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Generation of Localised Vertical Streams in Unstable Stratified Atmosphere

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Abstract: A new model of axially symmetric concentrated vortex generation was developed herein. In this work, the solution of a nonlinear equation for internal gravity waves in an unstable stratified atmosphere was obtained and analysed in the framework of ideal hydrodynamics. The related expressions for the velocities in the inner and outer regions of the vortex were described by Bessel functions and modified zeroth-order Bessel functions. The proposed new nonlinear analytical model allows the study of the structure and dynamics of vortices in the radial region. The formation of jets (i.e., structures elongated in the vertical direction with finite components of the poloidal (radial and vertical) velocities that grow exponentially in time in an unstable stratified atmosphere) was also analysed. The characteristic growth time was determined by the inverse growth rate of instability. It is shown that a seed vertical vorticity component may be responsible for the formation of vortices with a finite azimuthal velocity.

Keywords: vortex generation; nonlinear vortex structures; atmospheric instabilities

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1. Introduction

The existence of vortex structures in the atmosphere is one of the main factors that determine the weather and climate, as a result of the interaction of vortices of different topology and scale. In the variety of atmospheric vortex motions, mesoscale concentrated vortices are clearly defined and attract a lot of interest in both fundamental research and practice. Concentrated vortices (CVs) are non-stationary, vertically elongated vortex structures localised in space with a characteristic transverse scale from a few meters to hundreds of meters. CVs include dust devils (DDs), tornadoes (more intense and larger-scale vortices) [1–8], water jets (or waterspouts) which can be observed in the sea or large lakes [9–11] and fire tornadoes (fiery devils or fire whirls) which may suddenly appear in fires during calm weather [12–14]. In contrast to DDs, which carry dust particles, water jets involve water droplets into the vertical vortex motion. Despite the fact that vortices of this class arise in different media and are generated by different natural mechanisms, they all experience ascending helicoidal motion. The rotational speed in a CV reaches its maximum value at the characteristic radius of the vortex and tends towards zero at its periphery. DDs, as the simplest and easily observed CVs, are of particular interest for studying the entire class of CVs in the atmospheres of the Earth and Mars.

By analysing DD observational data, Sinclair [15,16] suggested that the necessary conditions for their occurrence are the presence of dust in the near-surface atmospheric layer and anomalously high ground temperatures. This is consistent with recent models [8,17,18] in which DDs are formed from convective cells (jets) in an unstable near-surface layer with a super-adiabatic temperature gradient. A number of observations [8,14] have shown that the generation of clockwise (anticyclonic) and counter-clockwise (cyclonic) eddies in an open area is equally probable. From the observed lack of correlation between the external vorticity, the generation time, and the vortex diameter, it follows that the external vorticity in the atmosphere alone is insufficient for the generation of DDs. Meteorological observations [16,19] served as the basis for the creation of the first thermodynamic model of DDs generation [20–22]. In this model, the warm air in a convectively unstable atmosphere rises and, later, undergoing cooling, descends. The proposed model is an analogue of a heat engine that draws energy from a hot surface layer.

Despite a substantial number of previous studies, the mechanism of generation and interpretation of the observed vortical structures remains uncertain. Recently, [23] proposed a hydrodynamic model for axially symmetric convective vortices (by assuming weak disturbances) in a convectively unstable atmosphere at the initial stage of generation. In the studies by, e.g., [24–29], this model was further developed for finite amplitudes of velocities with two-dimensional helical motion and different cases of stream functions and seed azimuthal velocities. However, these models were still restricted to the analysis of the radial and vertical velocity components of poloidal motion either in a very narrow central part or far at the periphery of the convective cell. The purpose of the current work was to expand the analytical model used to describe the dynamics of an axially symmetric vortex to an arbitrary radial distance from the centre. To achieve this, a solution was obtained in the form of Bessel functions (instead of power and exponential functions) using the method employed to find stationary large-scale dipole vortices of Rossby waves in a neutral atmosphere [30].

The paper is organised as follows: in Section 2, we derive the simplified equations for nonlinear internal gravity waves (IGWs) in an unstable stratified atmosphere; Section 3 discusses a new model of jet generation, and Section 4 examines the proposed model. In the conclusions, the main results of our study are discussed.

