Article

The Ontological Role of Applied Mathematics in Virtual Worlds

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Abstract: In this paper, I will argue that with the emergence of digital virtual worlds (in video games, animation movies, etc.) by the animation industry, we need to rethink the role and authority of mathematics, also from an ontological point of view. First I will demonstrate that the application of mathematics to the creation and description of the digital, virtual worlds behaves in many respects analogously to the application of mathematics to the description of real-world phenomena from the viewpoint of the history of science. However, from other aspects, the application of mathematics significantly differs in this virtual world from the application to real-world fields. The main thesis of my paper is that the role of mathematics in the digital animation industry can be ontologically different from its usual role. In the application of mathematics to digital virtual worlds, mathematical concepts are no longer just modelling tools, forming a subordinated, computational basis, but they can direct and organise, and even create non-mathematical theory, something that we can call, for example, digital physics and biology. I will study this new, creative role of mathematics through some concrete phenomena, specifically through gravity. Our conclusion is that the animation industry opens an entirely new chapter in the relationship between (digital) sciences and mathematics.

Keywords: applied mathematics; virtual worlds; virtual reality; animation industry; paradigm shift

1. Introduction

The common belief is that when we talk about applying mathematics, the application of mathematical principles and practices obviously presupposes some other non-mathematical field, the discipline to which we apply this knowledge. An area, a problem, that comes from an outside world compared to mathematics, to which mathematics can be applied. Without, for example, a real biological, sociological or engineering problem, there is no point in talking about application. Even behind the most theoretical level of physics, there is a flash of the real world, the real universe or the world of atomic particles.

In this paper, I study how mathematics behaves in a completely new application medium that still correlates with the real world in some way and significantly differs from other viewpoints—and this is the digital virtual world: the world of virtual reality, video games and animation movies.

As a general concept, when mathematics is applied, a certain link (called mapping) is thought to be established or found between the non-mathematical field and a suitable mathematical domain, but the role of the latter is evidently subordinated. This general view is questioned and extended in [1] where the author states that this mapping does not necessarily have to be created and limit the application of mathematics to (almost automatically) use only the findings of the correspondent structures in the non-mathematical area.

In the previously sketched, classical description of the characterisation of the application of mathematics [2], mathematics can practically not act as a creative force. Rizza suggests a critique of this and an extension of the concept of applied mathematics when he claims that "in many cases the use of mathematical theories is not conceptually subordi-

nated to the availability of qualitative theory but, on the contrary, directs and organises its emergence" [3].

In this paper, I will argue that with the advent of digital virtual worlds, we need to rethink the role and authority of applied mathematics. The main thesis of my paper is that applying mathematics in the animation industry, in the description of the digital virtual world(s), the above-mentioned—and somewhat richer—new concept of applied mathematics is very strongly validated. In this application the extended concept appears in its purest form, moreover, in this application mathematics not only directs and organises, but creates non-mathematical theory, therefore its role is ontologically different to what we normally think about applied mathematics. I will provide some concrete examples of this creative power.

The central role of the virtual but visually perceptible world(s) in our lives is unquestionable. For all the products of the animation industry, for all animation movies, computer games, 3D applications, serious games, etc. we have to build a visually authentic, perceptible and enjoyable virtual world. The virtual environment is similar to the real world in many aspects, but of course it can exceed it in both physical and visual terms, or at least it can definitely differ from it. This world is formed and realised on some display (screen, 3D glasses, etc.), which in turn is driven by a computer. This very fact evidently raises a number of questions from a philosophical point of view that go beyond the scope of this article and play only a marginal role in what we want to say, but we must address them briefly anyway. For a good overview of the problems of the ontology of virtuality see [4].

The first issue is the relationship between the virtual world and the real world. The computer is, after all, part of the real, physical world, with its circuits, display and other components. Can we describe the phenomenon of the emergence of video game properties on pure physical properties of the computer? Is it overall an intentional system? Do digital virtual worlds of video games and animation movies form causal worlds? For an exhaustive discussion of these issues see [5], where the authors argue that properties of that virtual (in their terms, diagenic) world cannot be fully explained by the emergence upon the physical properties of the computer.

Theories that intend to derive all aspects of the virtual world and video games purely from the physical operation of computer circuits and memory suggest a high level of physical determinism that, similarly to the analogous view that arises with the mind–body problem, is a very extreme standpoint. I argue against this view, especially because (a) these digital worlds are partly already created by artificial intelligence, the decisions of which cannot be simply derived from the hardware, and (b) on the other hand the player is obviously involved in the perception and shaping of video game worlds, which requires a much more complex explanation than the physical properties of circuits due to human–computer interaction. As Cogburn and Silcox aptly note, supported by several examples: "player's acts somehow exceed what the game's designer could have either explicitly or tacitly intended".

