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Lithium-Ion Battery Health Assessment Method Based on Double Optimization Belief Rule Base with Interpretability

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Abstract: Health assessment is necessary to ensure that lithium-ion batteries operate safely and dependably. Nonetheless, there are the following two common problems with the health assessment models for lithium-ion batteries that are currently in use: inability to comprehend the assessment results and the uncertainty around the chemical reactions occurring inside the battery. A rule-based modeling strategy that can handle ambiguous data in health state evaluation is the belief rule base (BRB). In existing BRB studies, experts often provide parameters such as the initial belief degree, but the parameters may not match the current data. In addition, random global optimization methods may undermine the interpretability of expert knowledge. Therefore, this paper proposes a lithium-ion battery health assessment method based on the double optimization belief rule base with interpretability (DO-BRB-I). First, the belief degree is optimized according to the data distribution. Then, to increase accuracy, belief degrees and other parameters are further optimized using the projection covariance matrix adaptive evolution strategy (P-CMA-ES). At the same time, four interpretability constraint strategies are suggested based on the features of lithium-ion batteries to preserve interpretability throughout the optimization process. Finally, to confirm the efficacy of the suggested approach, a sample of the health status assessment of the B0006 lithium-ion battery is provided.

Keywords: lithium-ion battery; belief rule base; health assessment; belief degree optimization; interpretability; modelling strategy; P-CMA-ES

Citation: Si, Z.; Shen, J.; He, W. Lithium-Ion Battery Health Assessment Method Based on Double Optimization Belief Rule Base with Interpretability. *Batteries* **2024**, *10*, 323. https://doi.org/10.3390/batteries10090323

Academic Editor: Rodolfo Dufo-López

Received: 20 July 2024
Revised: 10 September 2024
Accepted: 11 September 2024
Published: 12 September 2024

1. Introduction

Because of its high energy density, extended cycle life, and environmental friendliness, lithium-ion batteries find extensive application in a variety of fields, including aerospace, special equipment, and microgrid energy storage [1,2]. Lithium-ion batteries will progressively transition to an unstable state as a result of an internal chemical reaction with prolonged operation. Lithium-ion batteries have the potential to explode and destroy equipment if they are used in an unstable state. Lithium-ion battery health must, therefore, be evaluated in a precise, secure, and trustworthy manner [3].

In general, the micro-health parameters of lithium-ion batteries refer to microscopic indicators that can reflect the performance of the active substances and electrolytes inside the battery. Changes in these parameters are usually related to the chemical reaction of the battery, the aging of the material, and changes in the internal structure of the battery [4]. By monitoring changes in micro-health parameters, the internal health state of the battery can be determined. Current studies based on the health status assessment of lithium-ion batteries are mainly divided into the following three types: data-driven model, physical model, and hybrid model. The physical method of the lithium-ion battery health assessment is to determine the health status of the battery by analyzing its electrochemical reaction, internal structural changes, and electrical performance parameters. Amuta et al.

proposed a method for evaluating the health status of lithium-ion batteries based on voltage integration [5]. The method calculates the health status of a similar battery by integrating the voltage at a given ambient temperature and constant current charge. Su et al. proposed a method for evaluating the health status of lithium-ion batteries based on the distribution of relaxation time (DRT) [6]. Asiedu–Asante proposed a method for monitoring the health status of lithium-ion batteries via a frequency–domain reflectometry (FDR), which assesses battery aging by analyzing the correlation between the impedance measured by FDR and the equivalent series resistance (ESR) measured by EIS [7]. Wang et al. proposed a variable separation algorithm based on first- and second-order quasi-Newton methods to estimate and predict the health state of a battery by reducing the dimension and increasing the convergence rate [8]. However, the accuracy of physics-based techniques is limited because of intricate electrochemical reactions that occur inside lithium-ion batteries [9].

A data-driven approach enables accurate assessment of lithium-ion battery health through historical data and machine learning. Singh et al. proposed a deep machine learning prediction technique that utilizes encoders and decoders to extract important features and achieve high-precision Li-ion battery charge state prediction through time series analysis [10]. Teixeira et al. proposed a model for estimating the health state curve of lithium-ion batteries using gated cycle unit (GRU) neural networks, which provides a high-precision estimation of the state charge curve for smartphone battery exchange applications with low computational complexity and cost advantages [11]. Alwabli proposed a method for battery health analysis using logistic regression and convolutional neural networks, which further improved the performance indicators of convolutional neural networks through particle swarm optimization, a process that is significantly superior to other models [12]. Lin et al. suggested using feature optimization and convolutional neural networks to estimate the health state of lithium-ion batteries [13]. The method extracts a variety of electrical, thermodynamic and electrochemical characteristics; combines principal component analysis and convolutional neural network; and optimizes the feature dimension to achieve good experimental results. While most data-driven models can yield relatively accurate estimates, these methods are often black-box models and lack interpretability.

The hybrid approach to lithium-ion battery health status assessment is to improve the accuracy and reliability of the assessment by combining two or more assessment techniques and feature extraction methods for modeling. Wen et al. proposed an N-CatBoost hybrid framework for accurate estimation of lithium-ion battery health status and its uncertainty, demonstrating superior accuracy and interpretability to other machine learning algorithms [14]. Wang et al. proposed an MFE–GRU TCA hybrid model to accurately predict the health status, which showed root–mean–square errors (RMSE) of less than 0.832% and 0.614% on the NASA and CALCE datasets, respectively [15]. Yang et al. proposed an evidential inference rule health assessment method for lithium-ion batteries based on dynamic reference values [16]. This method improves the accuracy and robustness of the assessment through the dynamic adjustment and whale optimization algorithm (WOA), and the experimental results show that it has good generalization ability. Yuan et al. proposed a hybrid neural network based on variational mode decomposition and CNN–Transformer for efficient prediction of lithium-ion battery health, which performs well on CALCE datasets and has good generalization ability on NASA datasets [17]. However, based on variational mode decomposition and CNN–Transformer, it is a modeling method that combines the two models. This fusion method will not only increase the complexity of the algorithm, but also has higher requirements for the fusion strategy [18]. The DO-BRB-I model proposed in this paper can directly integrate expert knowledge and experience into the model, and has good interpretability. At the same time, because the BRB model adopts evidential reasoning (ER) for inference, its structure is simple, transparent, and easy to understand. This approach avoids the black-box characteristics of complex deep learning models and provides a more transparent decision-making basis when dealing with uncertainty and nonlinear problems of complex systems. The hybrid model is usually a balance of the physical model and data-driven model, which has better accuracy and interpretability [19].

The belief rule base (BRB) is a hybrid model based on data and knowledge [20]. It is a model proposed by Yang et al. in 2006 through the Dempster–Shafer (D–S) theory and IF–THEN rule [21]. A nonlinear modeling technique called BRB can represent several types of uncertain information, including ignorance and randomness [22]. In addition, because of its modeling method that combines expert knowledge and IF–THEN rules, BRB has strong causal reasoning ability and good interpretability [23]. Therefore, in recent years, many researchers have used BRB to evaluate the health status of lithium-ion batteries. For example, Han et al. proposed a lithium-ion battery health assessment model based on interpretability belief rule base (BRB-I), which addresses uncertainty and interpretability deficiencies and improves optimization results through WOA [3]. Zhao et al. proposed a method that combines an approximate belief rule base with a hidden Markov model to estimate lithium-ion battery capacity in orbit, using historical data and expert knowledge [24]. The model is validated with satellite battery performance data and has proved effective for in-orbit capacity estimation.

However, in these studies, parameters such as belief degree were directly divided by experts. While expert knowledge can provide broad and scientific guidance in one context, there may be limitations to the applicability of such guidance to more specific data sets. In addition, the problem that the randomness of the optimization algorithm in BRB will destroy the interpretability of BRB should be further studied. Therefore, this paper proposed a lithium-ion battery health assessment method based on the double optimization belief rule base with interpretability (DO-BRB-I). In this method, the belief degree is doubly optimized, and four interpretability constraint strategies are proposed in the optimization process, which further enhances the interpretability of the model. The main contributions of this paper are as follows:

(1) A belief degree optimization method for Gaussian membership function with Bayesian updating (GMF-B) is proposed. This method can optimize the belief degree according to the data distribution while maintaining the original expert knowledge.
(2) To improve the accuracy of the model, the projection covariance matrix adaptive evolution strategy (P-CMA-ES) is used to further optimize the other parameters, such as the belief degree, to form a double optimization.
(3) In view of the randomness of the optimization algorithm, four interpretability constraint strategies are proposed to constrain the interpretability based on the characteristics of lithium-ion batteries.