2. Reduced Equations

Meteorological observations were served as the foundation for the creation of the first thermodynamic models of vertical streams (convective cells) generation [18,22,31]. Currently, modern concepts of the generation of vertical streams are associated with unstable stratified atmospheres. The atmosphere is considered unstably stratified if the square of the Brunt–Väisälä or buoyancy frequency:

$$\omega_g^2 = g\left(\frac{\gamma_a - 1}{\gamma_a H} + \frac{1}{T}\frac{dT}{dz}\right), \tag{1}$$

characterising the IGWs is negative. Here, g is the gravitational acceleration, γ_a is the ratio of specific heats, H is the local scale of height of the atmosphere, T and dT/dz are the fluid temperature and temperature gradient in the vertical direction, respectively. Owing to the solar heating of the soil, the vertical temperature gradient (the second term in the Brunt–Väisälä frequency) is negative and its magnitude exceeds the first term. The latter corresponds to the famous Schwarzschild criterion for convective instability. In this case, the IGWs change to unstable, exponentially growing cells.

When deriving the governing equation, we will follow the works of [23,32]. As an initial set of equations, we used the ideal momentum Equation (neglecting viscosity) that can be written as

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{g}, \tag{2}$$

and the transport equation for the potential temperature, θ, that is a unique function of entropy which can be written as

$$\frac{d\theta}{dt} = 0, \tag{3}$$

where we neglected non-ideal effects such as thermal conduction and any additional heating/cooling processes. In the above equations, ρ and p denote the density and pressure, respectively, $\mathbf{v}$ is the velocity of the matter, $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ is the Euler (convective) time derivative, $\mathbf{g} = -g\hat{\mathbf{z}}$ is the gravitational acceleration, $\hat{\mathbf{z}}$ is the unit vector along the vertical axis, and $\theta = p^{1/\gamma_a}/\rho$. To complete our set of equations, we used the ideal gas law $p/\rho T = const$.

Following the procedure developed by [27,32–34], we can derive a reduced equation for nonlinear IGWs. To do this, we introduced a cylindrical coordinate system (r, ϕ, z) with the z axis in the vertical direction and assumed that $\partial/\partial\phi = 0$. The most general divergence-free flow velocity $\mathbf{v} = (v_r, v_\phi, v_z)$ can be decomposed into its poloidal $\mathbf{v}_p = (v_r, 0, v_z)$ and azimuthal $v_\phi \hat{\mathbf{e}}_\phi$ parts, i.e., $\mathbf{v} = \mathbf{v}_p + v_\phi \hat{\mathbf{e}}_\phi$. Here, $\mathbf{v}_p = \nabla \times (\psi \nabla \phi)$, $\hat{\mathbf{e}}_\phi$ is the respective unit vector and ϕ is the angle of the cylindrical set. The poloidal components of the velocity are related to the stream function $\psi(t, r, \phi, z)$ by means of the relations:

$$v_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}, \qquad v_z = \frac{1}{r}\frac{\partial \psi}{\partial r}. \tag{4}$$

According to [23–25,27,34], the reduced equation describing the evolution of nonlinear internal gravity waves (IGW) is given by

$$\left(\frac{\partial^2}{\partial t^2} + \omega_g^2\right)\Delta^* \psi + \frac{1}{r}\frac{\partial}{\partial t} J(\psi, \Delta^* \psi) = 0. \tag{5}$$

where $J(a, b) = (\partial a/\partial r)\partial b/\partial z - (\partial a/\partial z)\partial b/\partial r$ is the Jacobian and the operator Δ^* is defined as

$$\Delta^* = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right). \tag{6}$$

The Jacobian in Equation (5) corresponds to the so-called vector nonlinearity:

$$J(\psi, \Delta^* \psi) = [\nabla \psi \times \nabla \Delta^* \psi]_\phi.$$

If $\omega_g^2 < 0$, Equation (3) describes nonlinear dynamics IGW in unstably stratified atmosphere. Note that an equation similar to Equation (5) was previously obtained by [34] for the interpretation of behaviour of acoustic gravity vortices.

