philosophies

Article

Leibniz's Principle, (Non-)Entanglement, and Pauli Exclusion

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Abstract: Both bosons and fermions satisfy a strong version of Leibniz's Principle of the Identity of Indiscernibles (PII), and so are ontologically on a par with respect to the PII. This holds for non-entangled, non-product states and for physically entangled states—as it has been established in previous work. In this paper, the Leibniz strategy is completed by including the (bosonic) symmetric product states. A new understanding of Pauli's Exclusion Principle is provided, which distinguishes bosons from fermions in a peculiar ontological way. Finally, the program as a whole is defended against substantial objections.

Keywords: quantum mechanics; permutation invariance; entanglement; Leibniz's principle; Pauli's exclusion principle; symmetric product states

Citation: Friebe, C. Leibniz's Principle, (Non-)Entanglement, and Pauli Exclusion. *Philosophies* **2024**, *9*, 45. https://doi.org/10.3390/philosophies9020045

Academic Editors: Rafael A. Alemañ, Lorenzo Magnani and Marcin J. Schroeder

Received: 28 December 2023
Revised: 27 March 2024
Accepted: 27 March 2024
Published: 29 March 2024

1. Introduction

Bosons can be in symmetric product states, such as: [1]

$$|\text{prod}\rangle = |R, -1_z\rangle_1 \otimes |R, -1_z\rangle_2 \tag{1}$$

The straightforward reading of this state is obvious. There are two numerically distinct particles, labeled by the tensor-product indices "1" and "2", which share (also) their state-dependent properties. Both particles are spatially located at R, and both particles possess the spin-projection property of -1_z. The traditional Leibniz principle, according to which numerically distinct objects differ in some of those properties, is (clearly) violated.

This is a challenge for the "heterodox" views on identity and (in-)discernibility in quantum mechanics (QM). These views intend to defend Leibniz's principle of the identity of indiscernibles (PII), in some strong sense, against the "orthodox" views of QM. [2] Accordingly, even bosons *are* distinguished by state-dependent properties, namely (at least) in "GMW-non-entangled", non-product states, such as: [3]

$$|\text{non}\rangle = \frac{1}{\sqrt{2}}(|R, 1_z\rangle_1 \otimes |L, 0_z\rangle_2 + |L, 0_z\rangle_1 \otimes |R, 1_z\rangle_2) \tag{2}$$

According to the heterodox views, in this state there is one particle spatially located at R that possesses the spin-projection property 1_z, and there is a second particle spatially located at L and with spin property 0_z. [4]

However, this interpretation is adequate only if one rejects "factorism", the semantic view that the (original) tensor-product indices of the formalism refer as labels to the physical particles. [5] The particles labeled by "1" and "2" would share the same mixed state:

$$\hat{\rho}_{1;2} = \frac{1}{2}(|R, 1_z\rangle\langle 1_z, R| + |L, 0_z\rangle\langle 0_z, L|) \tag{3}$$

Particle 1 and particle 2 would still violate the (strong) PII. The PII is satisfied in this state only if one assumes anti-factorism: the tensor-product indices "1" and "2" are only mathematical symbols that denote Hilbert subspaces. The real particles, by contrast, are characterized by $(R, 1_z)$ versus $(L, 0_z)$, hence, qualitatively.

Unfortunately, this reading is apparently inadequate regarding the product state (1). Assuming anti-factorism does not help since both particles would qualitatively be characterized by $(R, -1z)$. Accordingly, Leibniz's principle would still be violated, (at least) for bosons in symmetric product states. As it seems, the PII can only be defended as a *contingent* principle that holds in some specific situations (states) but not in others.

However, this goes against the spirit of the traditional PII. [6] I suggest that, if one intends to defend the traditional PII, the principle must hold with (physical) necessity, i.e., in every situation (state) that is physically possible according to QM. Moreover, the PII should be considered as the *principle of individuation*. It is not sufficient that in every state the numerical distinctness of particles is accompanied by qualitative distinguishability. Rather, there must be numerically distinct particles *in virtue of* or because of qualitative exclusiveness. Otherwise, the ontology of the particles could still be such that their numerical distinctness is *primitive*. If the PII is not the principle of individuation, but only a further methodological principle for pragmatic purposes, there could well be bare particulars on the ontological groundfloor as the bearers of the physical properties. [7]

To definitely rule out this option, the Leibniz strategy should be the attempt to defend the PII in every QM-possible situation and as the principle of individuation, connected with the corresponding semantic view about how to refer to such Leibniz individuals. The main part has already been performed in previous work; I will give a summary of it (without discussion) in the following section. Then, Section 3 is devoted to the issue of how to defend the Leibniz strategy also for bosons in symmetric product states. It will be argued that fermions and bosons are ontologically on a par with respect to the PII. Nevertheless, they are ontologically distinguished in a peculiar way, to be specified. This provides also a new understanding of Pauli's Exclusion Principle. Finally, in Section 4, some substantial objections against this program will be addressed.

2. Identity and (In-)Discernibility in QM

Consider the following fermion-state:

$$|\text{perm.inv}\rangle = \frac{1}{\sqrt{2}}(|R, \uparrow_z\rangle_1 \otimes |L, \downarrow_z\rangle_2 - |L, \downarrow_z\rangle_1 \otimes |R, \uparrow_z\rangle_2) \tag{4}$$

The state satisfies the requirement of *permutation invariance* for similar particles in QM. Such a state is of particular interest because it describes a situation that is in some important respects close to the classical view. The properties of spatial location and spin projection are coupled so that the intuitive picture is reliable, according to which there is spin-up at the right hand and spin-down at the left one. One cannot violate any Bell inequality with such a state. Mathematically, such states can be obtained by (anti-)symmetrizing a product state, which characterizes the state as physically non-entangled, according to [4]. Nonetheless, the philosophical interpretations differ significantly already concerning this rather harmless state.

