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Hegel's Keplerian Revolution in Philosophy

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Abstract: In this paper, I approach Hegel's philosophy under the banner of a "Keplerian Revolution", the implicit reference being, of course, to Kant's supposed Copernican philosophical revolution. Kepler had been an early supporter of the Copernican paradigm in astronomy, but went well beyond his predecessor, and so is invoked here in an attempt to capture some of the important ways in which Hegel attempted to go beyond the philosophy of Kant. To make these issues more determinate, however, Hegel's Keplerian orientation will not be presented in its contrast to Kant's "Copernicanism" as such, but as contrasted with that of another early follower of Copernicus, Giordano Bruno, and this Brunian orientation will be used to characterize Kant's philosophy as seen from Hegel's rival Keplerian point of view. Interpreting Hegel as a philosophical Keplerian will require that we broach those worrisome aspects of Kepler's astronomy, namely his support for Plato's cosmology and the tradition of the "music of the spheres", but this will be shown to have connections to Hegel's own approach to logic. This in turn will help shed light on the meaning of Hegel's form of idealism and, in particular, on its usually unacknowledged Platonic dimensions.

Keywords: Hegel; Kepler; Plato; projective geometry; music of the spheres

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1. Introduction

Over the last half century, the views of the philosopher Georg Wilhelm Friedrich Hegel (1770–1831) have been subjected to much reinterpretation, especially concerning the status of his "absolute idealism" in relation to the "transcendental idealism" of his predecessor, Immanuel Kant. Sometimes Kant's philosophy has been summed up with a phrase that, although not used by Kant himself, had been widely seized upon to characterize his own philosophy: drawing on comments in which Kant alluded to parallels between his own method in philosophy and that of Nicholas Copernicus in astronomy, Kant is said to have initiated a "Copernican turn" or "Copernican revolution" in philosophy. For some interpreters of Hegel, keen to strike the right balance between what Hegel had taken from Kant and what he had criticized in him, it has been suggested that Hegel had attempted to *complete* Kant's incomplete Copernican revolution [1]. Unfortunately, this whole area has remained somewhat clouded and obscure: what Kant had actually meant by this phrase has never really found consensus and nor has agreement been reached among Hegel interpreters concerning the extent of that which he took from Kant and that of which he was critical.

In this paper I attempt to bring some clarity to these issues by returning to the vehicle of Kant's analogy, Copernicus's revolutionary astronomy. While it is generally agreed that in his reference to Copernicus Kant was trying to align his philosophical method in some way with that of the modern natural sciences, Copernicus can hardly be considered a clear representative of the modern scientific outlook. As Alexandre Koyré remarked, Copernicus himself was "not a Copernican" [2] (p. 65). In order to make the Copernican model somewhat more determinate, I extend its scope to include two early supporters and extenders of Copernicus's vision: Giordano Bruno and Johannes Kepler.

Rarely mentioning either Copernicus or Bruno in his written works [2], Hegel had written lengthy considerations of Kepler's astronomical work, starting with his ill-fated dissertation of 1801, *On the Orbits of the Planets* [5], in which he compared Newton negatively to Kepler. Here, Hegel had attracted derision for his apparent support of those aspects of Kepler's work that seemed out of step with modern science. These centered on Kepler's advocacy of Plato's approach to cosmology employing ratios and proportions from Pythagorean music theory to explain the relative sizes of the orbits of the planets, thereby aligning himself with the tradition of the "music of the spheres", which he defended in his *Harmonices Mundi* of 1619. While Hegel later dismissed aspects of what he had written on the orbits of the planets in the dissertation [3], his support for the scientific importance of Kepler over Newton would be continued in comments in his *Science of Logic* and especially in his *Lectures on the Philosophy of Nature* [3] (vol. 1, §270, rem. and add.).

Hegel would never renounce his early apparent enthusiasm for Kepler's Platonic "harmonic astronomy" and here I suggest this reluctance was founded in his logic. In retrospect, Kepler's harmonic model can be understood as an attempt to construe the space of the observable universe in terms of a form of non-Euclidean geometry suitable for attempts, like his own to survey it from a specific place within it—a form of geometry now known as projective geometry. In his transcendental idealism, Kant had transcendentally deduced the truth of Euclidean geometry, and the implicitly opposed attitudes to geometry of Kant and Hegel, I will suggest, would be reflected in differences between their respective logics. The "logical space" of judgments for Hegel, I will argue, incorporates "projective" as well as Euclidean features in contrast to Kant's homogeneous "Euclidean" logic.

Expanding the astronomical analogy used by Kant to include Bruno and Kepler thus helps overcome some of the indeterminacies of Kant's own self-characterization. Understanding Hegel's philosophy as "Keplerian" in contrast to the generally Brunian character of Kant's "Copernican" turn will allow us to obtain a clearer conception of the relation of Hegel's philosophy to that of Kant, but further than this, the link of Kepler's cosmology to that of Plato will allow us to appreciate those Platonic dimensions of Hegel's philosophy implicit in his move beyond Kant.

After comparing Copernican, Brunarian, and Keplerian cosmological revolutions in Section 2, in Section 3, I give an initial sketch of the contrast between Hegel's Keplerian and Kant's Brunian construal of logical space. In Section 4, Kepler's empirically based conception of the universe is contrasted to Bruno's, while in Section 5, Kepler's appeal to a harmonic cosmology is interpreted in terms of the projective space applied in his astronomy. Hegel's own path to this conception is shown to be via his interpretation of Plato's "most beautiful bond" responsible for the coherence of the cosmic animal in the *Timaeus*.

2. Cosmological Revolutions: Copernican, Brunian and Keplerian

In the Preface to the second, 1787, edition of the *Critique of Pure Reason*, Kant famously and controversially, compared the "change in the ways of thinking" that he was introducing into metaphysics to changes brought about in astronomy as initiated by Copernicus's reversal of the ancient cosmos [6] (p. Bxvi). Having regarded humans as residing on the surface of a stationary Earth at the center of the cosmos, Greek astronomers, such as Ptolemy had accepted the direct evidence of experience that the Sun, the planets and the rotating sphere of fixed stars all moved in their own ways about the Earth. In breaking with this picture, Copernicus thereby broke with the uncritical acceptance of immediate experience. To appreciate what our experience actually reveals, we have to imagine ourselves as located on a planet that rotates on its axis daily and annually circles the Sun, the real center of the cosmos. From then on, often highly counterintuitive theoretical accounts of the world would challenge those lived and unreflected-upon certainties that had dominated before the rise of science [4].

