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Article

Stochastic Equations of Hydrodynamic Theory of Plasma

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Abstract: Stochastic equations of the hydrodynamic theory of plasma are presented in relation to strong external fields. It is shown that the use of these stochastic equations makes it possible to obtain new theoretical solutions for plasma as a result of its heating in a strong external electric field. Theoretical solutions for the conductivity of turbulent plasma when heated in an external electric field of 100 V/cm are considered. Calculated values for the electron drift velocity, electron mobility, electron collision frequency, and the Coulomb logarithm in the region of strong electric fields are obtained. Here we consider experiments on turbulent heating of hydrogen plasma in the range of electric field strength of 100 < E < 1000. The calculated dependences of plasma conductivity are in satisfactory agreement with experimental data for heating plasma in a strong electric field. It is shown that the plasma turbulence in the region of strong electric fields E ~1000 V/cm is close to 100%. For the first time, it is confirmed that the derived dependences for collision frequency, drift velocity, and other values include the degree of turbulence of plasma, which makes it possible to correctly describe experimental data for heating plasma even with strong electric fields. In addition, it was determined that the scatter of experimental data may be associated with the variability of the function in the expression for the heat flux density. For the first time, it is shown theoretically that the experimentally determined fact of the possibility of the existence of an approximate constancy of plasma conductivity in the region E = 100–1000 V/cm can occur with an error of ~30%. The results show significant advantages of the stochastic hydrodynamic plasma theory over other methods that are not yet able to satisfactorily as well as qualitatively and quantitatively predict long-known experimental data while taking into account the degree of turbulence.

Keywords: stochastic equations; equivalence of measures; hydrodynamic theory of plasma

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1. Introduction

The stochastic equations of the hydrodynamic theory of plasma are presented in relation to strong external fields. The system of stochastic plasma equations is used for calculating the heating of turbulent hydrogen plasma in strong external electric fields. The problem of finding equations for solving the turbulence [1–4] is an important theoretical problem. The phenomenon of turbulence was tried to be solved using different ideas: (1) using the theory of attractors [5–11]; (2) on the basis of chaos theory [12,13]; (3) on the basis of the physics of nonlinear phenomena [14,15]; (4) on the basis of statistic theory [16–24]; (5) the theory of solitons [1]; (6) a quasi-periodic process [25–28]; (7) using the theory of self-organization [29,30]. Attempts to find solutions to this problem by numerical methods were made using DNS [31–45], LES, and RANS codes [46–51].

The processes of heating plasma by an electric field are connected with the turbulence in plasma [52–63]. The statistical theory of plasma physics [64–70] is used for describing these processes. But these processes can be determined using the theory of stochastic equations and equivalent measures [71–73]. In [74–80], the possibility of calculating critical Reynolds numbers was shown for isothermal flows [81–86] and for non-isothermal flows [87–90]. The profiles of averaged velocity and temperature were also determined in [91–93].The friction coefficients and heat transfer coefficients were obtained in [84,85].

Also, the second-order correlations were presented in [91,93–98]. The correlation dimension of the attractor in the boundary layer was calculated in [99,100]. The equation for the spectral function was shown in [101–106]. For non-isothermal flows, the Reynolds analogy was obtained in [104]. Then the formulas for friction coefficients on the wall of a flat plate and in a round tube during a laminar–turbulent transition [107,108] were derived.

2. Definition of Equivalence of Measures between Deterministic and Random Motions

It should be noted that the problem of transition from laminar to turbulent flow was discussed in [71–74,81–108]. As a result, fluid and gas flows around a cylinder, as well as in the boundary layer on a sphere, on a flat plate, in a pipe, and in a jet, were studied on the basis of stochastic theory. It is also known that a statistical apparatus developed for a continuous medium [32–35] is used for the hydrodynamic description of the turbulent plasma motion [32–35]. Here, on the basis of stochastic equations and equivalence relations of measures [71–73] developed for the continuum, the new results for turbulent plasma motion are presented. The correlator $D_{N,M}$ was derived in [71–74,99–104]; see also [105,108] as the definition of equivalence of measures between laminar (deterministic) and turbulent (random) motions. The application of the correlator $D_{N,M}$ for solving equations for mass, motion, and energy leads to sets of stochastic equations for four space–time areas: (1) the onset of generation (subscript 1, 0, or 1); (2) the generation (subscript 1, 1); (3) the diffusion (subscript 1, 1, 1) and (4) the dissipation of the turbulent fields. In the critical point $r_i \to r_c$; $\Delta\tau_i \to \tau_c$ for the parameter $m_i \to m_c$, and for each of four space–time regions of the correlator. The correlator $D_{M,N}(r_c; m_{cj}; \tau_c) = D_{1,0}(r_c; m_{cj}; \tau_c)$ [72–75,100–105] for the pair (M,N) = (1,0) gives the following equations $(d(\Phi)_{colst})_{1,0} = -R_{1,0}(\Phi_{st})$; $\left(\frac{d(\Phi)_{colst}}{d\tau}\right)_{1,0} = -R_{1,0}\left(\frac{\Phi_{st}}{\tau_{cor}}\right)$, here"d" is the full differential. Then the correlator for the pair (M,N) = (1,1), $D_{M,N}(r_c; m_{cj}; \tau_c) = D_{1,1}(r_c; m_{cj}; \tau_c)$ gives the following equations: $(d(\Phi)_{colst})_{1,1} = -R_{1,1}d(\Phi_{st})$, $\left(\frac{d(\Phi)_{colst}}{d\tau}\right)_{1,1} = -R_{1,1}\left(\frac{d\Phi_{st}}{d\tau}\right)$. Here Φ is the substantial quantity {mass (density ρ), momentum (ρU), and energy (E)}. Here $(\Phi)_{colst}$ is the deterministic component (subscript cost) having the zero stochastic component of measure, (Φ_{st}) is the stochastic component (subscript st), $\tau_{cor} = \frac{(U_{st})}{L}$ is the lifetime of Φ_{st}, L = 2π/k, and k is the wave number. The subscripts "cr" or "c" refer to the critical point r (x_{cr}, τ_{cr}) or r_c. The critical point is the space–time point of the onset of the interaction between the deterministic and random motions, which leads to the turbulence. It is important to emphasize that one of the main differences between statistical and stochastic theory is the number of regions inspace–time. The statistic theory envelops only three space–time areas: (2) the generation; (3) the diffusion; and (4) the dissipation of turbulent fields [16–24].

3. Stochastic Equations for Plasma

In accordance with [71–73,83–108], the stochastic equations were obtained without external and internal forces. These forces are taken into account for the stochastic plasma equations in accordance with [52–58]:

$$\frac{d(m_\alpha n_\alpha)_{col}}{d\tau} = -\frac{(m_\alpha n_\alpha)_{st}}{t_{cor}} - \frac{d(m_\alpha n_\alpha)_{st}}{dt}, \tag{1}$$

$$\begin{aligned}\frac{d(m_\alpha n_\alpha u_{i\alpha})_{col}}{d\tau} = &-\frac{\partial}{\partial x_j}(\tau_{\alpha ij})_{col} - \frac{\partial}{\partial x_j}(\tau_{\alpha ij})_{st} + (F_{int(\alpha,\beta)} + F_{k(\alpha i)})_{col} \\ &+(F_{int(\alpha,\beta)} + F_{k(\alpha i)})_{st} - \frac{(m_\alpha n_\alpha u_{i\alpha})_{st}}{t_{cor}} - \frac{d(m_\alpha n_\alpha u_{i\alpha})_{st}}{dt},\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{d(m_\alpha n_\alpha e_\alpha)_{col}}{d\tau} = &-\frac{\partial}{\partial x_j}(q_{\alpha i} + u_{\alpha i}\tau_{\alpha ij})_{col} - \frac{\partial}{\partial x_j}(q_{\alpha i} + u_{\alpha i}\tau_{\alpha ij})_{st} + (u_{\alpha i}[F_{int(\alpha,\beta)} + F_{k(\alpha i)}])_{col} \\ &+(u_{\alpha i}[F_{int(\alpha,\beta)} + F_{k(\alpha i)}])_{st} - \frac{(m_\alpha n_\alpha e_{\alpha i})_{st}}{t_{cor}} - \frac{d(m_\alpha n_\alpha e_{\alpha i})_{st}}{dt} \pm \left(q_{j_\alpha,Vrad}\right)_{col,1}.\end{aligned} \tag{3}$$