Next, consider a state that cannot be obtained by (anti-)symmetrizing a product state, such as:

$$|\text{EPR}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2) \tag{5}$$

Following GMW, this state is not only permutation invariant but represents also physical entanglement: one can violate Bell inequalities. Spatial locations and spin properties are no longer coupled such that the intuitive picture is no longer adequate. Unsurprisingly, such situations have always provoked philosophical dispute, e.g., about (non-)locality in nature. Notably, the orthodox views do not care about it, in the sense that the distinction between states of type (4) and of type (5) has accordingly no impact on the identity and (in-)discernibility of QM particles. The heterodox views, by contrast, hold that questions of discernibility (and individuality) should be approached with attention on what it means for similar particles to be entangled or not.

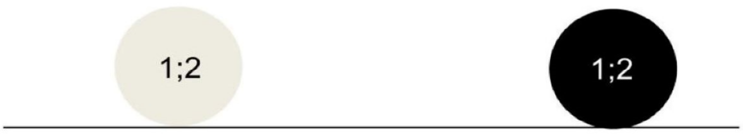
2.1. The Orthodox Views

Orthodoxy says that the traditional PII is violated already in state (4). In fact, it is violated in *every* physically accessible state for fermions and bosons alike, regardless of how many particles are considered. All electrons in the whole universe share the same mixed state, analogous to the mixed state associated with state (4):

$$\hat{\rho}_{1;2} = \frac{1}{2}(|R, \uparrow_z\rangle\langle\uparrow_z, R| + |L, \downarrow_z\rangle\langle\downarrow_z, L|) \tag{6}$$

However one may read the mixed state (ignorance interpretation is not allowed!); particles 1 and 2 are related to all state-dependent properties in exactly the same way.

This is remarkable because, qualitatively, the intuitive picture holds that there is spin-up at the right and spin-down at the left. However, according to the orthodox views, already in cases of non-entanglement, QM differs crucially from classically composed systems. Somehow, both labels must be associated with both spatial regions and with both spin projections (different colors in the figure) as it is shown in Figure 1.



$$|\Psi\rangle = {}^{1}/_{\sqrt{2}}\,(|R\rangle_1|\uparrow_z\rangle_1|L\rangle_2|\downarrow_z\rangle_2 - |L\rangle_1|\downarrow_z\rangle_1|R\rangle_2|\uparrow_z\rangle_2)$$

Figure 1. Orthodox view about non-entanglement.

Counterintuitive as it may be, assuming factorism, this interpretation is apparently unavoidable.

Orthodoxy assumes the semantic view that the tensor-product indices are labels that refer to the physical particles. Moreover, the standard labels "1" and "2" are proper names according to the *Direct Reference* approach. Correspondingly, such labels provide direct reference, i.e., independently of any property. "Gödel" refers to Gödel, whatever he did, e.g., also in the counterfactual situation in which Gödel did not prove the incompleteness theorem. So, "1" refers to particle 1, whatever this particle 'does', and "2" refers to particle 2, whatever this particle 'does'. Labels "1" and "2" refer to particle 1 and particle 2 even in the case in which they 'do' the very same thing regarding the state-dependent properties, i.e., even if they are indistinguishable with respect to these properties. [8]

There is a main difference between the orthodox views:

1. According to the so-called Received View, every interesting version of Leibniz's principle is violated, in every situation.
2. According to the Weak Discernibility approach, Leibniz's principle can be saved (at least in some situations) by weakening it, namely, by including physical, symmetric but irreflexive relations into the scope of the potentially distinguishing properties. [9]

Both views agree that the sentence "1 and 2 share the same (mixed) state" is QM-necessarily true. They disagree about whether the sentence "1 and 2 are in a relation that weakly discerns them" is (sometimes) true. In the case at hand, 1 and 2 are weakly discerned by being $R - L$ spatially apart from each other and by having opposite spin, according to Weak Discernibility.

However, this difference is metaphysically and semantically insignificant. Both views are still compatible with the following world view, i.e., no physical reasons have been presented, which rule out the following picture: There are two numerically distinct bare particulars on the ontological groundfloor. The numerical distinctness of objects is primitive. Further, there are the universals of mass, charge, spin(-projection), spatial location, etc., on the ontological groundfloor; there is some way to tie the bare particulars with the universals.

Singular terms (indices) refer directly to the bare particulars, and both bare particulars are (or have to be) tied with the universals in exactly the same way. Then, the views differ about whether the emerging complex objects are (sometimes) weakly discerned by symmetric but irreflexive relations.

2.2. The Leibniz Strategy

The Leibniz strategy is one of the heterodox views. Heterodoxy defends the PII in some strong version, i.e., not merely weakly, and hence rejects (direct) factorism. The Leibniz strategy, in particular, is an attempt to establish the PII as the principle of individuation for QM particles. Such particles should be numerically distinct in virtue of qualitative exclusiveness. Connected with this metaphysical view is *descriptivism*, the semantic view according to which particle labels must refer descriptively, via a definite description of their referents.