Already by 1804, Hegel's then colleague Friedrich Wilhelm Schelling had initiated what would turn out to be a long series of varying glosses on Kant's words. Like Copernicus, Kant had "reversed (*umkehrte*) the representation according to which the subject receives the object (in perception) inactively and calmly. . . a reversal which was carried over into all branches of knowledge" [8] (p. 599) [5]. Since then, many generally sympathetic to Kant have agreed with the parallel between Kant's modern challenge to traditional metaphysics and Copernicus's challenge to the ancient anthropocentric cosmos (e.g., [9] (p. 16)). Critics of Kant, however, would give to the analogy a reversed meaning, with Bertrand Russell, for example, bluntly asserting that Kant should have "spoken of a 'Ptolemaic counter-revolution', since he put man back at the center from which Copernicus had dethroned him" [10] (p. 9). Defenders of Kant could here reply that it is only the world *as experienced and known* that, in conforming to the structures of human knowledge, is anthropocentric in this way. It is the reality beyond appearance, the "world in itself", that is the philosophical equivalent to the modern decentered universe. This type of defense, however, in which "the things we know [*wissen*] are only appearances for us, and what they are in themselves remains for us an inaccessible world beyond this one [*Jenseits*]" [11] (§ 45 add.), would not be welcomed by Hegel. On the one hand, "it must be acknowledged as a very important result of the Kantian philosophy that it established the finitude of the merely experience-based knowledge of the understanding and designated its content as appearance". However, "insofar as reason is regarded in this way merely as stepping out beyond the finite and conditioned character of the understanding, by this means it is in fact itself downgraded to something finite and conditioned, for the true infinite is not merely on the far side of the finite, but instead contains the finite as sublated within it" [11] (§ 45 add.). In short, Hegel reacted against the price paid for Kant's version of the Copernican revolution in which, in Klaus Brinkmann's words, "theoretical philosophy seems henceforth confined to investigating the possibility and scope of empirical knowledge and thus becomes primarily a transcendental philosophy of science rather than a metaphysics" [1] (p. 49). The shape of Hegel's "infinite" in his "absolute" alternative to Kant's "transcendental" idealism, I suggest, might be revealed by looking closely at more specific variants of the modern "Copernican Revolution".

Compared to later transformations of the anthropocentric Greek cosmos, Copernicus's reversal had been comparatively conservative. The Greek cosmos was anthropocentric by virtue of its *geocentrism*, that is, by putting the Earth—our own planet—at the center of the cosmos, but Copernicus's picture remained anthropocentric by replacing the Earth with the Sun—that is, our Sun—at the center. But by the time of Copernicus, models of the universe were available that suggested more radical displacements of ourselves from its center, courtesy of the revival in the medieval period and Renaissance of Neoplatonist alternatives to the classical Ptolemaic picture of the world. Particularly striking in this regard was the image of the universe given by the fifteenth-century Catholic cardinal Nicholas of Cusa (1401–1464).

In the work *Of Learned Ignorance* of 1440, Cusa had offered a picture of the universe or "world machine" as infinite and without center or circumference [12] (bk. 2, ch. 12), although not on astronomical grounds (c.f., [13] (ch. 1), [14] (chs. 2, 3), [15] (pt. 3, ch. 2)). Rather, the picture of an infinite universe was offered as a metaphor to allow a better understanding of the nature of God himself. Moreover, for Cusa the universe is not infinite in the same sense that God is infinite: both God and the universe are "*maxima*", but God is an *absolute maxima* while the universe is a "contracted" one which confers on it a type of finitude [16]—in contrast to *God's* infinity, the world's is, in Karsten Harries phrase, a "finite infinite" [14] (p. 253). In works written by Giordano Bruno (1548–1600) in the last decades of the sixteenth century, however (e.g., [17]), Cusa's infinitist imagery, now blended with the very different influence of the ancient naturalist Lucretius, would reappear in a more robust cosmological form [13] (pp. 39–57), [14] (ch. 13). Bruno would reject the limitations Cusa had applied to the infinite cosmos, and this would result in a more unequivocally pantheistic picture [6], a picture in which the created universe was seen as "the complete and perfect expression of God" [15] (p. 140).

In Bruno's picture, the fixed stars are suns hypothesized to be like our own, with their own exoplanets perhaps supporting life, just as ours does. I suggest that, abstracted from particular theological connotations given to this picture by Bruno, it would be more Bruno's universe rather than that of Copernicus that would be adopted by the likes of Descartes and

Newton [13] (chs. 4, 5) [7]. Consistent with this, I suggest, Bruno's acentric infinite universe would have provided a more fitting model for Kant in his 1787 Preface than Copernicus's not-fully-modern reversal of ancient geo-centrism. Moreover, Bruno's decentered universe better fits with Kant's radically decentered account of the unity of human judgment and experience in the transcendental unity of apperception—a unity that Hegel would contest with his own distinct alternative. These opposed unities we might conceive as different accounts of the logical space of human judgment.

3. "Logical Space" in Kant's and Hegel's Accounts of Judgment

In his 1787 Preface to the second edition of the *Critique of Pure Reason*, Kant describes how the discovery of the axiomatic method of geometry in ancient Greece had established mathematics on a firm scientific basis [8], but it had taken longer for natural science as grounded on empirical principles to put itself on the scientific pathway [6] (pp. Bxi–xii). Nevertheless: "When Galileo rolled balls of a weight chosen by himself down an inclined plane, or when Torricelli made the air bear a weight that he had previously thought to be equal to that of a known column of water, or when in a later time Stahl changed metals into calx and then changed the latter back into metal by first removing something and then putting it back again, a light dawned on all those who study nature. They comprehended that reason has insight only into what it itself produces according to its own design; that it must take the lead with principles for its judgments according to constant laws and compel nature to answer its questions, rather than letting nature guide its movements by keeping reason, as it were, in leading-strings; for otherwise accidental observations, made according to no previously designed plan, can never connect up into a necessary law, which is yet what reason seeks and requires" [6] (pp. Bxii–xiii).