α is the component of plasma (particles of class α, electron–ion liquid). The forces resulting from the action of normal P_α stresses (pressure) and tangential stresses $\pi_{\alpha ij}$ are written as $\tau_{\alpha ij} = P_\alpha + \pi_{\alpha ij}$ and, then, it is possible to write that

$$-\frac{\partial}{\partial x_j}(\tau_{\alpha ij}) = -\frac{\partial P_\alpha}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_\alpha\left(\frac{\partial u_{i\alpha}}{\partial x_j} + \frac{\partial u_{j\alpha}}{\partial x_i}\right) - \delta_{ij}\mu_\alpha\frac{2}{3}\frac{\partial u_{l\alpha}}{\partial x_l}\right]. \quad (4)$$

Here, i, j, l = 1, 2, 3, the coefficients μ and ξ are the dynamic and second viscosities, respectively. The values u_i, u_j, u_l,x_i, x_j, x_l are the velocities and coordinates corresponding to i, j, l. The Kronecker delta is δ_{ij} = 1 for i = j, and δ_{ij} = 0 for i $\neq$ j. $F_{k(\alpha i)}$ is the sum of k external forces acting on particles of class «α» in the «i» direction. $F_{k(\alpha i)} = \sum_{i=1}^{3}\sum_{k=1}^{n}(F_k)_i$, $k = 1, 2, \ldots, n$; $i = 1, 2, 3$. For an electromagnetic field $F_{k(\alpha i)} = (j\{E + [u \times B]\})_{\alpha i} = (Z_\alpha e n_\alpha)\{E + [u \times B]\}_{\alpha i}$.

In addition, $F_{int(\alpha,\beta)}$ are the internal (intercomponent or interphase) forces caused by the interaction of liquid components with each other. In the case of a heterogeneous mixture, these are the interphase forces; in the case of plasma, these are the forces caused by the collision of particles of various kinds with each other (subscript int): $F_{int(\alpha,\beta)} = m_\alpha n_\alpha(\delta u_\alpha \backslash \delta\tau) = n_\alpha \sum_\beta \left(R_{\alpha\beta} + R^T_{\alpha\beta}\right)$ [53–57,65,69]. The force of relative friction, depending on the relative velocity of electrons and ions is $R_{i\alpha\beta} = \mu_{\alpha\beta}\nu_{\alpha\beta}(u_{i\alpha} - u_{i\beta})$ and the thermal force: $R^T_{\alpha\beta i} = \mu_{\alpha\beta}(u_{i\alpha})(\delta\nu_{\alpha\beta}) = \frac{T_{\alpha\beta}}{\nu_{\alpha\beta}}\frac{\delta\nu_{\alpha\beta}}{\delta T_{\alpha\beta}} grad T_{\alpha\beta}$. Here $\mu_{\alpha\beta}$, $T_{\alpha\beta}$, $\nu_{\alpha\beta}$ are the reduced particle mass, the effective temperature, and the collision frequency. The equation for the energy $(m_\alpha n_\alpha e_\alpha)$ per unit volume of the plasma grade α components (m_α and n_α are the mass and concentration of particles).

Here $-\frac{\partial}{\partial x_j}(u_{i\alpha}\tau_{\alpha ij}) = -\frac{\partial}{\partial x_j}\left[-u_{i\alpha}P_\alpha + u_{i\alpha}\mu_\alpha\left(\left(\frac{\partial u_{i\alpha}}{\partial x_j} + \frac{\partial u_{j\alpha}}{\partial x_i}\right) - \delta_{ij}\mu_\alpha\frac{2}{3}\frac{\partial u_{l\alpha}}{\partial x_l}\right)\right]$. In addition, the work caused by the collision of particles of different sorts with each other is determined by the dependence $u_{i\alpha}F_{int(\alpha,\beta)} = u_{i\alpha}m_\alpha n_\alpha(\delta u_\alpha \backslash \delta\tau) = u_{i\alpha}n_\alpha\sum_\beta\left(R_{\alpha\beta} + R^T_{\alpha\beta}\right)$ [52–56,64,68]. The work of relative friction forces depending on the relative velocity of electrons and ions is $u_{i\alpha}R_{i\alpha\beta} = u_{i\alpha}\mu_{\alpha\beta}\nu_{\alpha\beta}(u_{i\alpha} - u_{i\beta})$. The work of the thermal force is $u_{i\alpha}R^T_{\alpha\beta} = u_{i\alpha}\left[\mu_{\alpha\beta}(u_{i\alpha})(\delta\nu_{\alpha\beta})\right] = u_{i\alpha}\frac{T_{\alpha\beta}}{\nu_{\alpha\beta}}\frac{\delta\nu_{\alpha\beta}}{\delta T_{\alpha\beta}} grad T_{\alpha\beta}$ [52–56,64,68]. The work of external forces acting on particles of class «α» in the «i» direction is $u_{i\alpha}F_{k(\alpha i)} = u_{i\alpha}(j\{E + [u \times B]\})_{\alpha i} = u_{i\alpha}(Z_\alpha e n_\alpha)\{E + [u \times B]\}_{\alpha i} = u_{i\alpha}(Z_\alpha e n_\alpha)E$, $\left(q_{j_\alpha,Vrad}\right)$ is the braking radiation power per unit volume, $T_\alpha = m_\alpha e_\alpha$ is the particle energy. The written equations are defined as instantaneous provided that the hypothesis of plasma continuity is observed. At the same time, the direct consideration of equation for the collision frequencies of particles of the same grade and different grades among themselves determines, as is known, the limits of the hypothesis of continuity and the dualism when describing the motion of charged particles in plasma as a multicomponent fluid. This important aspect seems substantial for writing down the stochastic equations of conservation of plasma motion in an external electromagnetic field.

4. Equivalent Measures and Excitation of Plasma Turbulence by an Electric Field

In accordance with [72–75,100–109], for space–time area (1) of the onset of generation for a pair $(N, M)(1, 0)r_{c0}(x_c + \Delta x_0, \tau_c + \Delta\tau_0) - r_c$, the set of Equations (1)–(3) can be written as follows (see also [105,108]):

$$\frac{d(m_\alpha n_\alpha)_{col}}{d\tau} = -\frac{(m_\alpha n_\alpha)_{st}}{t_{cor}} \quad (5)$$

$$\begin{cases} \frac{d(m_\alpha n_\alpha u_{i\alpha})_{col,1,0}}{d\tau} = -\frac{\partial}{\partial x_j}(-\tau_{\alpha ij})_{col,1} + (F_{int(\alpha,\beta)} + F_{k(\alpha i)})_{col,1} - \frac{(m_\alpha n_\alpha u_{i\alpha})_{st}}{(\tau^u_{cor})_{1,0}} \\ or \\ \frac{d(m_\alpha n_\alpha u_{i\alpha})_{col,1,0}}{d\tau} = -\frac{(m_\alpha n_\alpha u_{i\alpha})_{st}}{(\tau^u_{cor})_{1,0}} \\ \frac{\partial}{\partial x_j}(-\tau_{\alpha ij})_{col,1} = (F_{int(\alpha,\beta)} + F_{k(\alpha i)})_{col,1} \end{cases}, \tag{6}$$

$$\begin{cases} \frac{d(m_\alpha n_\alpha e_\alpha)_{col,1,0}}{d\tau} = -\frac{\partial}{\partial x_j}(q_{\alpha i} + u_{\alpha i}\tau_{\alpha ij})_{col,1} + (u_{\alpha i}[F_{int(\alpha,\beta)} + F_{k(\alpha i)}])_{col,1} \pm \frac{\partial}{\partial x_j}\left(q_{j_\alpha, ЛУЧ}\right)_{col,1} \\ \qquad - \frac{(m_\alpha n_\alpha e_{\alpha i})_{st}}{(\tau^e_{cor})_{1,0}} \\ or \\ \frac{d(m_\alpha n_\alpha e_\alpha)_{col,1,0}}{d\tau} = -\frac{(m_\alpha n_\alpha e_\alpha)_{st}}{(\tau^e_{cor})_{1,0}} \\ \frac{\partial}{\partial x_j}(q_{\alpha i} + u_{\alpha i}\tau_{\alpha ij})_{col,1} = (u_{\alpha i}[F_{int(\alpha,\beta)} + F_{k(\alpha i)}])_{col,1} \pm \left(q_{j_\alpha, Vrad}\right)_{col,1} \end{cases} \tag{7}$$

For the case under consideration, the expression for the stress tensor of the flow of electrons and ions is determined as well as the expression for the relative friction force between electrons and ions and the equations for thermal forces for each plasma component (electrons and ions [53–57]).