In cases of non-entanglement, it is rather straightforward to identify the Leibniz particles. In the boson state (2), one particle is exclusively characterized by the bundle of state-dependent properties $(R, 1_z)$, and the second by the bundle $(L, 0_z)$. In the fermion state (4), the characterizing property bundles are $(R, \uparrow_z)$ versus $(L, \downarrow_z)$. In order to refer to these particles, one has to introduce *new*, descriptive labels, e.g., "1_L" and "2_L". Take two electrons (in state (4)), then "1_L" refers to the particle that possesses the properties $q_e, m_e, s = \frac{1}{2}; R, \uparrow_z$ (whichever particle that may be), and "2_L" refers to the particle that possesses the properties $q_e, m_e, s = \frac{1}{2}, L, \downarrow_z$ (whichever that particle may be).

The particles 1_L and 2_L are qualitatively distinguishable as shown in Figure 2, and this could not have been otherwise. Along this line, it is consistent to assume that 1_L and 2_L are Russellian bundles of universals, particles individuated by distinguishing properties, and "1_L" and "2_L" are the corresponding labels that have their referents mediated by descriptions.

$$|\Psi\rangle = {}^{1}/_{\sqrt{2}}\,(|R\rangle_1|\uparrow_z\rangle_1|L\rangle_2|\downarrow_z\rangle_2 - |L\rangle_1|\downarrow_z\rangle_1|R\rangle_2|\uparrow_z\rangle_2)$$

Figure 2. Leibniz strategy about non-entanglement.

Cases of entanglement are a challenge for the Leibniz strategy. Look, again at the given entangled fermion state: [10]

$$|\text{EPR}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2) \quad (8)$$

As stated, the properties of spatial location are no longer coupled with the spin properties; the intuitive picture is no longer adequate. There are no subsystems with complete bundles of properties; there are no labels available that uniquely refer to subsystems mediated by definite descriptions. [11] Apparently, the only descriptive proper name that refers adequately to a physical object refers to the system as a whole. "EPR_L" (say) refers to the physical object that possesses the properties $2q_e, 2m_e, R - L, \hat{S}^2 = 0; \hat{S}_z = 0$ (whichever object that may be). Correspondingly, the only PII individual is an *undivided* whole without subsystems as parts.

Thus, entanglement requires the *summing defense* strategy for the PII. [12] Summing is the physical process of entangling particles, namely of unifying these particles to create the undivided whole. Reversely, disentanglement, e.g., via EPR measurement, is dividing the whole, i.e., producing subsystems (particles as PII individuals). A property such as $\hat{S}^2 = 0$, which emerges by unification and vanishes by division, characterizes the whole as undivided. A property such as $\hat{S}_z = 0$, which survives division, indicates that the resulting non-entangled state still represents a whole but a composed whole. [13]

3. Pauli Exclusion and Symmetric Product States

As long as one considers purely permutation-invariant, non-product states and physically entangled ones, fermions and bosons are ontologically on a par. The (strong) PII can be saved and established as the principle of individuation. This requires anti-factorism, the introduction of new, descriptively referring labels, and the summing defense in cases of entanglement. Fermions and bosons are distinguished by Pauli's Exclusion Principle, which holds only for fermions so that for bosons symmetric product states are allowed. These states provide the hard problem for the Leibniz strategy.

3.1. Pauli Exclusion as an Ontological Problem

Consider a symmetric product state, such as (again):

$$|\text{prod}\rangle = |R, -1_z\rangle_1 \otimes |R, -1_z\rangle_2 \tag{9}$$

The straightforward reading is that there are two particles in this state, which are qualitatively indistinguishable. Particle 1 and particle 2 are both spatially located at R, and both possess the spin property -1 along the z-axis. However, then, not only would the Leibniz's principle be violated only in such a state but satisfied otherwise, but also the tensor-product indices "1" and "2" would (directly) refer to physical particles only in such states but be (physically) empty otherwise. In all other states, there only are Leibniz individuals that can be ontologically considered as Russellian bundles of QM properties, but in the product states, it seems as if bare particulars pop up as the primitively individuated bearers of the properties. Instead, one should be after a coherent view throughout every physically possible state.

The obvious problem is that newly introduced, descriptive labels such as "1_L" and "2_L" cannot refer to subsystems. Reasonably, there is only one descriptive proper name available that would refer to a subsystem, namely, the label that carries the description of $(m, q, R, -1_z)$. Yet, there is not only one single elementary particle in symmetric product states, for there is obviously twice the mass, charge, spin, or energy of a single particle in such product states.

The heavier problem is that, apparently, also the summing defense does not work in this case. One may argue that the adequately descriptive label to be introduced is a label that carries the description of the whole system, in particular, $(2; -2; 0)$, corresponding to the total spin operator $\hat{S}^2$, the global spin-projection operator $\hat{S}_z$, and the operator of spatial extension $\hat{Q}$. The whole would be undivided, the only (Leibniz) individual that exists in this case. But then, there would be tension with the physics. This would ontologically equate systems in symmetric product states with systems in entangled states. Physically, however, symmetric product states are *non-entangled* states; one cannot violate any Bell inequality with product states. Further, it is apparently not the case that the boson product emerges by unifying bosons in the way suggested, namely, not in the way that the (previously) distinguishing properties of the particles vanish and a non-supervenient property (such as $\hat{S}^2 = 0$, in the EPR-case) of the whole emerges.

Nonetheless, the Leibniz strategy should also apply the summing defense here. It will be a novel variant of it. The task is to distinguish undivided wholes in (bosonic or fermionic) entangled states from likewise undivided wholes in (bosonic) symmetric product states, and it is here where the ontological role of Pauli's Exclusion Principle comes into play. Formerly, Pauli exclusion was understood to concern numerically distinct particles. Correspondingly, it would mean that fermions cannot have all quantum numbers in common, while bosons could, which seemingly implies that fermions satisfy the PII, while bosons do not. Nowadays, the principle has technically been reformulated as stating that no $\otimes$-factor can occur more than once in any term of the state of the (so-called) composite system. The word "particle" does not even occur in this accepted formulation such that one can ontologically reformulate Pauli's principle as rather concerning *undivided wholes*.