In relation to such experimental practices, Kant invokes a type of judgment suitable for the modern scientific context and which is not dictated to by nature's immediate appearances. Now reason "compels nature to answer its questions" because the answers with which nature responds need to have the type of universal form that allow them to "connect up into a necessary law". In his *Prolegomena to Any Future Metaphysics* [19], written between the two editions of the *Critique of Pure Reason*, Kant had called such judgments *Erfahrungsurteile*, judgments of experience. These judgments constituted a subset of empirical judgments considered more generally in virtue of their having "objective validity" on the basis of being informed by concepts that "have their origin completely *a priori* in the pure understanding, and under which every perception must be first of all subsumed and then, by means of the same concepts, transformed into experience" [19] (§ 18). In that work Kant contrasts such judgments of experience with what he calls "mere judgments of perception" that "do not require a pure concept of the understanding, but only the logical connection of perception in a thinking subject" [19] (§ 18). Judgments of experience are, while judgements of perception are not, universally valid. "All of our judgments are at first merely judgments of perception; they hold only for us, e.g., for our subject, and only afterwards do we give them a new relation, namely to an object, and intend that the judgment should be valid at all times for us and for everyone else" [19] (§ 18). Examples of the former include "the air is elastic", and the latter, "the room is warm, the sugar is sweet, the wormwood repugnant" ([19] (§ 19). This distinction is similar to one implicit in one of Kant's pre-critical writings, *Attempt to Introduce the Concept of Negative Magnitudes into Philosophy* of 1763 [20].

There Kant addresses the problem of how to understand the puzzling phenomenon of negative numbers, but he extends the inquiry beyond mathematics to "consider this concept in relation to philosophy itself" [20] (p. 208) including how to think of negative *qualities*. In this context he addresses the repugnancy of wormwood that would later be referred to in relation to judgments of perception in the *Prolegomena*. Addressing the question, "Is displeasure simply the lack of pleasure?", Kant treats displeasure as "something positive in itself and not merely the contradictory of pleasure" [20] (p. 219). This type of negation Kant calls "real" and gives as an example, the negative sensation produced by the ingestion of wormwood. “What we have here is not a mere lack of pleasure, but something which is a true ground of the feeling which we call displeasure” [20] (p. 219). There are two types of conceptual negation—logical and real—and these can be distinguished by the role played by contradiction. “This opposition is two-fold: it is either *logical* through contradiction, or it is *real*, that is to say, without contradiction” [20] (p. 211).

The *Prolegomena*’s judgments of perception were clearly meant to capture the type of immediate, pre-reflective “lived” experiences—those “accidental observations, made according to no previously designed plan” [6] (pp. Bxii–xiii), such as concern the apparent daily movement of the Sun through the heavens. Science requires a different type of judgment, however, a type that allows those judgments to “connect up into a necessary law” and for this they need to be judgments that “should be valid at all times for us and for everyone else” [19] (§ 18), a prerequisite for obeying the law of non-contradiction. By the second edition of the *Critique of Pure Reason*, “judgments of perception” had effectively disappeared. From a *logical* point of view, judgments of experience seem to have become the only judgments worthy of the name, with the earlier purported “judgments” of perception now seemingly deprived of normative status and relegated to mere psychological significance as in the associationist doctrine of the empiricists. This is linked to the development in the second edition of Kant’s celebrated notion of the “transcendental unity of apperception”.

In the new “Second Section: Transcendental deduction of the pure concepts of the understanding” of the extensively rewritten “Of the Deduction of the Pure Concepts of the Understanding” in the second edition of *Critique of Pure Reason*, Kant would radically challenge any empiricist idea of cognition that talked of some combination of sensory representations: “the **combination** of a manifold in general can never come to us through the senses . . . for it is an act of the spontaneity of the power of representation” [6] (p. B130). In relation to such spontaneity, Kant introduces the idea of a “synthetic unity” of apperception or consciousness as an “objective condition for all cognition, not merely something I myself need in order to cognize an object but rather something under which every intuition must stand **in order to become an object for me**” [6] (p. B138). This in turn radically limits the conception of judgment: “If . . . I investigate more closely the relation of given cognitions in every judgment . . . then I find that a judgment is nothing other than the way to bring given cognitions to the **objective** unity of apperception” [6] (p. B141). The capacity to logically cohere within a totality of judgments thus defines what it is to be a judgment and there is, then, no place for “judgments” without fixed truth values and with predicates that stand to others in non-contradictory forms of opposition. That is, all *possible* judgments are defined by their capacity to belong to this new logical space, the transcendental unity of apperception. They will become actual judgments when sensory input is added in the form of empirical intuition, but this will simply “fill out” a pre-existing logical structure. The decentered nature of this complex unity, decentered because it is linked to a universalized “transcendental I” which is the “I” of no cognizing subject in particular, effectively mirrors Bruno’s radically decentered universe [9]. The logical space of Kantian judgments, we might say, is Brunian [10]. Hegel, however, would reinstate something like the “judgments of perception” of Kant’s *Prolegomena* with a resulting logical space that is more Keplerian.

Hegel’s treatment of judgment in Book III of *The Science of Logic* sits between chapters on “the concept” and “the syllogism”, and while the concept is shown to contain “the three moments of *universality*, *particularity*, and *singularity*” [21] (p. 529), the structure of the syllogism will be configured by various ordered triads of these three determinations. Universality, particularity, and singularity can then be expected to characterize the logical properties of subject and predicate terms of the judgments of the intervening chapter. Hegel’s use of these terms, while sometimes idiosyncratic, is nevertheless anchored in the logical tradition.

Hegel’s “universal” is initially presented in a generally Aristotelian manner as “the *soul* of the concrete” or the “essence” or “positive nature” of what it inhabits [21] (p. 531) while particulars form pluralities of diverse entities that give further determination to the universals that constitute their substances. Aristotle, however, had been somewhat vague as to the logical difference between particulars and singulars [11], while in modern logics these terms are often used interchangeably. The distinction between particularity and singularity will be crucial for Hegel's conception of judgment, however.