The structure of this paper is as follows: Problem formulation and construction of DO-BRB-I model are presented in Section 2. The optimization process of the model is presented in Section 3. Section 4 provides the reasoning process of the model. Section 5 contains a case study. Finally, the conclusion is given in Section 6.

2. Problem Formulation and DO-BRB-I Model Construction

This part first describes the problems existing in the health status assessment of lithium-ion batteries based on BRB. Then, the DO-BRB-I model is constructed.

2.1. Problem Formulation

This study aims to solve the following three problems in the health status assessment model of lithium-ion batteries based on BRB:

Problem 1: How to obtain a more reasonable belief distribution by combining the data distribution while maintaining the original expert knowledge.

Although experts can provide extensive and scientific guidance on the classification of belief degrees, this guidance is often based on the accumulation of experience and theory [25]. However, in the application of specific data sets, this guidance may have certain limitations. In particular, expert recommendations are often directed at general trends or phenomena, while in actual data, the true characteristics of the data may not be fully reflected due to environmental factors, differences in data distribution, or other complexities. Therefore, a new GMF-B method is proposed in this paper. This method can combine the reference values of the data set and the antecedent attributes, and then obtain a more reasonable belief distribution through Gaussian membership function and Bayesian updating. The process of the method can be described as follows:

$$\beta_r = g(A, \beta_e, x) \tag{1}$$

where A represents the reference value of the preceding attribute, β_e represents the initial reference value determined by expert knowledge, and x represents the input data. β_r represents a more reasonable belief degree after optimization, and $g(\cdot)$ represents a function of the GMF-B method.

Problem 2: How to get a more realistic belief distribution and then further enhance the accuracy of the model.

After solving problem 1, according to the data distribution, the confidence degree divided by experts is adjusted to obtain a more reasonable belief distribution. The initial belief degree of these optimizations provides a good basis for subsequent optimizations. The belief degree is optimized using P-CMA-ES to increase the model's accuracy even further. At the same time, in this step, rule weights and attribute weights, two important parameters that affect the model results, are optimized for the first time. The whole optimization process can be described as follows:

$$\Omega = \{\beta_r, \theta, \delta\} \tag{2}$$

$$\Omega_{best} = optimize(x, y, p, o) \tag{3}$$

where Ω represents the set of parameters in the optimization process, which includes belief degree β_r, rule weight θ, and attribute weight δ. x represents the input data, y represents the inference function, and o represents other parameters in the optimization process.

Problem 3: How to restrain the behavior that destroys interpretability in optimization process.

The P-CMA-ES adopted in problem 2 is a global optimization algorithm with randomness. During optimization, this randomness may destroy the interpretability of the initial parameters [20]. Therefore, it is necessary to propose a series of constraint strategies to limit the behavior that destroys interpretability. These strategies are described as follows:

$$Strategies : \{S | S_1, S_2, \ldots, S_t\} \tag{4}$$

where t represents the number of constraint strategies.

After adding constraints to the optimization algorithm, Equation (3) should be updated as follows:

$$\Omega_{best} = S - optimize(x, y, p, o) \tag{5}$$

2.2. Construction of DO-BRB-I Model

Lithium-ion battery health status assessment model based on DO-BRB-I supplemented the original BRB model with reasonable adjustment to the initial expert setting belief degree, and added interpretability constraint strategy in the optimization process. It is the k-th rule is described as follows:

$$\begin{aligned} R_k : & \ IF\ X_1\ is\ A_1^k \wedge X_2\ is\ A_2^k \wedge \ldots \wedge X_M\ is\ A_M^k, \\ & THEN\ y\ is\ \{(H_1, \beta_{1,k}), (H_2, \beta_{2,k}), \ldots, (H_N, \beta_{N,k})\} \\ & WITH\ rule\ weight\ \theta_1, \theta_2, \ldots, \theta_k \\ & AND\ attribute\ weight\ \delta_1, \delta_2 \ldots, \delta_M, \\ & IN\ S_1, S_2, \ldots, S_t \end{aligned} \tag{6}$$

where $X_i(i = 1, \ldots, M)$ refers to the indicators of lithium-ion battery health status assessment, $A_i(i = 1, \ldots, M)$ represents the reference value set of the assessment indicators, $H_i(i = 1, \ldots, N)$ is the N health status assessment results of DO-BRB-I, $\beta_i(i = 1, \ldots, N)$ represents the belief degree corresponding to each result under the belief rule k, $\theta_i(i = 1, \ldots, K)$ represents the rule weight of the i-th belief rule, and K represents the number of belief rules. $\delta_i(i = 1, \ldots, M)$ represents the attribute weight of the i-th assessment indicator, and M represents the number of assessment indicators. $S_1, S_2, \ldots, S_t$ stands for t interpretable constraint strategies.

The constructed DO-BRB-I model for lithium-ion battery health status assessment is shown in Figure 1. The whole framework can be divided into the following steps:

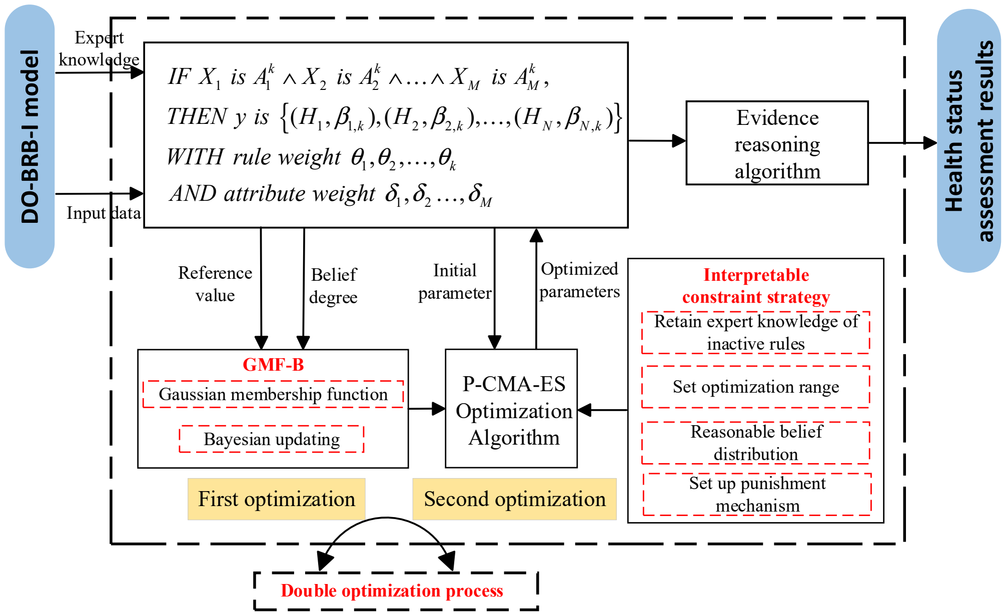


Figure 1. Lithium-ion battery health assessment model based on DO-BRB-I.

Step 1: Create the basic BRB model using the expert knowledge and input data.

Step 2: The GMF-B method constructed in this paper is used to optimize the belief degree constructed by the initial expert knowledge, and a more reasonable belief distribution is obtained.

Step 3: To further improve the progress of the model, the P-CMA-ES optimization algorithm is used to further optimize other parameters such as the belief degree. However, to prevent the interpretability of expert knowledge from being destroyed in the optimization process, four interpretability constraint strategies are added.

Step 4: According to the optimal parameters, the evidential reasoning (ER) algorithm is used to deduce the final lithium-ion battery health status results.

3. Optimization Process of DO-BRB-I Model

In this section, the initial optimization of the belief degree based on the data distribution is first performed in Section 3.1. The interpretability constraint strategy is then described in Section 3.2. The belief degree and other parameters are optimized twice in Section 3.3.

