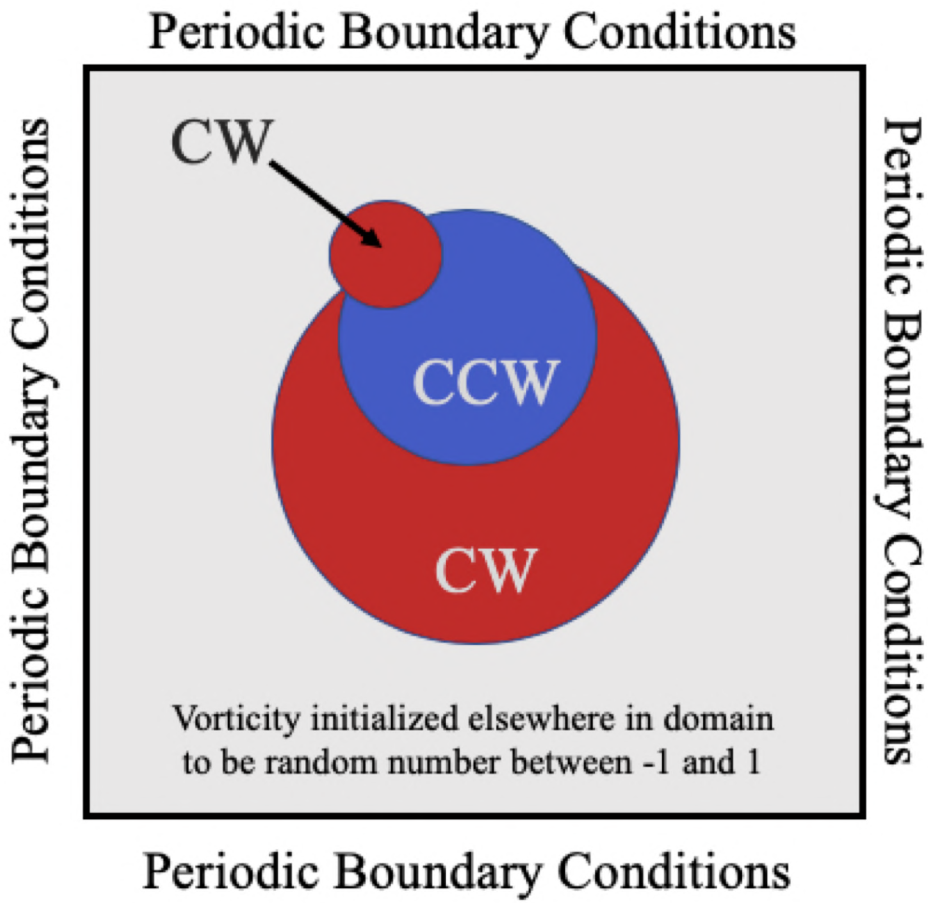
Examples:

1. Interacting Regions of Vorticity
2. **Overlapping Regions of Vorticity**
3. Evolution of vorticity from an initial vector velocity field

Overview of Numerical Scheme to follow

FFT: Overlapping Regions





- Vorticity is initialized into each circular region uniformly, as either a positive or negative vorticity value
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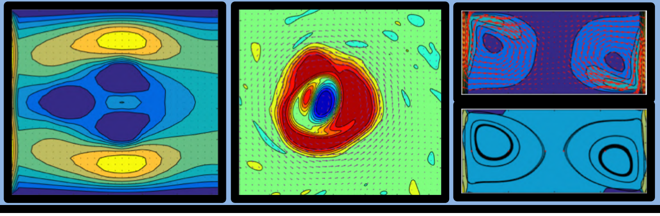
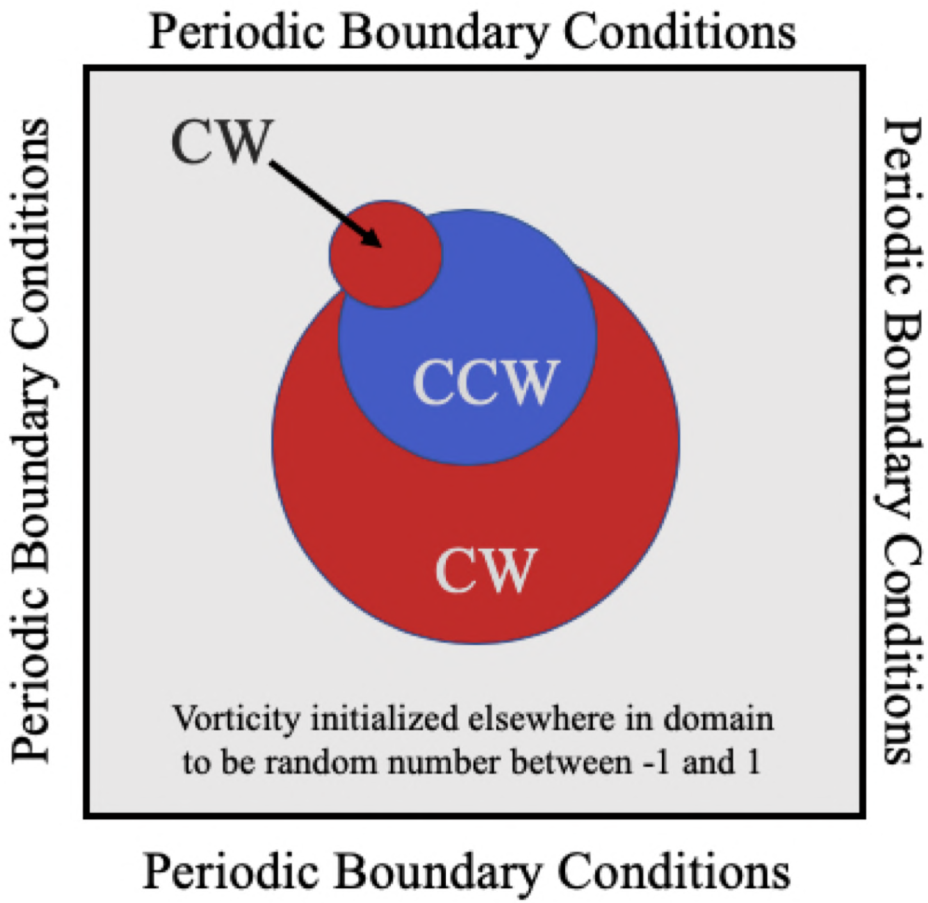
- Periodic boundary conditions on all sides of domain

FFT: Overlapping Regions





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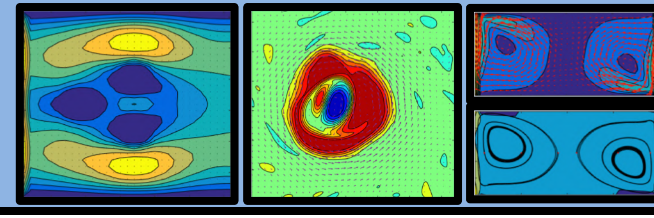

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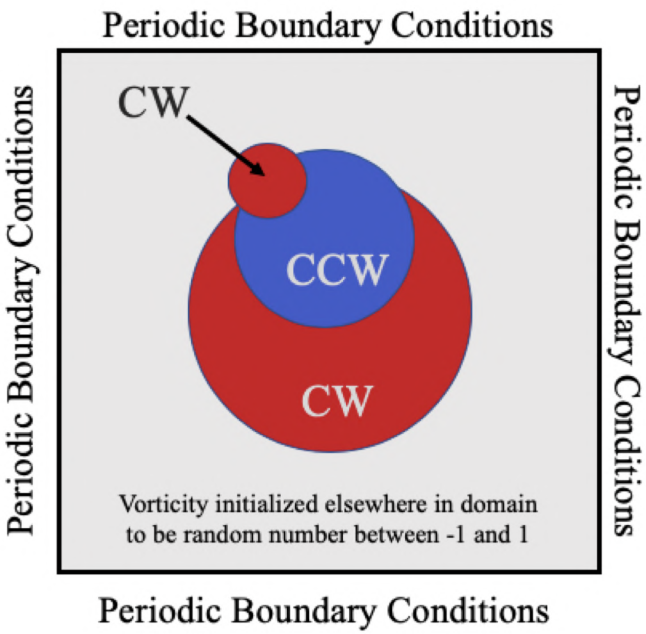
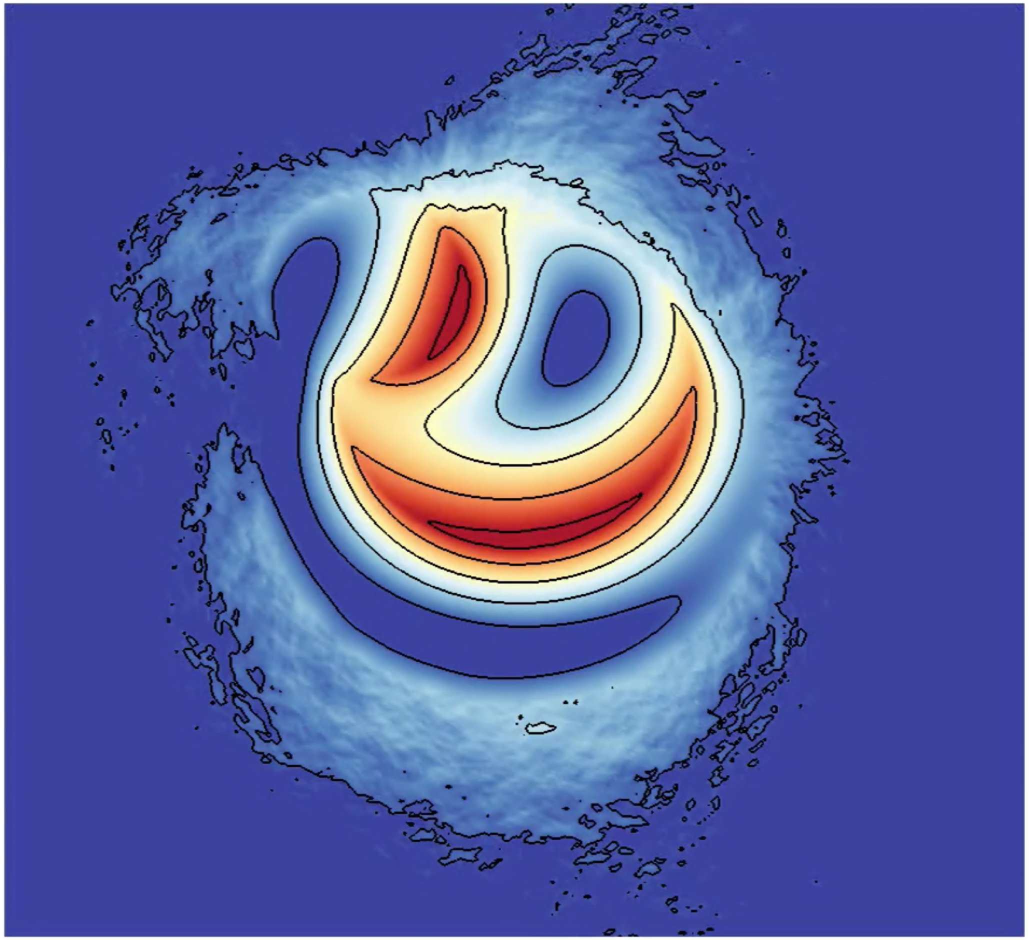
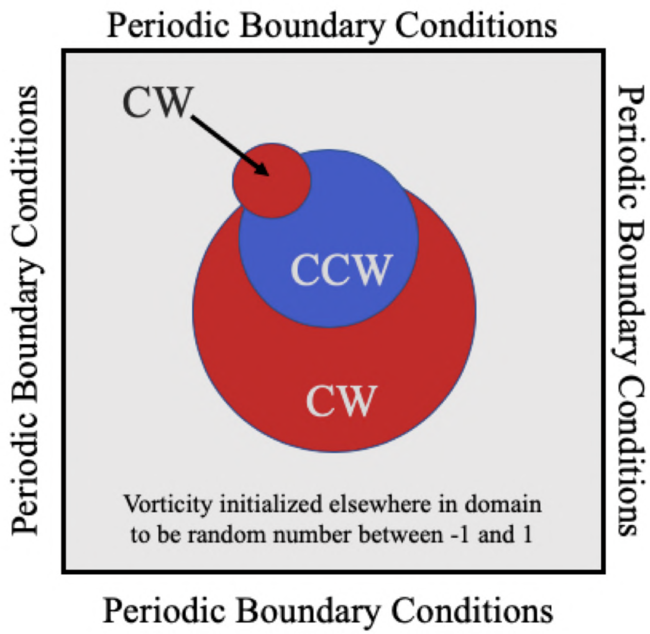
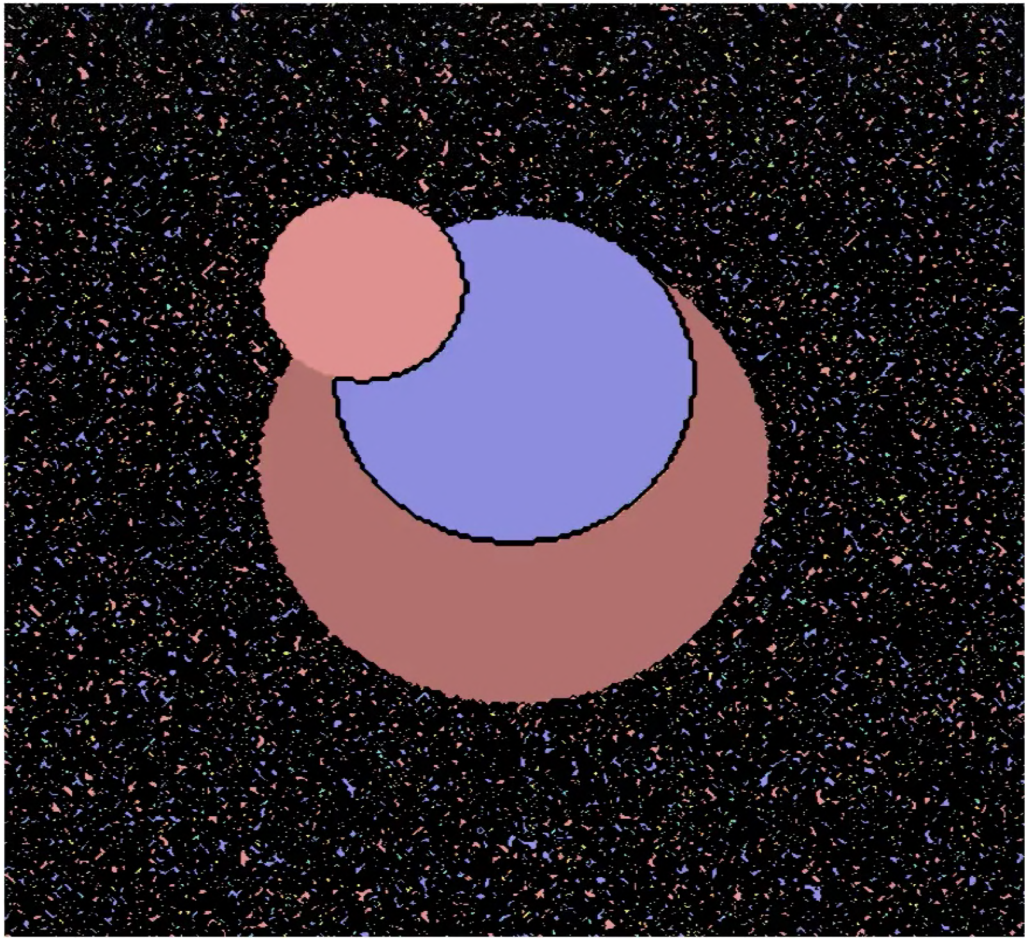