A particular species adds determination to its universal, a genus, just as in "human" the genus "animal" is made more determinate by the concept "rational", but the concept "human" can be further divided into, say, Greeks and non-Greeks. In contrast, "singularity is the determinate determinateness, differentiation as such, and through this reflection of the difference into itself, the difference becomes fixed" [21] (p. 548). Singularity is described both as "the absolute turning back of the concept into itself" and "the posited loss of [the concept] itself" [21] (p. 540)—a moment of conceptuality in which "the concept becomes *external to itself* and steps into actuality" [21] (p. 548). While in this latter sense, the singular stands in opposition to the concept as does Kant's "intuition" or Frege's "object", for Hegel, singularity nevertheless in another sense still remains within the sphere of the conceptual. It is *the concept itself* that has become external to itself. In its externality, the singular is associated with demonstrative reference. It is "a one which is qualitative, or a *this*" [21] (p. 548). But while Frege's concept–object distinction is ultimate and ontological, for Hegel "the *this is*; it is immediate [. . .] only in so far as it is pointed at", and in this sense closer to Kant's idea of the subject's "transcendental" constitution of its objects. Such "pointing at" he treats as occurring in the context of judgment: "The concept's turning back into itself is thus the absolute, originative partition of itself, that is, as singularity it is posited as judgment" [21] (p. 549). Hegel's account of judgment will, therefore, include judgments made about things considered in their singularity and not just things considered as instances of their kinds.

Hegel will, therefore, start his dynamic taxonomy of judgments with a form of sentence [*Satz*] having the structure "*the singular is universal*" [21] (p. 558), expressing a "positive judgment" [21] (p. 557–559). Unhelpfully, Hegel typically gives few examples, but here mentions "Gaius is learned", "the rose is red", and "the rose is fragrant" [21] (p. 558–559). And while he does not use the demonstrative with the example of the rose, it is clear from the context and the fact that it instantiates a *singular* that he has in mind the type of determinacy that could be expressed by the demonstrative "this rose" of a direct perceptual judgment and not any relatively indeterminate rose, "a rose" or "some rose" [12]. This simple form of judgment will be transformed into a richer form of judgment by a sequence of negations, and negation here introduces the relative indeterminacy of particularity. "The rose is *not* red" introduces the form "*The singular is a particular*", and in such a judgment "only the determinateness of the predicate is thereby denied and thus separated from the universality which equally attaches to it [. . .] if the rose is not red, it is nonetheless assumed that it has a colour, though another colour" [21] (p. 565). But while "the rose is not red" attributes *some* colour to the rose, one could not further determine that colour by an example. While one could say, pointing to a specific red square on a colour chart, "the rose is *this shade* of red", one could not do the same in order to exemplify some contrasting rose's "*non*-redness".

That the non-red rose is some other colour—pink, yellow, purple, or whatever—indicates that we are here in the sphere of judgments with "real negation" as found in Kant's early dual forms of judgment, the type of judgment abandoned with the second edition of the *Critique of Pure Reason*. In the evolution of Hegel's concept of judgment this form of negation is itself negated to produce judgments more like Kantian ones. Properly logical negation can be first seen in the peculiar "infinite judgment" which is the "negation of the negation" of the original positive judgment [21] (p. 567). These judgments, such as "the rose is not an elephant" or "the understanding is not a table", look more like corrections of category mistakes rather than judgments that apply to the world, but it is clear that the negation here is logical rather than real. Thus, the rose is not an elephant, *nor any other animal* and the understanding is not a table, *nor any other piece of furniture* [13]. The infinite judgment marks the transition of the judgment of existence of "inherence" into the judgment of reflection or "subsumption", and while Hegel's description of negation in this section is not clear, the strong suggestion is of judgments with a properly propositional content: "it is only in the judgment of reflection that we first have a *determinate content* strictly speaking, that is, a content as such" [21] (p. 568). In modern logic such a content would be described as a proposition with a fixed truth value. Nevertheless, singular sentences with real negation will recur within the cyclical process in which judgments are further determined. Crucially such a form of judgment will appear as the type of judgment that transitions into the syllogism, the evaluative "judgment of the concept", for which he gives the examples "this house is *bad*" and "this action is *good*" [21] (p. 583). Here, once more, we encounter an example about a specific house or act, a type of judgment made on the basis of features which are apparent to a perceiving judge, with the predication of terms, "good" or "bad", surely intended as standing in the relation of "real" negation.

Elsewhere, I have argued that Hegel's recurring dual judgment forms gives to his logic features analogous to those found in nineteenth-century *algebraic logic*, like that of George Boole and his followers, such as C. S. Peirce and W. E. Johnson [23] (chs. 8–10), and especially in *intuitionist* interpretations of such logics [24]. Similarly dualistic approaches to judgment form are also found in types of *modal logic* as revived in the second half of the twentieth century [25], containing a duality of "modal" as opposed to "classical" (i.e., essentially Fregean) sentence types. "Although both modal and classical languages talk about relational structures, they do so very differently. Whereas modal languages take an internal perspective, classical languages, with their quantifiers and variable binding, are the prime example of how to take an external perspective on relational structures" [26] (p. xiii). A judgment about a specific perceived concrete rose or house exemplifies an internal perspective on a "relational structure": a relation between the rose and its redness, or a house and what it is about it that makes it good or bad. In contrast, a judgment with propositional content, like Kant's "judgments of experience", captures facts, such as "that some rose or other is red", a fact understood from "nowhere in particular" or perhaps, to use a common metaphor, from an external "God's-eye view".

While standardly modern approaches to modal logic have treated "modal" sentences as derivable from "classical" or Fregean ones, an early advocate of "tense logic" (a form of modal logic), Arthur Prior, had insisted on the non-reducibility of such perspectival to non-perspectival judgments [27] [14], and in this respect, Prior's position is similar to Hegel's [15]. I take a somewhat offhand claim by Prior as providing help with respect to how to approach the logical space of Hegel's judgments in this regard. Speaking of tense logic as a form of modal logic, "we must develop" Prior says, "alternative tense-logics, rather like alternative geometries" [27] (p. 59). This appeal to alternative, presumably *non-Euclidean*, geometries, is suggestive in relation to Hegel's dual account of judgments, as his account had been formed in an historical context in which judgment forms were being linked to geometrical considerations concerning the perspectival representation of objects within perceptual experience. Crucial in this regard was the work of Gottfried Wilhelm Leibniz (1646–1716), whose logic was familiar to Hegel through the intermediary of the logic teacher at the Tübingen seminary during Hegel's time there, Gottfried Ploucquet [29].

Leibniz had clearly attempted to shape both his epistemology and logic in ways relevant to the broadly "perspectival" conception of perceptual judgment expressed in his *Discourse on Metaphysics*, and his concerns here had extended into his conception of space and its geometrical properties. While a finite monadic subject neither exists "in" space nor has extension, it nevertheless represents the universe, he writes, *as if* from a point of view, "rather as the same town is differently represented according to the different situations of the person who looks at it" [30] (§ 9). From this starting point, he would conceive of the perspectival features of judgments as being eliminated in stepwise fashion by the judging subject's iterated application of the principle of sufficient reason, thereby ascending a type of "Jacob's ladder" leading to an aperspectival God's-eye view [16].