3.1. The First Optimization Based on GMF-B

Initial belief degrees provided by experts are correct in the general direction, but may not be completely accurate for specific data sets and application scenarios, such as specific battery models. However, a good initial belief degree is an important guide for subsequent optimization. Therefore, it is very important to obtain a reasonable initial belief degree. The Gauss membership function provides a statistical and probabilistic representation of the membership degree of elements to fuzzy sets in fuzzy systems [26]. This membership degree reflects the influence of the data distribution on belief degree to some extent, but the initial expert knowledge reflects the expert's guidance and consideration in the overall direction. Therefore, the initial expert knowledge must be integrated when optimizing the belief degree. Bayesian updating is a method that uses new evidence to update prior probabilities, and it is very effective in many statistical and machine learning tasks. Therefore, a new GMF-B method is proposed in this section, which can be combined with the initial belief distribution and data distribution to optimize and obtain more reasonable belief degree, to further improve the accuracy and reliability of the model. The specific steps of this method are as follows:

Step 1: Preliminary preparation

Firstly, reference values of the preceding attributes and results of expert knowledge are A and H, respectively, which will be used to calculate the subsequent membership degree. Then, according to the initial belief degree defined by experts, a prior probability distribution matrix is generated, denoted as P.

Step 2: Calculate the membership of the reference value

The membership degree for the reference value of the preceding attribute and the membership degree for the result reference value are calculated, respectively, according to the calculation formula of the Gaussian membership degree function, as shown in the following formulas:

$$\phi^A(x_H, \sigma, H) = e^{-\frac{(x_H - H)^2}{2\sigma^2}} \tag{7}$$

$$\phi^H(x_A, \sigma, A) = e^{-\frac{(x_A - A)^2}{2\sigma^2}} \tag{8}$$

Equation (7) is used to calculate the membership degree between the reference value of the preceding attribute and the input data. Equation (8) is used to calculate the membership degree between the reference value of the result and the input data. σ stands for standard deviation based on the data distribution.

Step 3: Calculate the comprehensive membership degree

The comprehensive membership corresponding to the result can be obtained by multiplying the membership of the reference value of the preceding attribute in the current rule with the membership of the result. The formula is as follows:

$$\psi_i = (\phi_1^A \times \phi_2^A, \ldots, \times \phi_M^A)\phi_i^H, (i = 1, \ldots, N) \tag{9}$$

where ψ_i represents the comprehensive membership degree corresponding to the i-th result in the current rule, $\phi_1^A, \phi_2^A, \ldots, \phi_M^A$ represent the membership degree of M antecedent attribute reference values, and ϕ_i^H represents the membership degree of the i-th result.

Step 4: Perform Bayesian updates

The above three steps calculate the comprehensive membership degree of the result according to the input data and reference values. This membership can be used as the data's influence on belief degree. Therefore, the initial expert knowledge is used as the prior probability distribution, and the above membership degree is used as new evidence to update the prior probability by Bayesian method. This process can be expressed as follows:

$$P(H|C_i) = \sum_{i=1}^{N} \psi_i \tag{10}$$

$$P(C_i|H) = P(H|C_i)P(C_i) \tag{11}$$

where $P(H|C_i)$ is the likelihood function expressed as the sum of the comprehensive membership degree. $P(C_i)$ is the prior probability of the current combination and $P(C_i|H)$ is the posterior probability of the current combination.

Step 5: Normalized operation

Repeat the first four steps to calculate the posterior probability of each set of input data and each combination, and carry out normalization to get the final posterior probability distribution matrix. This posterior probability distribution matrix is the belief distribution of the expert knowledge optimized by the data distribution.

3.2. Interpretability Constraint Strategy

After completing the preliminary optimization of belief degree in Section 3.1, a more reasonable belief distribution is obtained. To further improve the accuracy of the model, it is beneficial to further optimize the belief degree and other parameters. However, the P-CMA-ES selected in this paper is a random global optimization, and, if it is not constrained, the interpretability of the original expert knowledge will be seriously damaged. Therefore, four interpretability constraint strategies are proposed in this section to constrain the second optimization process of the model according to the actual lithium-ion battery health status background. The following is a detailed introduction to these four strategies:

Strategy 1. Ensure that activated rules participate in optimization and reasoning

The interpretive BRB model incorporates every potential state combination for evaluating the health status of lithium-ion batteries. However, due to the possibility of inadequate observational data, not all rules may be activated by the input data [25]. Experts create a comprehensive rule base from an international standpoint, and data gathered from a particular lithium battery type may not activate all the rules, leaving some of them dormant and excluded from the BRB reasoning process. To keep valuable expert original information, certain non-activation rules should not be included in training. However, earlier research treated each parameter as an individual in a population and utilized a global optimization approach for model optimization. In global optimization, these individuals go through an evolutionary process, but for non-activation rules in BRB, this approach is not reasonable. Therefore, it is necessary to consider a reasonable way to identify non-active rules and retain their original relevant parameters as shown by the following:

$$\Omega = \left\{\theta_1, \ldots, \theta_k, \delta_1, \ldots, \delta_M, \beta_{1,k}, \ldots, \beta_{N,k}\right\} \tag{12}$$

Activation weights can be used to identify rules that are not activated. The activation of the k-th rule can be expressed as follows:

$$Q_k = \{w_1, w_2, \ldots, w_E\}, k = 1, 2, \ldots, L \tag{13}$$

where E indicates the number of input data, and $w_1, w_2, \ldots, w_E$ represents the active weight of the current rule for all input data. If all activation weights are 0, the current rule is never activated. Therefore, for this rule, the parameters in the Ω should be preserved.

Strategy 2. Set the optimization range of the parameter

Expert knowledge provides important guidance in the assessment of the health status of lithium-ion batteries [20]. However, when it comes to a specific battery type, it needs to be optimized according to the actual situation to ensure the accuracy of the model. To maintain the integrity of expert knowledge in the optimization process, it is necessary to set a reasonable optimization range for each parameter. This approach not only makes use of expert experience, but can also be effectively adjusted in the actual application of a specific battery model, thus achieving a balanced optimization result. This process can be described as follows:

$$\begin{aligned} &\Omega_{low} \le \Omega \le \Omega_{up} : \\ &\Big\{\theta_{k,low} \le \theta_k \le \theta_{k,up}, k = 1, 2, \ldots, L \\ &\delta_{m,low} \le \delta_m \le \delta_{m,up}, m = 1, 2, \ldots, M \\ &\beta_{n,k,low} \le \beta_{n,k} \le \beta_{n,k,up}, n = 1, \ldots, N\Big\} \end{aligned} \tag{14}$$

where Ω_{up} and Ω_{low} represent the upper and lower bounds of the optimization range, respectively.

Strategy 3. Ensure reasonable belief distribution

Due to the randomness of the optimization algorithm, some rules that do not conform to the health status of lithium ions may be generated to blindly obtain higher accuracy [25].

In Figure 2, several belief distributions that may be presented during the optimization process are shown, and, H1 to H4, respectively, represent several health states of lithium-ion batteries. The several scenarios shown in Figure 2b are unreasonable because the health of a lithium-ion battery cannot be both good and bad at the same time [3]. Therefore, a reasonable belief distribution should be either monotonic or convex. Therefore, the distribution of belief should be strictly constrained in the optimization process to make it conform to a reasonable distribution. The constraints on belief can be expressed as follows:

$$\begin{aligned} &\beta_i \sim K_i \ (\mathrm{i} = 1, 2, \ldots, L) \\ &K_i \in \{\{\beta_1 \le \beta_2 \le \ldots \le \beta_n\} \\ &\qquad or\{\beta_1 \ge \beta_2 \ge \ldots \ge \beta_n\} \\ &\qquad or\{\beta_1 \le \ldots \le max(\beta_1, \beta_2, \ldots, \beta_n) \ge \ldots \ge \beta_n\}\} \end{aligned} \tag{15}$$

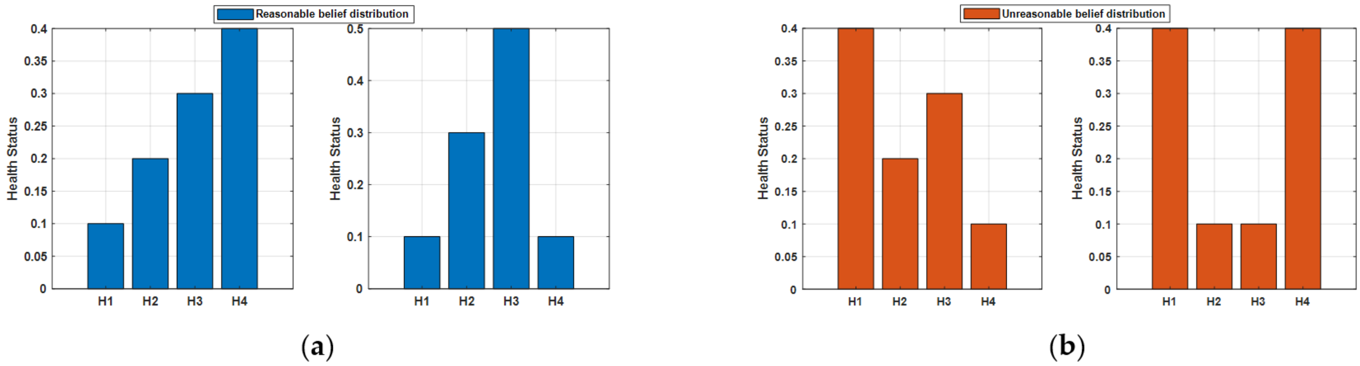


Figure 2. Belief distribution example. (**a**) Reasonable belief distribution; (**b**) Unreasonable belief distribution.