Not independently of this, a perhaps an even broader question is to what extent we consider objects and events in the virtual world to be real. There are two views here that can be called virtual fictionalism and virtual realism. The closest standpoint to my view on this issue is what Chalmers articulates in [6]: "I argue for virtual digitalism, on which virtual objects are real digital objects, and against virtual fictionalism, on which virtual objects are fictional objects. I also argue that perception in virtual reality need not be illusory, and that life in virtual worlds can have roughly the same sort of value as life in non-virtual worlds". I would like to emphasise once again, however, that these questions are not absolutely crucial to the views expressed in the following on the role of applied mathematics—those who are opposed to mine in these issues can also accept the central role of applied mathematics in the creation of these worlds.

Of course similar questions have already arisen a long time ago about mathematical objects themselves. Throughout the paper my approach is fundamentally based on a

non-Platonic view, that is I consider all the mathematical objects and operations as virtual ones. In the physical world there are no perfect lines, circles and further objects in the pure geometric sense. Therefore when we apply mathematics to the description of the real world, the model will inherently be imperfect. In terms of the application of mathematics in the virtual worlds, this relationship between the theoretical and the created objects (e.g., circles) have a more intimate relationship, where the imperfection, if any, follows from the intermediate layer of software and display techniques, rather than in the previous "classical" applications (e.g., physics, chemistry), where it stems much more from the imperfect modelling of an existing object or process.

Finally, I mention a problem that can also be related to virtual worlds: the different views articulated in terms of the so-called computer simulation experience. For a good overview of the state-of-play see [7,8]. The essence of the problem is whether computer simulation scientific experiences can or cannot be considered of equal importance and validation power to real-world experiences. The reason why this otherwise valid question is not detailed here is the fact that in virtual worlds we usually do not simulate any real situation from a mathematical point of view, so in an epistemological sense we do not want to get information and knowledge with the help of a computer, but we rather create information and knowledge with it. An exception to this may be a narrow segment of games that want to mimic real-life conditions as faithfully as possible (e.g., visualising real-world cars from their computer-aided design models in speed race-type games), to which we will briefly return in the next section.

2. Mathematics in Virtual Worlds

Mathematics has a clear central role to play in describing the virtual world. Studying this specific role and the analogies and differences between the application of mathematics to real-world problems or to virtual world problems, I will study two interrelated topics. In this chapter, on the one hand, I will argue that the application of mathematics to the creation and description of the virtual world behaves in many respects analogously to the application of mathematics to the description of the real world from the viewpoint of the history of science. On the other hand, I argue that from an ontological point of view, however, this new situation is just very different from previous applications of mathematics.

As for the first topic, I try to show that, perhaps surprisingly, the need to create a virtual world has involved and inspired the same mathematical disciplines over the past few decades, with similar—only accelerated—paradigm shifts, as we have used in the application of mathematics to the real-world problems in the past centuries, and what paradigm shifts we have witnessed in the last 3 millennia.

As for the second topic, in an ontological sense, a significant difference can be raised: the real world is largely not created by us, we only want to describe it with the help of mathematics, but the virtual reality is created entirely by us. This is true even if we compare this with the works of the engineering sciences. Although the engineer applies mathematics in such a way as to create something (e.g., a bridge), in the meantime they cannot disregard the coercive laws of physics. However, such laws—apart from the minimum physical limits of vision and display—should not be constrained by the creator of computer animation, and, moreover, as we shall see, it is mathematics that creates new laws of new, digital physics. Note, the when I use the term "digital" I mean "created by and for a computer", therefore it is not closely affected by the recent debate on the digital or analogue nature of the real world (cf. [9,10]).

Mathematics, in a sense, can use its creative power much more here, as it is less restricted by measured data. It does not have to create a model for facts and data originated from the real world provided by non-mathematical scientists, but can work in a much more creative way, I would say in an "original math mode" in this field.

While the previous sentences are primarily about creating the world of animation by mathematical methods, at the same time there are also efforts to imitate the representation of the physical world in virtual reality as faithfully as possible. This is obviously a challenge

to mathematics, how to create as quickly (with less computational cost) and as accurately as possible what they have just wanted to describe. Here I will argue that the mathematical description of a phenomenon and the mathematical creation of the same phenomenon in cyberspace does not necessarily coincide in terms of the application of mathematics. As an example, I will study gravitation in this context.

