

Article

# The Mathematics of Desert: Merit, Fit, and Well-Being

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**Abstract:** Here, we argue for a mathematical equation that captures desert. Our procedure consists of setting out principles that a correct equation must satisfy and then arguing that our set of equations satisfies them. We then consider two objections to the equation. First, an objector might argue that desert and well-being separately contribute to intrinsic goodness, and they do not separately contribute. The concern here is that our equations treat them as separate contributors. Second, our set of desert-equations are unlike equations in science because our equations involve multiple desert-equations with the applicable equation depending on how the variables are filled out. Neither objection succeeds.

**Keywords:** desert; merit; fit; well-being; intrinsic value; value

## 1. Background

We are assuming that desert is a property of the good rather than the right [1–4]. That desert is a property of the good can be seen in the purported ways it avoids the good having features that clash with our intuitions. These features include the way in which a welfarist values an incorrect distribution of pleasure and pain between saints and sinners, Repugnant-Conclusion populations, infinite lives, and utility monsters.<sup>1</sup> To see these, consider the following intuitions. The saints-and-sinners case tells us that it seems worse if saints are sad and sinners are happy than vice versa, even if the aggregate amount of happiness is the same in both cases. The Repugnant-Conclusion-type case tells us that it seems better if there are fewer people who exist and who are very happy (for example, 1 million people with ecstatic lives) than if there are far more people with lives barely worth living (1000 trillion people all of whom have only one moment of happiness per life). The infinite-life case tells us that it is better to enjoy a century of ecstasy than an infinite life each year of which is barely worth living. The utility monster case tells us that it does not seem better if one mediocre utility monster has incredibly large amounts of well-being and everyone else has lives that are barely worth living than if there is a more equal distribution of well-being. This is true even if the more equal distribution has less aggregate well-being. Welfarism asserts that the good is a function of, and only of, well-being. If the good is desert-adjusted well-being, then desert avoids the implausible results of these cases because the value of some person's well-being is discounted or transvalued (that is, having a value with an opposite valence) if it differs too much from what she deserves. There is an issue as to whether identifying the good as desert-adjusted well-being does avoid every one of these implausible results—for example, it is unclear whether it solves the Repugnant-Conclusion problem—but we sidestep these issues here.

Desert is sometimes set out as a geometric relation between well-being and intrinsic value (see [2,3,12–19]). The relation is depicted in a graph. In the graph, the vertical axis is intrinsic value (or, perhaps, intrinsic value from the perspective of desert) and the horizontal axis is well-being (or, perhaps, pleasure). Mathematical equations are clearer than graphs because the former explicitly specify the role of different factors. In exploring



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whether desert is part of the moral world and, if it is, how best to understand it, we consider two sets of desert-equations.

Our procedure is as follows. We set out some principles that a correct equation must meet. We then consider two sets of equations that might satisfy these principles. We argue that one of these sets is correct. We originally set out the principles verbatim elsewhere (see [20]). There, however, we argued for a different conclusion and considered different arguments than are done so here.

Our equations capture desert in the same way that the equation for force (force = mass  $\times$  acceleration) captures force. By 'capture' we mean 'describe'. An equation is not itself a moral property. At best, it describes the property.

## 2. The Three-Factor Account

The basic idea is that there are at least three factors that seem relevant to the intrinsic value,  $V$ , of a person's life, where  $V$  is the extent to which adding a given person with that life to the world, in itself, makes that world better or worse. One is the level of well-being,  $W$ , that the person enjoys. A second is the overall desert, or moral quality,  $M$ , of that person's life. A third is the degree of divergence,  $D$ , between the person's well-being,  $W$ , and the moral quality,  $M$ , of that person's life, where  $D$  is some function of  $W$  and  $M$ , rather than a quantity that can vary independently of  $W$  and  $M$ . Thus,  $W$  and  $M$  are independent variables, whereas  $D$  is a dependent variable.

Can one say anything about the relation between  $D$  and the independent variables  $W$  and  $M$ ? In what follows, we shall adopt the simplest view, according to which, first,  $D = 0$  when  $W = M$ , and secondly,  $D$  is equal to the absolute value of the difference between  $M$  and  $W$ , i.e.,  $D = |M - W|$ .

The goal, accordingly, is to find one or more equations that connect  $V$  with  $W$ ,  $M$ , and  $D$ . Remember, for a life, the variables stand for intrinsic value, well-being, desert/moral quality, and divergence between well-being and desert. The equations arrived at will be based upon general principles that either had some initial plausibility or that emerged in reflecting upon, discussing, and exchanging thoughts concerning specific cases.

We make the following assumptions. First, desert-adjusted well-being has intrinsic value. That is, desert and well-being together ground intrinsic value rather than either one independently doing so. Thus, desert-satisfaction (that is, fit) is not an independent ground of intrinsic value.<sup>2</sup> We do not here assume that desert-adjusted well-being is the only ground of intrinsic value, although we believe this to be true.<sup>3</sup> In addition, the intrinsic value of a life may be positive, zero, or negative.