Strategy 4. Punish behavior that goes beyond limits.

Although the boundary constraint is added in the optimization process, the step length setting of the optimization algorithm may not be suitable for all parameters and iteration processes. In the process of optimization algorithm exploration, there may still be a phenomenon that exceeds the boundary. Although this parameter may obtain higher precision, its transgression behavior has destroyed the original interpretability, so it needs to be punished for this behavior. The principle of punishment is to add an extra value to this parameter so that it will be eliminated in the subsequent screening process. This value is calculated as follows:

$$punishment = \sum_{i=1}^{g} \max(0, \Omega_i - \Omega_{i,up}) + \sum_{i=1}^{g} \max(0, \Omega_{i,low} - \Omega_i) \tag{16}$$

where Ω_i represents the initial population generated by the optimization algorithm.

3.3. The Second Optimization Based on P-CMA-ES

To improve the accuracy of the model, this part uses P-CMA-ES [27] to optimize the attribute weight, rule weight, and GMF-B adjusted belief degree of DO-BRB-I. This method can dynamically adjust the search strategy to better adapt to the characteristics of the current optimization problem. However, the original P-CMA-ES algorithm takes the form of random scattering points and iterates continuously to find the optimal parameter [20]. This type of random global optimization may result in optimized parameters that deviate excessively from the original parameters, erasing the original expert knowledge and making the model difficult to comprehend. Thus, the incorporation of interpretability constraint techniques improves the original P-CMA-ES method. The enhanced P-CMA-ES, the optimization method that emerged as a result, is best explained as follows:

$$output_{result} = \sum_{n=1}^{N} u(H_n)\beta_n \tag{17}$$

The DO-BRB-I modeling accuracy is represented by the mean square error (MSE). In this section, the rule weights, attribute weights, and belief degrees are optimized parameters, so the MSE can be expressed as follows:

$$MSE(\theta, \delta, \beta) = \frac{1}{T}\sum_{t=1}^{T}(output_{result} - output_{actual})^2 \tag{18}$$

where T is the training data volume, $output_{actual}$ is the actual output value and $output_{result}$ is the predicted output value of the system. Therefore, the optimization objective is to minimize the MSE through iterative refinement. The specific process is as follows:

Step 1 (Initial operation): Provide the initial parameters as follows:

$$\Omega^0 = \{\theta_1, \ldots, \theta_k, \delta_1, \ldots, \delta_M, \beta_1, \ldots, \beta_n\} \tag{19}$$

where $w^g = \Omega^0$ is the optimized parameter set.

Step 2 (Sampling operation): The initial population can be determined by the following:

$$\Omega_i^{g+1} \sim w^g + \varepsilon^g N(0, C^g) \ i = 1, \ldots, \lambda \tag{20}$$

where Ω_i^{g+1} represents the i-th solution of the (g+1)-th generation, ω represents the mean of the population, ε represents the step size, N represents the normal distribution, and C^g represents the covariance matrix of the g-th generation.

Step 3 (Constraint operation): In this step, the four constraint strategies introduced by Equations (12)–(16) are added to the optimization process of the model. These four constraint strategies not only retain the parameters of rules that have never been activated, but also impose detailed constraints on them. Ensuring that expert knowledge is not compromised.

Step 4 (Projection operation): The solutions produced by sampling operations may not satisfy the constraints, thus necessitating projection operations to ensure adherence to the constraints:

$$\begin{aligned}\Omega_i^{g+1}(1 + n_e \times (j-1) : n_e \times j) = \Omega_i^{g+1}(1 + n_e \times (j-1) : n_e \times j) - A_e^T \times (A_e \times A_e^T)^{-1} \\ \times \Omega_i^{g+1}(1 + n_e \times (j-1) : n_e \times j) \times A_e\end{aligned} \tag{21}$$

A hyperplane can be represented as $A_e\Omega_i^g(1 + n_e \times (j-1) : n_e \times j) = 1$, where n_e represents the number of equality constraint variables in solution Ω_i^g, $j = 1, \ldots, N+1$ represents the number of equality constraints in solution $\Omega_i^g w^{g+1} = \sum_{i=1}^{\tau} h_i \Omega_{i:\lambda}^{g+1}$, and $A_e = [1 \cdots 1]_{1\times N}$ represents a parameter vector.

Step 5 (Selection operation): Update the mean value by performing selection operations using the following formula:

$$w^{g+1} = \sum_{i=1}^{\tau} h_i \Omega_{i:\lambda}^{g+1} \tag{22}$$

where h_i represents the weight coefficient of the i-th equation. $\Omega_{i:\lambda}^{g+1}$ represents the i-th solution in the (g+1)-th generation. τ represents the subpopulation size.

Step 6 (Updating operation): Update the covariance matrix through adaptive operations and determine the population search range and direction. The calculation process is illustrated in the following formula:

$$C^{g+1} = (1 - a_1 - a_2)C^g + a_1 p_c^{g+1} \left(p_c^{g+1}\right)^T + a_2 \sum_{i=1}^{\tau} h_i \left(\frac{\left(\Omega_{i:\lambda}^{g+1} - \omega^g\right)}{\varepsilon^g} \right) \left(\frac{\left(\Omega_{i:\lambda}^{g+1} - \omega^g\right)}{\varepsilon^g} \right)^T \quad (23)$$

where a_1 and a_2 represent the learning rates. p_c^{g+1} represents the evolutionary path of the covariance, and 0 is the initial evolutionary path.

To sum up, the overall optimization process of DO-BRB-I is shown in Figure 3.

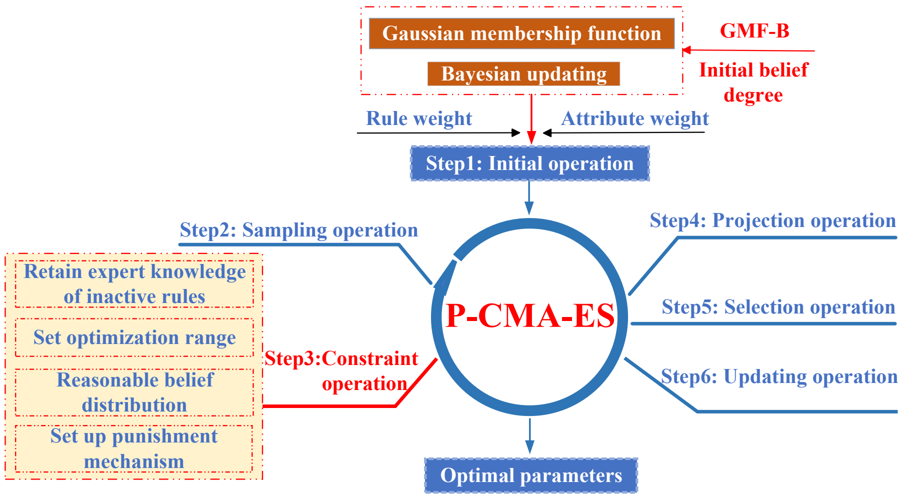


Figure 3. Complete optimization process for DO-BRB-I.