(a) The stress tensor for the flow of electrons and ions [53–57,65,69]

$$-\frac{\partial}{\partial x_j}(\tau_{\alpha ij}) = -\frac{\partial(P_\alpha + \pi_{\alpha ij})}{\partial x_i}; -\frac{\partial P_\alpha}{\partial x_i} = -\frac{\partial n_\alpha T_\alpha}{\partial x_i}; \left[\pi^{(e)}_{i,j}\right] = -0.73\frac{n_e T_e}{\nu_e}\omega^{(e)}_{i,j} = -0.73\frac{n_e T_e}{\nu_e}\left[\left(\frac{\partial u_{i(e)}}{\partial x_j} + \frac{\partial u_{j(e)}}{\partial x_i}\right) - \delta_{ij}\frac{2}{3}\frac{\partial u_{l(e)}}{\partial x_l}\right]$$
$$\left[\pi^{(e)}_{i,j}\right] = -0.96\frac{n_i T_i}{\nu_i}\omega^{(i)}_{i,j} = -0.96\frac{n_i T_i}{\nu_i}\left[\left(\frac{\partial u_{i(i)}}{\partial x_j} + \frac{\partial u_{j(i)}}{\partial x_i}\right) - \delta_{ij}\frac{2}{3}\frac{\partial u_{l(i)}}{\partial x_l}\right] \tag{8}$$

(b) The friction forces [53–57,65,69]:

(c) $R_{i\alpha\beta} = \mu_{\alpha\beta}\nu_{\alpha\beta}(u_{i\alpha} - u_{i\beta}) = -n_e m_e \nu_e\left(u_{i(e)} - u_{i(i)}\right)$ for $\omega_{pe} \gg \nu_e$,

$$R_{i\alpha\beta} = \mu_{\alpha\beta}\nu_{\alpha\beta}(u_{i\alpha} - u_{i\beta}) = -0,51 n_e m_e \nu_e\left(u_{i(e)} - u_{i(i)}\right) \text{ for } \omega_{pe} << \nu_e. \tag{9}$$

The thermal forces [53–57] are

$$R^T_{\alpha\beta} = -0.71 n_e \frac{\partial}{\partial x_l} T_e. \tag{10}$$

The external forces are $F_{k(ei)} = \left[eZn_{(ei)}E\right]$, where e is the elementary charge of electron, Z is the element number, and $n_{(ei)}$ is the concentration of negative and positive particles. Here $\nu_e = \Lambda\frac{\omega_{pe}}{3(2\pi)^{1,5}n_0}\left(\frac{\omega_{pe}}{\upsilon_{Te}}\right)^3$ is the electron collision frequency, $(\omega_{pe})^2 = \frac{ne^2}{\varepsilon_0 m_e}$ is the plasma frequency of electron oscillations taking into account the thermal motion $(\omega)^2 = (\omega_{pe})^2 + k^2{\upsilon_{pe}}^2$, the thermal velocity of electrons is $\upsilon_{Te} = \sqrt{(T_e/m_e)}$, T_e is the termal energy and Coulomb logarithm is $\Lambda = Ln\left[4\pi n_0\left(\frac{\omega_{pe}}{\upsilon_{Te}}\right)^{-3}\right]$, $r_d = \frac{\upsilon_{pe}}{\omega_{pe}}$, $\Lambda = Ln\left[\left(\frac{r_d}{r_s}\right)\right]$; $r_s = \left[\left(\frac{e^2}{{\upsilon_{pe}}^2 m_e \varepsilon_0}\right)\right]$ [109–118].

(d) For the energy equation, respectively, the work caused by the collision of particles of different sorts is $u_{i\alpha}F_{int(\alpha,\beta)} = u_{i\alpha}n_\alpha\sum_\beta\left(R_{\alpha\beta} + R^T_{\alpha\beta}\right)$ and

$$u_{i\alpha}F_{int(\alpha,\beta)} = u_{ie}n_e\left(R_{ei} + R^T_{ei}\right) = -n_e m_e \nu_e u_{i(e)}\left(u_{i(e)} - u_{i(i)}\right) - 0.71 u_{i(e)} n_e \frac{\partial}{\partial x_l} T_e, \tag{11}$$

The work caused by the external forces is

$$u_{(e)i}F_{k(ei)} = u_{(e)i}\left[\left[eZ_{ei}n_{(ei)}E\right]\right], \textit{ here } Z_{ei} = 1 \tag{12}$$

The equation for the heat flux is $q_{\alpha i} = -\delta \frac{n_e T}{\nu m_e} gradT_{\alpha\beta}$. For the flow electrons $\frac{\partial}{\partial x_j}(-q_{\alpha i}) = \frac{\partial}{\partial x_j}\left[-\delta \frac{n_e T_e}{\nu_e m_e} gradT_e\right]$, $\delta = (2.5 - 3.91)$ [52–56,64,68]. The equation for the braking radiation power $\left(q_{j_\alpha,\mathrm{Vrad}}\right)_{col,1} = 1.5 * 10^{-40} n_e n_i Z_{ei}^2 T_e^{0.5}$ [J/(c·m^3)] [52–56,117].

It should be noted here that the speed of sound in plasma, as is known, is determined by the formula $C_s = \left(\frac{T_e}{m_e} + \frac{5T_i}{3m_i}\right)^{0.5}$. Now it is possible to obtain theoretical equations for the critical value of external electrical field substituting the received expressions into the set of equations for the region of onset of turbulence. Here $t_{cor} = (\tau^u_{cor})_{1.0} = (\tau^e_{cor})_{1.0}$.

$$\frac{d(m_\alpha n_\alpha)_{col}}{d\tau} = -\frac{(m_\alpha n_\alpha)_{st}}{t_{cor}}, \tag{13}$$

$$\begin{cases} \frac{d(m_\alpha n_\alpha u_{i\alpha})_{col,1,0}}{d\tau} = -\frac{(m_\alpha n_\alpha u_{i\alpha})_{st}}{(\tau^u_{cor})_{1.0}} \\ \frac{\partial}{\partial x_j}(\tau_{\alpha ij})_{col,1} = \left(F_{int(\alpha,\beta)} + F_{k(\alpha i)}\right)_{col,1} \end{cases}, \tag{14}$$

$$\begin{cases} \frac{d(m_\alpha n_\alpha e_\alpha)_{col,1,0}}{d\tau} = -\frac{(m_\alpha n_\alpha e_\alpha)_{st}}{(\tau^e_{cor})_{10}} \\ \frac{\partial}{\partial x_j}(q_{\alpha i} + u_{\alpha i}\tau_{\alpha ij})_{col,1} - (u_{\alpha i}[F_{int(\alpha,\beta)}])_{col,1} \pm \left(q_{j_\alpha,\mathrm{Vrad}}\right)_{col,1} = (u_{\alpha i}[F_{k(\alpha i)}])_{col,1} \end{cases}. \tag{15}$$