It is important to note that from an ontological point of view, between mathematics and virtual reality there is a third, intermediate sphere: the software. The ontology of programming is a relatively extended field, and itself has a layered nature, from specification through algorithms to implementation and verification. The three main approaches are the rationalist, the technocratic and the empiricist views. The rationalist, which is somewhat the most rigorous approach, is that computer programs are fundamentally mathematical entities, the scripts are just alternatives of mathematical expressions [11].

The emipiricist approach is well summarised in [12] as "Computer science 'in the large' can be viewed as an experimental discipline that holds plenty of room for mathematical methods including formal verification". The technocratic (or engineering) approach actually says that computer science is not really a science. The computer scientist is a toolsmith, no more, no less [13], and the program code is just a bunch of data before running, although it evidently uses mathematical formulae and reasoning.

As we can see, all three approaches seek to relate to mathematics in one way or another. The rationalist approach is the closest to my view but the fact is that mathematics cannot fully model the programming and implementation phase with proof power, even if there are promising formal attempts to do so in recent times. The problem is that a computer game is a large-scale program that cannot be formally verified mathematically with our current tools—so we cannot be absolutely sure that the software, the implementation, does exactly what we intended in the specification. Overall, however, this is more of a technical problem, as with the development of formal verification methods (see e.g., [14,15]), it is hoped that in the near future we will reach a level where this verification can take place with complete certainty.

Complicating matters further is the blurring of the line between the real world and the virtual world, partly because with the advent of the 3D scanner and the pursuit of as much realism as possible, real objects and shapes also appear in the 3D virtual world. On the other hand, augmented reality is precisely about blurring the two worlds, where it is no longer always clear that we want to apply mathematics to describe or create a blended world. These issues are beyond the scope of the present paper, but definitely worth further investigation.

3. Paradigm Shifts in Mathematics of Animation—And Their Historic Analogues

In the following, I briefly summarise the great eras of the animation industry in terms of mathematical tools and with which I try to prove the above-mentioned parallels between these epochs and the great periods in the historical development of mathematics.

3.1. "Ancient Greek Era" of Video Games and Animation

The first computer games appeared in the early 1950s. Due to the limited abilities of displays, the rendering was quite rudimentary at that time. And from a mathematical point of view, games of that era did not go beyond the application of ancient Greek mathematical knowledge: basic (Euclidean) geometric elements, as well as elementary algebraic calculations, were used. Some mythical early games only worked with straight lines, sections, and points, such as Bertie the Brain (a kind of tic-tac-toe game). However, even Tennis for Two, accepted by many as the very first real (i.e., developed just for fun) game, did not go beyond using and displaying parabolic arcs.

We note here that ancient Greek mathematicians may not come to mind about early games simply because they used the mathematics Greeks were familiar with. This is because in the 1950–1960s we are in an age when the computer is basically for practical, computing purposes. With the creation of the aforementioned Tennis for Two and other

early games, it happens for the first time in computer science history that we use the computer and the mathematics behind it not for "work" but for entertainment, for our own pleasure—exactly the same way as we see with the Greeks in the study of mathematics for the first time in the history of mathematics. For the first time, a programmer may feel that he is not counting and thinking for scientific purposes, not applying math because he needs to achieve some practical goal, but simply for the pleasure of programming, gaming and thinking. For the first time, these games provide a l'art pour l'art experience for computer workers and users. Just as Greek mathematicians can be seen as the first scientists to study mathematics regularly for their own beauty, curiosity, and not for benefit, so the developers of the first games can be considered the first applied mathematicians in computer science to focus on leisure and joy instead of benefit.

3.2. "Medieval Age" of Video Games and Animation

Early home computer games and arcade games (such as Pac-Man, Donkey Kong, Super Mario) were realised in 2D, but from early on, geometric objects (vertical lines, points, parabolas in canonical position) and applied mathematical tools were shifted to algebra. Objects could be in various positions, intersections must be calculated many times throughout the game, and this means the use of classical (linear and low degree) algebraic tools. Just as the first great mathematical flourishing after the ancient Greeks was due to the algebraic studies of Arabic scholars, so in computer animation, it was the second major chapter in the application of mathematics.