One might wonder whether desert and well-being together ground intrinsic value rather than either one independently doing so. The idea that the desert and well-being together ground intrinsic value, rather than each separately doing so, is that, intuitively, it is not intrinsically good that a person gets what he deserves. If this were to independently ground intrinsic goodness, then there is a possible case in which a person has negative desert and negative well-being, but his life still makes the world intrinsically better because his desert-satisfaction outweighs his negative well-being. Intuitively, though, this seems wrong. It would also allow that a person has positive well-being and positive desert and yet his life makes the world intrinsically worse. Again, this would be because his desert-dissatisfaction outweighs his positive well-being. Intuitively, this seems wrong.

The way in which the factors combine explains why the different combinations of positive and negative well-being and desert-satisfaction and dissatisfaction combine to determine the intrinsic value of someone's life. For example, this explains why positive–positive makes the world intrinsically better and why negative–negative makes the world intrinsically worse.

Our approach here is consistent with retributivism. Even if the life of a person with negative well-being and negative desert has negative value, it still is the case that it is better than his life goes poorly than that it goes well. As a result, something that lowers the well-being of a person who has negative desert but positive well-being—for example, punishment—makes the world better.

There is a concern as to whether this is consistent with the general retributivist notion that it is intrinsically good that a person with negative desert have negative well-being. There is also the specific retributivist notion that it is intrinsically good that a wrongdoer be punished. Our theory is weaker than the general theory because it assumes that it is intrinsically better that the life of a person with negative desert go poorly—perhaps within a range—than that his life go well. It does not follow that the state of affairs involving a person with negative desert and negative well-being is intrinsically good. The same is true regarding a wrongdoer receiving the specific punishment he deserves.

Second, desert, well-being, and intrinsic value come in cardinal amounts and can be combined into equations that determine intrinsic goodness. More specifically, equations express relations that along with the things they relate determine intrinsic goodness. The equations, then, in part determine the intrinsic value of a basic intrinsic-value state. A basic intrinsic-value state is a state of affairs that is intrinsically valuable in virtue of, and only of, the facts that constitute it (see [22]).<sup>4</sup>

Third, for simplicity, we assume that desert depends on, and only on, virtue. For our purposes, virtue is the moral quality of an individual’s psychology. We further assume that what is deserved is a lifetime amount of well-being. Neither assumption is obvious.<sup>5</sup> Some desert theorists argue that desert is justified by desirable acts, for example, good, right, or virtuous acts.<sup>6</sup> In addition, one desert theorist argues that what a person deserves is an amount of well-being at, and only at, the time at which he has a ground of desert (that is, when he is virtuous or performs a desirable act). On this account, the basis for desert (ground) and what is deserved (object) are features of a momentary or short-term property rather than a long-term one, such as lifetime well-being (see [18]). Nothing here rests on these issues, so we sidestep them.

### 2.1. Principles

Among the general principles that initially seemed appealing, some very strongly, others less so, and also some that seem, upon investigation, problematic, are the following.

To see the intuitive appeal of the following principles, consider how God would view individuals’ lives, particularly in deciding whom to create. Our intuitions are that God would think that the life of a person whose life that goes well for him and who has positive desert makes the world a better place. The first principle—positive—tracks this intuition.

Similarly, God would think that a person whose life goes poorly for him and who has negative desert would make the world a worse place. The second principle—negative—tracks this intuition. A person whose life goes poorly for him and who has negative desert makes the world worse, but his life makes the world better than if his life had gone well.

As a side note, we are not assuming theism; rather, we are trying to obtain thought experiments as a means by which to explore our intuitions. Intuitively, God would think that the life of a person who has zero well-being (his life goes neither well nor poorly for him) and who has zero desert has zero intrinsic value. The third principle—neutral—tracks this intuition. We treat these intuitions as the data points that principles must satisfy (see Table 1).

**Table 1.** Monadic principles<sup>7</sup>.

N	Name	Content of the Principle
1	Positive	If a person has positive moral desert and positive well-being, then her life has positive intrinsic value.
2	Negative	If a person has negative moral desert and negative well-being, then her life has negative intrinsic value.
3	Neutral	If a person has zero moral desert and zero well-being, then her life has zero intrinsic value.
4	Well-being takes priority	If a person has positive moral desert and negative well-being, then her life has negative intrinsic value.

Desert ranks alignment between desert and well-being (positive/positive and negative/negative) as intrinsically better than non-alignment (positive/negative and negative/positive). This is the nature of the desert function. It does not so much explain why the alignment is good. Rather, it is a primitive feature of the good.