5. Plasma Conductivity

In accordance with the last set of Equation (15), for the electron flow with $Z_{ei} = 1$, it can be written that

$$\begin{gathered} \frac{\partial u_{i(e)} n_e T_e}{\partial x_i} - \frac{\partial}{\partial x_j}\left(-3.16\frac{n_e T}{\nu m_e} gradT_{\alpha\beta} - 0.73 u_{i(e)} \frac{n_e T_e}{\nu_e}\left\{\left(\frac{\partial u_{i(e)}}{\partial x_j} - \frac{\partial u_{j(e)}}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_{l(e)}}{\partial x_l}\delta ij\right\}\right) \\ -u_{i(e)_e} n_e \nu_e\left(u_{i(e)} - u_{i(i)}\right) + 0.71 u_{i(e)} n_e \frac{\partial}{\partial x_l} T_e - 1.5 * 10^{-40} n_e n_i 1_{ei}^2 T_e^{0.5} = u_{i(e)}[e n_e E] \end{gathered} \tag{16}$$

In the case of a plane directional motion $T_e = T_0 exp(-iwt + k_T r)$; $u_e = u_0 exp(-iwt + kr)$; $n_e = n_0 exp(-iwt + k_n r)$; and $d(q)/dx = -3.16(n/m\nu_e)T_e{}^2 k^2{}_T$, we have a parabolic equation, and for $u_{i(e)} >> u_{i(i)}$, the last equation can be written as

$$[E] = \frac{1}{u_{(e)i}(en_e)}\left|\left[\begin{matrix} -u_{(e)i} n_e T_e (k_{xT} + k_{x,u} + k_{xn}) - \left(0.73 u_{i(e)} \frac{n_e T_e}{\nu_e}\right) k_{y,u}(k_{yT} + k_{y,u} + k_{yn}) - \\ -0.71 u_{i(e)} n_e T_e k_{yT} - n_e m_e \nu_e u_{i(e)}\left(u_{i(e)} - u_{i(i)}\right) - 3.16 \cdot \left(k^2_{yT}\right)\frac{n_e}{m\nu_e} T_e^2 - 1.5 * 10^{-40} n_e n_i Z_{ei}^2 T_e^{0.5} \end{matrix}\right]\right.. \tag{17}$$

Let us consider two cases when: (1) the wave numbers $k_y = k_{yT} = k_{yn}$ $[E] = \left[\,\left|E_1^* + E_2^* + E_3^* + E_4^*\right|\right] = \frac{m_e}{e}\nu_e u_{1(e)} - \left(\frac{T_e}{e}\right)\left[-3k_x + 0.73(k_y)\left(3\frac{k_y}{\nu_e}u_{1(e)}\right) - 0.71(k_y) + 3.16\left(k^2_{yT}\right)\frac{n_e}{m\nu_e}T_e^2\right] - \frac{1.5*10^{-40} n_e n_i Z_{ei}^2 T_e^{0.5}}{u_{(e)i}(en_e)}$; (2) $\frac{u_{1(e)}}{u_{2(e)}} = \frac{k_y}{k_x}$, so $u_{1e} \gg u_{2e}$; $k_y >> k_x$, and

$$\begin{gathered} [E] = \left[\left|E_1^* + E_2^* + E_3^* + E_4^*\right|\right] \\ = \frac{m_e}{e}\nu_e u_{1(e)} - \left(\frac{T_e}{e}\right)\left[0.73(k_y)\left(3\frac{k_y}{\nu_e}u_{1(e)}\right) - 0.71(k_y) + 3.16\left(k^2_{yT}\right)\frac{n_e}{m\nu_e}T_e^2\right] - \frac{1.5*10^{-40} n_e n_i Z_{ei}^2 T_e^{0.5}}{u_{(e)i}(en_e)} \end{gathered} \tag{18}$$

Substituting the expression $\nu_e = \Lambda\frac{\omega_{pe}}{3(2\pi)^{1.5} n_0}\left(\frac{\omega_{pe}}{\upsilon_{Te}}\right)^3$, we obtain:

$$E_1^* = \frac{m_e}{e}\nu_e u_{1(e)} = \Lambda u_{1(e)}\frac{m_e}{e}\frac{\omega_{pe}}{2(2\pi)^{1.5} n_0}\left(\frac{\omega_{pe}}{\upsilon_{Te}}\right)^3 \tag{19}$$

Considering that $\Lambda = Ln\left[4\pi n_0\left(\frac{\omega_{pe}}{\upsilon_{Te}}\right)^{-3}\right] = Ln\left[4\pi n_0\left(\frac{\upsilon_{Te}}{\omega_{pe}}\right)^3\right]$, and $u_{1(e)} = u_e \approx \{\upsilon_{Te}; \upsilon_{dr}\}$.

Here, $\upsilon_{Te}; \upsilon_{dr}$ are the electron thermal speed and electron drift speed

$$E_1^* = \left(\frac{8}{7(2\pi)^{0,5}}\right)\frac{m_e \upsilon_{Te}\omega_{pe}}{e}\Lambda\left[\frac{1}{4\pi n_0}\left(\frac{\omega_{pe}}{\upsilon_{Te}}\right)^3\right] = \left(\frac{8}{7(2\pi)^{0,5}}\right)\frac{m_e \upsilon_{Te}\omega_{pe}}{e}\left(\frac{\Lambda}{e^{\Lambda}}.\right) \quad (20)$$

Here $(\omega_{pe})^2 = \frac{ne^2}{\varepsilon_0 m_e}$, $\varepsilon_0 = 8.85*10^{-12}$ ($C^2/(N*m^2)$).

$$E_2^* = 0.72(k_y)\left(\frac{T_e}{e}\right)\left(4.3k_x/k_y + 1 + 3\frac{k_y}{\nu_e}u_{1(e)}\right), \quad (21)$$

Let us introduce the notation

$$E_{2T}^* = 0.72k_y\left(\frac{T_e}{e}\right); \quad (22)$$

$$E_{2P}^* = 3k_x\left(\frac{T_e}{e}\right); \quad (23)$$

$$E_{2,e-i}^* = 0.72k_y^2\left(\frac{T_e}{e}\right)\frac{\upsilon_{Te}}{\omega_{pe}}\frac{7(2\pi)^{0.5}}{8}\frac{e^{\Lambda}}{\Lambda} = 1.57 \cdot k_y^2 \cdot r_d \cdot \left(\frac{T_e}{e}\right) \cdot \frac{e^{\Lambda}}{\Lambda}. \quad (24)$$

The third term is

$$E_3^* = -3.16\left(k_T^2\right)\frac{n_e}{m\nu_e}\frac{1}{u_{(e)1}(en_e)}T_{(e)1}^2 = -3.16\left(k_T^2\right)\frac{T_{(e)}}{(e)}\frac{T_{(e)}}{\upsilon_{(e)1}m\nu_e} \quad (25)$$

$$E_4^* = -\frac{1.5*10^{-40}n_e n_i Z_{ei}^2 T_e^{0,5}}{u_{(e)i}(en_e)} = -\frac{1.5*10^{-40}n_e n_i Z_{ei}^2 T_e^{0.5}}{u_{(e)i}(en_e)} \quad (26)$$

Further, the value determined by Equation (24) is not taken into account due to Equation (17).