3.3. "Modern Age" of Video Games and Animation

The transition from 2D to 3D in computer graphics was a groundbreaking novelty of the field. From an applied mathematical point of view, this course was accompanied by the application of spatial geometry, vector algebra and more robust calculation methods. This is also the time of birth of the first animation movies. The current elementary, mechanical, schematic motion of the characters is replaced by the differential calculus-based motion calculation—the parallel with Newton's results is evident, so much so that a separate expression is made for the totality of the physical (looking) parameters and relationships that appear in the animations: game physics (for a good technical overview see e.g., [16]). We will return to this in the next chapter. This is the time of free-form spline curves and surfaces with heavy use of approximation theory and analysis. One example: smoothly joining four simple, rectangle-like neighbouring surface patches described by two-variable vector-valued functions, the continuity of their second-order cross partial derivatives in the common point must be assured.

City-building and agriculture-simulation games apply axonometric view, more precisely isometric projection. This was originally described in the modern age of mathematics in [17]. Isometric mapping (informally called 2.5D due to its limited realism) was commonly used in video games since the geometry of this mapping required low computational costs and was achievable with the constrained resources of computers of that era. Later, with the development of computational capacity, the general axonometric and finally the real perspective mapping and view will appear.

3.4. The 21st Century of Video Games and Animation

The new period is the time of discretisation at a theoretical level. Scientists realised that the use of deep analysis is somewhat meaningless in the discrete representation of shapes on a pixel-based display. When the first full-length animation movies were released, the animation industry used mathematical surface modelling tools that were not originally developed for this purpose, but for the automotive industry. Computer-aided geometric modelling, which plays a central role in engineering and design, has used so-called free-form spline surfaces (and still uses them, see [18]). Ready-made mathematical tools and software were developed for these surfaces, so—for lack of a better—animation professionals also started their work with them.

However, it soon became clear that mathematical tools that were suitable for designing a car could not necessarily be used effectively to create animated characters. The flexible, user-friendly design of free-form surfaces was an advantage in industry, however, too complex shapes cannot be created with a single surface. In topological terms, the body parts of even the most expensive and sophisticated cars are basically made of rectangles, compared to which even the simplest anthropomorphic character is amazingly complex.

The other huge difference is the requirement for the analytical properties of the surface. The high degree of smoothness of surfaces required in the automotive industry can be met by the high order (continuous) differentiability of free-form surfaces. For this purpose, various sophisticated mathematical methods such as Bézier surfaces, B-spline surfaces and NURBS surfaces have been developed [18]. However, this makes little sense in the animation industry—no one wants to continuously twice derive the surface shape of Shrek. In addition, this degree of smoothness of the surface is unnecessary due to the limited capabilities of the displays as well, as pixels can only display discrete mathematical information in the end. Of course one can discretise the original analytical description of shapes, but the real breakthrough is a method that is inherently discrete, and cannot really be applicable in other circumstances than animation. In shape modelling this is the subdivision surface, which launched a new process that continues to this day—or, if you like, a paradigm shift: the virtual world has shifted from an analytical approach to a discrete one.

4. From Analytical to Discrete

In the application of mathematics, a dilemma that frequently arises is the use of mathematically exact analytical models versus the use of numerical methods. As several authors note, numerical methods are often preferred by researchers even when there is an otherwise well-known exact formula for solving the given problem [19,20]. The main reasons for preferring numerical methods are their simplicity and universality. As the authors suggest in [20]: "numerical methods provide quantitative predictions more easily than analytical solutions. In other words, ease of making quantitative predictions sometimes prevails over the exactness of solutions". Indeed, the necessary conditions for applying numerical methods are almost always fulfilled in the applications, thus, their application is a much more universal tool for solving problems, while accuracy is in most cases well within the margin of acceptable error. What happens is that although modelling the problem basically requires a continuous approach, such as describing a process using a differential equation, we already use discrete tools to solve it: the numerical solution replaces differential equations with difference equations (for a nice example see [20]).

While in computer simulations and computationally intensive mathematical models, the above situation may be advantageous but not automatically necessary, in case of video games and animation movies the discrete approach is a must, due to the pixel-based visualisation. The usual mathematical model can discretise an analogue world of real physical, chemical, and engineering problems, for the advantages of which we accept the limits of its accuracy. In the mathematical description of the visualisation of the virtual world, however, the model is about a world that has no continuous counterpart, the world is inherently discrete, thus, translation from the former analytical approach to discrete is in fact a wrong, constrained direction. It is only applied because we use models to describe the virtual world, originally created to describe similar problems in the real world. Mathematical descriptions of virtual shapes can be much more effective if we use a discrete model in the first place since in the end, we do not have to worry about reversal, we do not have to realise the final result in any analytical world.