Look (again) at the given fermionic entangled state:

$$|\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2) \tag{10}$$

According to the Leibniz strategy, it represents an undivided whole characterized by the property bundle $(0;0;R-L)$, corresponding to the total spin operator $\hat{S}^2$, the global spin-projection operator $\hat{S}_z$, and the operator of spatial extension $\hat{Q}$. Then, Pauli exclusion says that certain combinations of eigenvalues (properties)—such as $(1;1;0)$ or $(1;-1;0)$ for $\hat{S}^2$, $\hat{S}_z$, and $\hat{Q}$—are *excluded* for fermionic undivided wholes. Thus, the principle provides constraints on the definite bundle of properties that fermionic wholes can possess. In contrast, for bosonic undivided wholes, all combinations are allowed. Regarding the running example state, in particular, the eigenvalue set $(2;-2;0)$ of $\hat{S}^2$, $\hat{S}_z$, and $\hat{Q}$ is physically accessible.

This has ontological consequences. In order to *apply* Pauli's principle, one has to distinguish fermions from bosons. However, if one assumes, as the Leibniz strategy does, that fermionic and bosonic entangled wholes are undivided, i.e., they do not contain numerically distinct particles, the question arises: what characterizes such a whole as "fermionic" or "bosonic"? [14] The only reasonable answer, on this assumption, seems to be that an undivided whole is fermionic (bosonic) precisely in the case in which it has been produced out of numerically distinct fermions (bosons) and/or it can be divided into numerically distinct fermions (bosons). The very meaning of "Pauli exclusion", when applied to undivided wholes, has to be imbedded into the Leibniz strategy of unifying and dividing physical systems.

3.2. *The Peculiar Summing Defense for Bosons*

A new version of the summing defense is required. The idea is that entangled states represent undivided but unstable wholes, whereas product states represent fairly stable wholes. Entangled systems can easily be disentangled, e.g., by a rather simple EPR measurement. Within such states, there are immediately dividable wholes. By contrast, product systems, in particular Bose–Einstein condensates, are immediately *undividable* wholes. They must firstly be modified in order to become dividable: they must be *extended* into entangled wholes. Whereas fermionic wholes are necessarily extended of some magnitude (Fermi pressure), bosonic wholes can be unified in a peculiar sense to become non-extended (Bose–Einstein condensate).

Dividing a whole implies difference in number: only one object at the beginning (In) and two objects as the result (Out). In product states, however, there is no room for differential interaction (by measurement or unitary evolution), hence no room for dividing the system. The product whole can only be *changed* in its entirety. Change only requires difference in property, not in number, of things. Regarding the given system with $(2;-2;0)$ as the definite bundle of global properties, one can manipulate it in a way that (only) this property bundle will be replaced with another, likewise possessed by a single undivided whole. [15] Hopefully, one can change the undivided whole to obtain an entangled system, e.g., $(2;-2;0)$ will be replaced with $(2;0;R-L)$ in this state:

$$|\mathrm{ent}\rangle = \frac{1}{\sqrt{2}}(|-1_z\rangle_1 \otimes |1_z\rangle_2 + |1_z\rangle_1 \otimes |-1_z\rangle_2)(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2) \tag{11}$$

There is still an undivided whole. However, now it *is* immediately dividable.

In a second step, it can indeed be divided (by disentanglement); hence, numerically distinct and qualitatively distinguishable bosons can be produced (represented with a non-entangled state). This is occurring in experimental physics: a Bose–Einstein condensate is a stable ground state, in the sense that much more effort than in EPR-like measurements is needed to 'destroy' the condensate. [16] What matters here is that the second sense of "unification" must be the reversed process of the change that transforms a product whole into an entangled system. How exactly is this change to be characterized?

For this aim, look at the ground-state wave function of a Bose–Einstein condensate. A single-particle state is multiply occupied. [17] This means that its spatial spread is exactly the same as if there were only a single, doubtlessly undividable particle. The distance between the (allegedly) many bosons in the condensate is zero, or, the expectation value to measure a single particle location is the same for each boson produced out of the condensate. The distance between the expectation value of the position equals 0. In some representation of the Bose–Einstein ground state, i.e., in some spin–spatial–distance space, the condensate should hence be representable by a point-like peak. Therefore, it is in some sense 'unextended', as much as a single elementary particle is unextended in comparison with a typical many-particle state. By contrast, a Fermi ground state is, in virtue of Pauli exclusion, QM-necessarily extended of some magnitude (Fermi pressure)—it is extended in every way of its representation, because of the fact that it is QM-necessarily distinguished from a single-particle state, i.e., from an undividable entity. [18] The change one seeks to understand is therefore of a characteristic sort: *extending*.

In more experimentalist's terms, one can say that, in order to 'destroy' a Bose–Einstein condensate, one has to *excite* the ground state before one can 'cut' the excited state in separated part(icle)s. The Fermi ground state is—in comparison to its Bose–Einstein analogue—already in an excited condition. To excite a ground state is to increase the degrees of freedom; these can be minimal in the boson case but are necessarily non-minimal in the fermion case. Increasing the degrees of freedom, in turn, means extending the system. Then, in this case, an excited state is an entangled state, such as the $1s2s$ configuration of a He atom (with $\hat{S}^2 = 0$):

$$|1\text{s}2\text{s}\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle_1 \otimes |\uparrow\rangle_2 - |\uparrow\rangle_1 \otimes |\downarrow\rangle_2)(|E_0\rangle_1 \otimes |E_1\rangle_2 + |E_1\rangle_1 \otimes |E_0\rangle_2) \quad (12)$$

but not a non-entangled state, such as:

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|E_0, \uparrow_z\rangle_1 \otimes |E_1, \downarrow_z\rangle_2 - |E_1, |\downarrow_z\rangle_1 \otimes |E_0, \uparrow_z\rangle_2). \quad (13)$$

The former state is the analogue of the undivided but dividable whole that results by exciting the Bose–Einstein ground state: the system within such a state is still bounded (e.g., in a trap). The latter is the analogue of the 'destroyed' condensate in which distinguishable particles have been produced.