Visualization of Mag. of Velocity w/ its Contours

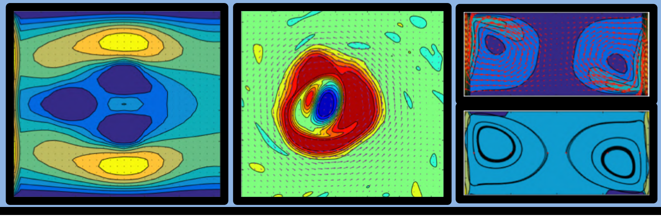
FFT: Overlapping Regions

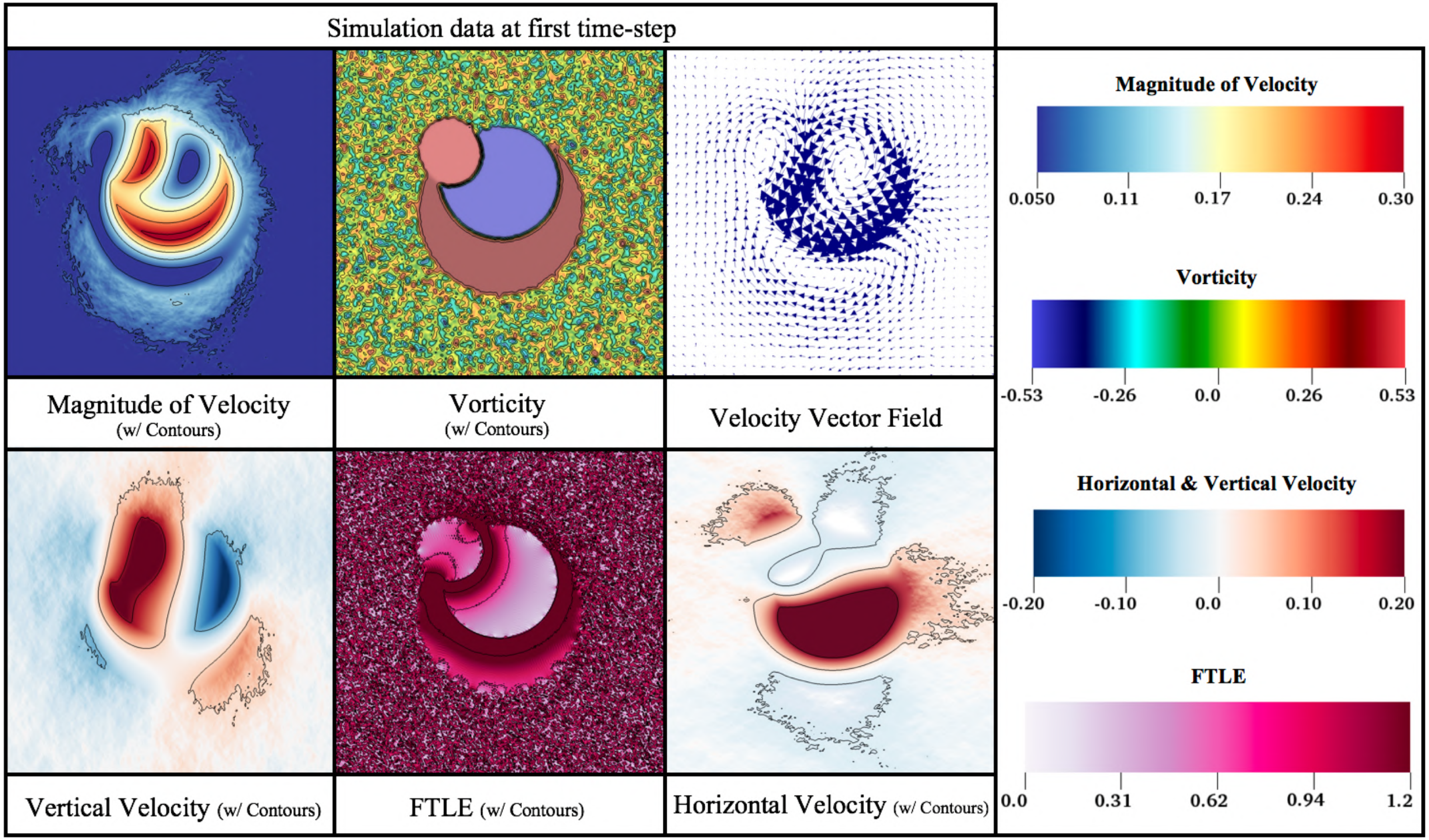




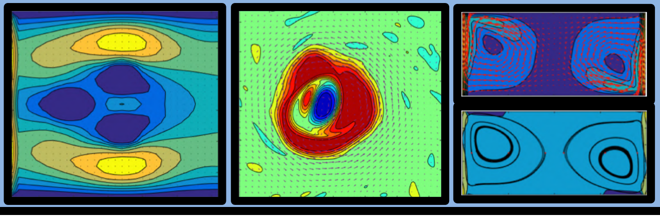
Visualization of Vorticity w/ its Contours

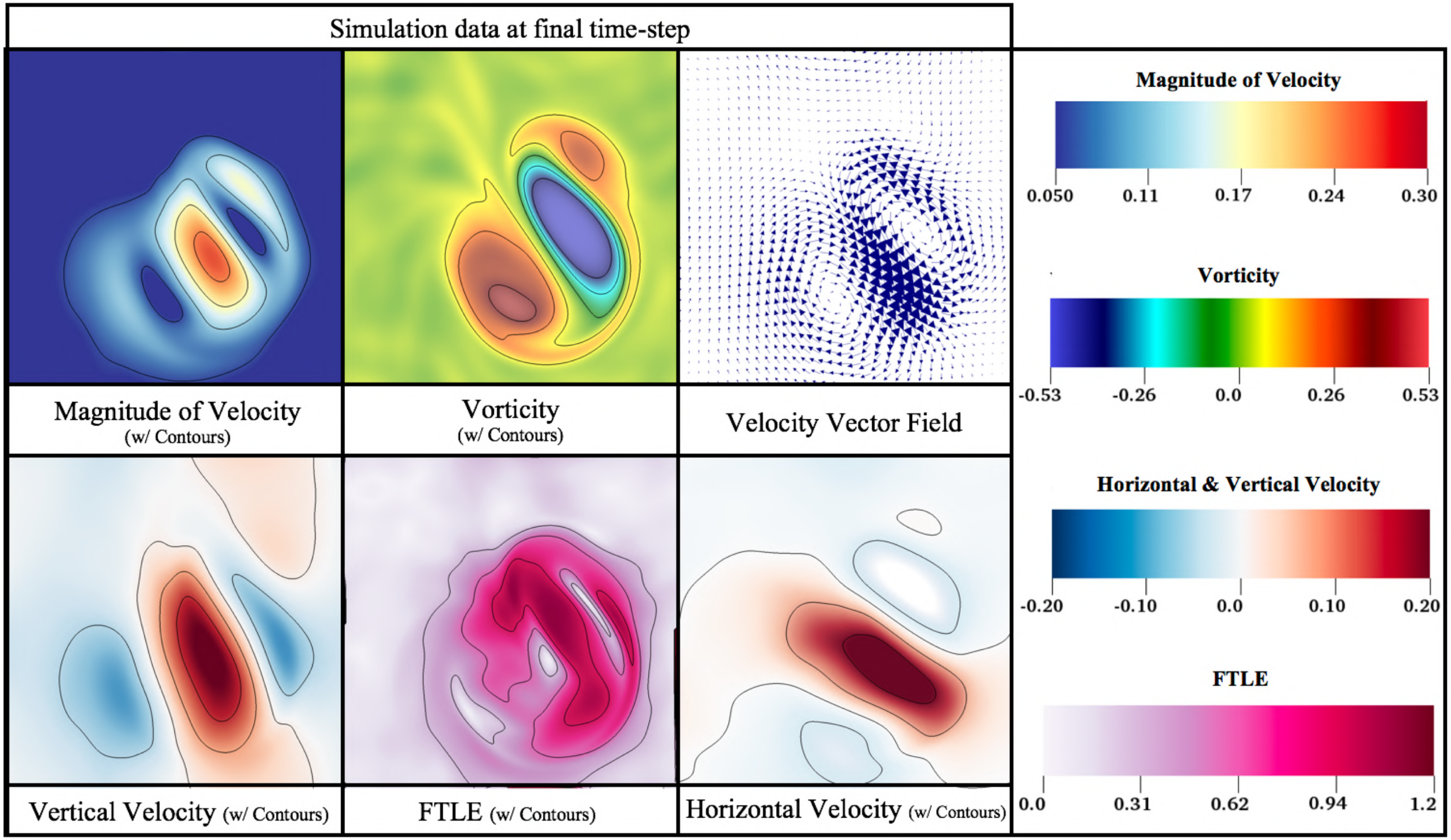
FFT: Overlapping Regions

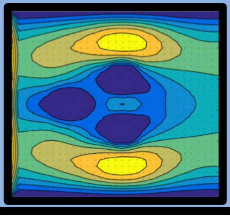


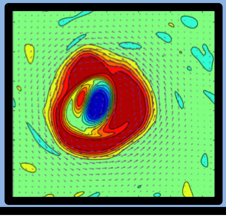


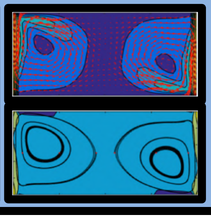
FFT: Overlapping Regions











Spectral Method (FFT)

Examples:

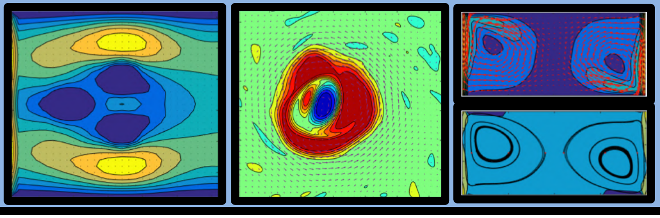
1. Interacting Regions of Vorticity

2. Overlapping Regions of Vorticity

3. **Evolution of vorticity from an initial vector velocity field**

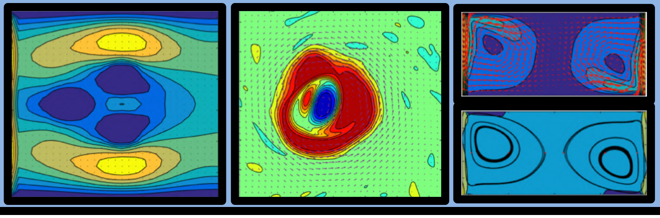
Overview of Numerical Scheme to follow

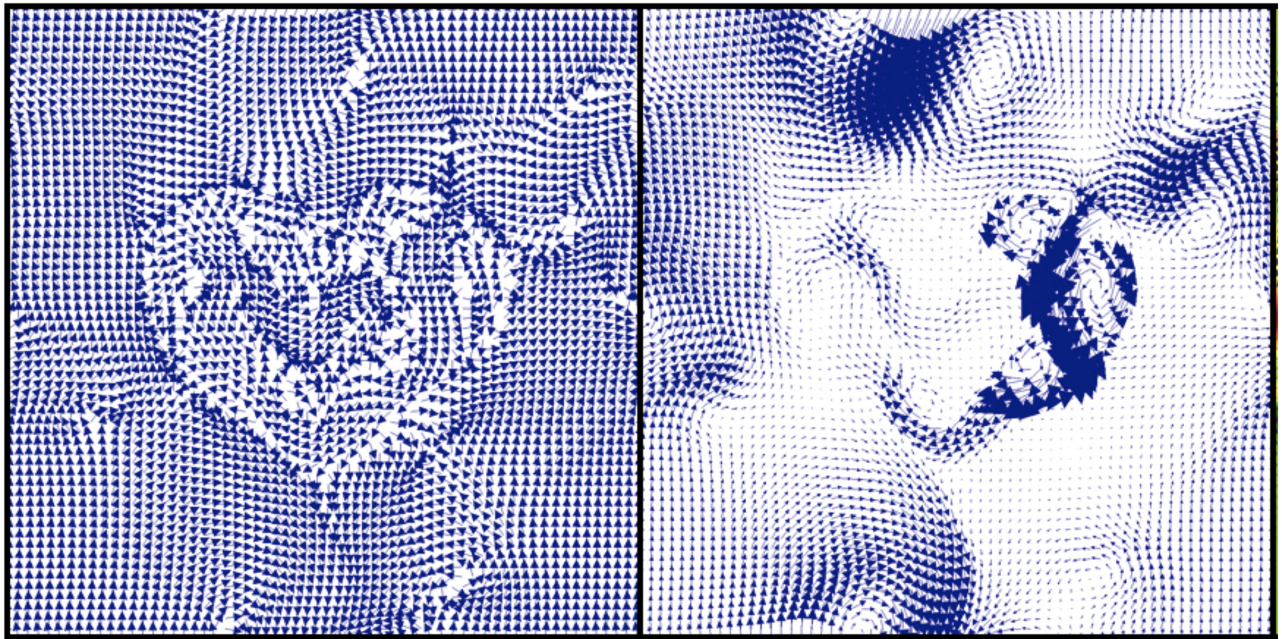
FFT: Evolve Vorticity from Initial Velocity Field



The **initial vorticity** for this simulation will be **computed from a velocity field** from an independent simulation and ***then evolved forward in time***!