To see the next set of principles, consider which of two individuals God would prefer to create. For example, if two people have lives that go equally well for them (for example, +10 utils) and the first is more deserving than the second, then God would prefer to create the first over the second. Similarly, if two people are equally deserving, both have lives that go well for them, and the first has a life that goes better for him than does the second, God would prefer to create the first. The 5th and 6th principles—Superiority 1 and Superiority 2—track these intuitions. The other principles rely on similar intuitions.

To see the idea behind Superiority 1 and Superiority 2, consider what happens when a good person obtains more pleasure than he deserves as opposed to obtaining the pleasure he deserves. The extra pleasure makes his life have more intrinsic value, less intrinsic value, or equal intrinsic value than if he were to get what he deserves. It is not equal intrinsic value because this would likely indicate that desert-satisfaction has no effect on intrinsic value. If desert is feature of the good-maker, this is incorrect.

If desert-satisfaction were to lessen intrinsic value, then there could be scenarios in which one person is more deserving and has more well-being than a second person and yet the second person’s life has more value. Intuitively, this is implausible. One way to see this is that it is hard to see why a neutral observer—for example, God—would prefer it. It might also be the case—again depending on the value-function—that a person might have positive desert and well-being and yet his life makes the world intrinsically worse. Again, this is implausible. If the above points are correct, then it is plausible that if two people are equally deserving, both have lives that go well for them, and the first has a life that goes better for him than does the second, God would prefer to create the first (see Table 2).

**Table 2.** Dyadic principles.

N	Name	Content of the Principle
5	Superiority 1	If one person has greater desert than a second, and they have equal well-being, then the first person’s life is more valuable than the second person’s life.
6	Superiority 2	If one person’s desert is equal to a second person’s, but the first has greater well-being, then the first person’s life is more valuable than the second person’s life.
6 *	Superiority 2 Modified	If one person’s desert is positive (or zero) and equal to a second person’s, but the first has greater well-being, then the first person’s life is more valuable than the second person’s life. If one person’s desert is negative and equal to a second person’s, but the first has well-being that is greater, but not greater than his desert, then the first person’s life is more valuable than the second person’s life.
7	Double Superiority	If one person has greater desert and greater well-being than a second person, the first person’s life is more valuable than the second person’s life.
7 *	Double Superiority Modified	If one person has greater positive (or zero) desert and greater well-being than a second person, the first person’s life is more valuable than the second person’s life. If one person’s desert is negative and greater than a second person’s, but the first has well-being that is greater, but not greater than his desert, then the first person’s life is more valuable than the second person’s life.
8	Equality	If one person’s desert and well-being are equal to a second person’s desert and well-being, then their lives are equally valuable.

\*: The Modified version is similar to the original one, but covers a different range of cases.

Consider Double Superiority. We intuit that God would prefer to create a person who is more deserving and whose life goes better than someone who is less deserving and whose life goes less well (although it still goes well for him). The seventh principle—Double Superiority—tracks this intuition.

Next, consider how God would view two people who have equal desert and whose lives go equally well for them. He would view their lives as having equal intrinsic value. The 8th principle—Equality—tracks this intuition.

The third set of principles also rely on intuitions about what God would prefer to create. The reader should consider them via the same type of thought experiments with which he or she considered the previous principles.

Consider how God would view two people each with a well-being level of 20 and whose desert levels are 20 and 21, respectively. He would view the second person’s life as only slightly better or worse than the second. The ninth principle—Small Difference—tracks this intuition. The 10th principle—Small Difference Modified—narrows Small Difference to cases in which the person’s desert does not change valence, that is, change between positive, neutral, and negative.

The 10th principle—Opposite—addresses symmetry in value. It says that if two people have desert and well-being of opposite valence, but the same absolute magnitude, then they have the same absolute value. An absolute value of something is its value independent of the valence—positive or negative—of the value. Therefore, for example, the life of a person with a desert and well-being level of 10/10 should have the same absolute intrinsic value as a person with a desert and well-being level of  $-10/-10$ . The underlying idea here is that the intrinsic-value function should operate in a mirrored way—that is, the opposite—with regard to equally good and bad people who have equally good and bad lives respectively (see Table 3).

**Table 3.** Metaphysical principles.

N	Name	Content
9	Small Difference	If a person’s desert were only slightly more (or less) than it actually is, then, holding well-being constant, the value of her life would be only slightly more (or less) than it actually is.
9*	Small Difference Modified	If a person’s desert were only slightly more, (or less), than it actually is, but without changing from a negative value to a positive value, or to zero, or from a positive value, or zero, to a negative value, then, holding well-being constant, the value of her life would be only slightly more, (or less), than it actually is.
10	Opposite	If one person has positive desert and positive well-being, a second has negative desert and negative well-being, and their desert and well-being have the same absolute magnitude (that is, independent of whether they are positive or negative, the magnitude of desert and well-being are the same), then the values of their lives have the same absolute value.

\*: The Modified version is similar to the original one, but covers a different range of cases.