In order to distinguish extending the point-like peak from dividing the excited state, one therefore has to say that dividing acts on something that is already extended, while extending acts on something that is non-extended. Thus, "extending" and "dividing" have in common a sense of "increasing". Dividing increases the *number* of objects, which is desired to obtain distinguishable particles. Extending, in contrast, increases only *dimension* (degrees of freedom). The spin–spatial–distance point of the Bose–Einstein condensate firstly has to be extended before one can divide it to obtain distinguishable bosons.

Pauli's Exclusion Principle can now be *ontologically* reformulated: fermionic wholes are QM-necessarily extended in spin–spatial–distance space, i.e., independently of any physical force, fermionic wave functions are always (in every representation) extended. Therefore, fermionic wholes are immediately dividable by disentanglement. Reversed direction: PII-individuated fermions can always be unified. Numerically distinct, extended, and sufficiently discriminated regions transform into a still extended region without containing sufficiently discriminated proper subregions. "Unifying"—which has a sense of "decreasing"—(only) means with respect to fermions: decreasing the number of entities (discriminated regions/distinguishable particles).

For bosonic wholes, no such constraint holds. In spin–spatial–distance space, it is QM possible that their representations are point-like. Thus, bosons can be unified also in a different way that must be the reversal of extending. In this way, unification acts on *one single* extended region without an inner structure and transforms it into a non-extended peak. In the first sense—the only sense that is available for fermions—unification reduces the number of objects; in the second sense—available only for bosons—unification decreases the dimension (minimizes the degrees of freedom) of objects.

4. Discussion

Several objections can be (and have been) raised against the Leibniz strategy. I will address the most important ones in what follows, in systematic order from arguments defending orthodoxy up to disagreements among anti-factorists.

Why should orthodoxy be impressed?

(Direct) factorism is the most natural semantic view about QM, close to the mathematical formalism. And, with (direct) factorism, the orthodox view that similar particles QM-necessarily share the same (mixed) state in every situation, even in cases of non-entanglement, is unavoidable. The Leibniz strategy is apparently motivated on purely metaphysical grounds (to avoid a certain undesired ontology), and spends (too) much effort, such as performing the summing defense, only to satisfy some philosophical prejudice.

Response: The Leibniz strategy is, on the contrary, motivated on physical grounds. The point of departure is the observation that the orthodox views do not respect the important distinction between pure permutation invariance and physical entanglement. In more detail, the argument goes as follows. Consider again the given (fermionic) non-entangled state:

$$|\text{non}\rangle = \frac{1}{\sqrt{2}}(|R,\uparrow_z\rangle_1 \otimes |L,\downarrow_z\rangle_2 - |L,\downarrow_z\rangle_1 \otimes |R,\uparrow_z\rangle_2) \tag{14}$$

Both particle 1 and particle 2 are, as said before, in the same mixed state:

$$\hat{\rho}_{1;2} = \frac{1}{2}(|R,\uparrow_z\rangle\langle\uparrow_z,R| + |L,\downarrow_z\rangle\langle\downarrow_z,L|) \tag{15}$$

Now, consider the following symmetric, i.e., allowed, operator:

$$\hat{O} = (\hat{R}\hat{s}_y) \otimes \hat{1} + \hat{1} \otimes (\hat{R}\hat{s}_y) \tag{16}$$

Let it act on the given state. At spatial location R, a spin measurement in the y direction is performed. The eigenvalue–eigenvector link being assumed, the resulting state will be, for example, this one:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|R,\uparrow_y\rangle_1 \otimes |L,\downarrow_z\rangle_2 - |L,\downarrow_z\rangle_1 \otimes |R,|\uparrow_y\rangle_2) \tag{17}$$

Apparently, the subsystem on the right side has changed its spin property, whereas the subsystem on the left side has not been touched by the measurement. In contrast to this strong intuition, both particle 1 and particle 2 would have changed their common mixed state. They would be, afterwards, in this new but still common state:

$$\hat{\rho}_{1;2} = \frac{1}{2}(|R,\uparrow_y\rangle\langle\uparrow_y,R| + |L,\downarrow_z\rangle\langle\downarrow_z,L|) \tag{18}$$

Although the incoming state is not a physically entangled state, a measurement acting only at R would touch and change both the particles labeled by "1" and "2". This holds likewise according to the Received View and according to Weak Discernibility; the difference between the orthodox views is only that the latter would add that both particles also have changed their (common) relation that weakly discerns them. So, both orthodox views treat the particles as if they were entangled, yet the incoming state is purely permutation invariant. Respecting this distinction requires banning the given labels.

Why are distinguishing properties the individuating properties?