The subdivision surface mentioned above is perhaps the first high-level product of this idea, which also transforms and discretises the concept of the smooth surface itself. For centuries, we thought about the (curved) smooth surface in the same manner, which we tried to capture with continuous functions and algebraic formulae, with a potential use of approximate methods and numerical solutions in technical calculations. The subdivision

surface actually looks like an analytical, smooth, continuous curved surface, but in fact it is a polyhedron: it consists of triangles, but the number of faces, edges, and vertices can sometimes be hundreds of thousands or millions, separately. It is very simple to produce: the faces of a coarse polyhedron consisting of a few triangular facets are subdivided and curved with more and more iterations into smaller and smaller triangles until they appear sufficiently smooth and continuous. The basis of the subdivision is that we define a new division point for each edge by a weighted affine combination of the surrounding vertices, and instead of the original edge, we use the two new sections connecting its two endpoints to the new point (these will usually not be correspondent to the original edge). The resulting new edges also define new facets, typically forming four new smaller triangles from each original triangle, but not in the plane of the original triangle. If we do this for each edge, we say that we performed a subdivision iteration, so then the number of facets quadrupled, but of course we get a polyhedron with the same topology as the original, only "looking smoother" due to the smaller triangles. After performing three to five iterations, the polyhedron is perfectly suited to display it in the virtual world, namely, by seeing it as a smooth surface, i.e., not perceiving its polyhedron nature. Additional shading techniques (Gouraud-shading, Phong-shading) help in the perception of smoothness of this sight.

In our discussion, the point is that in this subdivision technique, we do not want to create a smooth, differentiable surface patch that has to be discretised to display, but we build a polyhedron that looks sufficiently smooth, and this polyhedron is supposed to be a smooth surface. Assuming an infinite number of iterations, of course one can find a continuous, differentiable surface as a limit surface of this procedure, but this is actually irrelevant. This mathematical description thus uses from the outset a tool that was developed not for the discretisation of analytical descriptions but for the original needs of the visual authenticity of the discrete world. In every video game and every animated film in the 21st century, every smooth-looking shape is in fact an "angular" polyhedron, even if we do not notice it. This is nothing else than a new concept of a "smooth" surface specifically developed for the virtual world. It is worth noting that there is already a set of "discrete differential" geometry tools for these surfaces—a phrase with an apparent contradiction in our usual mathematical view.

A special advantage of this new model is that it is freely scalable (through the number of iterations) compared to the previous analytical, algebraic surface description methods, depending on what physical display and at what resolution the shape is presented, which is also theoretically important for video games [21].

5. Mathematical Creation of New Physics—The Case Study of Gravity

As mentioned, the virtual world is a visually authentic, perceptible and enjoyable world that may resemble the real world in many ways, but could be (and indeed is) different in physical and visual terms. Notice that, depending on the wishes of the creators, the task of mathematics is actually to create new physics in the virtual world, animation movies, and video games.

As an example, let us consider gravity. This physical phenomenon fundamentally defines our everyday lives. Physics has been trying for centuries to pinpoint the nature of gravity and give an accurate description of it. Several branches of mathematics have helped with this, from simple quadratic functions to Riemannian geometry. One of the simplest manifestations of gravity can be, for example, the dropping of an object (even from the Leaning Tower of Pisa). There is a well-known mathematical description of this at the level of high school physics, but we should always be aware that this description is only valid under extremely sterile conditions. Under realistic circumstances, however, we can never fully describe the dropping of a particular object at a particular location. The careless little movement of our hand at the starting moment of the fall, the actual shape of the object and the derived air resistance associated with the fall, the turbulences, the resurrection of the wind, in general the atmospheric conditions make each drop absolutely unique. The above mentioned model, known from high school, can approach it within some error tolerance,

but even top physics and the entire mathematical apparatus that helps it cannot describe it precisely, because of its contingency and uniqueness. Bouncing a basketball, in general, can be modelled at some level, but it is practically impossible to describe the bouncing of a particular basketball on a specific field with mathematical tools.