The next principle concerns a vicious individual. Consider, for example, Hitler. The intuition is that if God is deciding whether to give Hitler a life that goes poorly for him or an ecstatic life, God would not be indifferent. That is, he would not flip a coin to decide which life to give to Hitler. The 11th principle—Not Equally Good: Negative Desert Version—tracks this intuition. A similar intuition is that God would not be indifferent about whether Saint Francis’ life goes very poorly or is ecstatic. The 12th principle—Not Equally Good: Positive Desert Version—tracks this intuition (see Table 4).

**Table 4.** A relevance principle.

N	Name	Content
11	Not Equally Good—Negative Desert Version	For a person with any negative desert, M, there is no quantity of well-being, W, such that the value of that person’s life is the same regardless of whether W is positive or negative.

One additional principle is suggested by Principle 11—Not Equally Good, namely, Not Equally (see Table 5).

**Table 5.** Good—positive desert version.

N	Name	Content
12	Not Equally Good—Positive Desert Version	For a person with positive desert, it is not equally good if he were to have a life that goes very well or very poorly for him.

Similar to the Hitler case, consider what life God would like to see a good person have. Consider, for example, Paul Newman. Intuitively, God would not be indifferent as to whether Newman had a (prudentially) horrible life or an ecstatic life.

The following, additional principles follow from similar intuitions to the ones given above. Again, we used God-creation thought experiments to explore our intuitions. God would think that if two people have the same positive desert, but the first’s life goes better than the second, this would be a life with greater intrinsic value. For example, if two people deserve a well-being of 20 and the first has a life with a well-being level of 15 and the second has a life with a well-being level of 14, God would think the first life has greater intrinsic value. This is also true if the well-being levels for both lives are greater than what they deserve. The 13th principle—Positive Desert and Greater Well-Being—tracks this intuition (see Table 6).

**Table 6.** Same Positive Desert and Greater Well-Being.

N	Name	Content
13	Positive Moral Desert and Greater Well-Being	If two people have the same positive (or zero) level of moral desert, but the first has greater well-being than the second, the life of the first has either more value than, or less disvalue than, the life of the second.

God would also think that if one person has greater positive desert than a second (20 and 10, respectively) and the first has less positive well-being than the second (10 and 20, respectively), there is a limit to the extent the second person’s life outweighs the first. The limit occurs because both lives have positive finite value (see Table 7).

**Table 7.** Moral Desert, Well-Being, and the Value of a Life.

N	Name	Content
14	Moral Desert, Well-Being, and the Value of a Life	If one person has less positive well-being than another, but greater moral desert, there is a limit to the extent to which the second difference can outweigh the first.

None of the preceding principles explicitly mention divergence. Here are two principles that do refer to divergence. The 15th principle—Negative Moral Desert and Divergence—tracks the intuition that it is better that people with negative well-being get what they deserve than some other amount of well-being (see Table 8).

**Table 8.** Negative Moral Desert and Divergence.

N	Name	Content
15	Negative Moral Desert and Divergence	If two people have the same negative level of moral desert, but the first person’s well-being diverges more from that person’s moral desert than the second person’s does, and in the same direction, then the life of the first has more disvalue than the life of the second.

These principles—see, for example, Principles 10 and 15—capture the Hitler case. Specifically, they entail that the life of a person with negative desert has greater value if he gets what he deserves—a specific level of negative well-being—than if he does not get this and, instead, obtains positive well-being. In addition, Superiority 2 tells us that if one person has positive desert, his life is more valuable if it goes very well for him than if it merely goes moderately well for him.

The 16th principle—Zero Divergence—stipulates that the value for a person getting what he deserves is equal to two of what he deserves (see Table 9). This forms the baseline value for other principles. Because value and well-being units are in some sense arbitrary—consider, for example, feet or meters—the baseline may be set in this way. We assume that desert is a relation between intrinsic value and well-being. This allows the units on both sides of the equation to be the same.

**Table 9.** Zero Divergence.

N	Name	Content
16	Zero Divergence	If a person’s moral desert is equal to that person’s well-being, then $V = W + M = 2W = 2M$

First of all, it would seem that increases in the level of a person’s moral desert should increase the intrinsic value of adding such a person to the world. Doing so, of course, may increase the extent of divergence, which, itself, is a negative factor, but it seems plausible, for example, that if the positive addition to the value of V due to the increase in the person’s moral desert is accompanied by any associated loss because of an increase in divergence, the gain should outweigh the loss. If so, the equations should entail Superiority 1. It says: If one person has greater desert than a second, and they have equal well-being, then the first person’s life is more valuable than the second person’s life.

Next, what about W? Is there a parallel principle to the effect that an increase in the level of a person’s well-being should always increase the intrinsic value of adding such a person to the world? If one considers the case of a thoroughly evil individual who enjoys positive well-being, it seems clear that the answer is no: increasing the well-being of such a person would surely not itself make the world a better place.