4. Reasoning Process of DO-BRB-I

The ER is an inference rule for evidence composition. The method is mainly used to solve uncertainty and fuzziness problems, and it uses the concepts of Dempster–Shafer theory and Bayesian reasoning. The essence of the ER algorithm is to integrate information from multiple independent pieces of evidence to form a comprehensive and reliable conclusion or prediction [28]. This fusion improves the reliability and accuracy of decision-making and achieves good results, so ER is often used in BRB reasoning. The DO-BRB-I model developed in this study is derived from the fundamental BRB model. The ER has been selected as the inference engine for this model. Its detailed process is described as follows:

Step 1: Calculate the degree of matching, which indicates the flexibility of the rules, between the input sample information and belief rules. The matching degree of the k-th rule for the i-th input is calculated as follows:

$$a_i^k = \begin{cases} \frac{A_i^{l+1} - x_i}{A_i^{l+1} - A_i^l}, k = l, A_i^l \le x_i \le A_i^{l+1} \\ 1 - a_i^k, k = l + 1 \\ 0, i = l \ldots K, k \ne l, l + 1 \end{cases} \quad (24)$$

where a_i^k represents the degree of matching, A represents the reference value of the prerequisite attribute, and x represents the input data. This membership function ensures that at least one rule can be activated for each input of data.

Step 2: Determine the activation weight through the following calculation:

$$w_k = \frac{\theta_k \prod_{i=1}^{M} \left(a_i^k\right)^{\delta_i}}{\sum_{i=1}^{K} \theta_l \prod_{i=1}^{M} \left(a_i^l\right)^{\delta_i}} \quad (25)$$

where $\delta_i(i = 1, \ldots, M)$ represents the attribute weight for the kth index.

Step 3: Calculate the ultimate belief degree by applying rule inference with the ER analysis algorithm using the following formula:

$$\beta_n = \frac{\mu \times \left[\prod_{i=1}^{L} \left(w_l \beta_{n,l} + 1 - w_l \sum_{i=1}^{N} \beta_{i,l} \right) - \prod_{l=1}^{L} \left(1 - w_l \sum_{i=1}^{N} \beta_{i,l} \right) \right]}{1 - \mu \times \left[\prod_{l=1}^{L} (l - w_l) \right]} \tag{26}$$

$$\mu = \frac{1}{\sum_{n=1}^{N} \prod_{l=1}^{L} (w_l \beta_{n,l} + 1 - w_l \sum_{i=1}^{N} \beta_{i,l}) - (N-1) \prod_{l=1}^{L} (1 - w_l \sum_{i=1}^{N} \beta_{i,l})} \tag{27}$$

where β_n represents the belief degree from the final belief distribution.

Step 4: Calculate the expected utility value. The final belief distribution result and utility conversion formula are as follows:

$$y = \{(H_n, \beta_n), n = 1, \ldots, N\} \tag{28}$$

$$\mu(S(A')) = \sum_{n=1}^{N} \mu(H_n)\beta_n \tag{29}$$

where A' is the actual input vector, $\mu(H_n)$ is the utility of H_n, and $\mu(S(A'))$ is the expected utility at the end. $S(\cdot)$ is a set composed of belief distributions. This methodology, which combines the IF–THEN rule-based and utility-based methods, enhances the reliability of initial information and facilitates logical adjustments to belief structures during the inference process.

5. Case Study

This section will use a case study of a real-world lithium-ion battery health assessment to verify the effectiveness of the DO-BRB-I approach. The data used in this paper are from the NASA Ames Prognostics Center of Excellence and contain information on aging of 18650 LiCoO2 batteries. The aging data of 167 B0006 lithium-ion batteries are selected in this experiment, of which 112 are training data. All data are test data.

5.1. Construction of the DO-BRB-I Model

This article identifies the constant current (CC) and constant voltage (CV) stages of lithium-ion batteries as the first characteristics. The CC stage involves charging lithium-ion batteries with a steady current of 1.5 A until the battery voltage reaches 4.2 V. In the CV stage, the charging process occurs at a consistent voltage of 4.2 V until the current decreases to 20 mA. In a complete charging stage, if the CC stage is longer and the CV stage is shorter, it indicates that the health status is the best. Based on the operating process and historical knowledge of the battery, set four reference points for each attribute, namely Long (L), Normal (N), Little Short (LS), and Short (S). Attribute weights are also set for each attribute. These values are displayed in Table 1. The capacity of the battery as a test result is divided into safe (S), normal (N), small bad (LB), and very bad (VB), as shown in Table 2. The changes of CC and CV stages of the battery in the data set are shown in Figure 4a, and the changes of battery capacity are shown in Figure 4b [29]. The part highlighted in the pink circle is due to the instability of the chemical reactions that take place inside lithium-ion batteries. Using the Cartesian product to construct the initial BRB, the expert assigns belief degree to each rule result, and the initial belief rule base is shown in Table A1 in the Appendix A. In addition, the rule weight for each rule is initialized to 1.

Table 1. The reference values of antecedent attribute.

Attribute	Attribute Weight	VL	L	N	S
PC-time-CC	1	0.93	0.72	0.48	0.22
PC-time-CV	1	0.53	0.48	0.42	0.34

Table 2. The reference value of health status.

Health Status	S	N	LB	VB
Reference value	2.04	1.67	1.38	1.14

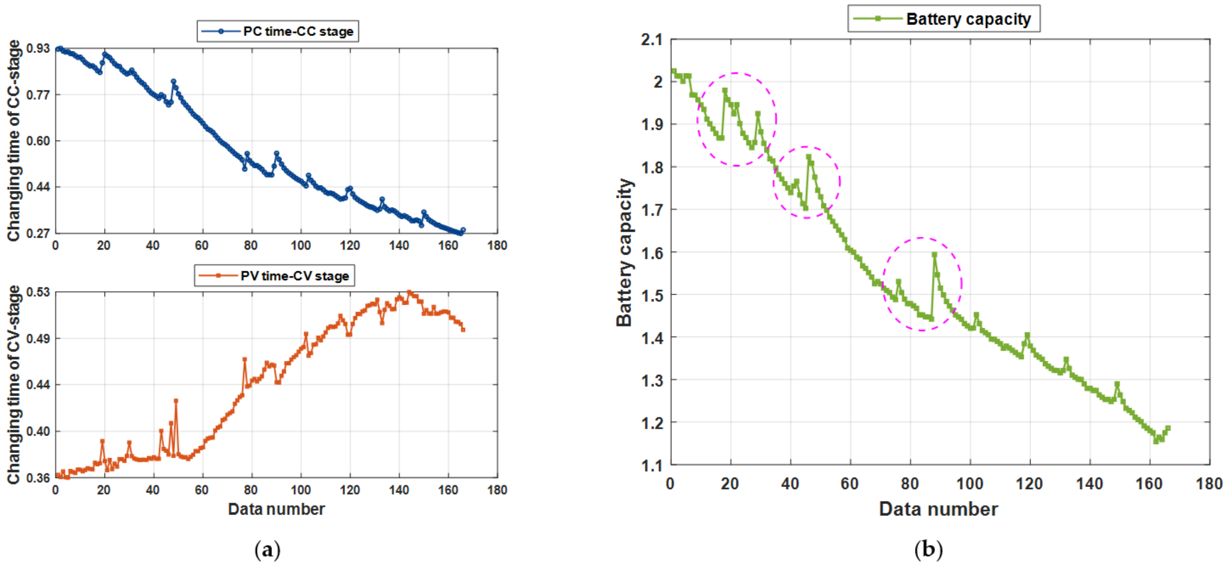


Figure 4. Data distribution of lithium-ion batteries. (**a**) The data distribution of the two input attributes of the lithium-ion battery in the PC time-CC stage and the PV time-CV stage; (**b**) The capacity variation distribution of lithium-ion batteries.

After this process is performed, the initial BRB model for lithium-ion battery health assessment is constructed. Then, the GMF-B belief degree optimization method introduced in Section 3.1 is used for the first optimization of belief degree, and the P-CMA-ES optimization algorithm with four constraint strategies introduced in Section 3.3 is used for the second optimization of other parameters such as the belief degree. Finally, ER is used to deduce the model, and the final evaluation result of lithium-ion battery health status can be obtained.

5.2. Experimental Analysis of DO-BRB-I

5.2.1. Analysis of Experimental Results

In the previous section, a DO-BRB-I based lithium-ion battery health status assessment model was constructed. The DO-BRB-I evaluation results have an MSE of 0.0007, while the initial BRB model built on expert knowledge has an MSE of 0.0144. Figure 5 shows the comparison of the true value, estimated value by DO-BRB-I, and estimated value by expert knowledge. As can be seen from the figure, DO-BRB-I has very accurate evaluation results, and DO-BRB-I strictly follows the four interpretability constraint strategies proposed in Section 3.2 in the optimization process. Therefore, the model constructed in this paper greatly improves the accuracy of the model on the premise of maintaining good interpretability.