3. Jet Generation

The scalar stream function that can generate the components of the velocity will be chosen in the form

$$\psi(t, r, z) = v_0 r^2 (z/L) \exp(\gamma t) \Psi(\delta R), \tag{7}$$

where $v_0 = const$ is the characteristic vortex velocity; $\gamma = |\omega_g|$; $R = r/r_0$, $L = const$ is the characteristic spatial scale in the vertical direction such that $L \ll H$; Ψ is a function that depends on the radial distance (subsequently determined) and $\delta = const$. Of course, the choice of the stream function in this form is not unique, however, the function has to satisfy the conditions that the three components of the velocity as well as pressure are regular on the symmetry axis of the vortex. Moreover, for analytical progress, we also require that the function has a separable form. With this stream function, Equation (5) is reduced to:

$$J(\psi, \Delta^* \psi) = 0. \tag{8}$$

The nonlinear solution of Equation (8) can be reduced to the linear solution of the form

$$\Delta^* \psi = A\psi, \tag{9}$$

where the quantity A is a constant. The stream function considered here has to remain localised in the radial direction, therefore, it must satisfy the conditions:

$$\left(\psi,\ \frac{\partial\psi}{\partial r}\right) \to 0 \tag{10}$$

when $r \to 0$ and $r \to \infty$, i.e., the function has to be regular along the symmetry axis of the cylinder and vanish at infinity. To find a solution to Equation (8) satisfying these boundary conditions, we used the method proposed by [30,33] to study large-scale stationary vortices. Applying the Δ^* operator on the stream function given by Equation (7), we obtain:

$$\Delta^*\psi = v_0 r_0^2 (z/L)\exp(\gamma t)\left(\delta^2 R^2 \frac{d^2\Psi}{dR^2} + 3\delta R \frac{d\Psi}{dR}\right). \tag{11}$$

Making use of Equations (9) and (11), this results in the following linear equation for the function Ψ:

$$\delta^2 R^2 \frac{d^2\Psi}{dR^2} + 3\delta R \frac{d\Psi}{dR} = \pm\delta^2 R^2 \Psi. \tag{12}$$

The solution of the above equation can be given in terms of Bessel functions. It can be easily shown that regular solutions at $R = 0$ can be obtain only when $\delta = 3$. In this case, Equation (12) reduces to the Bessel or modified Bessel equations of zeroth order. In order to satisfy conditions (10), we seek a solution for Equation (12) by means of the method of joining two continuous solutions in the internal ψ_{int} $(r < r_0)$ and external ψ_{ext} $(r > r_0)$ regions. On the boundary of the vortex at $r = r_0$, the continuity condition:

$$\left(\psi,\ \frac{\partial\psi}{\partial r}\right)_{int} = \left(\psi,\ \frac{\partial\psi}{\partial r}\right)_{ext}, \tag{13}$$

must be satisfied. In the external region, we seek a solution to Equation (12) in the form:

$$\Psi_{ext}(R) = \frac{K_0(3R)}{K_0(3)} \tag{14}$$

that satisfies the conditions (10). In the internal region, the solution will be sought in the form:

$$\Psi_{int}(R) = m - (m-1)\frac{I_0(3R)}{I_0(3)}, \tag{15}$$

where $I_0(z)$ and $K_0(z)$ are the Bessel and modified Bessel functions and m is a constant parameter. Hence, the solutions (14) and (15) satisfy the condition $\psi_{ext} = \psi_{int}$ when $r = r_0$. From the second continuity condition given by Equation (13) for $\partial\psi/\partial r$ at the interface $r = r_0$, we have:

$$\frac{K_1(3)}{K_0(3)} = (m-1)\frac{I_1(3)}{I_0(3)}. \tag{16}$$

This equation allows us to determine the value of the constant m, so that:

$$m = 1 + \frac{K_1(3)}{K_0(3)}\frac{I_0(3)}{I_1(3)} \simeq 2.4271.$$

The spatial dependence of $\Psi(R)$ and its "smoothness" at the boundary $R = 1$ are shown in Figure 1.