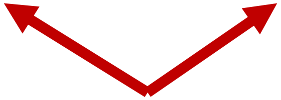
FFT: Evolve Vorticity from Initial Velocity Field





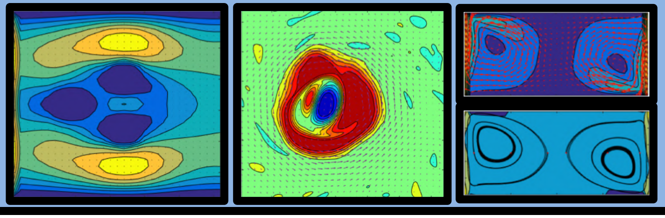
Velocity Vector Field
(not scaled by magnitude)

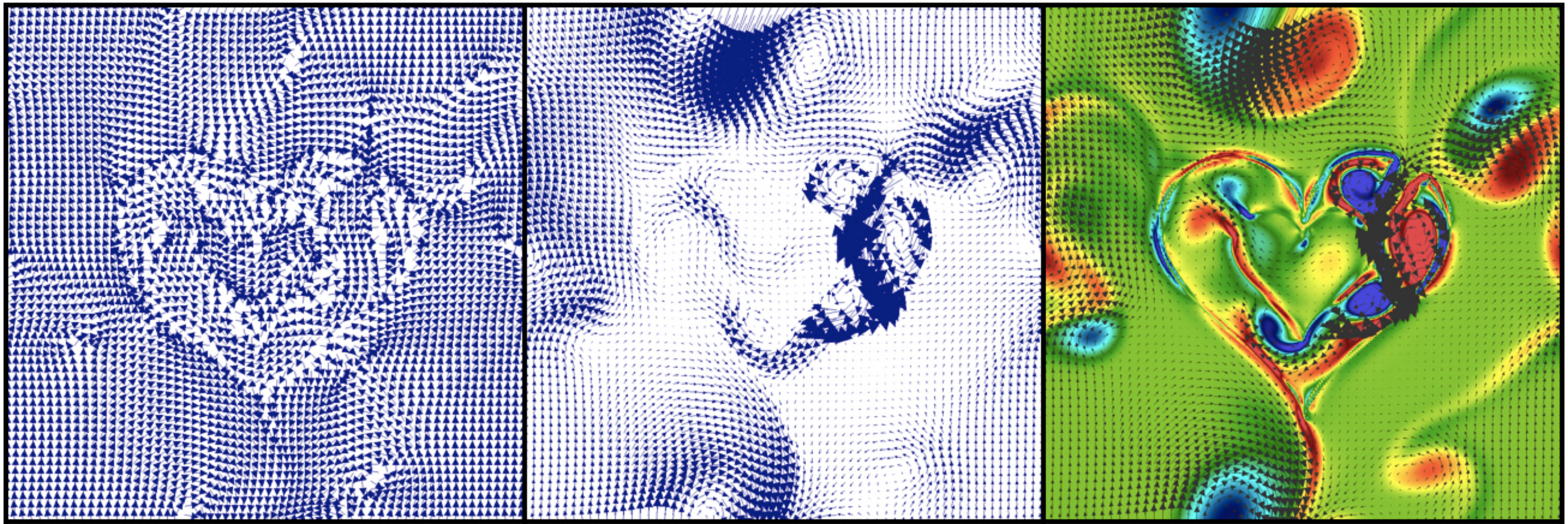
Velocity Vector Field
(scaled by magnitude)



Initial Velocity Field taken from a timepoint for an independent fluid simulation

FFT: Evolve Vorticity from Initial Velocity Field



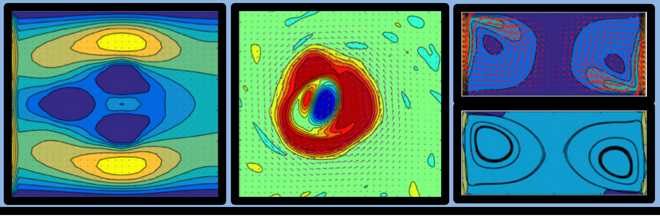


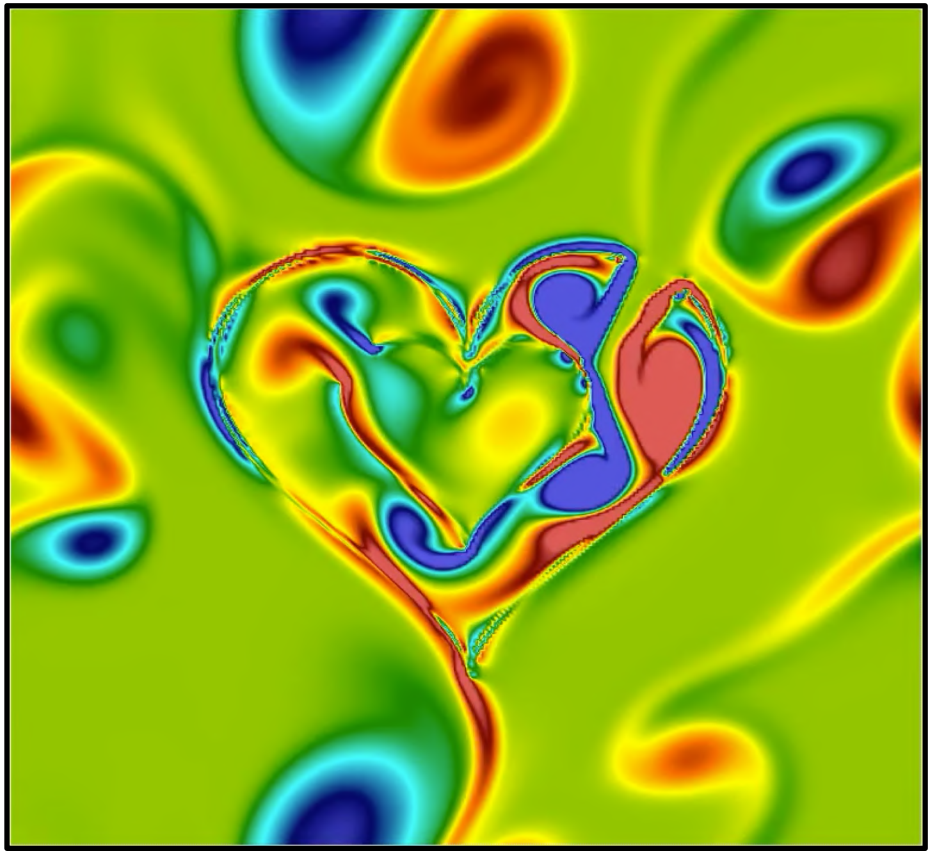
Velocity Vector Field *(not scaled by magnitude)*	Velocity Vector Field *(scaled by magnitude)*	Vorticity Colormap and (scaled) Velocity Vector Field

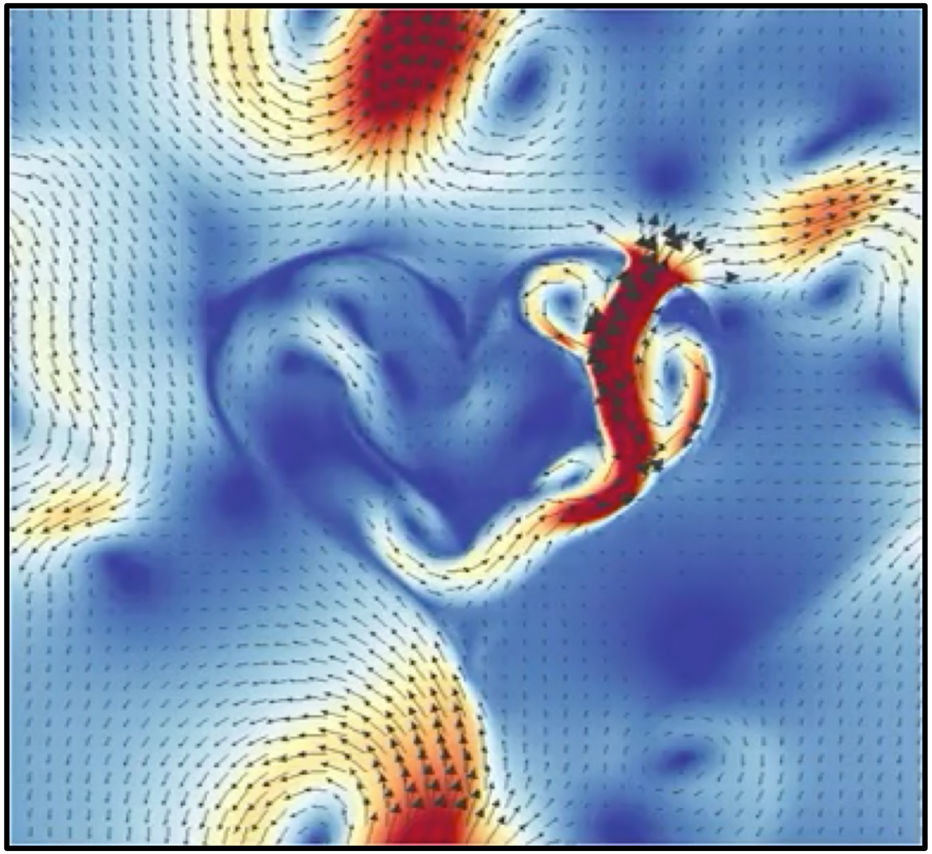
Initial Velocity Field taken from a timepoint for an independent fluid simulation

Initial Vorticity associated with such velocity field

FFT: Evolve Vorticity from Initial Velocity Field



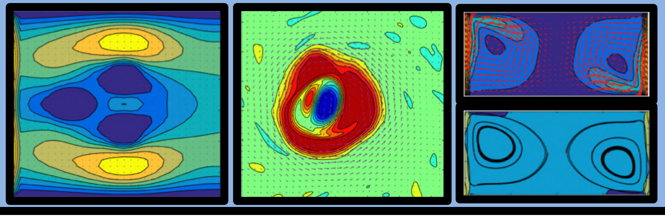


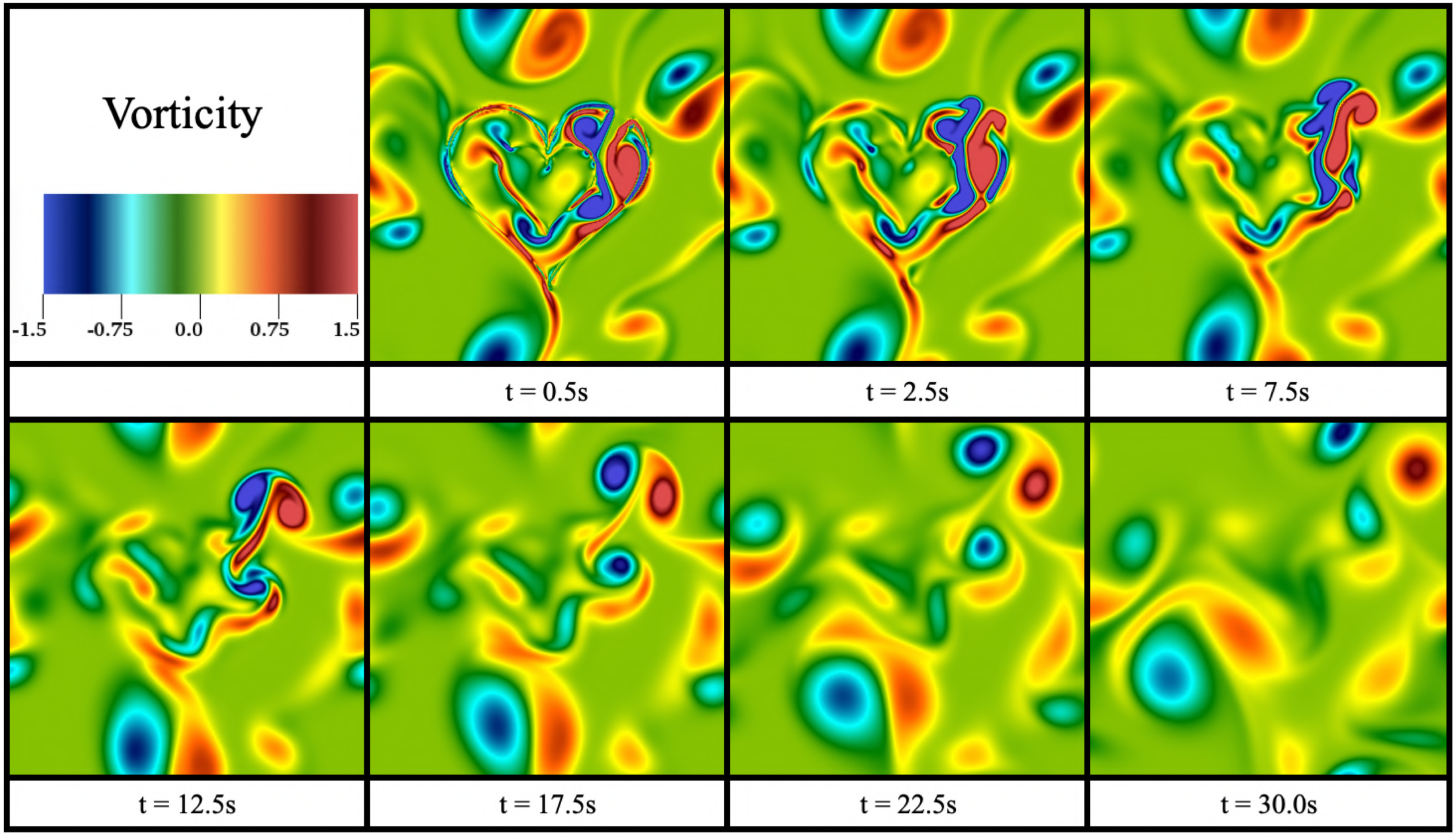


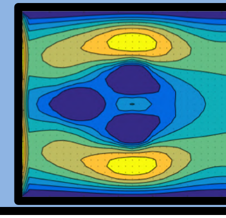
Vorticity

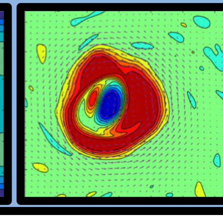
Magnitude of Velocity w/ Velocity Field (scaled)

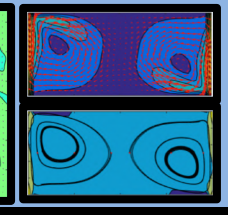
FFT: Evolve Vorticity from Initial Velocity Field











Spectral Method (FFT)

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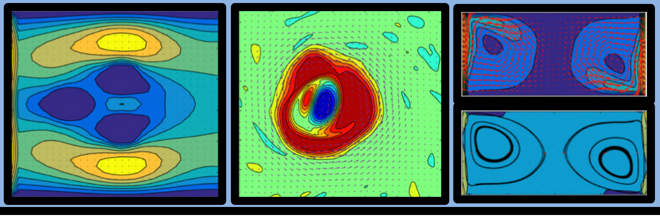
Overview of Numerical Scheme to follow*

Spectral Methods (FFT)



- **First put into vorticity formulation of Navier-Stokes**

$$\rho\left(\frac{\partial \mathbf{u}(\mathbf{x},t)}{\partial t} + \mathbf{u}(\mathbf{x},t)\cdot\nabla\mathbf{u}(\mathbf{x},t)\right) = -\nabla p(\mathbf{x},t) + \mu\Delta\mathbf{u}(\mathbf{x},t)$$

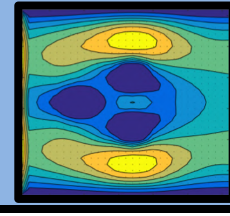
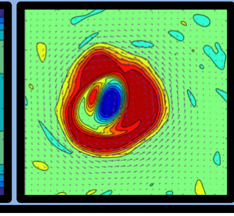
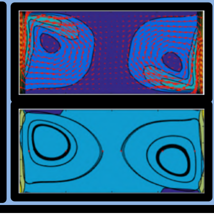
Define vorticity:

$$\boldsymbol{\omega} = \nabla\times\mathbf{u}$$

Vorticity Form. Of NS:

$$\frac{\partial\omega}{\partial t} + \nabla\times(\boldsymbol{\omega}\times\mathbf{u}) = \nu\Delta\omega$$