On the other hand, if a person has positive moral desert, or neutral moral desert, then it seems plausible that increasing the well-being of such a person would intrinsically make the world a better place. We reject—and our principles reflect this—the notion that the life of a person with positive desert and well-being can make the world worse even if his well-being is much greater than his desert (see Table 10). This can be seen by considering whether God would view such a person as making the world worse. This is consistent with there being an important relation between what someone deserves and what he receives. This is to say that a good person getting more than what one deserves does not make the world intrinsically worse than his getting what he deserves.

In the case of a person with neutral or positive moral desert, then, one has the following principle, listed earlier:

**Table 10.** Positive Moral Desert and Well-Being.

N	Name	Content
13	Positive Moral Desert and Well-Being	If two people have the same positive (or zero) level of moral desert, but the first has greater well-being than the second, the life of the first has more value than the life of the second.

What about the case of a person with negative moral desert? Does increasing that person’s well-being always make the world a worse place? The answer is surely no, since such a person could, for example, be suffering at a much greater level than seems appropriate given that person’s negative level of moral desert. Thus, we have to distinguish between the case where such a person’s well-being is less than that person’s moral desert, and the case where it is greater. We have, then, the two principles displayed in Table 11:

**Table 11.** Negative Desert Principles.

P	Name	Content
17	Increases in Well-Being Up to the Level of Negative Moral Desert	If two people have the same negative level, <i>M</i> , of moral desert, with levels of well-being <i>W</i> and <i>W+</i> , respectively, where both <i>W</i> and <i>W+</i> are less than <i>M</i> , and where $W < W+$ , then the life of the second person has greater value than the life of the first person.
18	Increases in Well-Being Beyond the Level of Negative Moral Desert	If two people have the same negative level, <i>M</i> , of moral desert, with levels of well-being <i>W</i> and <i>W+</i> , respectively, where both <i>W</i> and <i>W+</i> are greater than <i>M</i> , and where $W < W+$ , then the life of the second person has less value than the life of the first person.

2.2. Equations

A crucial principle to think about is Principle 4 (see Table 12).

**Table 12.** Well-Being Takes Priority.

N	Name	Content
4	Well-Being Takes Priority	If a person has positive moral desert and negative well-being, then her life has negative intrinsic value.

We think this principle is correct. It is hard to imagine that God would think that he made the world a better place by adding a person whose life goes poorly for her even if she is virtuous. What likely explains why some people reject this principle is that they assume that a virtuous person with negative well-being would view her life as acceptable because she has a global desire—a desire regarding how her life goes as a whole—to be a virtuous person. This might explain why someone might prefer that she be unhappy and very virtuous person to her being minimally happy and minimally virtuous. We think this preference is mistaken—it is neither good for the person nor the world—but such a preference would explain why someone mistakenly think the above principle is false.

In any case, we suggest that it is best not to take a stand on Principle 4, and to consider instead what equation seems best, depending on whether that principle is correct or not. Assume, first of all, that Principle 4 is not sound, and that positive moral desert can outweigh negative well-being.

The result would then be the following four equations that we call Option 1 (see Table 13).



**Table 13.** Four Equations.

#	Desert	Well-Being	Equation
1	Positive	Positive	Equation (1): If $M \geq 0$ and $W \geq 0$ , then $V = M + W - (D/2)$
2	Negative	Negative	Equation (2): If $W \leq M \leq 0$ , then $V = M + W - (D/2)$
3	Negative	Positive	Equation (3): If $M < 0$ and $W \geq M$ , then $V = 2M - D = 3M - W$
4	Positive	Negative	Equation (4a): If $M \geq 0$ and $W < 0$ , then $V = M + W - (D/2)$

For reader ease, we have left out when desert or well-being are zero. These can be seen in the equations.

Here is a condensed version of these four equations (see Table 14).

**Table 14.** Condensed Equations.

Equation I: $V = M + W - (D/2)$ , provided that it is not the case that $M < 0$ and $W \geq M$
Equation II: If $M < 0$ and $W \geq M$ , then $V = 2M - D = 3M - W$

Assume, on the other hand, that Principle 4 is sound, and that the combination of positive moral desert,  $M$ , no matter how great, together with the negative factor of the constantly increasing divergence,  $D$ , cannot outweigh the negative well-being  $W$ . The result is that if Principle 4—Well-Being Takes Priority—is sound, as we believe it is, then we have the following four equations. Again, for reader ease, we have left out when desert or well-being are zero. Here, then, is Option 2 (see Table 15).

**Table 15.** Four Equations Assuming Principle 4.

#	Desert	Well-Being	Equation
1	Positive	Positive	Equation (1): If $M \geq 0$ and $W \geq 0$ , then $V = M + W - (D/2)$
2	Negative	Negative	Equation (2): If $W \leq M \leq 0$ , then $V = M + W - (D/2)$
3	Negative	Positive	Equation (3): If $M < 0$ and $W \geq M$ , then $V = 2M - D = 3M - W$
4	Positive	Negative	Equation (4b): If $W < 0$ and $M \geq 0$ , then $V = \frac{3W}{2} \left( \frac{1}{1+M} \right)$

Regarding Equation (4b), we used the number 1 in both the numerator and denominator for simplicity. The correct number might be another positive number. The correct number would then be used in the below equation (see Table 16).