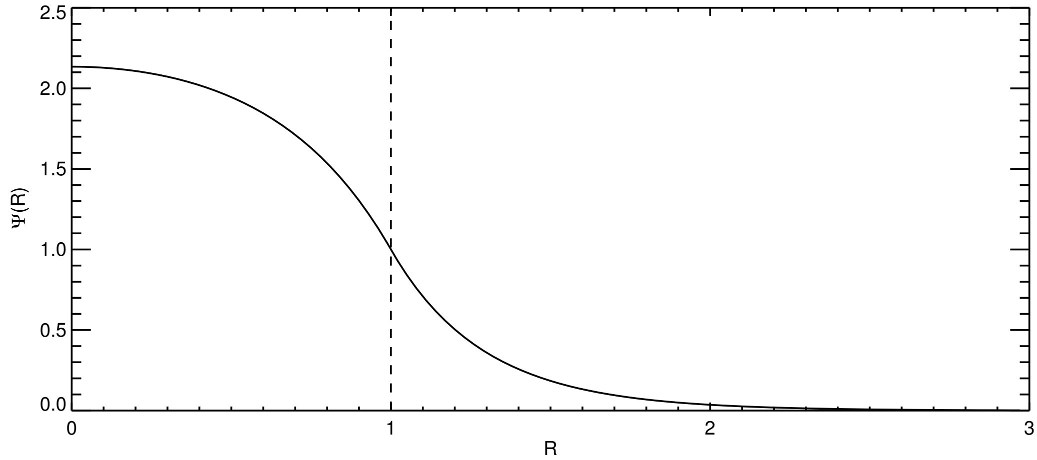

Figure 1. The variation of the stream function $\Psi(R)$ with the dimensionless radial distance, R.

Then, making use of Equations (4), (7) and (14) or (15), the radial velocity in the internal ($0 \le R < 1$) and external regions ($1 \le R < \infty$) take the form:

$$v_r^{int} = -v_0 \frac{r_0}{L} R \exp(\gamma t) \left[m - (m-1) \frac{I_0(3R)}{I_0(3)} \right], \tag{17}$$

$$v_r^{ext} = -v_0 \frac{r_0}{L} R \exp(\gamma t) \frac{K_0(3R)}{K_0(3)}. \tag{18}$$

The radial variation of v_r in the internal and external regions (in units of v_0) is shown in Figure 2 for different values of the exponential increment term, γt, for the particular value of $r_0/L = 0.1$. This demonstrates that the solutions and their derivatives are continuous at the boundary and v_r is regular at the symmetry axis and vanishes far away from the structure.

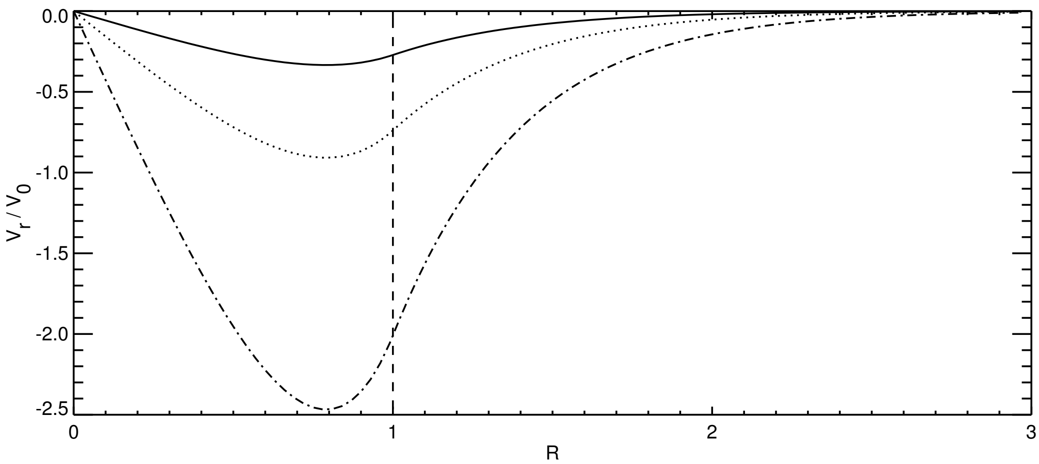

Figure 2. $V_r/V_0(R)$. Solid, dot and dash–dot lines correspond to $\gamma t = 1, 2, 3$, respectively. $r_0/L = 0.1$.