Spectral Methods (FFT)

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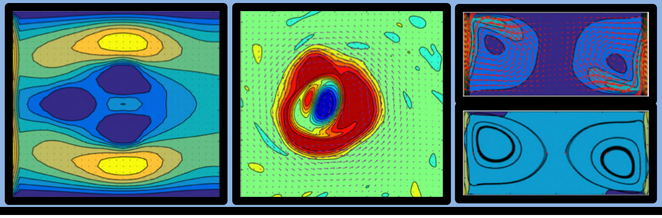
Spectral Methods (FFT)



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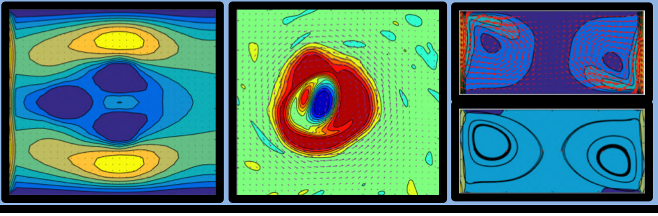
Spectral Methods (FFT)



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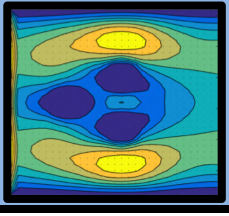
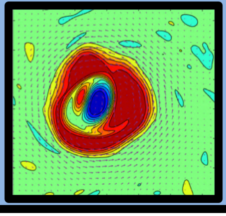
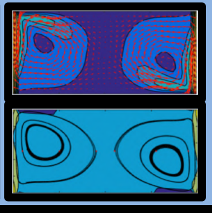
Spectral Methods (FFT)

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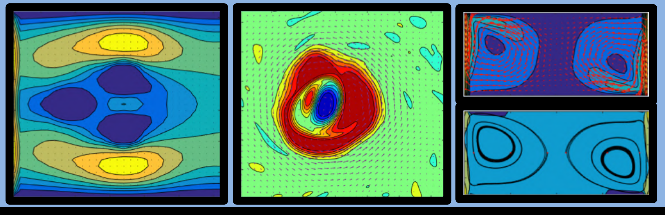
Spectral Methods (FFT)



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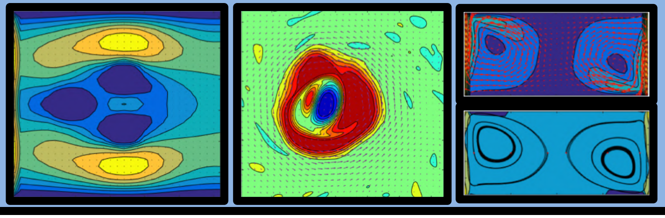
B/c only considering 2D flows!

Spectral Methods (FFT)



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$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \nu \Delta \boldsymbol{\omega}$$

Define *"Vector Potential"*, aka *Stream-function*:

$$\mathbf{u} = \nabla \times \psi \hat{k}$$

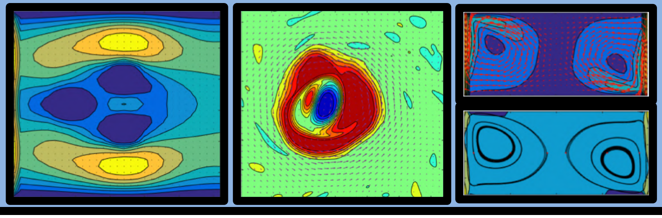
$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

Hence we have the Poisson problem for the stream-function,

$$\Delta \psi = -\boldsymbol{\omega}$$

Spectral Methods (FFT)



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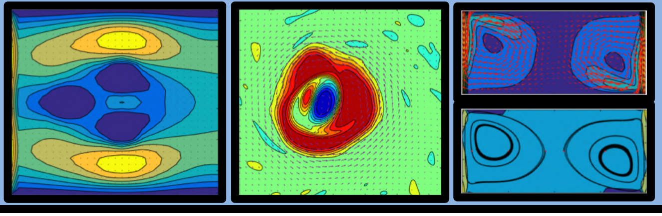

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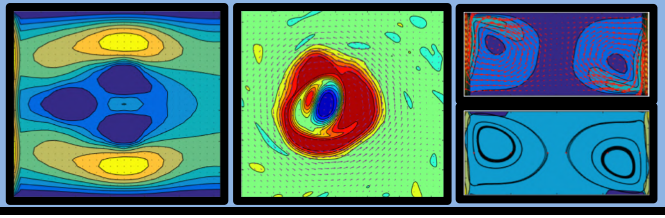

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$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \Delta \omega$$

Define *"Vector Potential"*, aka *Stream-function*:

$$\mathbf{u} =$$

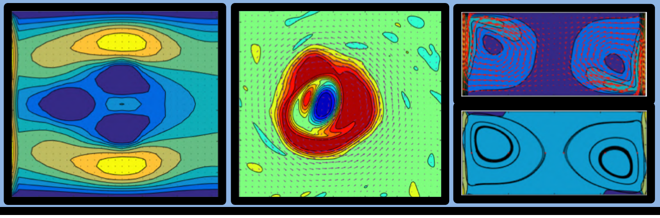
So if we can find the vorticity…we can find the velocity field and it will be automatically incompressible (divergence free)

$$v = -\frac{\partial \psi}{\partial x}$$

Hence we have the Poisson problem for the stream-function,

$$\boxed{\Delta \psi = -\boldsymbol{\omega}}$$

Spectral Methods (FFT)



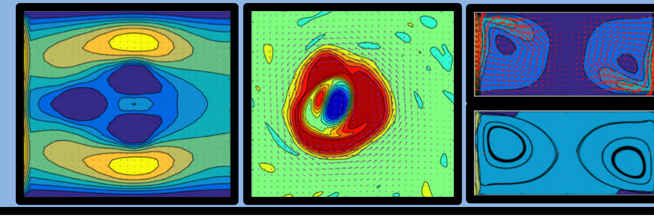
- **Steps For Algorithm: (EVERYTHING IN FREQUENCY SPACE!)**

1. Solve Poisson Problem for Steam-Function from previous time-step vorticity

2. Compute x,y derivatives of stream-function and vorticity **(in real space)**, then compute advection term, and finally transform it into frequency space

3. Crank-Nicholson (**semi-implicit**: explicit for nonlinear advection term, implicit for viscous term)

Spectral Methods (FFT)



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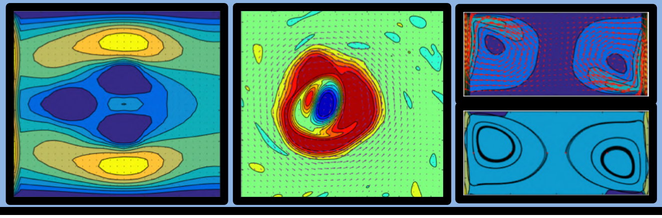

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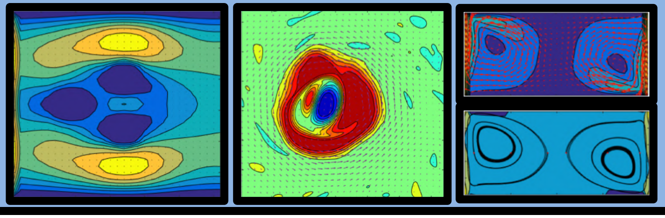

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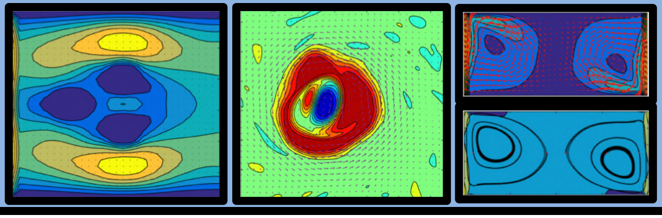

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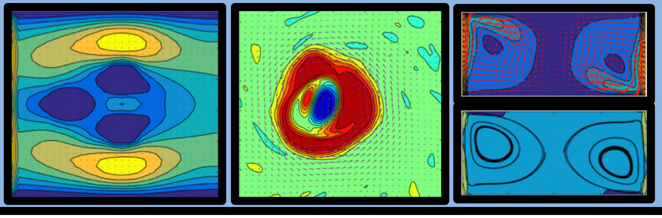
  1. Solve Poisson Problem for Stream-function from previous time-step vorticity

$$\hat{\psi}_{ij}^{n} = \frac{\omega_{ij}^{n}}{k_{X_i}^2 + k_{Y_j}^2}$$

Spectral Methods (FFT)



- **Steps For Algorithm: (EVERYTHING IN FREQUENCY SPACE!)**

2. Compute x,y derivatives of stream-function and vorticity **(in real space)**, then compute advection term, and finally transform it into frequency space

$$u_{ij}^{n} = \mathscr{F}^{-1}\left\{K_Y \hat{\psi}_{ij}^{n}\right\}$$

$$v_{ij}^{n} = \mathscr{F}^{-1}\left\{-K_X \hat{\psi}_{ij}^{n}\right\}$$

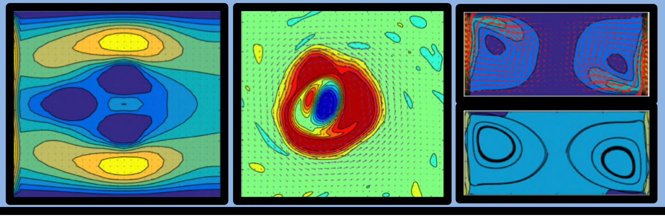
$$\omega_{x_{ij}}^{n} = \mathscr{F}^{-1}\left\{K_X \hat{\omega}_{ij}^{n}\right\}$$

$$\omega_{y_{ij}}^{n} = \mathscr{F}^{-1}\left\{K_Y \hat{\omega}_{ij}^{n}\right\}$$

$$F_{\mathrm{adv}_{ij}}^{n} = u_{ij}^{n} \cdot \omega_{x_{ij}}^{n} + v_{ij}^{n} \cdot \omega_{y_{ij}}^{n}$$

$$\hat{F}_{\mathrm{adv}_{ij}}^{n} = \mathscr{F}\left\{F_{\mathrm{adv}_{ij}}^{n}\right\}$$

Spectral Methods (FFT)



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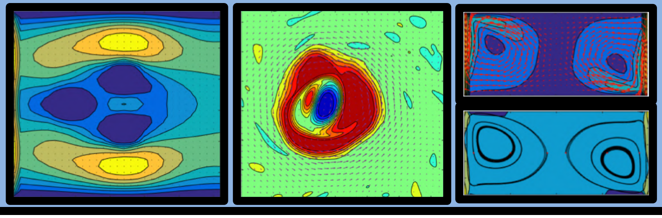
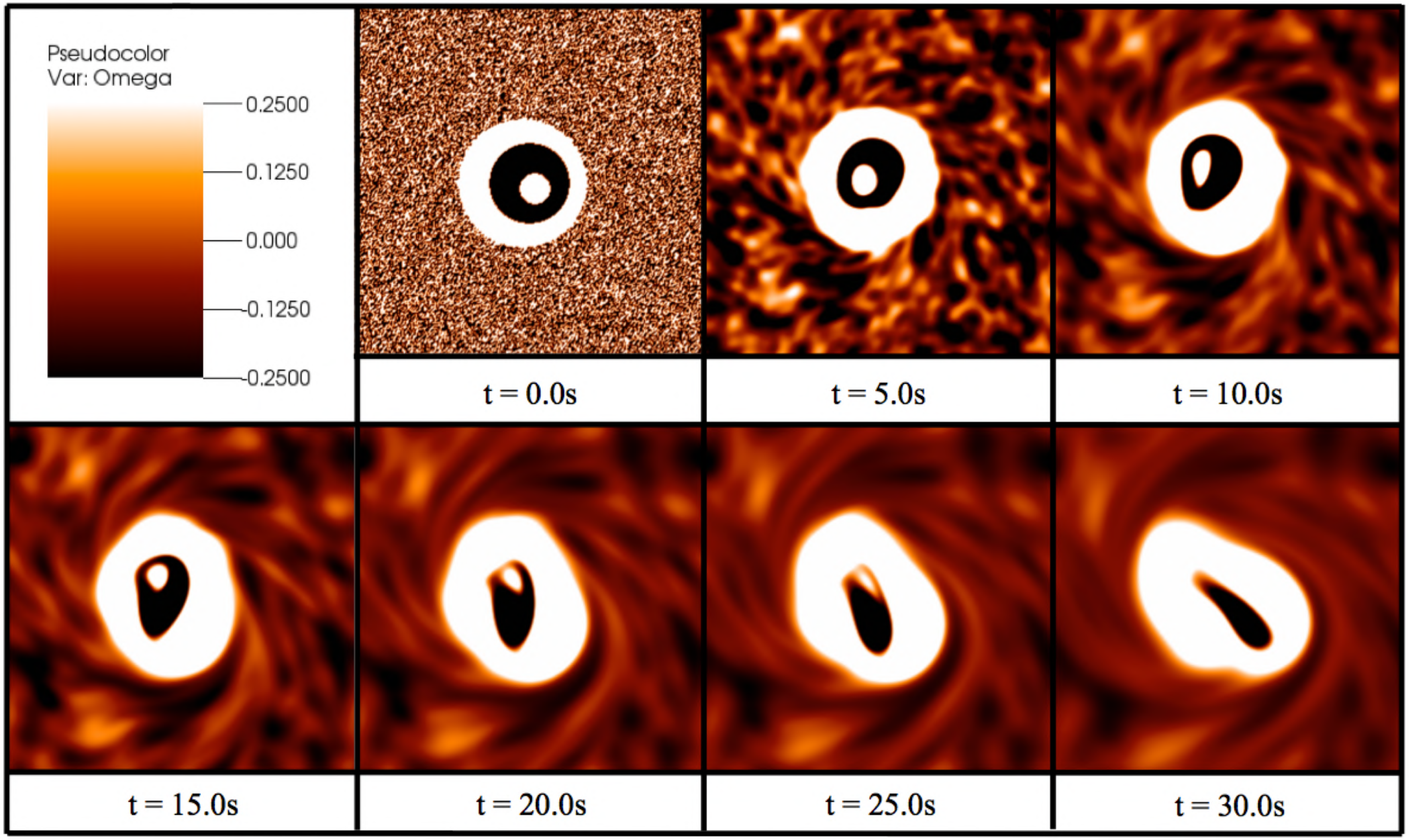
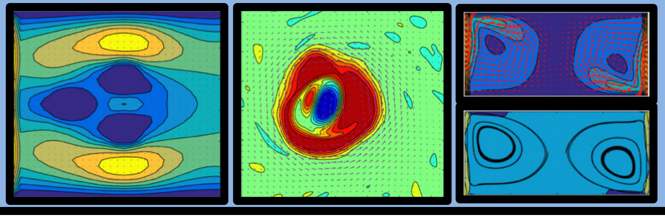
$$\hat{\psi}_{ij}^{n+1} = \frac{\overbrace{\left[1 + \frac{\nu\Delta t}{2}\left(k_{X_{ij}}^2 + k_{Y_{ij}}^2\right)\right]\hat{\psi}_{ij}^{n}}^{\text{implicit}} - \overbrace{\Delta t\ \hat{F}_{\mathrm{adv}_{ij}}^{n}}^{\text{explicit}}}{1 - \frac{\nu\Delta t}{2}\left(k_{X_{ij}}^2 + k_{Y_{ij}}^2\right)}$$