**Table 16.** The Fourth Equation with a Variable Rather Than a Constant.

#	Desert	Well-Being	Equation
4	Positive	Negative	Equation (4c): Alternative: If $W < 0$ and $M \geq 0$ , then $V = \frac{3W}{2} \left( \frac{X}{X+M} \right)$

Large values of  $X$  would have the result that  $V$  approaches 0 more slowly as  $M$  increases.

### 2.3. Units

We assume that the units of desert ( $M$ ) and well-being ( $W$ ) occur in a ratio scale. A ratio scale has a true zero point and equal-sized intervals. This allows mathematical functions—such as addition and multiplication—to apply to amounts of these factors. As a result, we can say that differences and ratios express the actual relations of these factors and not just the relations given an artificial way of measuring them. This is similar to non-moral

variables—for example, density, mass, and volume—and moral variables—for example, utils. Underlying this ratio-scale assumption with regard to  $M$  and  $W$  is the assumption that these are natural properties and, thus, the units that measure them can be expressed in a ratio scale.

Our axiology has several advantages over existing axiologies. First, our equation does not assume that desert-satisfaction by itself makes the world intrinsically better.<sup>8</sup> As discussed above, if desert-satisfaction by itself were to make the world better, then it is possible that the life of a person with negative well-being and negative desert would have positive intrinsic value. This is implausible. To see this, consider whether God would want to add this person to the world.

Second, our theory is a series of equations rather than graphs. This adds something to the graphing approaches because it allows us to see the way that desert, well-being, and the difference between them affect intrinsic value in a way quite different from a graph. A set of equations also tells us just how complex the relations between them are.

Third, our approach is better than existing equations. Consider, for example, Bradford Skow's equation for desert [18]. His equation is as follows.

$$C = k D \log (e/2 [1 + W/D])$$

Here, 'C' means 'contribution to value', 'W' means 'individual welfare', 'D' means 'desert-level', 'log' means 'logarithm', and 'k' and 'e' are constants. Factor  $k$  is analogous to Boltzmann's constant—energy/temperature—that is used in the equation relating to the energy of an ideal gas. Factor  $e$  is a constant that normalizes the equation. The equation runs into difficulty for some negative values for  $W$  or  $D$  because the logarithm of a negative number is undefined. Thus, for example, if  $D$  is negative and  $W > -D$ , then one is taking the log of a negative quantity. The log of a negative number is undefined. Similarly, if  $W$  is negative, while  $D$  is positive and the absolute value of  $W$  is greater than the absolute value of  $D$ ,  $W/D$  will be greater than  $-1$ , and the result is a log of a negative number. Again, this is undefined. It also obtains the wrong result when  $D = 0$  and  $W \neq 0$  because the equation tells us that this state of affairs has zero intrinsic value. Intuitively, it does not. Intuitively, for example, if a person has neutral desert and  $-1000$  well-being, his life makes the world worse.

Skow never intended that his equations apply to cases in which desert is zero or negative or to cases in which well-being is zero or negative. Thus, this is not a criticism of his article. Rather, it is to say that our equations are more general.

### 3. Argument for the Equations

In the Appendix A, we summarize our findings that Option 2 equations satisfy most of the principles set out above. In particular, they satisfy the principles that—on reflection—seem correct.

### 4. Concerns

The two sets of equations raise concerns. A concern regarding Equation (1) is that it makes desert and well-being independent good-makers. Equation (1) [ $V = M + W - (D/2)$ ] entails that desert and well-being are independent good-makers. This is because each by itself makes the world better. This is mistaken, however, because we cannot tell whether the value of the world is negative or positive (or even an other-things-being-equal value) if we know only one of  $M$  and  $W$ .

By analogy, here is an account of well-being:  $W = H + O$ . Here  $W$  = well-being,  $H$  = amount of pleasure, and  $O$  = amount of objective-list goods. Our suspicion is that, from this account alone, we can infer that pleasure and objective-list goods each, on its own, makes someone's life go better. If this is correct, then Equation (1) suggests that well-being and desert each make the world a better place.

We respond by asking that one to consider this case (see Table 17).

**Table 17.** Addition and Multiplication.

	M	W	M + W	M × W	(M × W)/(M + W)
Life 1	10	10	20	100	5
Life 2	20	20	40	400	10
Life 3	30	30	60	900	15
Life 4	40	40	80	1600	20

Multiplication better expresses the combined effect of M and D than does addition. Still, addition produces numbers that are too small, and multiplication produces numbers that are too large. In addition, neither addition nor multiplication considers how close M and W are to each other, that is, how well they fit each other. If fit matters, this is a problem. And, as mentioned above, there is a concern that some of the equations make M and W independent good-makers.