According to the Leibniz strategy, there is not only qualitative distinguishability with respect to state-dependent properties (in non-entangled states), but these properties are considered to be the individuating properties, i.e., there are two numerically distinct particles in virtue of qualitative exclusiveness. Now, the objector accepts that distinguishable particles emerge in cases of non-entanglement but rejects the further claim that these properties are individuating. The descriptive labels apparently are not unique, at least not in the case of fermionic non-entanglement. [19]

In detail: Every non-entangled (fermion) state is overdetermined in the way that it is a common eigenstate of (infinitely many) non-commuting operators. In particular, the running example state:

$$|\text{non}\rangle = \frac{1}{\sqrt{2}}(|R,\uparrow_z\rangle_1 \otimes |L,\downarrow_z\rangle_2 - |L,\downarrow_z\rangle_1 \otimes |R,\uparrow_z\rangle_2) \tag{19}$$

can also be written, for example, in the following way:

$$|\Gamma\rangle = \frac{1}{\sqrt{2}}(|R\rangle_1 \otimes |\uparrow_z\rangle_2 + |L\rangle_1 \otimes |\downarrow_z\rangle_2) \qquad |\Lambda\rangle = \frac{1}{\sqrt{2}}(|R\rangle_1 \otimes |\uparrow_z\rangle_2 - |L\rangle_1 \otimes |\downarrow_z\rangle_2) \tag{20}$$

As it seems, one must argue that—also!—there is a particle with properties associated with $|\Gamma\rangle$ and there is a particle with properties associated with $|\Lambda\rangle$. The joint state fails to determine its very constituents because the mathematical physics does not tell which (bundles of) distinguishing properties really are the *individuating* properties. In the case at hand, both $(R,\uparrow_z)$ versus $(L,\downarrow_z)$ as well as (Γ) versus (Λ) (and infinitely many more pairs of distinguishing property-bundles) can perform the individuating job.

Response: According to the Leibniz strategy, a state should never be considered as a fundamental or primitively given state. Every physical situation has been produced, namely, by unifying particles or by dividing an undivided whole. A mathematical state description alone cannot uniquely determine which Leibniz individuals there are. In the case at hand, the state presumably emerges by dividing an entangled whole so that the joint state *taken together with the interaction that has produced it* determines what the individuating properties of the particles are. [20] If the dividing process is spatial-spin-like, the produced particles will be individuated by the properties of spatial location and spin projection, but if the dividing process is Γ-Λ-like, the individuating properties of the produced particles will be associated with $|\Gamma\rangle$ and $|\Lambda\rangle$. This response suggests that the Leibniz strategy not only should be accepted regarding non-entangled states but shows its virtue taken as a whole.

Why can factorism not be defended?

Still concerning non-entangled states, the objector now accepts unique distinguishability, apparently even in its individuating sense, but rejects anti-factorism. Recall that the Leibniz strategy assumes anti-factorism and introduces new, descriptive labels, which then is the motivation for the summing defense regarding entangled states and for the peculiar summing defense regarding bosonic symmetric product states. However, the objection goes, only *direct* factorism must be rejected, while *descriptive* factorism can be defended. Granted that the indices "1" and "2" of the standard formalism, if considered to be referring to physical particles, would be directly referring, and granted that this supports the orthodox views, there still can be an alternative factorization of the composite Hilbert space in which the factor-space indices can be considered as labels that refer descriptively. This interpretation is closer to the formalism than the Leibniz strategy and probably avoids such bold moves like the summing defenses.

Muller and Leegwater [5] (2022, sec. 3) construct, for the given fermion case, a four-dimensional proper subspace with descriptively referring $\otimes$-factor Hilbert-space labels, which they call "snapshot Hilbert space", for a short time t. As a basis, they choose the following four states:

$$|\uparrow_z\uparrow_z;t\rangle = \frac{1}{\sqrt{2}}(|L,\uparrow_z;t\rangle_1 \otimes |R,\uparrow_z;t\rangle_2 - |R,\uparrow_z;t\rangle_1 \otimes |L,\uparrow_z;t\rangle_2) \tag{21}$$

$$|\uparrow_z\downarrow_z;t\rangle = \frac{1}{\sqrt{2}}(|L,\uparrow_z;t\rangle_1 \otimes |R,\downarrow_z;t\rangle_2 - |R,\downarrow_z;t\rangle_1 \otimes |L,\uparrow_z;t\rangle_2) \tag{22}$$

$$|\downarrow_z\uparrow_z;t\rangle = \frac{1}{\sqrt{2}}(|L,\downarrow_z;t\rangle_1 \otimes |R,\uparrow_z;t\rangle_2 - |R,\uparrow_z;t\rangle_1 \otimes |L,\downarrow_z;t\rangle_2) \tag{23}$$

$$|\downarrow_z\downarrow_z;t\rangle = \frac{1}{\sqrt{2}}(|L,\downarrow_z;t\rangle_1 \otimes |R,\downarrow_z;t\rangle_2 - |R,\downarrow_z;t\rangle_1 \otimes |L,\downarrow_z;t\rangle_2) \tag{24}$$

Then, they factorize this snapshot space $\mathcal{H}(t)$ into $\mathcal{H}_\mathrm{l}(t)$ and $\mathcal{H}_\mathrm{r}(t)$ such that the basis can be written as:

$$|\uparrow_z\uparrow_z;t\rangle = |\uparrow_z;t\rangle_l \otimes |\uparrow_z;t\rangle_r, \qquad |\uparrow_z\downarrow_z;t\rangle = |\uparrow_z;t\rangle_l \otimes |\downarrow_z;t\rangle_r, \qquad etc. \tag{25}$$

Consequently, the running (fermionic) non-entangled state turns out to be a non-symmetric product state, which corresponds to distinguishability regarding state-dependent properties:

$$|non\rangle = |\downarrow_z;t\rangle_l \otimes |\uparrow_z;t\rangle_r \tag{26}$$

Now, the indices refer descriptively to the particles: "l" refers to the particle spatially located at L, and "r" refers to the particle spatially located at R. Thus, factorism can be defended.