Leibniz had alluded to similar types of issues in relation to an analysis of spatial relations: "I believe that, so far as geometry is concerned, we need still another analysis which is distinctly geometrical or linear and which will express *situation* [*situs*] directly as algebra expresses *magnitude* directly" [31] (pp. 248–249). This form of analysis he called "*analysis situs*" or "analysis of situation" [32], and it would mesh with the relativistic conception of space that he would oppose to Newton's absolute space [32] (p. 256) [17]. With this idea of an alternative form of geometric analysis he was tapping into a tradition that had started in the Renaissance exploring the laws of perspectival representation in relation to the development of painting [33] and this had brought his interests into relation to the "projective geometry" of the French mathematician and engineer, Girard Desargues (1591–1661) and, especially, his follower Blaise Pascal (1623–1662). I have explored aspects of Hegel's relation to this projective geometric tradition elsewhere [23,34], but here I want to focus on the role of Kepler in relation to these developments as Kepler himself had anticipated some of the main features of Desargues's geometry in his 1604 study of optics, *Ad Vitellionem paralipomena* [35], a study that was relevant to the methodology employed in his astronomical work.

4. Kepler: Measuring Distances within the Projective Space of a "Finitely Infinite" Universe

As a devout Christian, Johannes Kepler (1571–1630) had believed that God had created the universe according to geometrical archetypes [18], but while in his early work he apparently "believed God could create matter to perfectly instantiate" those archetypes, in later works, more like Cusa, he "spoke as though the material imposes limitations of its own. Matter could not conform to a pure geometrical archetypal model if it was to 'take on the organs necessary to life'" [37] (pp. 140–141). Consistent with this, I suggest, for Kepler neither would the *space* within which matter existed in the cosmos "perfectly instantiate" those geometric archetypes in the mind of God. The geometric properties of the space around us would not be Euclidean but rather projective.

In comparison to Bruno's cosmological picture, and, by implication, those of Descartes and Newton, Kepler's has generally been regarded as more conservative, a conservativism often put down to religiously based Aristotelian views (e.g., [13] (p. 58)). For example, in the work of 1606, *De Stella Nova*, Kepler cites and agrees with Aristotle's arguments against the view of that "sect of philosophers" for whom the "depths of nature . . . extend to an infinite altitude" (quoted in [13] (p. 59)). Kepler retains similar arguments against this sect's modern equivalents, and in particular, "the unfortunate" Giordano Bruno [19], who had "made the world so infinite that (he posits) as many worlds as there are fixed stars" [13] (p. 60). With such a view, suggests Kepler, Bruno "misuses the authority of Copernicus as well as that of astronomy in general, which proved—particularly the Copernican one—that the fixed stars are at an incredible altitude" [13] (p. 61) but not an infinite one. However, I will suggest that Koyré here misses the point of Kepler's resistance to the Brunian infinite universe when he treats Kepler as merely repeating Aristotle's criticisms of ancient accounts of the infinite [20], and that "in his conception of being, of motion, though not of science, Kepler, in the last analysis, remains an Aristotelian" [13] (p. 87). In contrast, I will argue that rather than being grounded in Aristotelianism, Kepler's resistance to Bruno had reflected a greater fidelity to Cusan Neoplatonism. Instead of regressing to the "closed cosmos" side of Koyré's closed cosmos/infinite universe opposition, Kepler should be seen as offering a way beyond this dichotomy. For him, the "closed" or manifest universe itself is characterized by a type of "contracted" infinity. Hegel, himself opposed to the conventional finite–infinite distinction, had been, I suggest, one of the few to grasp this [21].

Koyré, it must be admitted, does not attribute Kepler's resistance to Bruno's infinite universe entirely to an adherence to traditional Aristotelianism and mentions those "purely scientific reasons" and even his anticipation of "some present-day epistemologies" which declare the infinite view to be "scientifically meaningless" [13] (p. 58). Kepler had claimed that "by admitting the infinity of the fixed stars" thinkers, like Bruno, had "become involved in inextricable labyrinths" (*De Stella Nova*, quoted in [13] (60)). Koyré quotes Kepler that "this very contagion carries with it I don't know what secret, hidden horror; indeed one finds oneself wandering in this immensity, to which are denied limits and center and therefore also all determinate places" [13] (61). Kepler's "hidden horror" here suggests that without some determinate place to stand *within* the universe—a place from which one could perform one's measurings—one could never discover order. But as we will see, while this rules out what we might call Bruno's acentric "indeterminate" infinite, not all conceptions of worldly infinity are thereby excluded.

Koyré spells out Kepler's reservations as following from two premises, first, the principle of sufficient reason, and next, the empirical character of astronomy which "has to deal with observable data, that is, with the appearances (*phainomena*)", appearances to which "it has to adapt its hypotheses [. . .] and that it has no right to transcend them by positing the existence of things that are either incompatible with them, or, even worse, of things that do not and cannot 'appear'" [13] (p. 62). "Appearances" may hide reality, but without them the possibility of the knowledge of this reality must be abandoned.

With this last point, Koyré alludes to limitations of empirical experience that go beyond the familiar "problem of induction". Empirical examination of the universe is always carried out under specific circumstances which rule out the very possibility of experiencing certain parts or aspects of it [22]. For Kepler, such impossibility commences at the boundary of the outer celestial sphere, a boundary beyond which we cannot extend our observations and measurements. Such a Keplerian reason for rejecting the Brunian universe as a proper object of any empirically based astronomy has been underlined recently by Christopher Graney: "[I]n the early seventeenth century, science reveals the Copernican universe to consist of exactly that which Kepler describes in the sixteenth chapter of *De Stella Nova*: a vast shell of huge but dull stars, surrounding a tiny but brilliant sun and its lively planets . . .. A universe of sun-like stars, on the other hand, is the creation of those who do not do their science carefully enough. . .. It is not the universe observed by careful astronomers" [39] (p. 166) [23]. As Koyré reminds us, Kepler's criticisms of Bruno in *De Stella Nova* reflect a time "before the enlargement of the observable data by the discovery and the use of the telescope" that would open up further "aspects of the world that we *see*" [13] (p. 62), but this does not affect the general point concerning the boundedness of observation and measurement by some horizon. Each development of instruments that augment perception will reveal aspects of the universe that earlier observation could not detect, but the inverse of this is that only some *subsequent* development will reveal the epistemic limits that apply at any particular time. From an empirical point of view, we can never be sure that our instruments are fully adequate to revealing the totality of what exists "out there".