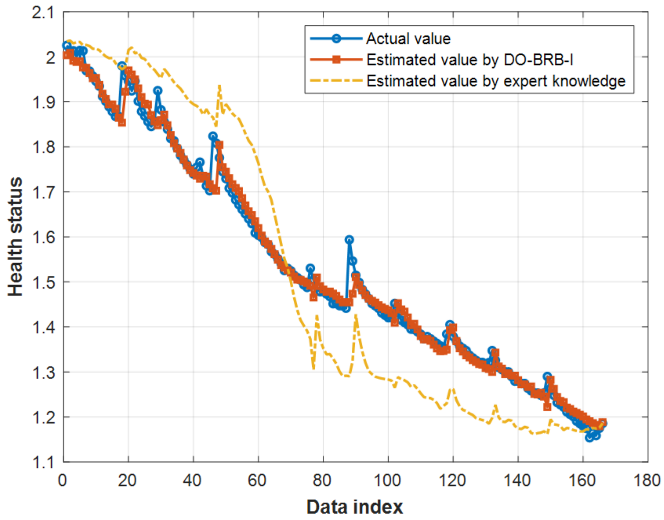


Figure 5. Comparison of experimental results.

5.2.2. The Effectiveness Analysis of the Optimization Process

The DO-BRB-I model constructed in this paper has a dual optimization process. The initial optimization is mainly for the belief degree of expert division. Because the belief degree given by experts is not necessarily applicable to the current specific data set, it is not targeted. Therefore, the GMF-B proposed in Section 3.1 of this paper is used for the initial optimization of belief degree, which can update the belief degree according to the data set while maintaining the original expert knowledge to obtain a more reasonable belief distribution. This process plays a good guiding role in the subsequent optimization. The secondary optimization is mainly to further improve the accuracy of the model, and further optimize the belief degree obtained from the initial optimization, as well as the attribute weights and rule weights. The optimization information of attribute weights and rule weights is shown in Tables 3 and 4. Information about belief levels is presented in Table A1 in the Appendix A. Table 5 shows the comparison of MSE, root–mean–squared error (RMSE), mean absolute error (MAE), mean absolute percent error (MAPE) of the model after two optimizations, and the initial BRB model, where GMF-B-BRB represents the model optimized once by GMF-B. The table shows that every optimization raises the original BRB mode's accuracy.

Table 3. The optimization information for the attribute weights.

Attribute	Initial Value	Constraint Range	The Optimized Value
PC-time-CC	1	0.6~1.0	0.9438
PC-time-CV	1	0.2~0.8	0.5070

Table 4. The optimization information for the rule weights.

Rule Number	Initial Value	Constraint Range	The Optimized Value
1	1	0.2~1.0	0.2854
2	1	0.4~1.0	0.6305
3	1	0.4~1.0	0.6995
4	1	0.6~1.0	1
5	1	0.4~1.0	0.5169
6	1	0.6~1.0	0.8144
7	1	0.4~1.0	0.5580
8	1	0.6~1.0	1
9	1	0.6~1.0	0.9978
10	1	0.6~1.0	0.9986

Table 4. *Cont.*

Rule Number	Initial Value	Constraint Range	The Optimized Value
11	1	0.4~1.0	0.7850
12	1	0.6~1.0	0.9279
13	1	0.6~1.0	1
14	1	0.6~1.0	0.8592
15	1	0.6~1.0	0.9673
16	1	0.4~1.0	0.6865

Table 5. The comparison of various indicators in the optimization process.

Model	MSE	RMSE	MAE	MAPE
Initial BRB	0.0144	0.1199	0.1080	0.0712
GMF-B-BRB	0.0049	0.0700	0.0624	0.0409
DO-BRB-I	0.0007	0.0265	0.0153	0.0096

5.2.3. Interpretability Analysis of the Model

The P-CMA-ES optimization algorithm used in the second optimization is a random global optimization. Therefore, to keep the original interpretability of the model from being destroyed, this paper proposes four interpretability constraint strategies to constrain the interpretability of the model. In this section, these four strategies will be analyzed in the context of actual lithium-ion battery health assessment.

Strategy 1: Figure 6 shows the activation of all the rules in this experiment. As can be seen from the figure, the activation weights of rules 4, 8, and 13 are always 0 for all input data, which means that these three rules have never been activated. Therefore, according to interpretability strategy 1, expert knowledge in these three rules should be retained. If expert knowledge is not constrained, the situation shown in the circle in Figure 7 occurs. The original expert knowledge is severely damaged. It can be seen from Table 4 that their rule weights are retained, and the belief degrees of these three rules in Figure 8 are also retained, as shown in the green ellipse.

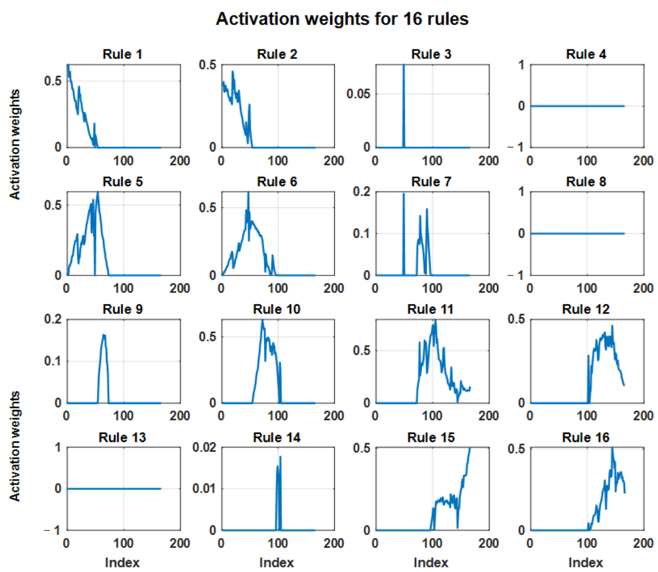


Figure 6. The activation weights for 16 rules.

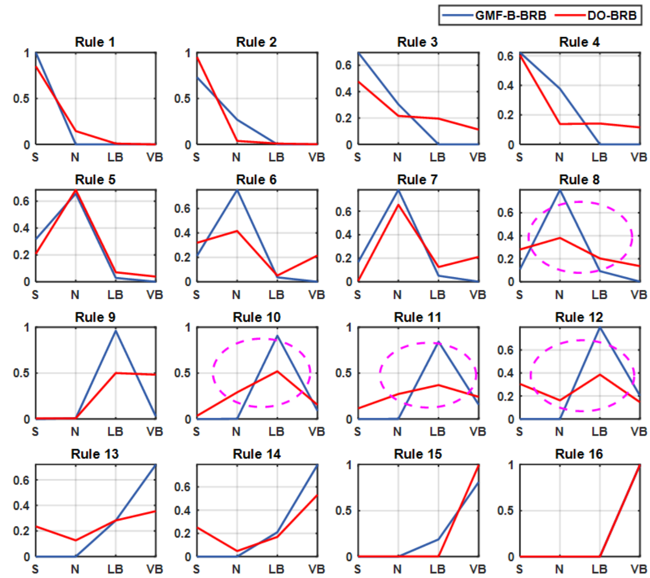


Figure 7. The comparison of belief distributions without interpretability constraints.

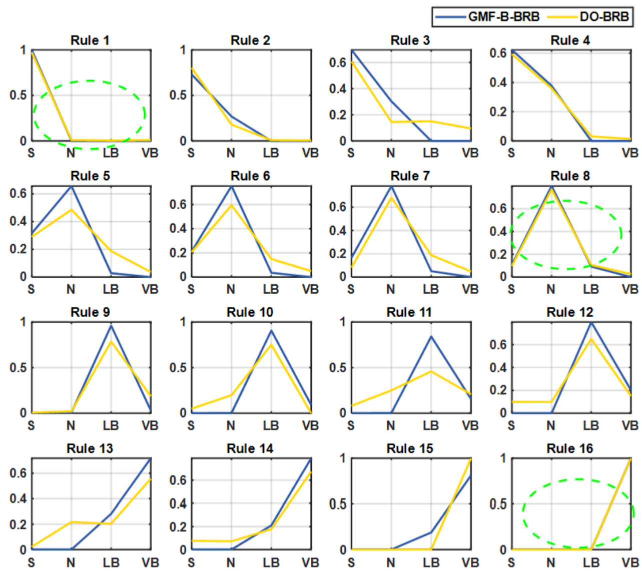


Figure 8. The comparison of belief distributions after interpretability constraints.