Now drop an object or bounce a basketball in an animation movie. In the software that produces the given scene of the movie, we can use a formula well known from high school and calculate the fall of the object based on it, and display it on the screen accordingly. However—and this is key to what I have to say—we can also use a different formula in the film. Gravity can be stronger, weaker, it can even work radically different than "real" gravity. According to the concept of the directors of the animation movie, the bouncing ball can be compressed much more strongly when it collides with the ground to create an exaggerated, more funny or more realistic effect on the viewer. This latter one is actually a specific effect called "squash and stretch" in the twelve principles of animation [22].

A movie character rushing beyond the edge of the canyon rift can take a few more steps in the air and then float in one spot to realise her position before falling at an incredible starting speed that is completely contrary to the laws of "real" physics. All of these gravity laws are described by mathematical tools, but these formulas, as well as their effects, can differ radically from the mathematical descriptions used so far in the real world.

From an ontological point of view, applied mathematics is given a different status here. In the physical descriptions so far, mathematics has tried to support understanding of the real world as precisely as possible, but has created nothing but a limited model. However, in the case of animation physics, mathematics appears as a genuine creator. This world is physically created by mathematics, and the world will function exactly as described by the given mathematical formulae. Even the small random deviations often incorporated into formulae for a more realistic view are mathematically controlled. These are actually pseudo-random deviations created by mathematics since they are generated by the random number generator of a computer.

In the created world, the application of mathematics is not of explanatory nature. On the contrary, mathematics creates the world, with all its digital, physical, chemical and biological theories, laws and parameters. Yes, it is not just about the physics mentioned in the previous example, but also about other areas of science. Plants can grow and dry out at lightning speed, the weather can change at will, fictional materials can explode and unexpectedly spray glitter on the actors. All of the aspects of virtual nature have to be defined by mathematics, and that will behave exactly and throughout the way the theory created by mathematics prescribes. In my view, this is thus the quintessence of the case mentioned by Rizza when applied mathematics directs and organises the emergence of qualitative theories.

Of course, mathematics applied here is not necessarily entirely new. In some cases, mathematical theories and tools that have developed over the centuries—and are subordinated to qualitative theories—are being used in the virtual world as well. Even then, however, the relationship between applied mathematics and the phenomenon it describes is significantly different than before. The mathematical description of a digital phenomenon, either purposefully created to be different from the real world or simply inherited from the earlier real-world models, is in absolute terms valid in the virtual world. There are no exceptions, there are no deviances, and there are no inevitable abstractions in modelling. This digital world behaves exactly as mathematical theory predicts. In the case we use earlier models, this world, created and then displayed in a mathematical way, is an embodiment of the "ideal" physics, chemistry and biology that have only existed in our minds so far. We are in a world where the gravitational constant is not a measured value, not an empirical physical constant, but a digital–physical constant predefined by us, and all the gravitational effects of our world occur accordingly. There is no point in talking about the accuracy of the value. The computer floating-point arithmetic is not a limitation of the mathematical description, but part of it. There is no meaning of relative uncertainty.

In other cases, the new rules described by mathematics do not even follow the models originally designed to describe the real world. To stay with gravity: it is clear that Super Mario's gravitational acceleration has nothing in common with the measured value at any point on Earth. He is able to jump up to 5 times higher than his height (note, that the value of gravitational acceleration may change from edition to edition of Super Mario). The gravitational acceleration is an absolute constant in the game, does not depend on altitude and latitude, and it is evidently defined by the mathematician to pass it on to the software developer.

Overall, this also leads to a consequence of reconsidering the phrase and meaning of applied mathematics. There is no doubt that those who create these digital, physical or biological laws and virtual worlds, are mostly considered applied mathematicians. However, if we accept the narrative of mathematising "as a virtuous practice in its own right" [23], then one can realise that those who create these virtual laws and spaces, frequently independent of our real-life experience, are, perhaps, not "applied" mathematicians any more.

6. Conclusions

The virtual world is a field of application of mathematics that is significantly different from all previous applications in both ontological and epistemological terms. In this paper, I have tried to show that this field of application is the purest form of the case when mathematics is not subordinated to the theory of the field where it is intended to be applied, but it directs and organises its emergence. The virtual world can have many analogies to the real world, but its specific nature allows mathematics to step up in a creative way and provide new theories and paradigms. In many ways, similar problems emerge here as in real-world science, therefore the interdisciplinary approach is crucial [24]. This analogy can also be seen in action from the historical aspect of the application of mathematics. However, unlike what happens in real-world science, here mathematics has an ontological role, creating new (digital) physical and chemical principles, which, moreover, are fulfilled with deadly accuracy. This, in an ontological sense, opens a new chapter in the interconnectivity of applied mathematics and digital sciences.

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