When based upon the empirical nature of astronomy, the claim that the universe be considered to have a finite limit differs from the thesis of the "closed world" understood in an *Aristotelian* sense. Aristotle's criticisms of the infinite universe had been conceptually based in his rejection of the very *idea* of a void outside the celestial sphere [24]. In contrast, we can never convince ourselves on the basis of observation that the universe has a fixed horizon in Aristotle's sense. Whether there was more of it beyond the sphere of the fixed stars must for Kepler have been a question for *further* empirical discovery [25]. In this spirit, unlike many of his Aristotelian contemporaries, he welcomed Galileo's telescope, quickly adopting his own version and, against Aristotelian sceptics, theorized about how this instrument was able to extend the scope of observation [26].

The universe we can learn about is the limited observable universe and, for Kepler the universe so conceived is heliocentric with an outer edge at the sphere of the fixed stars, but this does not preclude later conceptions of a more encompassing universe. Something like the difference between the subject-centered observable world and an indeterminate world of possibilities beyond it would be reflected in Hegel's own logical account of different types of egocentric and centerless versions of logical space. Judgments of existence would apply to the range of things we can observe, while the more Kantian judgments of reflection would extend to indeterminate possibilities about which we might at some later stage learn the truth. For the moment, however, let us pursue the idea of a type of egocentric geometry appropriate for the observable universe. Kepler had grasped, I have suggested, that the appropriate geometry for the observable universe was not geometry as traditionally understood, that is, not Euclidean geometry. Moreover, the form of geometry he would explore would for him be linked to the thesis of *musica mundana*. This geometry was what would come to be known as "projective geometry", and it would introduce a sense of "finite" or contracted and determinate infinity that could be opposed to the indeterminate infinity of Bruno.

5. Projective Geometry, the Musical Ratios of Kepler's Universe, and Hegel's Syllogism

Projective geometry is a geometry of relations among points and straight lines on a plane [27]. It had briefly appeared in the seventeenth century in the work of Girard Desargues [44] and his follower Blaise Pascal, but after a period of neglect it would have its golden age in the nineteenth century. Desargues approached the "conic sections" studied in antiquity—circle, ellipse, parabola, and hyperbola—not as different geometrical figures as would be understood within Euclidean geometry, but as internally related to each other via the process of "central projection" [43] (p. 3). Here we might think of the way that, say, a circular coin might be perceived by a viewer from a particular angle as elliptical. Grasping these shapes as internally linked in this way, or as "projectively equivalent", amounted to treating these shapes as essentially versions of the *same object.* Understood in this way, Desargues's geometry was meant to apply to the practical problems involved in perspectival representation and was linked with the types of theories and techniques that had been elaborated by painters, such as Leon Batista Alberti and Leonardo da Vinci, in the fifteenth century [33]. Thirty years before Desargues, however, Kepler in his optics had studied the relations among the conic sections in much the same way, treating the ellipse, the parabola, and the hyperbola as all variations on the circle [45,46] (pp. lvi–lix).

In antiquity, Apollonius of Perga had conceived of the conic sections as generated by sectioning a three-dimensional cone by a plane at different angles [28]. Kepler, however, sought principles linking these shapes when all were considered as existing on a single plane, finding this in the fundamental "principle of analogy". According to this principle, shapes could be considered as transformable into one another by a continuous series of gradual transformations as when, for example, one thinks of an ellipse as resulting from the stretching of a circle [46] (pp. lvii–viii) [29]. Considered as internally related in this way, the shapes were, thus, individuated differently to the way as conceived in Euclidean geometry.

In relation to such transformations, Kepler had posited an idea that would also independently appear in Desargues's geometry as well, that of "points at infinity" [45]. The idea of analogous or projectively equivalent shapes linked by continuous transformations had to address problems where different conic sections might be seen as having different parts. For example, while a circle has a center, an ellipse has two foci, and a parabola a single focus. But in thinking of a circle as a "squashed" ellipse or an ellipse a "stretched" circle, the two foci of an ellipse might come to be thought of as overlapping in the circle. Accepting such types of equivalences would lead to one of the central ideas of planar projective geometry, that of imaginary points at infinity, as a parabola with its one focus will presumably be thought of as a figure resulting from the *infinite* stretching of an ellipse such that one of its foci comes to exist at an infinite distance from the other. Preserving some sense of the determinate relation existing between two foci of an ellipse, this new notion of infinity will no longer be entirely indefinite.

Developed within Desargues's planar projective geometry, these points at infinity would formally distinguish the "projective plane" from the Euclidean plane, and by analogy, projective three-dimensional space from its Euclidean equivalent, and it can be easily appreciated how a geometry capable of "points at infinity" might be applied to the types of issues with which the theory of perspectival painting had been wrestling. This Renaissance tradition had its own conception of infinitely distant "vanishing points" at which perspectivally represented lines that are "objectively" parallel—the representation of tiling patterns on floors, for example—when extended appear to converge at some point at or near the horizon. In fact, Desargues, although not Kepler, had used the notion of a *line* at infinity made up of all the points at infinity on which differently oriented sets of parallel lines converged [43] (p. 109). Such a line at infinity could thus be grasped as something like a horizon encircling a landscape.

A major non-Euclidean consequence could be drawn from the recognition of such points at infinity in a planar geometry—the idea that they were points at which parallel lines in a plane could be considered to meet [46] (p. lxii) [30], contradicting Euclid's "fifth postulate", which has parallel lines never meeting [48] (post. 5). In the nineteenth century, the idea of internally coherent geometries without Euclid's questionable fifth postulate would appear in the form of specifically *non*-Euclidean geometries.

The type of stretching and squashing transformations undergone by projectively equivalent figures in projective geometry mean that it is a *non-metrical* geometry in the sense that line-lengths and the angles between them are not considered fixed or "invariant" as they are in Euclidean geometry and Descartes's "analytic" coordinate geometry based upon it. But a geometry must be based on some values that remain constant, and what replaces the invariants of Euclidean geometry in its projective counterpart are certain double-ratios holding among line-lengths and angle sizes. The main invariant in this regard is the so-called "cross-ratio", a particular double-ratio holding among line-segment-lengths defined by distances between two pairs of points on a line, as in Figure 1 [31].