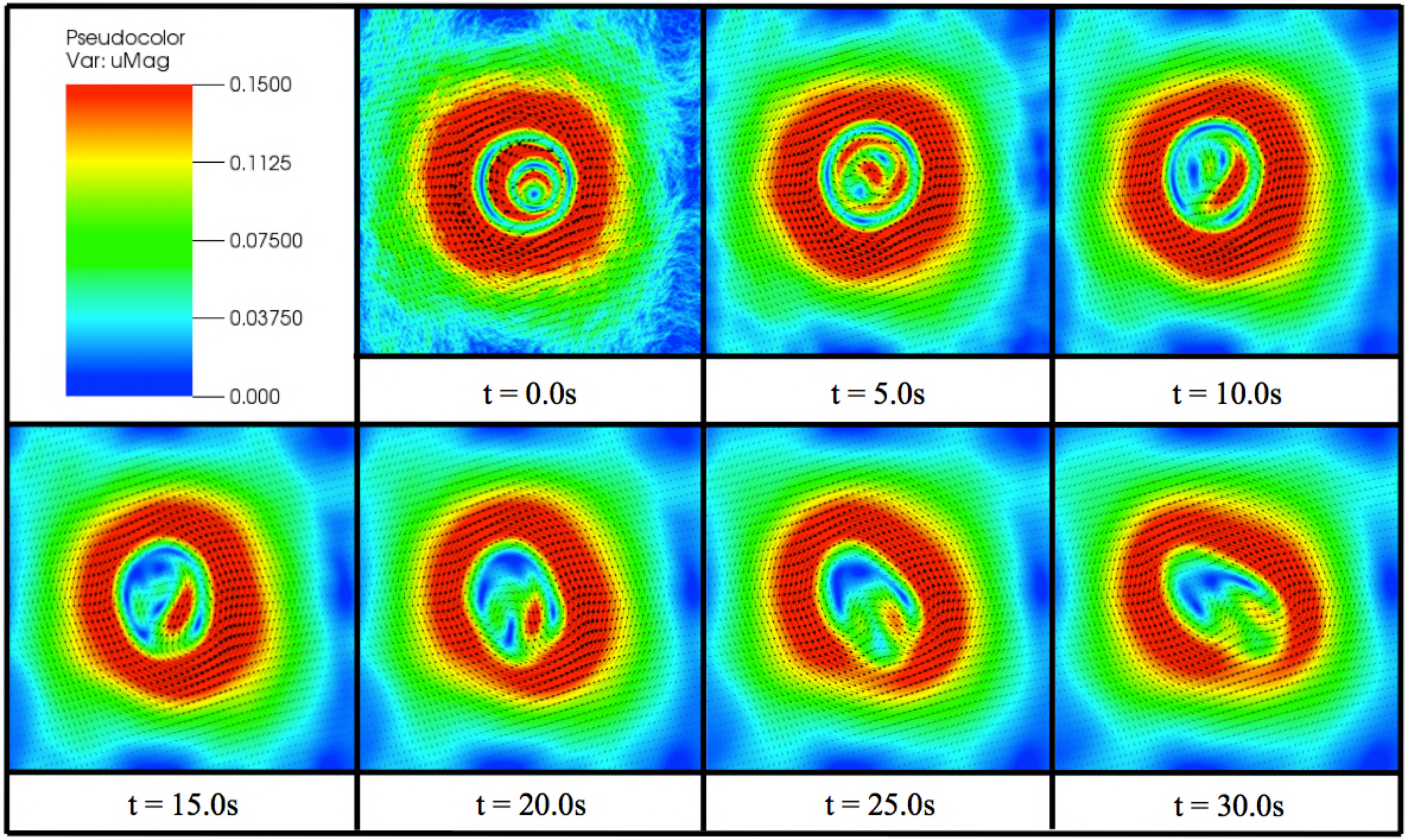
Spectral Methods (FFT)

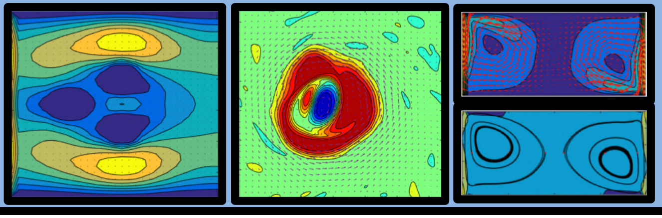




Spectral Methods (FFT)







Lattice Boltzmann Method (LBM)

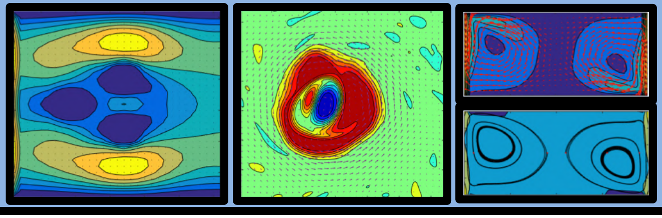
Examples:

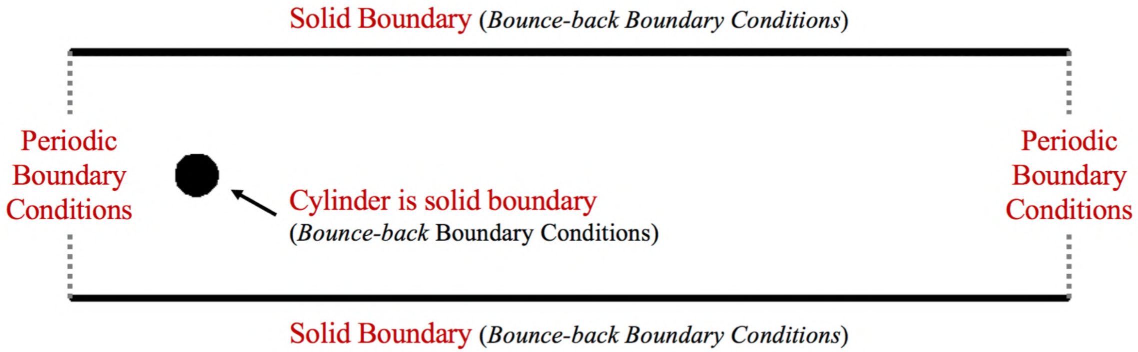
1. **Flow around one or multiple cylinders**

2. Flow around porous cylinder

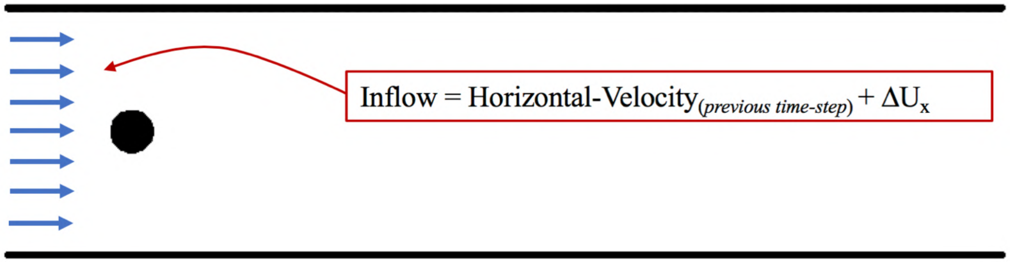
Overview of Numerical Scheme to follow

LBM: Flow around cylinders



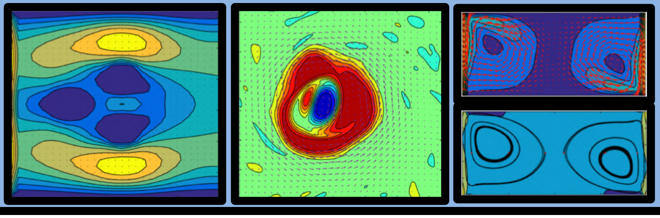


Boundary conditions along domain and on cylinder(s)



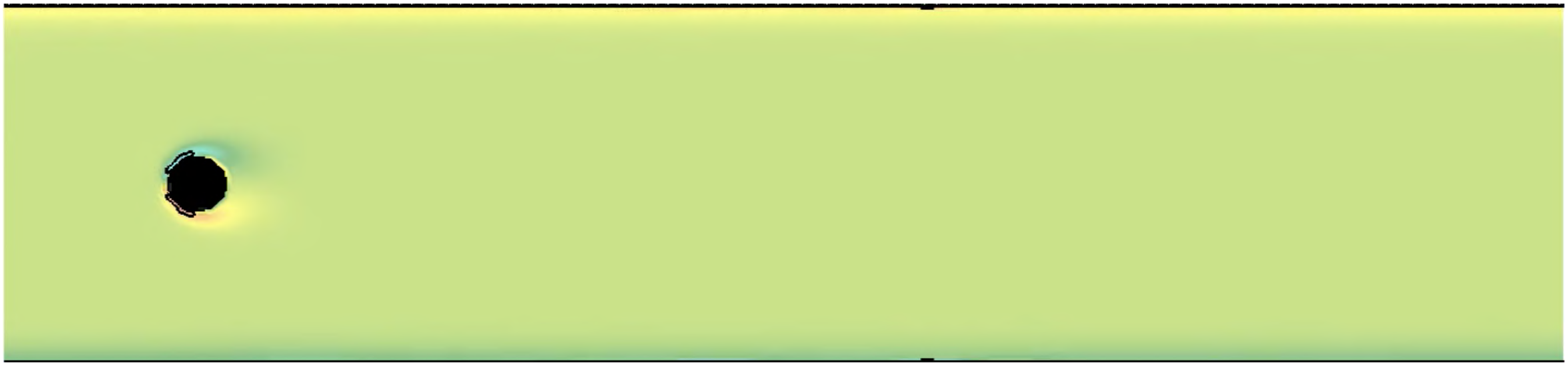
Inflow: left to right, gets ramped up during simulation

LBM: Flow around cylinders



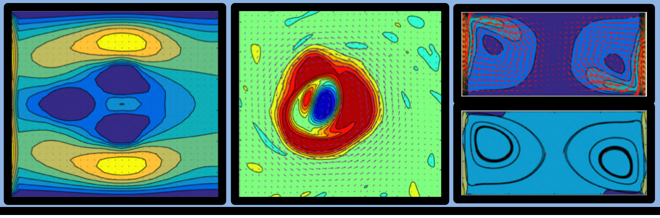
Flow around a single cylinder:

Vorticity + contours



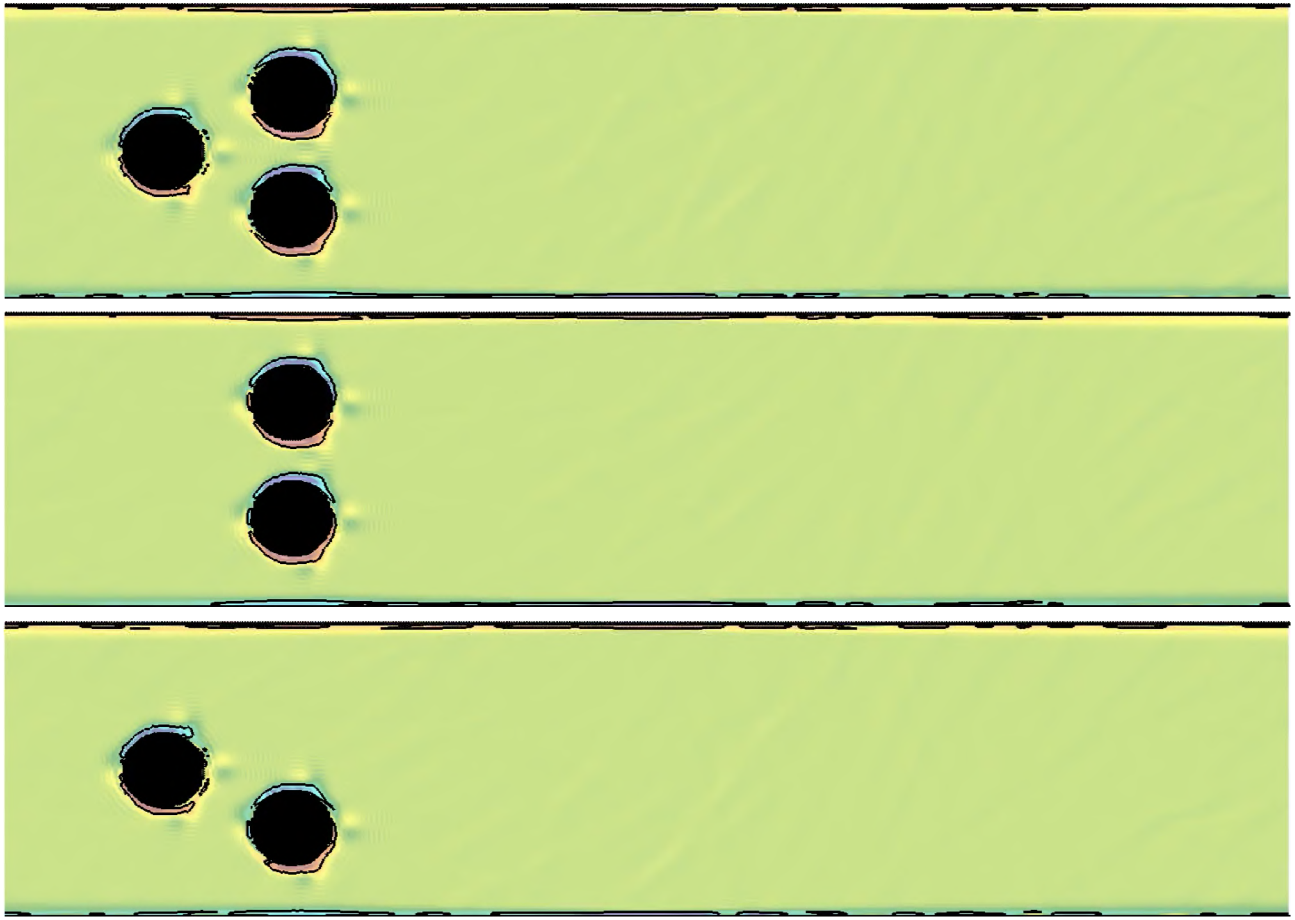
- Flow remains symmetric until instability forms in which there is a *transition from smooth flow to vortical flow patterns* and
- Vortices begin to be *shed* off of cylinder (*e.g., vortex shedding*)