Similarly, the expressions for the vertical velocity in the internal ($0 \le R < 1$) and external regions ($1 \le R < \infty$) can be written as

$$v_z^{int} = v_0 \frac{z}{L} \exp(\gamma t) \left[2m - 2(m-1) \frac{I_0(3R)}{I_0(3)} - 3(m-1) R \frac{I_1(3R)}{I_0(3)} \right], \tag{19}$$

$$v_z^{ext} = v_0 \frac{z}{L} \exp(\gamma t) \left[2 \frac{K_0(3R)}{K_0(3)} - 3R \frac{K_1(3R)}{K_0(3)} \right]. \tag{20}$$

Such a structure of the poloidal fluid motion of convective cells describes exponentially growing vertical streams (or jets) in time. The variation of the vertical component of the velocity component (in units of v_0 with respect to the dimensionless radial distance from the centre, $R = r/r_0$) is illustrated in Figure 3 for three values of the exponential increment γt.

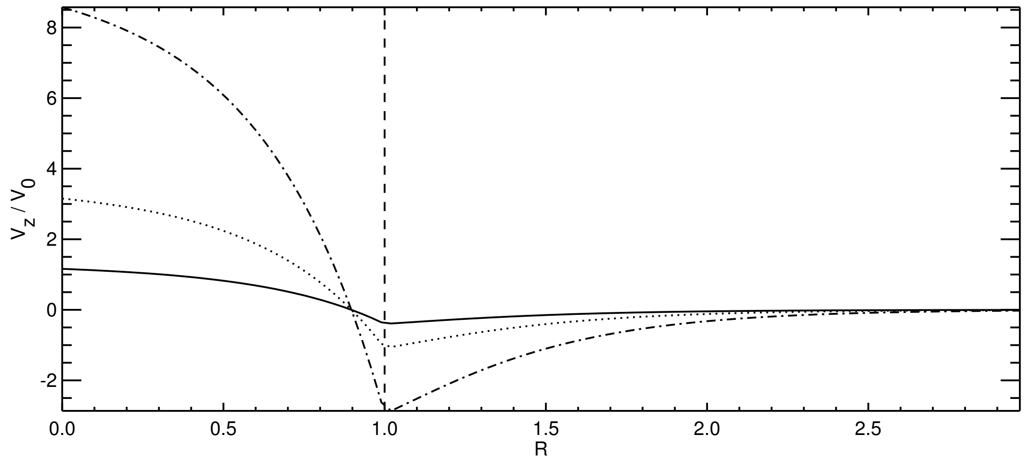

Figure 3. The variation of the dimensionless vertical component of the velocity, V_z/V_0, in terms of the dimensionless radial distance $R = r/r_0$. The solid, dot, and dash–dot lines correspond to γt = 1, 2, 3, and $z/L = 0.1$, respectively.

4. Vortex Model Generation

To study the generation of the vortex motion, we used the azimuthal component of the momentum Equation (taking into account the fact that $\partial/\partial\phi = 0$):

$$\frac{\partial v_\phi}{\partial t} + \frac{v_r}{r}\frac{\partial}{\partial r}(r v_\phi) + v_z\frac{\partial v_\phi}{\partial z} = 0, \tag{21}$$

where the radial and vertical components of the velocity are given by Equations (17–20). To determine the temporal and spatial evolution of the azimuthal component of the velocity, we provided a seed azimuthal velocity of the form:

$$v_{\phi 0}(t = 0, r, z) = v_{\phi 00}\frac{r}{a}. \tag{22}$$

Here, $v_{\phi 00}$ and a are the characteristic azimuthal velocity and spatial scale of the seed azimuthal velocity. Using Equation (21), the expression for azimuthal velocity in the internal ($0 \le R < 1$) and external ($1 \le R < \infty$) vortex region become:

$$v_\phi^{int} = \frac{2v_{\phi 00}}{\gamma a}\frac{r_0}{L}R\exp(\gamma t)\left[m - (m-1)\frac{I_0(3R)}{I_0(3)}\right], \tag{23}$$

$$v_\phi^{ext} = \frac{2v_{\phi 00}}{\gamma a}\frac{r_0}{L}R\exp(\gamma t)\frac{K_0(3R)}{K_0(3)}. \tag{24}$$