Response: The first worry with this argument concerns the physics. It is apparently unjustified to exclude the states with equal spatial locations. Instead, one should expect that the proper subspace includes the following two states as its additional basis vectors:

$$|RR;t\rangle = \frac{1}{\sqrt{2}}(|R,\uparrow_z;t\rangle_1 \otimes |R,\downarrow_z;t\rangle_2 - |R,\downarrow_z;t\rangle_1 \otimes |R,\uparrow_z;t\rangle_2) \tag{27}$$

$$|LL;t\rangle = \frac{1}{\sqrt{2}}(|L,\uparrow_z;t\rangle_1 \otimes |L,\downarrow_z;t\rangle_2 - |L,\downarrow_z;t\rangle_1 \otimes |L,\uparrow_z;t\rangle_2) \tag{28}$$

Obviously, the argument does not go through with these states on board. However, for their exclusion, i.e., for the reduction of the six-dimensional proper subspace to a four-dimensional one, one needs some sort of superselection rule that characterizes the given four-dimensional subspace. Why are the other states, outside the constructed four-dimensional space, inaccessible?

The second part of the reply regards the semantics. Two significantly different things should be kept apart, namely, how the reference of a referring term has been *fixed* from how the referring term *works*. It might well be the case that the reference of "Hesperus" had been fixed mediated by the description 'lights up in the evening', but from this alone, it does not follow that "Hesperus" refers descriptively. Counterfactual reasoning is required in order to address the question of how the referring terms work. But [5] do not show how "l" and "r" behave in other situations.

It has been overlooked that the standard reading has two different implications. It is not only the case that "1" and "2" refer to particle 1 and particle 2, whatever these particles 'do' (i.e., "1" and "2" refer directly), but it is also the case that the reference of "1" and "2" had been fixed independently of any state-dependent property. According to the orthodox views, there is no possible *baptize* situation in which the QM particles 1 and 2 can be baptized via pointing or mediated by a description. So, the question arises of what exactly [5] have shown: Has it been shown that there really are labels available with the formalism, which refer descriptively? Or, rather, has it merely been shown that one can construct a situation in which the reference of the labels can be fixed with the help of descriptions? In my opinion, "snapshot Hilbert space" is nothing more than a baptize situation. By contrast, the newly introduced labels by the Leibniz strategy really are descriptive labels, as they perform their referring job with descriptions.

Why one needs the summing defense?

Regarding the entangled states, other heterodox views do not apply the summing defense strategy. Correspondingly, there are *two distinguishable* particles also in cases of entanglement. Look again at the given (fermionic) entangled state:

$$|\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2) \tag{29}$$

These heterodox views argue that in this state, there is a particle spatially located at R and a second particle spatially located at L. None of these particles possesses a definite property of spin projection (this is what entanglement implies), so their discerning properties are not maximally specific, i.e., cannot be represented by one-dimensional projectors. Nonetheless, the (not only weak) PII is satisfied by two numerically distinct particles, also in this case. [21]

Response: The reply has a physical, a metaphysical, and a semantic aspect. From the physical point of view, it is questionable whether there is any internal spatial structure before the measurement of (local) spatial locations. If one 'sees' a particle located at R (and another at L), one has (virtually) performed a position measurement. After such a measurement, also the Leibniz strategy would say that the whole has partially been divided, i.e., subsystems have partially been produced (without spin division). Yet, before such a measurement, there is apparently only one single object, without an internal structure and being $R - L$ large.

Metaphysically, the Leibniz strategy defends the PII as the principle of individuation. This is closely connected with Russell's ontology of objects as bundles of universals, i.e., with constituting objects by bundling properties. It can be performed with maximally specific subsystems, as one has it within non-entangled states via coupling state-dependent properties. It is questionable, again, whether incomplete bundles could support qualitative distinguishability as being individuating. Apparently, these other heterodox views are happy with the PII as a merely contingent principle that is helpful for pragmatic reasons in some situations. For this purpose, it may suffice to have distinguishability that is not maximally specific [22] [17].

From the semantic side, the question arises of how to refer to these particles. Analogous to the argument in the previous paragraph, the situation within entangled states seems to be (at best) a baptize situation, in which one can fix the referents via pointing. This does not imply that the labels would work descriptively via spatial locations. Counterfactual reasoning would be required in order to understand how the referring terms perform their job. Connected with this, the question should be addressed of whether, for example, the particle located at R is diachronically identical to the particle at R in the non-entangled states.

The Leibniz strategy already has an answer to such questions. Within entangled states, the only (Leibniz) individual is the undivided whole, maximally specified by global properties—in the present case, $2q_e, 2m_e, R - L, \hat{S}^2 = 0; \hat{S}_z = 0$—being referred to by a (global) descriptive proper name, e.g., "EPR_L".

Why are product wholes undividable?

Finally, with regard to the symmetric product states, other heterodox views do not apply the summing defense. Accordingly, the (strong) PII cannot be saved in such states, but it is sufficient that the principle holds contingently, in most of the states but not in all. Further, proponents of other heterodox views may argue that it is physically not justified that a system in a symmetric product state can change its state immediately only with an entangled state. Mathematically, a direct transition from product states to non-entangled, non-product states is possible but inconsistent with the Leibniz strategy.