Thus, the first relation is obtained from the second equation of set (15)

$$\left\{\frac{\partial}{\partial x_j}\left(q_{\alpha i} + u_{\alpha i}\tau_{\alpha ij}\right)_{col,1} - \left(u_{\alpha i}\left[F_{int(\alpha,\beta)}\right]\right)_{col,1} = \left(u_{\alpha i}\left[F_{k(\alpha i)}\right]\right)_{col,1}\right. .$$

Then, from the first equation of set (15) $\left\{\frac{d(m_\alpha n_\alpha e_{\alpha i})_{col,1,0}}{d\tau} = -\frac{(m_\alpha n_\alpha e_{\alpha i})_{st}}{(\tau_{cor}^e)_{10}}\right.$, we obtain the following relations ($E_{th} = T_e$ is the electron termal energy):

$n_e\left(\frac{m_e u_e^2}{2} + E_{th}\right)\nu_T = -\frac{E_{st}}{\tau_{st}} = \frac{-E_{st}(u_{st})}{L} = -E_{st}\left(\frac{E_{st}}{n_e m_e}\right)^{0.5} \cdot k_y/2\pi\Delta$; $L = \frac{2\pi}{k_y} \cdot \Delta$; Δ is taking in accordance with [102] for the region of generation of turbulence:

$$\Delta \simeq \left(\frac{E_{st}}{n_e T_e}\right)^l, l = \left(\frac{7}{2} - \frac{9}{2}\right). \quad (27)$$

Then, $n_e\left(\frac{m_e u_e^2}{2} + E_{th}\right) = \left(\frac{E_{st}}{n_e m_e}\right)^{0.5} \cdot [E_{st}k_y/(\nu_T 2\pi\Delta)]$ and, finally, $\left(u_e^2\right) + \frac{T_e}{m_e} = -2\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}$ $[\frac{E_{st}}{n_e m_e} \cdot \frac{k_y}{\nu_T}1/2\pi\Delta]$.

Also, from the first equation of set (15) $\left\{\frac{d(m_\alpha n_\alpha u_{i\alpha})_{col,1,0}}{d\tau} = -\frac{(m_\alpha n_\alpha u_{i\alpha})_{st}}{(\tau_{cor}^u)_{10}}\right.$, it is possible to obtain the following relations

$$n_e(m_e u_e)\nu_T = -n_e m_e\frac{-(u_{st})^{0.5}}{\tau_{st}} = -n_e m_e\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}}{L} = -E_{st} \cdot k_y/2\pi\Delta \text{ or } (u_e) = \frac{E_{st}}{n_e m_e} \cdot \frac{k_y}{\nu_T}\frac{1}{2\pi\Delta}.$$

Then, using Equation (27), it is possible to obtain the following relations:

$$\left(\frac{E_{st}}{n_e m_e}\cdot\frac{k_y}{\nu_T}\frac{1}{2\pi\Delta}\right)^2 - 2\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}\left(\frac{E_{st}}{n_e m_e}\cdot\frac{k_y}{\nu_T}\frac{1}{2\pi\Delta}\right) + \frac{T_e}{m_e} = 0, \tag{28}$$

$$\upsilon_{\rm dr} \approx \left(\frac{E_{st}}{n_e m_e}\cdot\frac{k_y}{\nu_T}1/2\pi\Delta\right) = \left(\frac{E_{st}}{n_e m_e}\right)^{0.5} - \left(\frac{E_{st}}{n_e m_e} - \frac{T_e}{m_e}\right)^{0.5}. \tag{29}$$

It is easy to see that when $E_{st} = E_{\rm th}$, the stochastic energy equals to the thermal energy; then, $v_{pe}^2 = \frac{T_e}{m_e} \approx \frac{E_{st}}{n_e m_e}$ and $u_e \approx \upsilon_{\rm dr} \approx v_{pe}$. For the wave number, we obtain

$$k_y = 2\pi\Delta\frac{(\upsilon_{\rm dr})\nu_T}{\left(\frac{E_{st}}{n_e m_e}\right)}. \tag{30}$$

In Equations (17)–(29), $u_{1(e)} = \upsilon_{\rm dr}$. Here, $\nu_T = \nu_e[(\Lambda)_{\rm dr}/\Lambda)](\upsilon_{\rm pe}/\upsilon_{\rm dr})^3$; $(\Lambda)_{\rm dr} = {\rm Ln}\left(\frac{(r_{\rm d})_{\rm dr}}{r_s}\right)$; $(r_{\rm d})_{\rm dr} = \upsilon_{\rm dr}/\omega_{\rm pe}$; $\Lambda = {\rm Ln}\left[4\pi n_0\left(\frac{\omega_{\rm pe}}{\upsilon_{\rm pe}}\right)^{-3}\right]$; $r_{\rm d} = \frac{\upsilon_{pe}}{\omega_{\rm pe}}$; $\Lambda = {\rm Ln}\left[\left(\frac{r_{\rm d}}{r_{\rm s}}\right)\right]$; $r_{\rm s} = \left[\left(\frac{e^2}{\upsilon_{\rm pe} m_e \varepsilon_0}\right)\right]$.

The value of critical electrical strength [67–70] may be obtained from Equation (17) as

$$[E] \approx 1.5\frac{1}{u_{(e)i}(en_e)}\left[3.16\cdot\left(k_{yT}^2\right)\frac{n_e}{m\nu_e}T_e^2\right] \approx 15\pi\left(\frac{E_{st}}{n_e T_e}\right)^{l-1}({\rm k_{yT}})\left(\frac{\rm T_e}{\rm e}\right)\ [{\rm V/m}] \tag{31}$$

$$[E]cr \approx 15\pi\left(\frac{E_{st}}{n_e T_e}\right)^{l-1}({\rm k_{yT}})\left(\frac{\rm T_e}{\rm e}\right) \approx 44,2({\rm k_{yT}})\left(\frac{\rm T_e}{\rm e}\right)\ [{\rm V/m}]. \tag{32}$$

Then, in according to Ohm's law $\gamma = j/E$ given that $j = en\upsilon_{\rm dr}$ in the general case, the conductivity is

$$\gamma = \frac{\rm j}{[\rm E]} = \frac{en\upsilon_{\rm dr}}{\left[\left|E_1^* + E_2^* + E_3^* + E_4^*\right|\right]}. \tag{33}$$

Let us estimate an effect of E_4^* on the electron conductivity. From data [119]: T_e = 100 Ev, n = 10^{18}, $Z_{\rm ei}^2 = 1$.

$$E_4^* = \frac{1.5*10^{-40} n_e n_i Z_{\rm ei}^2 T_e^{0.5}}{u_{(e)i}(en_e)} = \frac{1.5*10^{-40} n_e n_i Z_{\rm ei}^2 T_e^{0.5}}{\upsilon_{\rm dr}(en_e)} \sim \frac{1.5*10^{-40} n_e n_i Z_{\rm ei}^2 T_e^{0.5}}{\upsilon_{\rm dr}(en_e)} \sim \frac{10^{-3}*T_e^{0.5}}{\upsilon_{\rm dr}} \sim \frac{1.2}{\upsilon_{\rm dr}} \sim 10^{-6}$$

Therefore, this value is neglected further for the conditions of hydrogen plasma [118].

6. Comparison of Calculation Results with Experiments

Here, we present below the calculations for conductivity in plasma and for the value of the critical electrical field using the theory of stochastic equations and equivalence of measure. In Tables 1–4, the results of the calculation are presented for the energy of electrons with densities n = 10^{18} m^{-3} and Te = 100–10,000 eV; Δ is taken in accordance with Equation (27). For calculating the drift speed of electrons, we used Equations (28) and (29), for the wave number, Equation (25); for E_1^*, E_2^*, E_3^*, E_4^*, Equations (17)–(26); and for conductivity, Equation (33) are used. Tables 3 and 4 show the results of calculations according to the initial data indicated above in the tables and the sequence of calculations according to points (1)–(9).The presented algorithm for calculating the electronic conductivity for hydrogen-plasma experiments is based on the theoretical solution of stochastic equations for turbulent plasma.

Table 1. $(\omega_{pe})^2 = \frac{ne^2}{\varepsilon_0 m_e} = n * (3178,7)$.

n	$\omega_{pe}\ c^{-1}$
$10^{11}cm^{-3}$ $(10^{17}m^{-3})$	0.178×10^{11}
$10^{12}cm^{-3}$ $(10^{18}m^{-3})$	0.561×10^{11}
$10^{13}cm^{-3}$ $(10^{19}m^{-3})$	1.78×10^{11}

Table 2. $\upsilon_{Te} = \sqrt{(T_e/m_e)}; r_s = \left[\left(\frac{e^2}{\upsilon_{pe}{}^2 m_e \varepsilon_0}\right)\right] = \left(\frac{3178{,}7}{\upsilon_{pe}{}^2}\right)$.