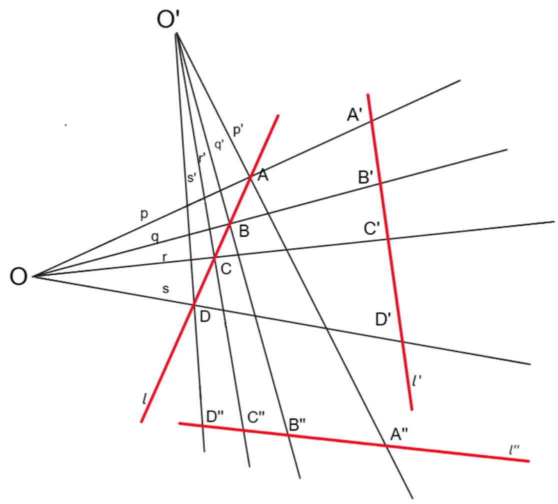

Figure 1. The invariance of the cross-ratio across different lines sectioning pencils of rays.

Here, a "pencil" of four rays, namely p, q, r, and s, radiating from point O, is sectioned by the line l to form a "range" of four points, namely A, B, C, and D, on that line. This pencil is said to "project" this range, from point O, onto a range of corresponding points, i.e., A', B', C', and D', on a further sectioning line l', in such a way that the "cross-ratio" holding between the two ratios $AB{:}BC$ and $AD{:}DC$ will have the same value as that holding among the equivalent double ratio among points A', B', C', and D' on line l'. That is, the ratio between ratios $AB{:}BC$ and $AD{:}DC$ is equal to that between $A'B'{:}B'C'$ and $A'D'{:}D'C'$, or, expressed in fractions, $\frac{\frac{AB}{BC}}{\frac{AD}{DC}} = \frac{\frac{A'B'}{B'C'}}{\frac{A'D'}{D'C'}}$. Furthermore, if a different pencil of rays, i.e., p', q', r', and s', radiating from a different point O' projects the original range onto a *further* sectioning line, l'', dividing it at A'', B'', C'', and D'', then that cross-ratio has the same value as well.

When expressed in the above form, the cross-ratio can appear to depend upon the lengths of line-segments, in this sense making projective geometry dependent upon Euclidean geometry with its metrical features [32]. But this hides the deeper sense in which the compound ratios of projective geometry can be given values independently of any given metric as supplied by Cartesian coordinates. From this point of view, the cross-ratio relation can be seen as a generalization of one particular cross-ratio called the "harmonic cross-ratio", which can be constructed within an entirely non-metrical geometry from certain constructed figures defined in terms of lines or points [33]. In the harmonic cross-ratio, the cross-ratio has a value of 1, that is, the ratio *AB*:*BC* in the figure above will be equal to that of *AD:DC* [34]. Under conditions in which this harmonic cross-ratio is combined with the equality of *AB* and *BC*, point *D* will come to stand at an infinite distance from the other three points: it will be a "point at infinity".

At the end of the nineteenth century, Felix Klein would raise the question of "the sense in which it seems psychologically justified to construe projective geometry before metrical geometry and to regard it as the very basis of the latter" and would suggest an answer: "We can distinguish between mechanical and optical properties of space. The former find their mathematical expression in the free mobility of solid bodies, the latter in the grouping of the straight lines that run through space (the rays of light, or the lines of sight emitted by the rays). The question here is not how our idea of space comes about (or has come about over the course of generations): no one will doubt that mechanical and optical experiences work together or have worked together. The question is whether the properties of one or the other type should be given priority in the methodical construction of spatial science. For Mr. von Helmholtz, as we have examined in detail, the mechanical properties are preferred: but one can also begin with the optical properties. The developments can be viewed side by side and each of them has its own particular advantages" [49] (p. 570).

Desargues's geometrical investigations had related to Renaissance theories of perspectival representation in painting, and we might say that from the point of view of a painter attempting to portray spatial relations as they are seen from a particular viewpoint *within* that space, projective geometry might be treated as primitive. In the nineteenth century, it would be revived in France by military engineers [35], and much the same could be said for the military engineer, calculating distances among objects in a landscape from some particular location within it. In contrast, for the development of a science of mechanics seeking general laws, one will presumably take "centerless" Euclidean space as primitive. Moreover, we might conceive of the opposition between the conceptions of space as differentiated according to its optical or mechanical properties as similar to that found in Hegel's conception of the different logical spaces occupied by judgments of existence and subsumption, respectively.

I have suggested that Kepler anticipated some of the features of Desargues's projective geometry, but he does not seem to have anticipated the harmonic cross-ratio. In contrast, Kepler would explore the relations among moving objects in space in terms of a geometry that, he believed, manifested the ratios that were responsible for the harmonies of music. Concerning this "music" of the spheres, he would write: "I grant that no sounds are given forth but I affirm and demonstrate that the movements are modulated according to harmonic proportions" [41] (p. 6). Might there not be some relation between the "harmony" existing within the harmonic cross-ratio and Kepler's harmonies? I suggest that there is.

The "harmonic cross-ratio" would be so named as in classical Greece it had first appeared as a structure known as the "musical *tetraktys*" or "*harmonia*" that had been a double-ratio between four points on a line given values 1, 4/3, 3/2 and 2 (or, for convenience, when multiplied by 6: 6, 8, 9, and 12), and used for dividing the musical octave (*diapason*), represented by the interval between 1 and 2 (or 6 and 12), into 2 complementary consonant intervals, the *diapente* (now known as the perfect fifth, as in the interval C to G), having the value 3/2 (or 9), and the *diatessaron* (the perfect fourth, as in C to F) having the value 4/3 (or 8) [51] (pp. 198–200).

These values had been worked out by experiments with the monochord—a single stringed instrument equipped with a dividing bridge and a measuring tape—recording the patterns of consonance and dissonance among the evoked tones. These results had been formalized by Pythagorean mathematicians around the time of Plato. The intervals of the *diapason*, *diapente*, and *diatessaron* were determined as corresponding to the three dividing "means" or "middle terms" of the interval: the geometric, the arithmetic, and the harmonic. Of these, the geometric mean was regarded as being particularly special.