Response: Firstly, note that the objection comes up within heterodoxy, and I will not argue that the orthodox views are not consistent with their treatment of symmetric product states. However, it *is* inconsistent to defend the (strong) PII in other states but to reject the purported summing defense for product wholes. Besides the already mentioned metaphysical and semantic worries with a contingent PII, this case makes it physically plainly inconsistent. If there really are utter indiscernibles in symmetric product states, then no interaction—i.e., neither by measurement (perturbation) nor by a unitary evolution—can have distinguishable effects on these particles. One simply has no differential access. Thus, it is not comprehensible how it could work to have indistinguishable particles at the beginning (In) but distinguishable ones afterwards (Out).

A product whole can only be changed in its entirety (as it would be with orthodoxy). The only physical way to produce distinguishability is dividing. Dividing, in turn, requires extension, but product wholes are not extended, or at least not in the convenient way. So, heterodoxy should buy that, out of symmetric product states, a plurality of distinguishable bosons can only emerge by initially extending the whole and then dividing it.

5. Conclusions

Based on the defense of the PII for non-entangled, non-product states and physically entangled states of similar particles, the Leibniz strategy has been performed as a completed view that includes symmetric product states for similar bosons. The summing defense is more subtle here: product states are crucially distinguished from entangled states by the fact that systems in entangled states can immediately be divided (e.g., by EPR-like measurements) to produce qualitatively distinguishable particles, whereas systems in symmetric product states cannot be immediately divided. Such states are point-like, and a point has to be (dimensionally) extended to become dividable. Due to Pauli exclusion, fermions can be unified only into entangled states, while bosons can also be unified in a quite different way, namely, by merely reducing the dimension (degrees of freedom), instead of the particle number. Thus, concerning bosons, the summing defense of the PII requires (also) this other sense of "unification": decreasing dimension (instead of number). Bosons are ontologically discerned from fermions in the way that they can be unified in a peculiar way in which fermions cannot be unified (Bose–Einstein condensation versus Fermi pressure).

Nonetheless, bosons and fermions are ontologically on a par with respect to the PII. They are PII individuals throughout all physically available types of states. The completed view is both ontologically as well as semantically unified.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The author declares no conflicts of interest.

Notes

1 I will always consider similar particles, i.e., particles of the same kind that share all state-independent properties such as mass and charge, and I will always consider situations in which one usually assumes two numerically distinct particles (generalizations to more than two particles are straightforward). Bosons have integer spin, fermions have half-integer spin, para-particles are out of consideration.

2 See [1], chapter 2 for the "road to orthodoxy", and chapter 6 for the "heterodox approach". The PII comes in different grades, but in this paper the "strong" sense is only opposed to the *Weak Discernibility* approach, which traces back to [2].

3 The terminology "GMW-(non-)entangled" is borrowed from [3]. It refers to the distinction of purely permutation invariant states and physically entangled ones, introduced by [4].

4 In this state, the properties of spatial location and spin projection are coupled.

5 The notion "factorism" has been introduced also by [3]. Some important distinctions concerning (anti-)factorism are provided by Muller and Leegwater [5] (2022, sec. 1).

6 For the modal status of the historical PII, see Rodriguez-Pereyra [6] (2014, chap. 2).

7 Other heterodox views do not apply the (strong) PII in this metaphysically committing way. For a discussion, see the final section.

8 Direct Reference traces back to [7]. In his own version, a baptize situation is required, in which the directly referring terms have to be introduced via pointing or even mediated by descriptions. This is not necessary, and it is not the case in standard QM. It is (only) imperative that "1" and "2" perform their referring job independently of any property. For further discussion, see Friebe [8] (2022, Section 3), and here, Section 4.

9 For a defense of the Received View, see [9] (there are variants with differences that do not matter for the present purposes). For a defense of Weak Discernibility, see [10], and regarding bosons in symmetric product states, see [11].

10 An entangled boson-state is this:

$$|\text{ent}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1 \otimes |1\rangle_2 + |0\rangle_1 \otimes |0\rangle_2)(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2) \tag{7}$$

It requires the same strategy.—Partially entangled states will for simplicity not be considered here.

11 For a discussion of this point, see Section 4.

12 For an analysis of the debate(s) on identity and (in)discerniblity, see [12]. She distinguishes three defense strategies for the proponents of the PII in light of some counterexample: identity, discerning, and summing defense.

13 This view traces back to [13]. In particular, see therein p. 274 for application to the case of similar particles.

14 It would obviously be circular to claim that the fermionic whole is characterized by the fact that certain combinations of properties are excluded.

15 Thus, the interaction concerns one individual only: no need for differential interaction where no difference is.

16 "Destroying" the condensate is not intended to mean to absorb or annihilate the particles but to obtain a non-product state.

17 For more on Bose–Einstein condensation, Fermi pressure, etc., see Cohen-Tannoudji and Guery-Odelin [14] (2011, chap. 21).

18 This holds also for states with no spatial distance, such as:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|R, \uparrow_z\rangle_1 \otimes |R, \downarrow_y\rangle_2 - |R, \downarrow_y\rangle_1 \otimes |R, \uparrow_z\rangle_2)$$

which is extended by 'spin-distance'.

19 See the discussion in Caulton [3] (2014, sec. 7) and [15].

20 I suggest that it is irrelevant whether the interaction takes place by a measurement or is a unitary evolution: in the latter case, the Hamiltonian determines which constituents will emerge.

21 See Bigaj [1] (2022, Section 6.2), which goes back to [3]. See also Friebe [8] (2022, Section 5) for the view that a Kantian solution of QM individuation, i.e., individuation via spatial differences, accepts the ontology of objects not being maximally specific.

22 However, this view suffers from an "intra-basis ambiguity", which is an even stronger kind of undertetermination than the "inter-basis ambiguity" regarding fermionic overdetermination (discussed above). For this argument, see Brautigam [16] (2024, Section 4.2).

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