Strategy 2: Interpretability constraint strategy 2 mainly constrains the boundary range of the optimized parameter. In this experiment, the optimized parameters are attribute weight, rule weight, and belief degree. Therefore, Table 3, Table 4, and Table A1 in Appendix A give the constrained ranges and optimized results of these parameters, respectively. Figure 8 shows the comparison results before and after optimization with belief degree constraints, while Figure 7 shows the comparison results before and after optimization without constraints. It can be seen from the figure that the two curves in Figure 8 are more consistent, while the randomness of the optimization algorithm in Figure 7 obviously damages the original distribution form; several rules drawn in the pink ellipse seriously damage the interpretability of the original rules.

Strategy 3: This strategy is mainly restricted for the occurrence of unreasonable belief distribution, which seriously violates the normal distribution of lithium-ion battery health. After reduction, all the belief distributions are convex or monotonic, as shown in Figure 7.

Strategy 4: This strategy is mainly aimed at the fact that there are still many transgressive behaviors in the offspring after the boundary constraint of strategy 2. As shown in Figure 9, the red and blue lines are the upper and lower bounds, respectively. The population values produced in the offspring were then counted, and the points that crossed the line had been marked pink. According to strategy 4, such points should be punished, so that these points are eliminated in the subsequent screening. After being constrained by strategy 4, as shown in Tables 3 and 4, and Table A1 in the Appendix A, all optimized parameters are strictly within the constrained range.

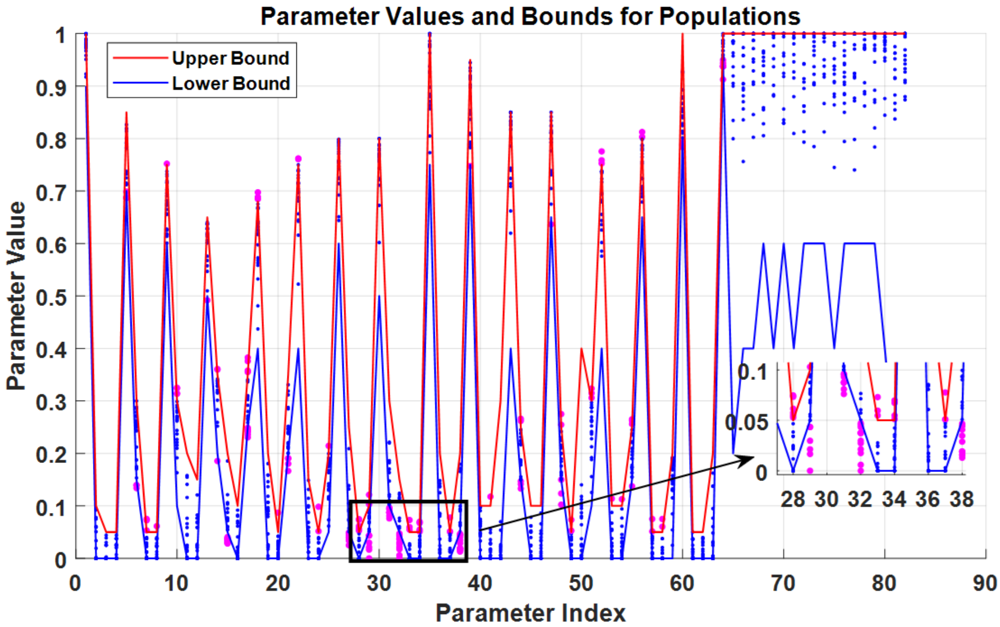


Figure 9. Parameter values and bounds for populations.

5.3. Comparative Study

This section will compare the DO-BRB-I model with other BRB models based on different optimization algorithms and other data-driven approaches, focusing on evaluating its accuracy and interpretability. The accuracy of the model is measured by the MSE, while interpretability is measured by DMSE. The calculation formula of MSE is given in Equation (18). The calculation method of DMSE was proposed by Han et al., as follows [3]:

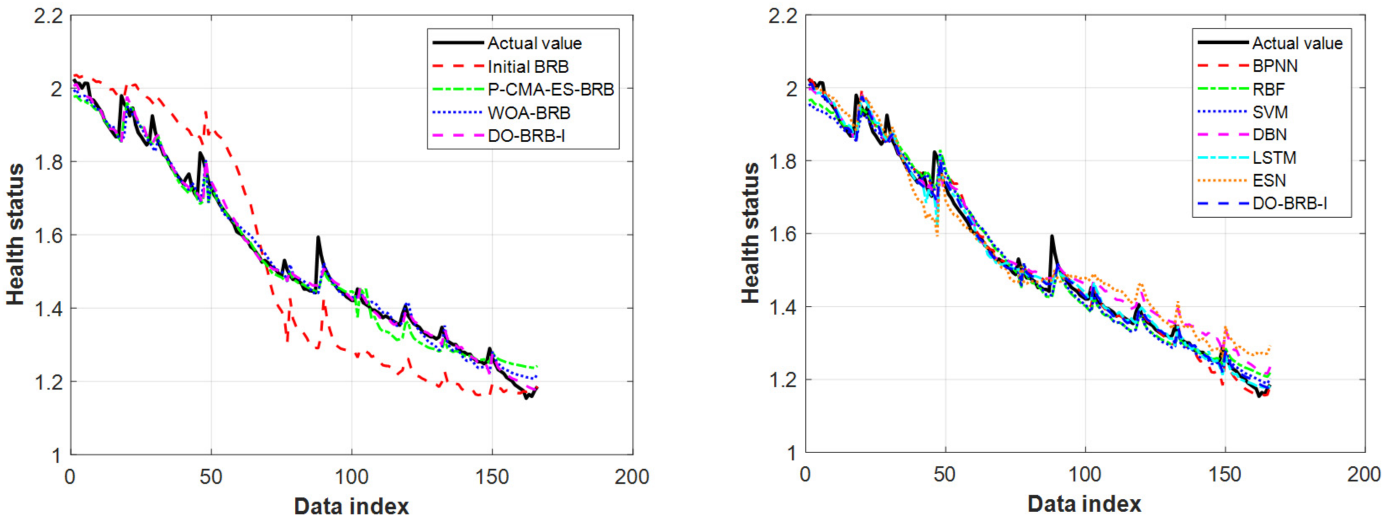
$$DMSE = \frac{MSE_{ex} - MSE_{op}}{\rho(h_n, h_n{}')} \tag{30}$$

where MSE_{ex} represents the MSE value of the initial BRB model constructed by expert knowledge and MSE_{op} represents the MSE value of the optimized model. The difference in Euclidean distance between the original and improved parameters created by experts is represented by $\rho(h_n, h_n{}')$. A higher DMSE value means that the model has a better balance of interpretability and accuracy.

As shown in Table 6, in the comparison with BRB model, the BRB model based on P-CMA-ES and WOA optimization algorithm is selected. In comparison with other data-driven methods, back propagation neural network (BPNN), radial basis function (RBF), support vector machine (SVM), DBN, long short-term memory (LSTM), and echo state network (ESN) are chosen. In Table 6, DMSE represents the degree to which the optimized model and the original model expert knowledge are preserved in the BRB model. Therefore, the original BRB model and other non-BRB models cannot calculate DMSE, which is indicated by "*" in the table. Figure 10 shows a comparison of these methods.

Table 6. The comparison results of various indicators of each model.

Part	Model	MSE	DMSE	RMSE	MAE	MAPE
Part I	Initial BRB	0.0144	*	0.1199	0.1080	0.0712
	P-CMA-ES-BRB	0.0011	0.0072	0.0337	0.0192	0.0117
	WOA-BRB	0.0009	0.0035	0.0297	0.0207	0.0134
	DO-BRB-I	0.0007	0.0106	0.0265	0.0153	0.0096
Part II	BPNN	0.0009	*	0.0294	0.0194	0.0121
	RBF	0.0012	*	0.0363	0.0251	0.0153
	SVM	0.0011	*	0.0344	0.0259	0.0165
	DBN	0.0017	*	0.0424	0.0331	0.0221
	LSTM	0.0009	*	0.0306	0.0182	0.0138
	ESN	0.0028	*	0.0528	0.0390	0.0244



(**a**) (**b**)

Figure 10. Comparison of experimental results. (**a**) Comparison with other BRB methods; (**b**) Comparison with other advanced methods.