LBM: Flow around cylinders



Flow around multiple cylinders:

Vorticity + contours



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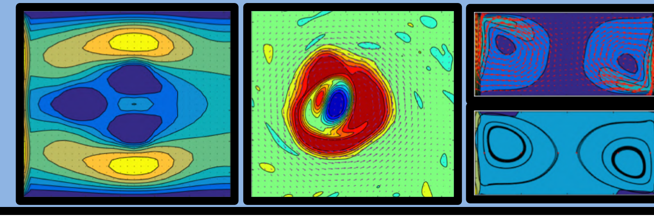

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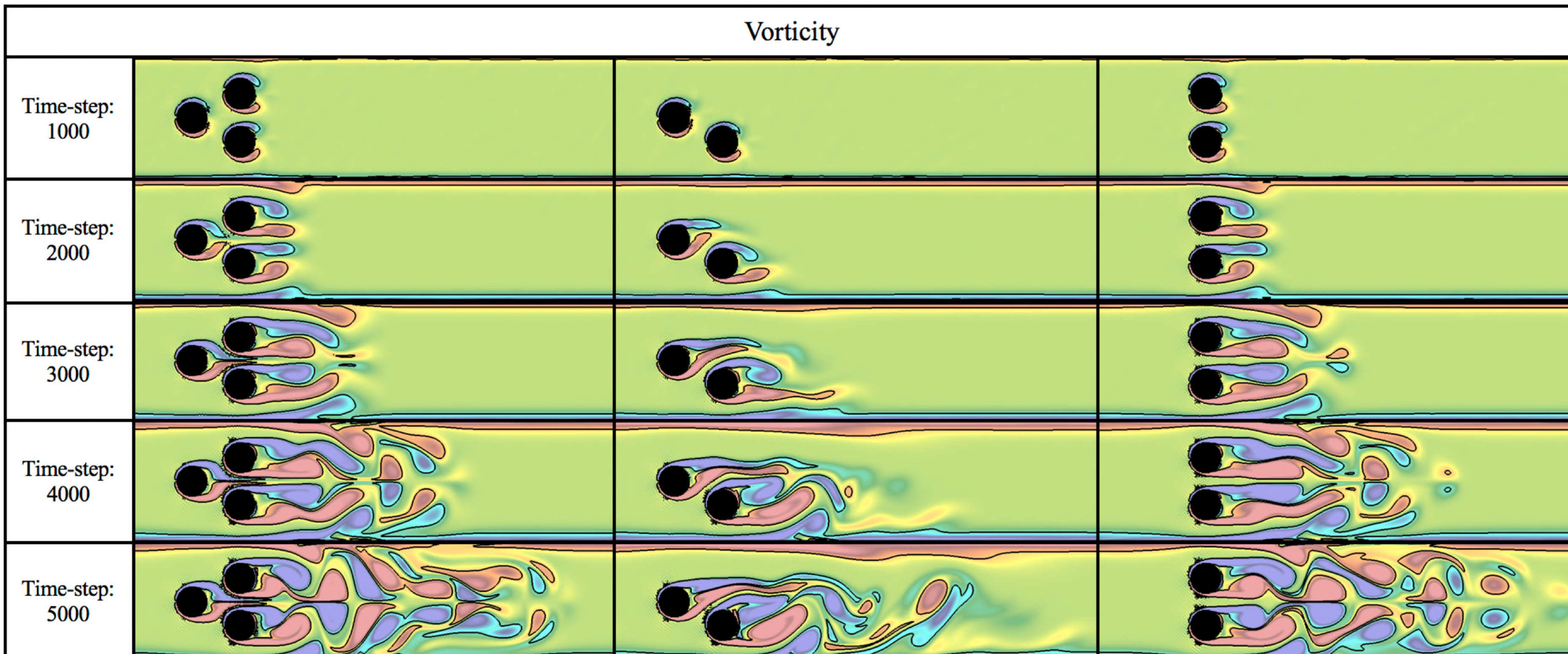

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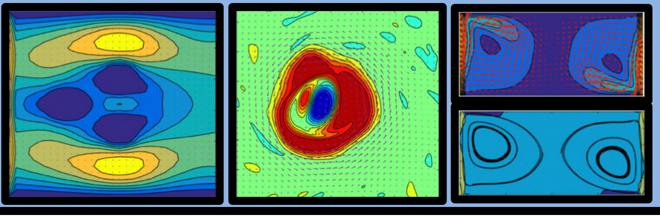
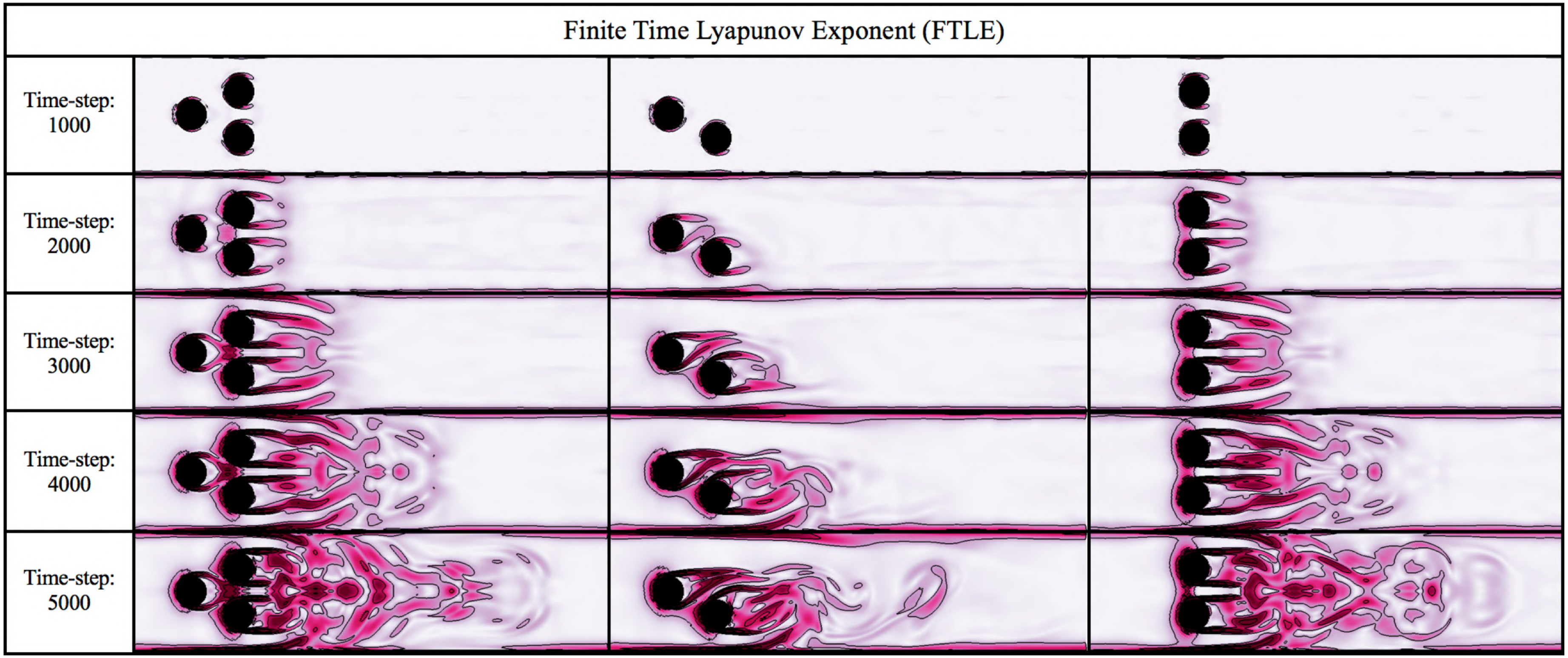
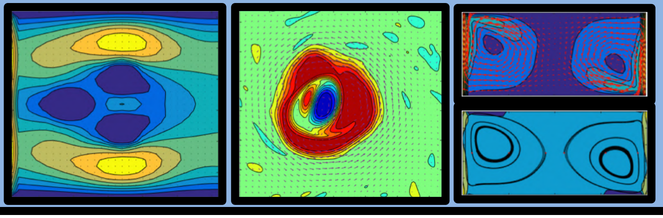


Flow around multiple cylinders:

FTLE + contours





Lattice Boltzmann Method (LBM)

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Overview of Numerical Scheme to follow

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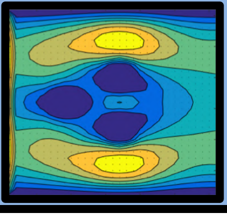
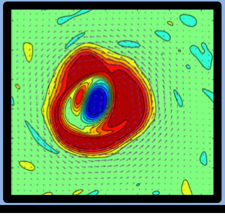
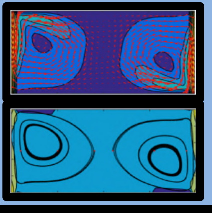
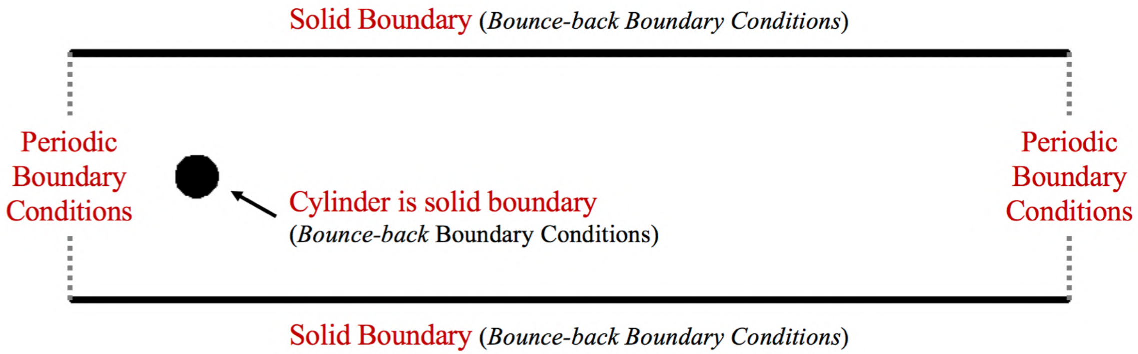




SAME BOUNDARY CONDITIONS AND INFLOW CONDITIONS AS PREVIOUS LBM CASES!



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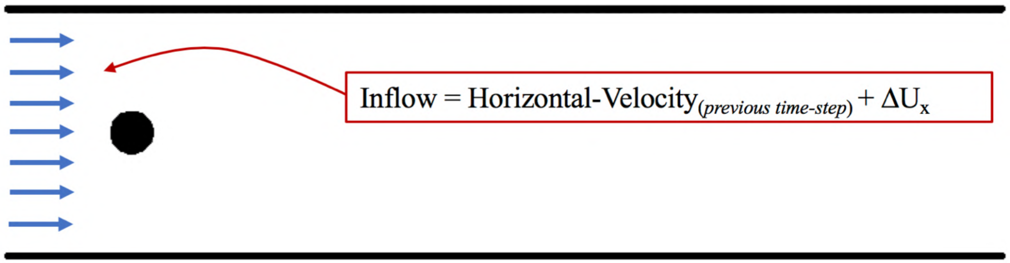

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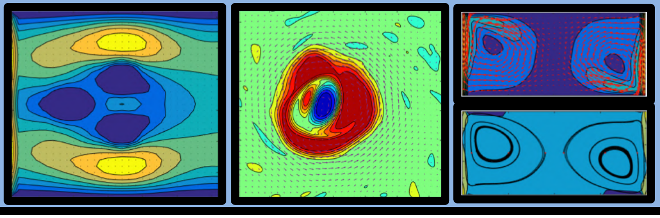
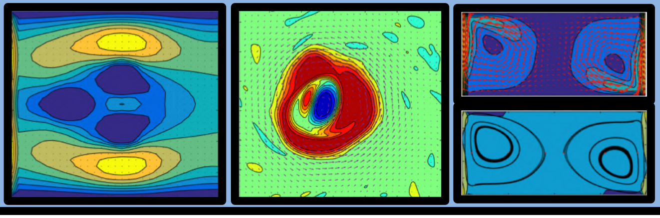


Porous Cylinder Geometries

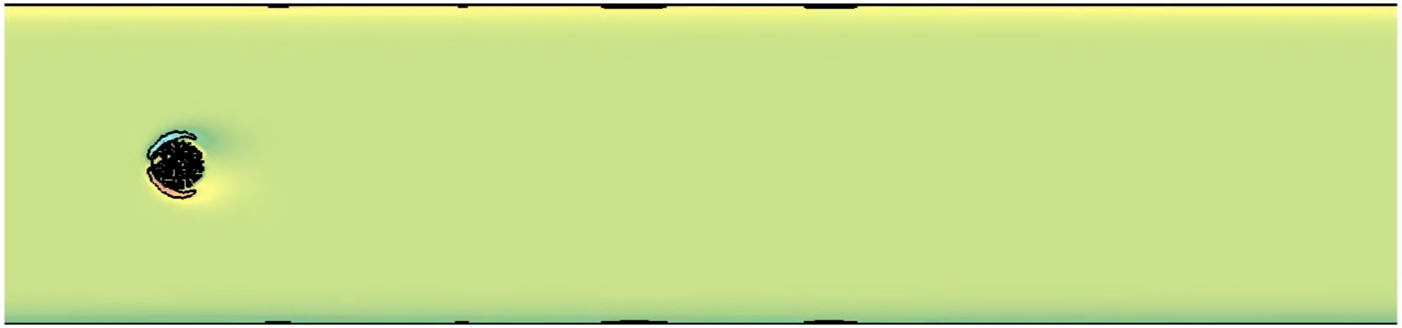
- **VF = void fraction**
- Higher VF = lower amount of grid cells blocked out for geometry
- ***Porosity is modeled randomly***; no particular void networks through cylinder.