Te	υ_{pe}	r_s
100 Ev $(160 \times 10^{-19}$ J)	4.193×10^6	0.18×10^{-9}
200 Ev $(320 \times 10^{-19}$ J)	5.929×10^6	0.09×10^{-9}
10^4Ev $(16{,}000 \times 10^{-19}$ J)	4.193×10^7	0.18×10^{-11}

Table 3. Results of calculation of the electronic conductivity for data [118]: n = 10^{18} $[m^{-3}]$; $\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}}{\upsilon_{pe}} = 1.3$; Te = 100 Ev$(160^{-19}$ J) ; $r_d = \upsilon_{pe}/\omega_{pe}$; $r_s = 0.18*10^{-9}$ [m]; $\upsilon_{pe} = 4.193*10^6$ [m/c], $\Delta = \left(\frac{E_{st}}{n_e T_e}\right)^{3.9} = 7.74$.

ω_{pe} $[c^{-1}]$	υ_{dr} [m/c]	r_d*10^{-5}[m]	$(r_d)_{dr}*10^{-5}$ [m]	Λ = $Ln(r_d/r_s)$	$(\Lambda)_{dr}$	ν_e $[c^{-1}]$	ν_T $[c^{-1}]$	k_y $\left[m^{-1}\right]$	E_1^* [V/m]	$\mid E_2^* \mid$ [V/m]	$\mid E_3^* \mid$ [V/m]	$[E]$ [V/m]	γ $[c^{-1}]$ CGSE
$0.561*10^{11}$	$1.967 \cdot 10^6$	7.474	3.506	12.94	12.18	3.5841×10^4	$3.272 \cdot 10^5$	1.05842	3.67	1374.8	9669.56	11,044.4	$0.26 \cdot 10^{12}$

Table 4. Results of calculation of the electronic conductivity for data [118]: n = 10^{18} $[m^{-3}]$; $\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}}{\upsilon_{pe}} = 2$; Te = 10,000 Ev$(160^{-19}$ J); $r_d = \upsilon_{pe}/\omega_{pe}$; $r_s = 0.18 \times 10^{-11}$ [m]; $\upsilon_{pe} = 4.193 \times 10^7$ [m/c], $\Delta = \left(\frac{E_{st}}{n_e T_e}\right)^{3.9} = 222.86$.

ω_{pe} $[c^{-1}]$	υ_{dr} [m/c]	$r_d \times 10^{-4}$ [m]	$(r_d)_{dr} \times 10^{-4}$ [m]	Λ = $Ln(r_d/r_s)$	$(\Lambda)_{dr}$	ν_e $[c^{-1}]$	ν_T $[c^{-1}]$	k_y $\left[m^{-1}\right]$	E_1^* [V/m]	$\mid E_2^* \mid$ [V/m]	$\mid E_3^* \mid$ [V/m]	$[E]$ [V/m]	γ $[c^{-1}]$ CGSE
0.561×10^{11}	1.12×10^7	7.474	1.996	19,843	18.52	56.65	$2.748 \cdot 10^3$	0.00606	0.187	1033.87	66,050.8	67,084.08	$0.24 \cdot 10^{12}$

(1) $\left(\frac{E_{st}}{n_e m_e}\right)^{0.5} \approx 1.3 v_{pe}$ and $\left(\frac{E_{st}}{n_e m_e}\right) = 1.69 v_{pe}^2$

(2) $\upsilon_{dr} \approx \left(\frac{E_{st}}{n_e m_e} \cdot \frac{k_y}{\nu_T} 1/2\pi\Delta\right) = \left(\frac{E_{st}}{n_e m_e}\right)^{0.5} - \left(\frac{E_{st}}{n_e m_e} - \frac{T_e}{m_e}\right)^{0.5} \cdot \upsilon_{pe} = 0.469 \cdot \upsilon_{pe} = 1.967 \cdot 10^6$ [m/c]

(3) $\Delta \simeq \left(\frac{E_{st}}{n_e T_e}\right)^l = (1.69)^{3.9} = 7.74$; $(r_d)_{dr} = \upsilon_{dr}/\omega_{pe} = 3.506*10^{-5}$; $r_d/r_s = 41.52*10^4$; $\frac{(r_d)_{dr}}{rs} = 194{,}791$; $\nu_T = \nu_e[(\Lambda)_{dr}/\Lambda)]\ (\upsilon_{pe}/\upsilon_{dr})^3 = 3.58 \cdot 10^4 \cdot 0.943 \cdot 9.69 = 3.272 \cdot 10^5\ [c^{-1}]$

(4) $k_y = 2\pi\Delta \frac{(\upsilon_{dr})\nu_T}{\left(\frac{E_{st}}{n_e m_e}\right)} = 2\pi\Delta \frac{(\upsilon_{dr})\nu_T}{\left(\frac{E_{st}}{n_e m_e}\right)} \approx 1.058\left[\frac{1}{m}\right]; \rightarrow k_y{}^2 = 1.12\left[1/m^2\right]$

(5) $E_1^* = \frac{m_e}{e} \nu_t u_{1e}; E_2^* = 0.72(k_y)\left(\frac{T_e}{e}\right)\left(\frac{4.3k_x}{k_y} + 1 - \frac{k_y}{\nu_e} u_{1\epsilon}\right); E_3^* = 3.16\left(k_{yT}^2\right)\frac{n_e}{m\nu_e} T_e^2$

(6) $\left[E_1^* + E_2^*\right] = \frac{m_e}{e}\nu_t u_{1(e)} + 0.72(k_y)\left(\frac{T_e}{e}\right)\left[\left(1 - 3\frac{k_y}{\nu_e} u_{1(e)}\right)\right] = 3.67 + 76.176 - 76.176*19.0964$
$= 3.67 + 76.176 - 1454.6873664 = 1374.84$ [B/m]

(7) $E_3^* = -3.16 \cdot \left(k_y^2\right)\frac{n_e}{m\nu_e}\frac{1}{u_{(e)1}(en_e)} T_{(e)1}^2 = -3.16\left(k_y^2\right)\frac{1}{m\nu_e}\frac{1}{\upsilon_{(e)1}(e)} T_{(e)1}^2 =$
$-3.16\left(k_y^2\right)\frac{1}{9.1*10^{-31}*3.272*10^5*1.967*10^6*1.6*10^{-19}} T_{(e)1}^2 = -3.16 * 1.12 * \frac{1}{9.37*10^{-38}}$
$(25{,}600 * 10^{-38}) = \frac{-3.91*0.8573*2{,}56*10^4}{9.37} = \frac{-3.16*1.1200008573*2.56*10^4}{9.37} = 9669.56$ [V/m]

(8) $[E] = \left[E_1^* + E_2^* + E_3^*\right] = 1374.6 + 9669.56 = 11{,}044.19$ [V/m] = [110.44] [V/cm]

(9) $\gamma = \frac{j}{E} = \frac{en\upsilon_{dr}}{E} = \frac{1{,}6*10^{-19}*10^{18}*1{,}96715*10^6 env}{11.044*10^3} = 0.288 * 10^2$ [1/(Om*m)]

In CGSE $\gamma = \frac{j}{E} = \frac{j}{E} = \frac{env}{E} = 0.388 * 10^2 * 9 * 10^9 = 0.26 * 10^{12}\ [c^{-1}]$

(1) $\left(\frac{E_{st}}{n_e m_e}\right)^{0.5} \approx 2.v_{pe}$ and $\left(\frac{E_{st}}{n_e m_e}\right) = 4.v_{pe}^2$