An objector might note that the presence of addition in Equation (1) shows that desert and well-being are distinct good-makers. However, on our interpretation of this equation, there is an organic-unity-type effect. On this account, desert, well-being, and the difference between them do not contribute separately but rather combine to determine the intrinsic goodness of someone’s life. The three factors together act as an organic-unity good-maker. Both have to be present to some degree—even if it is a zero degree—and thus do not make independent contributions to the good. Again, the underlying problem is that if desert is expressed in terms of multiplication, then whenever one variable—whether desert or well-being—has a value of zero, the person’s life would have a value of zero. This is incorrect. Consider, for example, a case when a person deserves a well-being of 0 but has a well-being of −100. If, instead, desert is expressed in terms of addition, then the problem of independent contribution seems to arise. The organic-unity interpretation avoids this result.

There is another concern. Our equation sets have multiple equations and which equation applies depends on the value of the variables. Scientific equations—or, if they are different, accounts—do not have these features. Table 18 shows some examples.

**Table 18.** Some examples of scientific equations.

Momentum = $m \times v$	(mass × velocity)
Force = $m \times a$	(mass × acceleration)
Boyles’s Law $pv = k$	(For a given quantity of gas, pressure times volume equals a constant)

If the absence of a single equation or even a set of physics-like equations—or, if they are different, a single account or set of accounts—is evidence against an equation set tracking reality, then our proposed equation set is problematic. Perhaps the queerness of the morality—J. L. Mackie’s sense of queerness—explains why the correct desert-equations differ from scientific equations in significant ways [29].

**5. Conclusions**

If we assume that Principle 4 is false (that is, positive moral desert can outweigh negative well-being), then Option 1 is likely correct because it satisfies the other desert-principles (see Table 19).

**Table 19.** Four Equations If Principle 4 is False.

#	Desert	Well-Being	Equation
1	Positive	Positive	Equation (1): If $M \geq 0$ and $W \geq 0$ , then $V = M + W - (D/2)$
2	Negative	Negative	Equation (2): If $W \leq M \leq 0$ , then $V = M + W - (D/2)$
3	Negative	Positive	Equation (3): If $M < 0$ and $W \geq M$ , then $V = 2M - D = 3M - W$
4	Positive	Negative	Equation (4a): If $M \geq 0$ and $W < 0$ , then $V = M + W - (D/2)$

Table 20 is a condensed version of these four equations.

**Table 20.** Condensed Equations If Principle 4 is False.

Equation I: $V = M + W - (D/2)$ , provided that it is not the case that $M < 0$ and $W \geq M$
Equation II: If $M < 0$ and $W \geq M$ , then $V = 2M + D$

If, on the other hand, that Principle 4 is true, as we believe it is, then Option 2 is likely correct (see Table 21).

**Table 21.** Four Equations If Principle 4 is True.

#	Desert	Well-Being	Equation
1	Positive	Positive	Equation (1): If $M \geq 0$ and $W \geq 0$ , then $V = M + W - (D/2)$
2	Negative	Negative	Equation (2): If $W \leq M \leq 0$ , then $V = M + W - (D/2)$
3	Negative	Positive	Equation (3): If $M < 0$ and $W \geq M$ , then $V = 2M - D = 3M - W$
4	Positive	Negative	Equation (4b): If $W < 0$ and $M \geq 0$ , then $V = \frac{3W}{2} \left( \frac{1}{1+M} \right)$

In the Appendix A below, we note that Option 2 largely coheres with the first set of desert-principles (see [20] (pp. 124–134)).

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### Appendix A. Option 2 Satisfies the Desert-Principles Listed in Part 4

Here we summarize our claim that Option 2 equations satisfy most of the principles. These equations are also consistent with our rejection of some of the Part Two principles. We summarize our results in Table A1.

**Table A1.** Result Summary.

Number	Name	Content	Do the Desert-Equations Satisfy It?
1	Positive	If a person has positive moral desert and positive well-being, then her life has positive intrinsic value.	Yes
2	Negative	If a person has negative moral desert and negative well-being, then her life has negative intrinsic value.	Yes
3	Neutral	If a person has zero moral desert and zero well-being, then her life has zero intrinsic value.	Yes
4	Well-Being Takes Priority	If a person has positive moral desert and negative well-being, then her life has negative intrinsic value.	Yes (Option 2)