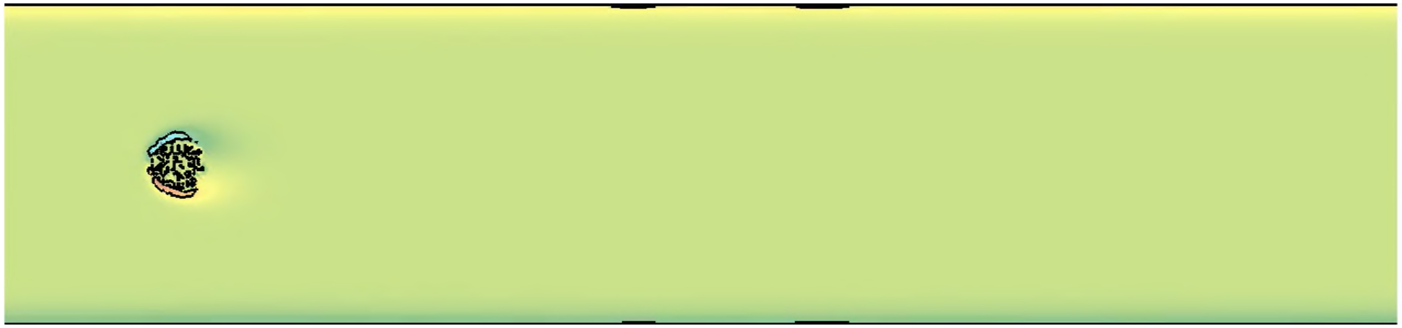
Flow around POROUS cylinders:

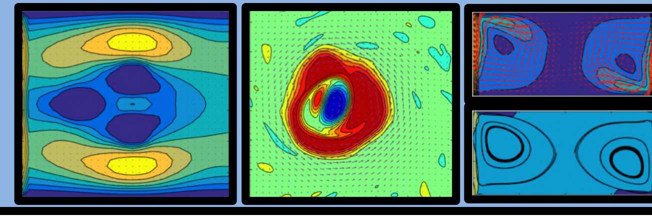
VF = 0%
(solid)

VF = 25%

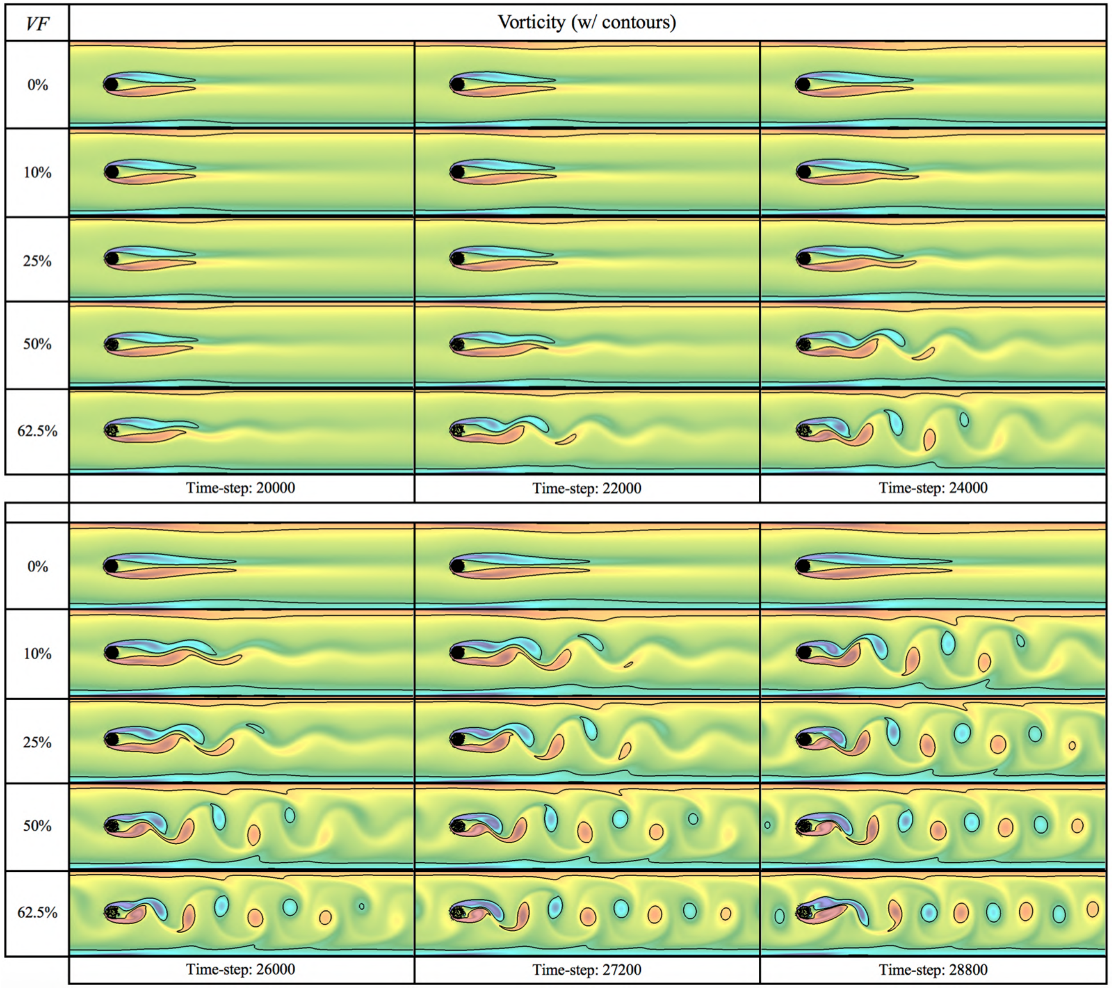


VF = 62.5%

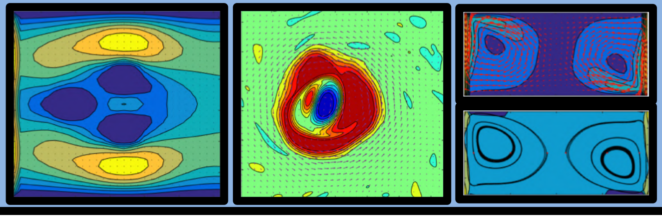




Flow around POROUS cylinders:



- There is a quicker transition to vortex shedding in all cases of porous cylinders when compared to the solid cylinder
- Cases with higher VF, see such transition earlier than lower cases



Lattice Boltzmann Method (LBM)

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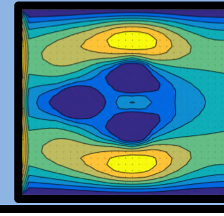
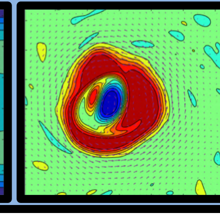
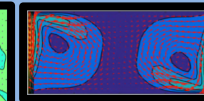
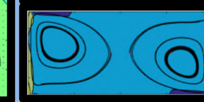
***Overview of Numerical Scheme ***

Lattice Boltzmann

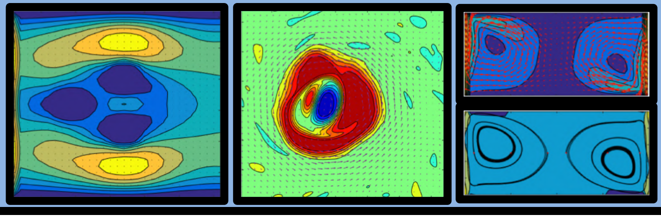






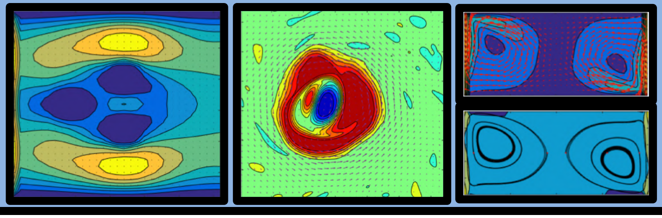


Lattice Boltzmann



- **Uses discrete Boltzmann equations to model fluid dynamics (does not explicitly solve Navier-Stokes)**
  - Think of it as tracks fictitious particles of fluid in the flow
  - Then we have a constitutive propagation relation and consider collision processes over a discrete lattice mesh

Lattice Boltzmann



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$$\frac{\partial f}{\partial t} + \vec{u} \cdot \nabla f \quad = \quad \Omega$$

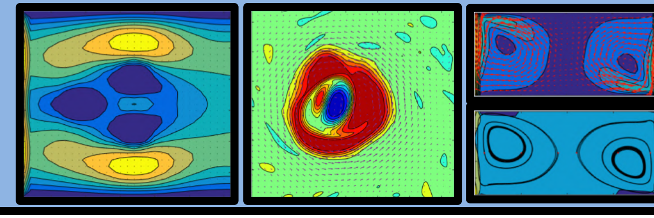
Collision Operator

Boltzmann Transport Eq.

$$f_i(\vec{x}, t)$$

Particle Distribution Function
(probability density)

Lattice Boltzmann



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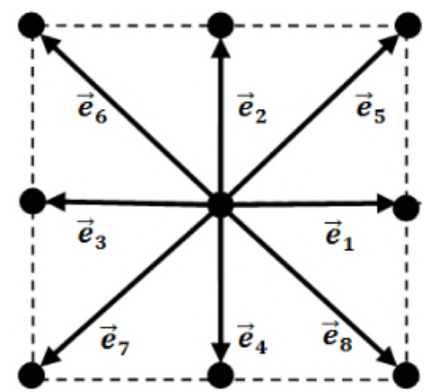

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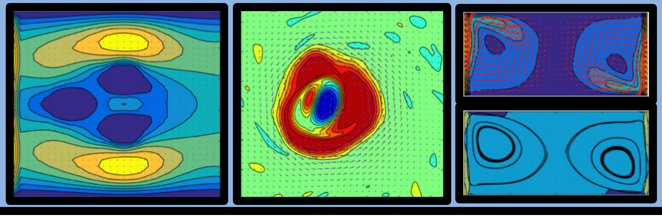
Lattice Boltzmann



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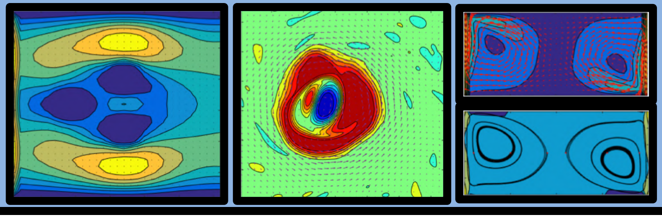
$$\rho(\vec{x},t) = \sum_{i=0}^{8} f_i(\vec{x},t) \qquad \vec{u}(\vec{x},t) = \frac{1}{\rho}\sum_{i=0}^{8} c f_i \vec{e}_i$$

- **Steps:**

  1. Stream the particle densities propagate in each direction
  2. Find an equivalent "equilibrium" density
  3. Relax the densities towards that EQ. state, in proportion governed by τ ('viscosity')

Lattice Boltzmann



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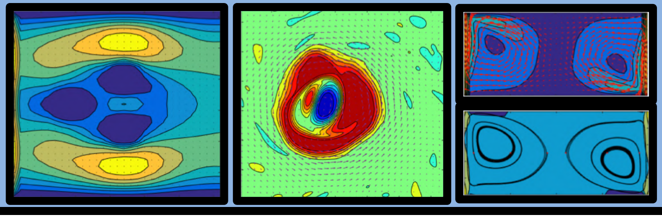
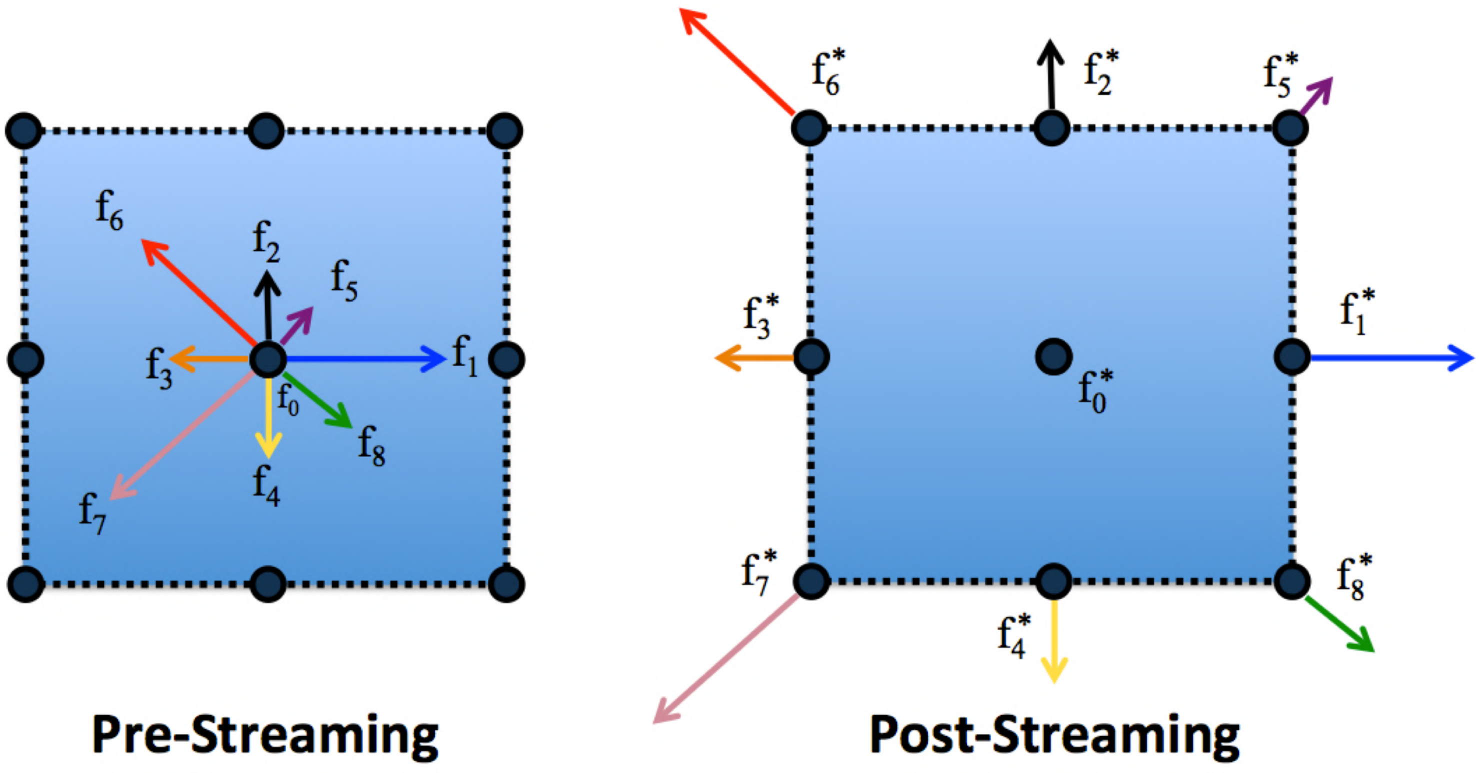
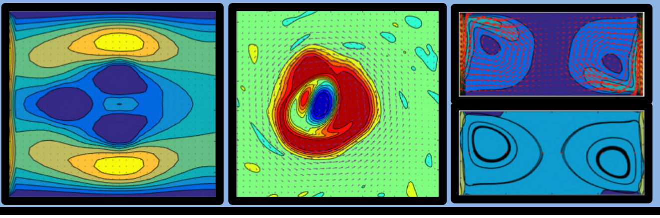