(2) $\upsilon_{\rm dr} \approx \left(\frac{E_{st}}{n_e m_e}\cdot\frac{k_y}{\nu_T}1/2\pi\Delta\right) = \left(\frac{E_{st}}{n_e m_e}\right)^{0.5} - \left(\frac{E_{st}}{n_e m_e} - \frac{T_e}{m_e}\right)^{0.5}\cdot\upsilon_{pe} = 0.268\cdot\upsilon_{pe} = 1.12\cdot10^7$ [m/c]

$\nu_e = \frac{\Lambda\omega_{pe}}{3(2\pi)^{1.5}n_0}\left(\frac{1}{r_d}\right)^3 = \frac{19.843*0.561*(2.39506)}{47.21}10^{11-18+(9)} = 56.68*10^0$

(3) $\Delta \simeq \left(\frac{E_{st}}{n_e T_e}\right)^l = (4)^{3.9} = 222,86; (r_{\rm d})_{\rm dr} = \upsilon_{\rm dr}/\omega_{\rm pe} = 1.996*10^{-4}; {\rm r_d}/{\rm r_s} = 4.149*10^8;$

$\frac{(r_{\rm d})_{\rm dr}}{\rm rs} = 1.11*(10)^8; \nu_T = \nu_e[(\Lambda)_{\rm dr}/\Lambda)](\upsilon_{\rm pe}/\upsilon_{\rm dr})^3 = 56.65*0.93\cdot51.98 = 2.7489431852\cdot10^3$ [c^{-1}]

(4) $k_y = 2\pi\Delta\frac{(\upsilon_{\rm dr})\nu_T}{\left(\frac{E_{st}}{n_e m_e}\right)} = 2\pi\Delta\frac{(\upsilon_{\rm dr})\nu_T}{\left(\frac{E_{st}}{n_e m_e}\right)} \approx 6.28*222.86\frac{(1.12\cdot10**7)2.7489431852\cdot10**3}{(4*17.5810**14)} \approx 0.006104\left[\frac{1}{\rm m}\right];$

${\rm k}_y{}^2 = 3.72*10^{-5}\left[1/{\rm m}^2\right]$

(5) ${\rm E}_1^* = \frac{\rm m_e}{\rm e}\nu_{\rm e}{\rm u}_{1e}; {\rm E}_2^* = 0.72({\rm k_y})\left(\frac{\rm T_e}{\rm e}\right)\left(\frac{4.3{\rm k_x}}{\rm k_y} + 1 + \frac{\rm k_y}{\nu_{\rm e}}{\rm u}_{1\epsilon}\right);\ {\rm E}_3^* = -3.16\left(k_T^2\right)\frac{T_{(e)}}{(e)}\frac{T_{(e)}}{\upsilon_{(e)1}m\nu_e}$

(6) $\left[E_1^* + E_2^*\right] = \left|\frac{m_e}{e}\nu_e u_{1(e)} + 0.72(k_y)\left(\frac{T_e}{e}\right)\left[\left(1 - 3\frac{k_y}{\nu_e}u_{1(e)}\right)\right]\right| = |0.187 + 43.63 + 43.63*24.7|$ $= |0.187 + 43.63 - 1077.6873664| = 1121.28 * 1.018 = 1033.87$ [B/m]

(7) $E_3^* = -3.16\cdot\left(k_y^2\right)\frac{n_e}{m\nu_e}\frac{1}{u_{(e)1}(en_e)}T^2_{(e)1} = -3.16\left(k_y^2\right)\frac{1}{m\nu_e}\frac{1}{\upsilon_{(e)1}(e)}T^2_{(e)1} = -3.16$

$\left(k_y^2\right)\frac{1}{9.1*10^{-31}*2.7489*10^3*1.12*10^7*1.6*10^{-19}}T^2_{(e)1} = -3.16*3.66(3.72)*\frac{1*10^{-5}}{44.826*10^{-40}}$

$\left(2.56*10^8*10^{-38}\right) = \frac{29.607*10^5}{44.826} = 0.660508*10^5 = 66{,}050.8$ [V/m]

(8) $[E] = \left[E_1^* + E_2^* + E_3^*\right] = 1033.87 + 66{,}050.8 = 67{,}084.6703$ [V/m] = [671.72] [V/cm]

(9) $\gamma = \frac{j}{E} = \frac{en\upsilon_{\rm dr}}{E} = \frac{1{,}6*10^{-19}*10^{18}*1{,}1296715*10^7}{6.7172*10^4} = 0.256*10^2$ [1/(Om*m)]

In CGSE $\gamma = \frac{j}{E} = \frac{j}{E} = \frac{env}{E} = 0.256*10^2*9*10^9 = 0.24*10^{12}$ [c^{-1}]

7. Discussion

The authors of [118] showed the results for strong electric fields E from 100 to 700 V/cm. As can be seen, the predicted results presented in Figure 1 are consistent with experimental data [118] for this electric field region.

Also, the presented results of calculations based on the stochastic equations confirm that there is an electric-field region with approximately constant conductivity of ~30%. That is, in the equivalence region, there is a distribution corresponding to the experimentally determined in [118] for 100 < E < 700 V/cm.

This is the second region with approximately constant conductivity. This region was first experimentally determined in [118]. In this study, the measurements were carried out in a wide range of electric-field strength of 0.5 < E < 700 V/cm, while, in the region of 0.5 < E < 90 V/cm, there were only three measurements. Nevertheless, the authors of [118] confirmed the presence of a region with a constant conductivity in the range of 0.2 < E < 20 V/cm with a subsequent decrease in conductivity down to the voltage E = 100, which was first determined in [119].

The conductivity and current of a non-turbulent plasma are determined by well-known ratios: $j = en_e\upsilon_{\rm Te} = en_e\mu_{\rm Te}E = \gamma E$; $\gamma = \frac{j}{[E]} = \frac{en_e\upsilon_{\rm Te}}{[E]}$, $\upsilon_{\rm Te} = \mu_{\rm Te}E$; the formula for the conductivity is $\gamma = en_e\mu_{\rm Te} = \frac{{\rm n}_e e^2}{\nu_e{\rm m}_e}$. The mobility of electrons is $\mu_{\rm Te} = \frac{e}{\nu_e{\rm m}_e}$. The known formula for the collision frequency $\nu_e = \Lambda\frac{\omega_{\rm pe}}{3(2\pi)^{1{,}5}{\rm n}_0}\left(\frac{\omega_{\rm pe}}{\upsilon_{\rm Te}}\right)^3$ includes $\upsilon_{\rm Te}$. The calculations show that the value of the thermal velocity $\upsilon_{\rm Te}$ in the case of the plasma turbulence gives no correct magnitudes of electrical conductivity of the turbulent plasma. Thus, it is important to calculate the effective collision frequency ν_T. The solutions of stochastic equations enabled us to derive the formula for the drift velocity (see Equation (24)) $\upsilon_{\rm dr} \approx [(\frac{E_{st}}{n_e m_e})\cdot(\frac{k_y}{\nu_T})\left(\frac{E_{st}}{nT_e}\right)^{3.9}1/2\pi]]$. Then it is possible writing $j = en_e\upsilon_{\rm dr} = en_e\mu_{\rm dr}E = \gamma E$; $\gamma = \frac{j}{[E]} = \frac{en_e\upsilon_{\rm dr}}{[E]}$, $\upsilon_{\rm dr} = \mu_{\rm dr}E$; here, the mobility of electrons is $\mu_{\rm dr} = \frac{e}{\nu_T{\rm m}_e}$, and the formula for the conductivity is $n\gamma = en_e\mu_{\rm dr} = \frac{{\rm n}_e e^2}{\nu_T{\rm m}_e}$. It is easy to see that, now, the drift velocity of a turbulent plasma depends also on the intensity of plasma turbulence, its temperature (thermal energy), and the scale of turbulence of the turbulent plasma flow, instead of only on the frequency of collisions of electrons ν_e. New Formulas (29) and (30) may be converted into Equation (33) for a classical neutral plasma with Langrumov inhomogeneity. The calculations of each of the motion components depending on the external field are presented as $[E] = |E_1^* + E_2^* + +E_3^*|$. Here E_1^* is the value of the electric-field strength corresponding to the force of relative friction dependent on the relative velocity of electrons and ions $E_2^* = E_{2T}^* + E_{2P}^* + +E_{2,e-i}^*$. E_{2P}^* is the value of the electric field strength, corresponding to the gas pressure of telectrons and ions. Here $E_{2,e-i}^*$ is the value of the electric field strength corresponding to the voltage tensor caused by the viscosity of the flow of electrons and plasma ions. The value of E_{2T}^* is the electric-field strength corresponding to the thermal force, and E_3^* is the electric-field strength corresponding to the heat flow. Tables 3 and 4 show that $|E_1^*| < |E_2^*| < |E_3^*|$. The obtained Formulas (17)–(33) for the Coulomb logarithm, the wave number (turbulence scale), the collision frequency, the current, and the drift velocity enable us to achieve agreement between the predicted and experimental data for the conductivity *j* of turbulent plasma in a strong electric field of 100 < E < 1000 V/cm. The development of the theory for calculating plasma heating was undertaken in [120–128].