**Table A1.** *Cont.*

Number	Name	Content	Do the Desert-Equations Satisfy It?
5	Superiority 1	If one person has greater desert than a second, and they have equal well-being, then the first person’s life is more valuable than the second person’s life.	Yes
6	Superiority 2	If one person’s desert is equal to a second person’s, but the first has greater well-being, then the first person’s life is more valuable than the second person’s life.	No (Equation (3))
6 *	Superiority 2—Modified	If one person’s desert is positive (or zero) and equal to a second person’s, but the first has greater well-being, then the first person’s life is more valuable than the second person’s life.	Yes
7	Double Superiority	If one person has greater desert and greater well-being than a second person, the first person’s life is more valuable than the second person’s life.	No (Equation (3))
7 *	Double Superiority—Modified	If one person has greater positive (or zero) desert and greater well-being than a second person, the first person’s life is more valuable than the second person’s life. If one person’s desert is negative and greater than a second person’s, but the first has well-being that is greater, but not greater than his desert, then the first person’s life is more valuable than the second person’s life.	Yes
8	Equality	If one person’s desert and well-being are equal to a second person’s desert and well-being, then their lives are equally valuable.	Yes
9	Small Difference	If a person’s desert were only slightly more (or less) than it actually is, then, holding well-being constant, the value of her life would be only slightly more (or less) than it actually is.	No (Equation (4a–c))
9 *	Small Difference Modified	If a person’s desert were only slightly more, or less, than it actually is, but without changing from a negative value to a positive value, or to zero, or from a positive value, or zero, to a negative value, then, holding well-being constant, the value of her life would be only slightly more (or less) than it actually is.	Yes
10	Opposite	If one person has positive desert and positive well-being, a second has negative desert and negative well-being, and their desert and well-being have the same absolute magnitude (that is, independent of whether they are positive or negative, the magnitude of desert and well-being are the same), then the values of their lives have the same absolute value.	No (Equations (1) and (2))
11	Not Equally Good—Negative Desert Version	For a person with negative desert, it is not equally good if he were to have a life that goes very well or very poorly for him.	No (Vagueness)
11 *	Not Equally Good—Negative Desert Version Modified	For a person with any negative desert, M, there is no quantity of well-being, W, such that the value of that person’s life is the same regardless of whether W is positive or negative.	Nearly Always Yes
12	Not Equally Good—Positive Desert Version	For a person with positive desert, it is not equally good if he were to have a life that goes very well or very poorly for him.	Yes
12 *	Not Equally Good—Positive Desert Strengthened	For a person with positive desert, it is not equally good if he were to have a life that goes well or poorly for him.	Yes
13	Positive Moral Desert and Well-Being	If two people have the same positive (or zero) level of moral desert, but the first has greater well-being than the second, the life of the first has either more value than, or less disvalue than, the life of the second.	Yes

Table A1. Cont.

Number	Name	Content	Do the Desert-Equations Satisfy It?
14	Moral Desert, Well-Being, and the Value of a Life	If one person has less positive well-being than another, but greater moral desert, there is a limit to the extent to which the second difference can outweigh the first.	No (Equations (1) and (3))
15	Negative Moral Desert and Divergence	If two people have the same negative level of moral desert, but the first person's well-being diverges more from that person's moral desert than the second person's does, and in the same direction, then the life of the first has more disvalue than the life of the second.	Yes
16	Zero Divergence	If a person's moral desert is equal to that person's well-being, then $V = W + M = 2W = 2M$	Yes (Equations (1) and (3)) No (Equation (1) Alternative and 3 Alternative)
17	Increases in Well-Being Up to the Level of Negative Moral Desert	If two people have the same negative level of moral desert, with levels of well-being $W$ and $W+$ , respectively, where both $W$ and $W+$ are less than $M$ , and where $W < W+$ , then the life of the second person has greater value than the life of the first person.	Yes
18	Increases in Well-Being Beyond the Level of Negative Moral Desert	If two people have the same negative level of moral desert, with levels of well-being $W$ and $W+$ , respectively, where both $W$ and $W+$ are greater than $M$ , and where $W < W+$ , then the life of the second person has less value than the life of the first person.	Yes

\*: The Modified version is similar to the original one, but covers a different range of cases.

In summary, Option 2 Equations (1)–(4c) satisfy most of the principles.

## Notes

- 1 For the problem of infinite lives, see [5–7]. For the problem with the Repugnant Conclusion, see [8]. For the problem with a utility monster, see [9]. For the distribution of utility between good and bad people, see [10,11].
- 2 For the notion that desert-satisfaction by itself is intrinsically good, see [2,3,15,16]. This is the fit theory of desert. For the notion that desert-adjusted well-being is intrinsically good, see [1,14,21]. This is the merit theory of desert. For the notion that merit alone is intrinsically good, see [19].
- 3 For an argument for this position, see [1].
- 4 A problem arises if there can be an infinite amount of desert or well-being. If infinity is not a number, the problem goes away. For the notion that infinity is not a number, see [22].
- 5 For arguments that people can deserve things other than an amount of well-being, see [4,23–28].
- 6 See, for example, [4,14,24,25].
- 7 The tables in the article come from [20].
- 8 For Axiologies that do this see [2,3].

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