Analyzing the data in Table 6 can be summarized as follows:

(1) The DO-BRB-I adds a double optimization and interpretability constraint strategy to the original BRB. The DO-BRB-I has the lowest MSE, RMSE, MAE, MAPE, and the highest DMSE compared to other BRB models. The results show that the proposed model not only has outstanding performance in prediction accuracy, but also has better error control ability and overall stability, and has stronger adaptability. These characteristics further illustrate the effectiveness and advantages of the proposed method, which can better complete the health status assessment of lithium-ion batteries. Although the accuracy of WOA–BRB is close to that of DO-BRB-I, the DMSE values are very different. These differences show that although WOA–BRB has good accuracy, the interpretability is seriously damaged in the optimization process.

(2) Compared with other data-driven methods, although the original BRB model based on P-CMA-ES optimization algorithm also has better accuracy, it is still slightly inferior to BPNN and LSTM. However, the method proposed in this paper makes a double optimization of the belief degree and further improves the accuracy of the model. It is, therefore, ahead of the models listed in Part II in terms of accuracy. Furthermore, the model developed in this study can be traced back to its sources, the modeling process is transparent, and it employs the ER analysis algorithm for reasoning. Additionally, during the optimization phase, the interpretability of the model is constrained using four different interpretability constraint strategies. Therefore, the DO-BRB-I model

constructed in this paper is highly interpretable. However, DBN and BPNN models are black-box models, the decision-makers are not clear about their internal working principle, and they are not interpretable.

In summary, the DO-BRB-I model developed in this paper has a strong ability in the assessment of the health status of lithium-ion batteries.

6. Conclusions

This paper presents a lithium-ion battery health assessment method based on DO-BRB-I. This approach can accurately and completely analyze the health status of lithium-ion batteries while maintaining high interpretability of the model. The innovation of the method proposed in this paper mainly includes the following points:

(1) A new belief degree optimization method for GMF-B is proposed. This method can adjust the belief degree delimited by experts according to some prior data to obtain a more reasonable belief distribution.
(2) Based on the features of the lithium-ion battery health assessment background, four interpretability constraint strategies are proposed for the second optimization. These four strategies strictly constrain the behaviors that destroy interpretability in the optimization process.

The method proposed in this paper is to optimize the belief degree independently according to the data distribution before the P-CMA-ES optimization algorithm, which further improves the accuracy and interpretability of the model. However, the BRB model based on the original P-CMA-ES is still slightly inferior to other models. Therefore, it is the future research direction to further explore the tuning methods of belief degree and other parameters in P-CMA-ES algorithm to achieve better accuracy under single optimization. In addition, the DO-BRB-I proposed in this paper is a general method, and it is also meaningful to extend the application of this model in more backgrounds.

Author Contributions: Conceptualization and methodology, Z.S., J.S. and W.H.; investigation, J.S.; writing—original draft preparation, Z.S.; writing—review and editing, Z.S., J.S. and W.H. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the Harbin Normal University Graduate Student Innovation Program under Grant No. HSDSSCX2024-41, in part by the Open Foundation of Key Laboratory of the Ministry of Education on Application of Artificial Intelligence in Equipment under Grant No. AAIE-2023-0102, in part by the Shandong Provincial Natural Science Foundation under Grant No. ZR2023QF010.

Data Availability Statement: The research data in this article were obtained in part from the NASA Diagnostic Centre of Excellence. For the source URL of the lithium-ion battery dataset in the Baidu AI Studio platform, please visit: https://aistudio.baidu.com/datasetdetail/171099 (accessed on 10 September 2024).

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Table A1. Belief degree optimization information.

No.	Attribute CC ∧ CV	Initial Belief Degree	Initial Optimization Belief Degree	The Range of Belief Degree Optimization	Second Optimization Belief Degree
1	VL ∧ S	{1.00, 0.00, 0.00, 0.00}	{1.00, 0.00, 0.00, 0.00}	{0.90~1.00, 0.00~0.10, 0.00~0.05, 0.00~0.05}	{0.95, 0.01, 0.04, 0.00}
2	VL ∧ N	{0.80, 0.20, 0.00, 0.00}	{0.73, 0.27, 0.00, 0.00}	{0.70~0.85, 0.15~0.30, 0.00~0.05, 0.00~0.05}	{0.83, 0.15, 0.02, 0.00}
3	VL ∧ L	{0.70, 0.20, 0.10, 0.00}	{0.69, 0.31, 0.00, 0.00}	{0.60~0.75, 0.10~0.30, 0.00~0.20, 0.00~0.15}	{0.68, 0.12, 0.16, 0.04}
4	VL ∧ VL	{0.65, 0.25, 0.10, 0.00}	{0.62, 0.38, 0.00, 0.00}	{0.50~0.65, 0.20~0.35, 0.05~0.20, 0.00~0.10}	{0.68, 0.32, 0.00, 0.00}
5	L ∧ S	{0.60, 0.30, 0.10, 0.00}	{0.31, 0.65, 0.04, 0.00}	{0.25~0.35, 0.40~0.68, 0.00~0.20, 0.00~0.05}	{0.28, 0.50, 0.19, 0.03}
6	L ∧ N	{0.50, 0.40, 0.10, 0.00}	{0.21, 0.75, 0.04, 0.00}	{0.20~0.35, 0.40~0.75, 0.00~0.15, 0.00~0.05}	{0.24, 0.58, 0.15, 0.03}

Table A1. *Cont.*

No.	Attribute CC ∧ CV	Initial Belief Degree	Initial Optimization Belief Degree	The Range of Belief Degree Optimization	Second Optimization Belief Degree
7	L ∧ L	{0.40, 0.40, 0.10, 0.10}	{0.17, 0.78, 0.05, 0.00}	{0.05~0.20, 0.60~0.80, 0.05~0.25, 0.00~0.05}	{0.15, 0.68, 0.15, 0.02}
8	L ∧ VL	{0.30, 0.45, 0.15, 0.10}	{0.11, 0.80, 0.09, 0.00}	{0.05~0.10, 0.50~0.80, 0.10~0.30, 0.05~0.15}	{0.11, 0.80, 0.09, 0.00}
9	N ∧ S	{0.00, 0.20, 0.60, 0.20}	{0.00, 0.01, 0.96, 0.03}	{0.00~0.05, 0.00~0.05, 0.75~1.00, 0.00~0.20}	{0.00, 0.00, 0.91, 0.09}
10	N ∧ N	{0.00, 0.10, 0.50, 0.40}	{0.00, 0.00, 0.91, 0.09}	{0.00~0.05, 0.05~0.20, 0.75~0.95, 0.00~0.10}	{0.05, 0.20, 0.75, 0.00}
11	N ∧ L	{0.00, 0.10, 0.40, 0.50}	{0.00, 0.00, 0.84, 0.16}	{0.00~0.10, 0.00~0.20, 0.65~0.85, 0.15~0.25}	{0.04, 0.08, 0.65, 0.23}
12	N ∧ VL	{0.00, 0.00, 0.40, 0.60}	{0.00, 0.00, 0.80, 0.20}	{0.00~0.10, 0.00~0.10, 0.65~0.85, 0.15~0.25}	{0.10, 0.10, 0.65, 0.15}
13	S ∧ S	{0.00, 0.15, 0.25, 0.60}	{0.00, 0.00, 0.28, 0.72}	{0.00~0.05, 0.00~0.40, 0.10~0.30, 0.40~0.75}	{0.00, 0.00, 0.28, 0.72}
14	S ∧ N	{0.00, 0.10, 0.20, 0.70}	{0.00, 0.00, 0.21, 0.79}	{0.00~0.10, 0.00~0.10, 0.15~0.25, 0.65~0.80}	{0.02, 0.02, 0.20, 0.76}
15	S ∧ L	{0.00, 0.00, 0.20, 0.80}	{0.00, 0.00, 0.19, 0.81}	{0.00~0.05, 0.00~0.05, 0.00~0.20, 0.80~1.00}	{0.00, 0.00, 0.00, 1.00}
16	S ∧ VL	{0.00, 0.00, 0.00, 1.00}	{0.00, 0.00, 0.00, 1.00}	{0.00~0.05, 0.00~0.05, 0.00~0.20, 0.90~1.00}	{0.00, 0.00, 0.00, 1.00}

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