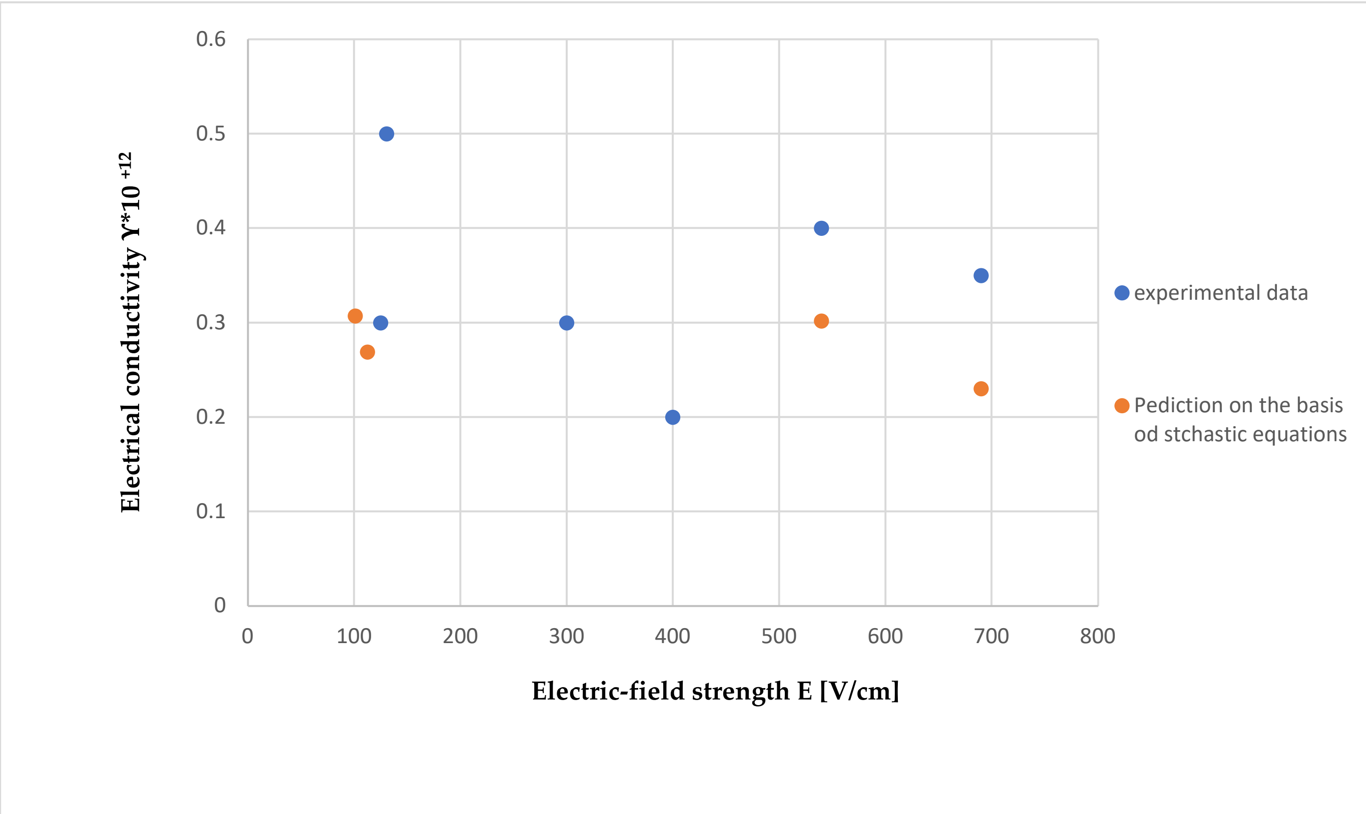

Figure 1. Plasma conductivity as a function of electric-field strength for hydrogen according to data [118].

It should be noted that, for the data of the experiments [118], no calculations with using the theory of ion-sound instability are known in the literature. Nevertheless, even if there are results of prediction using the theory of ion–sound instability, it only means that there are currently several theoretical tools, one of which is presented in this article and represents the stochastic theory of turbulent plasma.

It should also be noted that, usually, the Boltzmann equation with the Fokker–Planck collision term takes into consideration only a dynamical frictional force coming from the many-body collisions through the Coulomb force [128]. In [129], the electron transport under the effect of two kinds of friction in an electron–deuteron plasma was discussed.

Equations (31) and (32) present new formulas for the value of the critical electric field strength. We also mention data from [130–132].

The presented theory and obtained results of calculations show that, for high level of turbulence of plasma, $\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}}{v_{pe}} > 1.3$, the conductivity is $\Upsilon = 0.26 \cdot 10^{12}$ [c^{-1}]. For $\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5}}{v_{pe}} = 2$, the conductivity is $\Upsilon = 0.22 \cdot 10^{12}$ [c^{-1}], see Tables 3 and 4. However, the spread of experimental data for 100 < E < 700 V/cm is ~30%.

8. Conclusions

The obtained results show that the stochastic turbulence theory based on stochastic differential equations and equivalence of measures between deterministic and random fields is valid also for the turbulent plasma during heating by the strong electric field {Equations (1)–(15)}. It is shown that, after the onset of plasma turbulence, the existing experimental data have a certain spread of ~30% for a strong electric field of 100 < E < 1000 V/cm; see data in [118] and Figure 1. The energy balance was discussed for the plasma with the temperature Te~10,000 Ev in the region of the external strong electric field of 100 < E < 1000 V/cm. It is theoretically shown that the energy of the external electric field compensates also the energy costs due to heat flux E_3 and the total shear-stress tensor of the plasma particles E_2, which are the functions of the collision integral, instead of only the energy costs due to the forces between the particles E_1. Thus, the plasma becomes more turbulent with constant conductivity, and a plateau is formed even in the region of the external strong electric field of 100 < E < 1000 V/cm. It is theoretically determined that, starting from a voltage of about 100 V/cm, the plateau exists, and the energy of the deterministic field continues to pass mainly only into a random turbulent field.

In addition, on the basis of stochastic equations for the experimental values of electron density and temperature, the plasma current, and conductivity, we theoretically determined also the drift velocity, collision frequency, Coulomb interval, and the wave number (turbulence scale). Besides, we determined theoretically the level of turbulence of plasma $\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5} - v_{pe}}{v_{pe}} * 100\% = 30\%$ for electrical field E = 100 V/cm and $\frac{\left(\frac{E_{st}}{n_e m_e}\right)^{0.5} - v_{pe}}{v_{pe}} * 100\% \sim 100\%$ for electric field E = 700 V/cm.

As a result, the correct application of stochastic theory in the range of 100–1000 V/cm was confirmed. It should be noted that both the development of stochastic theory for plasma and the theoretical tool is proposed for the calculation of turbulent heating by the strong electric field in the range of 100 < E < 1000 V/cm. Stochastic theory for plasma processes can apparently lead to the development of tools for the numerical methodology [102,106,107] of direct theoretical–numerical simulation.

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