- **Steps:**
    1. Stream the particle densities propagate in each direction



Lattice Boltzmann

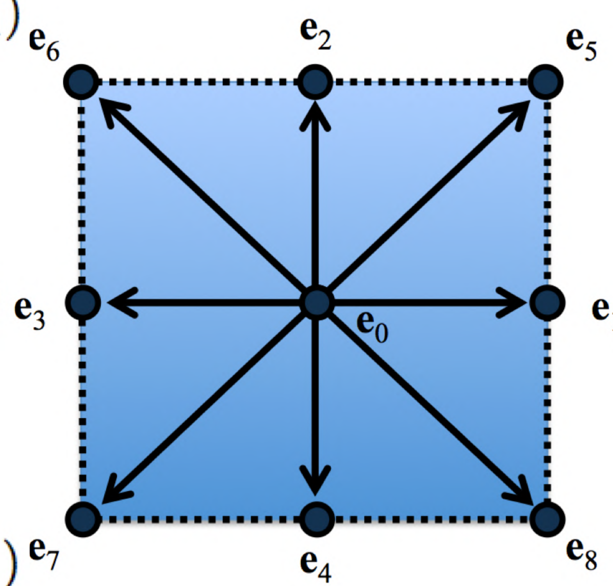


- **Steps:**
    1. Stream the particle densities propagate in each direction

$$f_2^*(x_i, y_j) = f_2(x_i, y_{j-1})$$

$$f_6^*(x_i, y_j) = f_6(x_{i+1}, y_{j-1})$$

$$f_5^*(x_i, y_j) = f_5(x_{i-1}, y_{j-1})$$



$$f_3^*(x_i, y_j) = f_3(x_{i+1}, y_j)$$

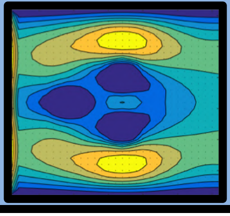
$$f_1^*(x_i, y_j) = f_1(x_{i-1}, y_j)$$

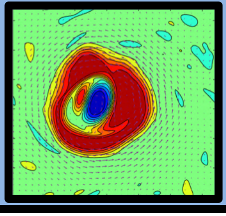
$$f_7^*(x_i, y_j) = f_7(x_{i+1}, y_{j+1})$$

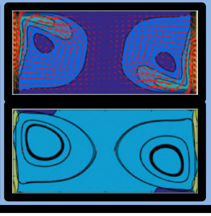
$$f_8^*(x_i, y_j) = f_8(x_{i-1}, y_{j+1})$$

$$f_4^*(x_i, y_j) = f_4(x_i, y_{j+1})$$

Lattice Boltzmann







- **Steps:**

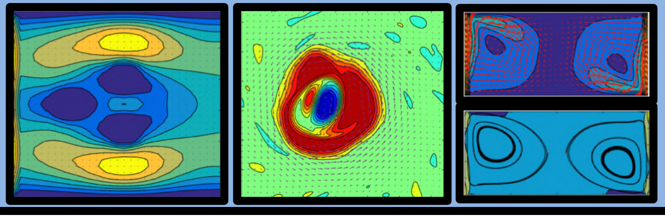
  2. Find an equivalent "equilibrium' density (BGK Collision Model, PRL 1954)

$$f_i^{eq}(\vec{x},t) \quad = \quad w_i\rho + \rho s_i(\vec{u}(\vec{x},t))$$

$$s_i(\vec{u}) \quad = \quad w_i\left[3\frac{\vec{e}_i\cdot\vec{u}}{c} + \frac{9}{2}\frac{(\vec{e}_i\cdot\vec{u})^2}{c^2} - \frac{3}{2}\frac{\vec{u}\cdot\vec{u}}{c^2}\right] \qquad c = \frac{\Delta x}{\Delta t}$$

$$w_i \quad = \quad \begin{cases} 4/9 & i=0 \\ 1/9 & i=1,2,3,4 \\ 1/36 & i=5,6,7,8 \end{cases}$$

Lattice Boltzmann



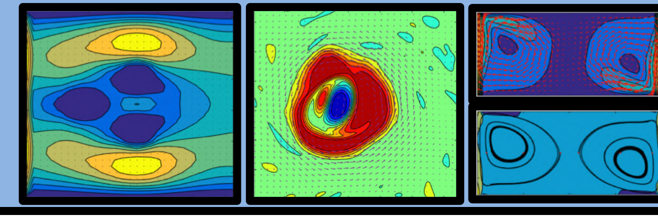
- **Steps:**

  3. Relax the densities towards that EQ. state, in proportion governed by τ, (BGK Collision Model, PRL 1954)

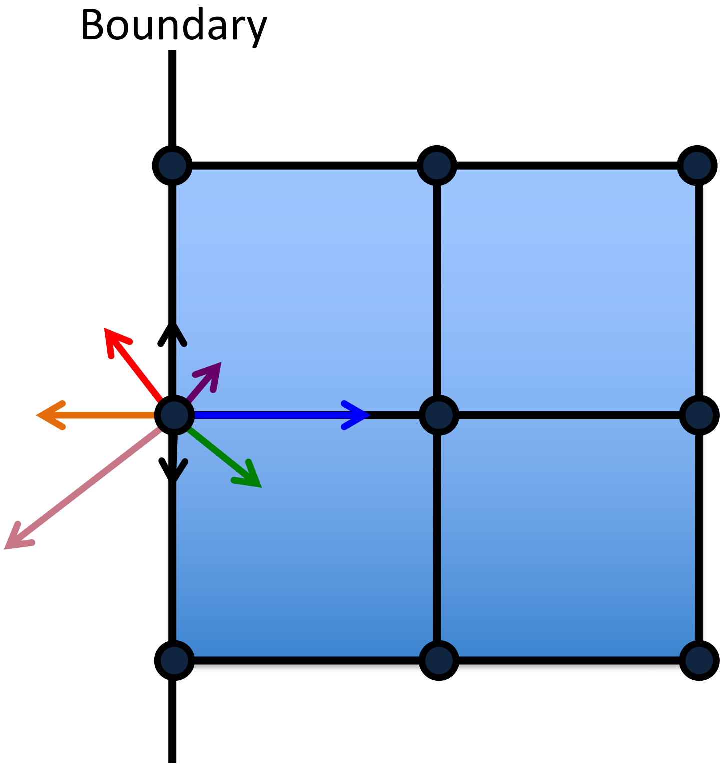
$$f_i = f_i^* - \frac{f_i^* - f_i^{eq}}{\tau}$$

$$\nu = \frac{2\tau - 1}{6} \frac{(\Delta x)^2}{\Delta t}$$

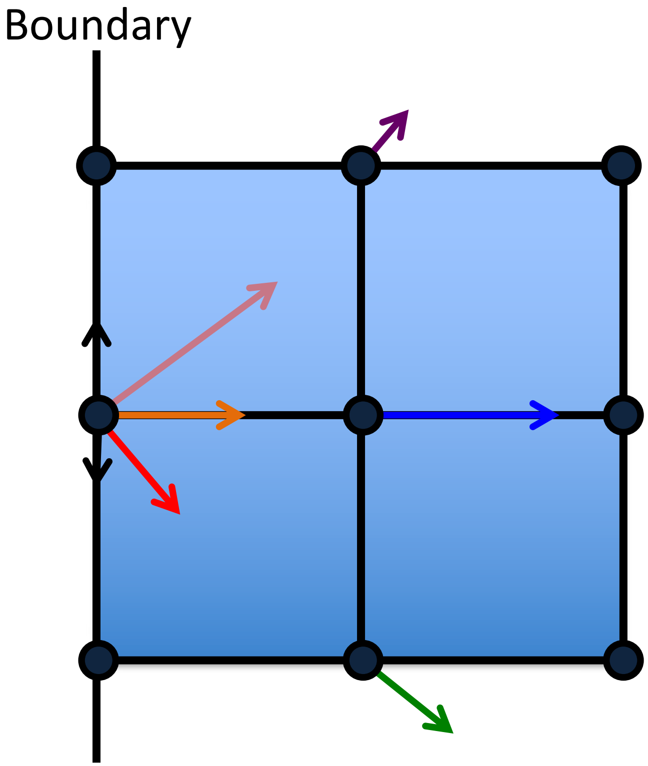
Lattice Boltzmann



Boundary Conditions: “*Reflective Conditions*”

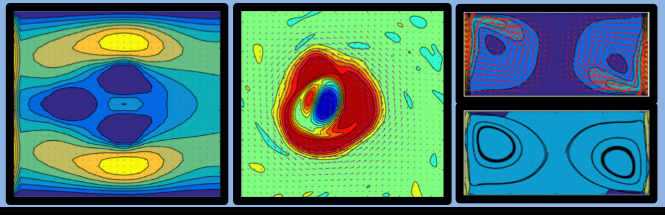


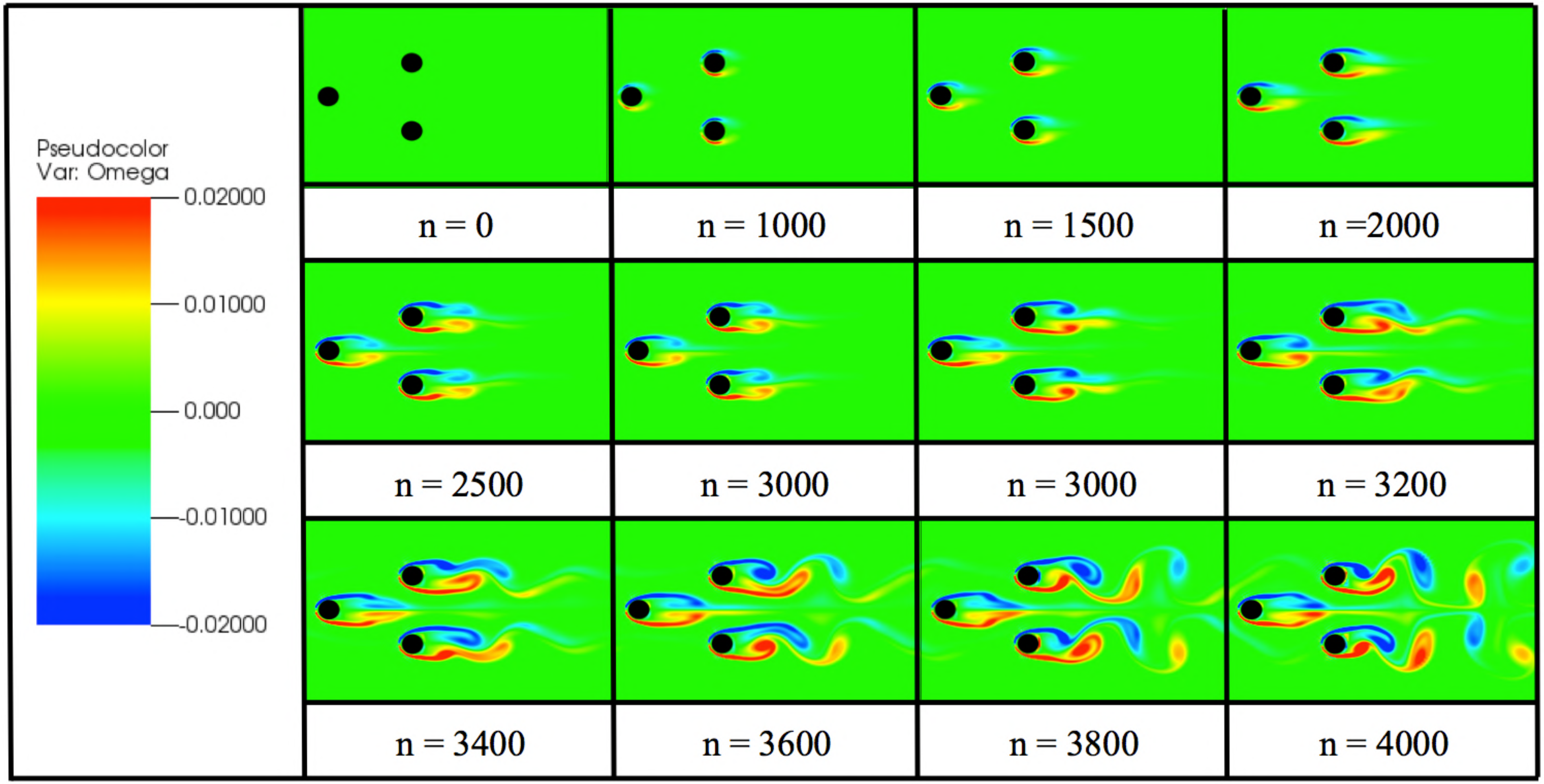
Pre-Streaming



Post-Streaming

Lattice Boltzmann





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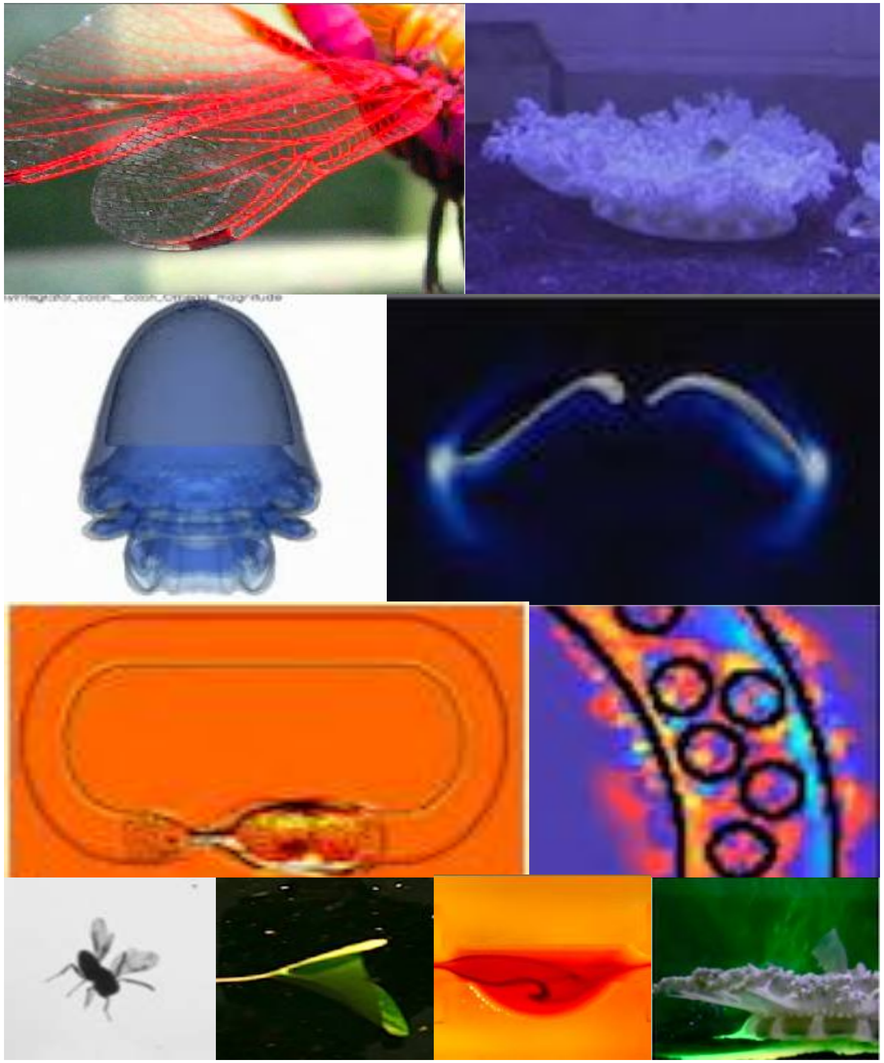

Questions?

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