Figures 2–4 demonstrate the exponential localisation of the flow in the radial direction. In particular, Figure 2 shows the dependence of the radial component of the normalised flow velocity (v_r/v_0) as a function of the dimensionless quantity R for three different values of γt and $r_0/L = 0.1$. The radial velocity converges at the axis of symmetry and reaches the maximum value at a radial distance of $R \approx 0.8$. Figure 3 shows the dependence of normalised axial flow velocity (v_z/v_0) with respect to the same dimensionless quantity R for $z/L = 0.1$ and different values of γt which increment in accordance with Equations (19) and (20). It can be seen that v_z/v_0 reaches its maximum value at the centre of the jet. In the region $R \approx 0.8$, the axial velocity component vanishes, and at the region $R > 0.8$, the ascending flow in the centre of the jet transforms into a descending one, reaching maximum values at $R = 1$. Figure 4 illustrates the dependence of the azimuthal velocity component v_ϕ / v_0 with respect to R for three different values of the γt increment at accepted values of $v_{\phi 00}/\gamma a$ (as can be seen, e.g.,in Equations (23) and (24)). It can be seen that the azimuthal velocity reaches its maximum values at $R \approx 0.8$.

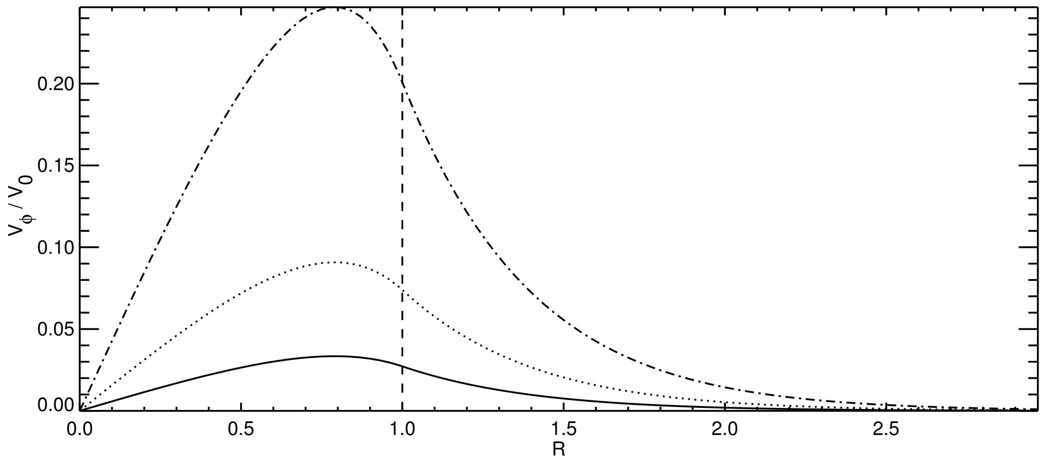

Figure 4. V_ϕ/V_0. Solid, dot, and dash–dot lines correspond to $\gamma t = 1, 2, 3$, respectively, $2V_{\phi 00}/\gamma a = 0.1$, $r_0/L = 0.1$.

5. Conclusions

In the present study, we obtained a nonlinear equation for IGWs in an unstable stratified atmosphere for axially symmetric structures exponentially growing in time in the framework of ideal hydrodynamics. It was shown that this equation can be reduced to a simpler equation that still contains a vector nonlinearity. However, the proposed stream function, $\Psi(R)$, allows the reduction in the nonlinear equation to a modified Bessel equation. By matching solutions at the boundary of the convective cell, separating the internal and external regions with their own dynamics, an analytical solution was obtained for the entire radial distances R. Therefore, the proposed model makes it possible to analyse exponentially localised structures of poloidal fluid motion, e.g., exponentially growing vertical jets in time, for any value of R. The region of applicability of the proposed model is only limited to a relatively thin atmospheric layer, where convective instability develops but can be expanded to explain the generation of high-speed astrophysical jets or jets in the solar corona; however, this will be the subject of future research.

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Conflicts of Interest: The authors declare that they have no conflict of interest.

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