In a geometric sequence of, say, three line lengths a, b, and c, the ratio of the first to the mean is the same as the mean to the last, $a{:}b = b{:}c$. However, while the geometric sequence applied to the sequence of octaves, applied *within the* octave, the geometric mean in fact produced the *most dissonant* note, the "tritone" (C to F#). More importantly, however, as the Greek number system was limited to whole, positive numbers, no numerical representation could be given to the geometric mean of 1 and 2, which we now think of in terms of the "irrational number", $\sqrt{2}$. In contrast, the two most consonant intervals within the octave were found to be given by the arithmetic mean (the mid-point of two values a and b or $\frac{a+b}{2}$), and its converse, the harmonic mean, calculated as the reciprocal of the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$, which reduces to the simpler formula, $\frac{2ab}{a+b}$. Basic algebra will show the musical *tetraktys* to be an instance of the harmonic cross-ratio as represented below in Figure 2, but more than this, the harmonic mean and the arithmetic mean within an interval defined geometrically could be used to provide approximations for a "number" the Greeks could not represent, $\sqrt{2}$ [36].

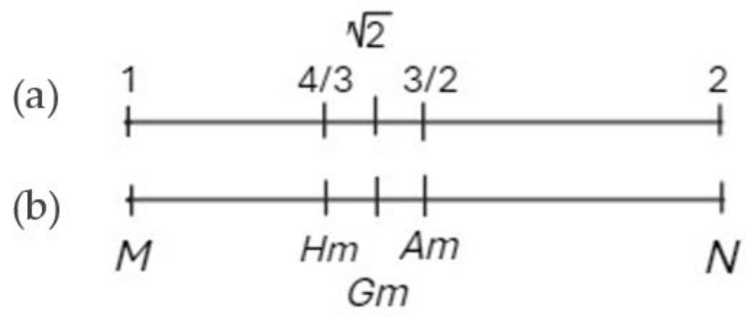

Figure 2. (**a**) The musical *tetraktys*; (**b**) the division of interval *MN* by the harmonic, geometric, and arithmetic means.

While, as we have seen, in his optics Kepler had anticipated two major features of Desargues's projective geometry—the idea of the "projective equivalence" of the conic sections and that of imaginary "points at infinity"—he seems to have *not*, however, in any way anticipated the device of the cross-ratio with its capacity for determining relative distances among objects in three-dimensional space. Nevertheless, it is not entirely absent from his astronomy as it was the mathematical infrastructure of the harmonic cross-ratio, that would be at the center of his thesis of *musica mundana*. Thus, in *Harmonice Mundi*, Book III, he invokes the "Pythagorean *Tetractys*" of "12, 9, 8, 6" that relates the *diapason*, the *diapente* and the *diatessaron* and that "was held by the Pythagoreans to be as worthy of consideration and admiration", so much so that it was transferred from music to "natural philosophy" [52] (pp. 134–135) [37].

We might, thus, describe Kepler's optics as pregnant with projective geometry, but did he apply this geometry to the world in his astronomy? While Kepler rejected the idea that the distance of the outer sphere of the fixed stars from the sun and its planets was infinite in the sense of Bruno, nevertheless, in contrast to earlier estimations of its distance he stressed its *immensity*—that is, its being *beyond measure*. Might this not be regarded as a type of finite, contracted infinity? The idea of points at infinity raises the conception of the observable universe as bounded by imaginary points at infinity analogous to the way points at infinity on the horizon bound a landscape. Kepler does not seem to have extended the idea imaginary points in this sense—for example, unlike Desargues, he did not think of a totality of imaginary points as forming an imaginary line that might be related to the horizon [43] (p. 109). Nevertheless, such an answer seems to be in the spirit of Kepler's approach.

Judith Field has noted how in Kepler's work of 1596, *Mysterium Cosmographicum*, for the purpose of calculating those "insensibly small" distances of the planets from the sun and from each other, the space between the planets and the fixed stars can be considered as being "like infinity" [36] (pp. 41–43). The same idea is found later in *Epitome Astronomicae*

Copernicanae of 1618–1621 [41], where Kepler attempts to buttress his case for his harmonically conceived astronomy. Replying to a direct question from an imaginary interlocutor on the size of the universe, Kepler again raises the idea of how the distance of the stars from the sun and planets in comparison to the distances among those objects seems "like infinity": "Even if the reasons of Copernicus do not extend to determining by observation the altitude of the sphere of the fixed stars: so that the altitude seems to be like infinity: for in comparison with this distance the total interval between the sun and the Earth. . . is imperceptible: nevertheless reason, making a stand upon the traces found, discloses a footpath for arriving even at this ratio" [41] (p. 42). Such a "footpath", I suggest, is provided via the principle of analogy in the form of the application of the harmonic cross-ratio.

With his harmonic astronomy, Kepler had appropriated the approach found in the neo-Platonic tradition and originating in Plato's dialogue, *Timaeus*. There Plato, via his fictional Pythagorean astronomer, Timaeus of Locri, had described the parts of the body and mind of the "cosmic animal" as unified by a "most beautiful bond" [53] (31b–32a)—a structure that various neo-Platonic commentators had identified as the musical *tetraktys* [54] (pp. 284–285), [55] (pp. 174–177). In his *Lectures on the History of Philosophy*, Hegel paraphrases Plato: "This brings into play in the most beautiful way the proportion [*die Analogie*] or the continuing geometric ratio [*das stetige geometrische Verhältnis*]. If the middle one of three numbers, masses or forces is related to the third as the first is to it and, conversely, it is related to the first as the third is to it (*a* is to *b* as *b* is to *c*), then, since the middle term has become first and last and, conversely, the last and the first have become the middle term, they have then all become one" [4] (pp. 209–210). Hegel then adds in his own voice: "With this the absolute identity is established. This is the syllogism [*der Schluss*] known to us from logic. It retains the form in which it appears in the familiar syllogistic, but here it is the rational" [4] (p. 210). Hegel, I suggest, was here following the neo-Platonists in modelling Plato's "most beautiful bond", and, hence, Plato's syllogism itself, on the inverted ratios of the musical *tetraktys*, a structure that would be later rediscovered as the core invariant of projective geometry.

In his own Platonic-based syllogism, as noted, Hegel unites the determinacies of singularity, particularity, and universality into a whole; it is "the completely posited concept" [21] (p. 588), the concept "which is determined and is truly in possession of its determinateness, namely, in that it differentiates itself internally and is the unity of its thus intelligible and determined differences. Only in this way does reason rise above the finite, the conditioned, the sensuous, [. . .] and is in this negativity *replete with content*, for as unity it is the unity of determinate extremes" [21] (p. 589). If the musical *tetraktys* plays for Hegel the role of model for the structure of the syllogism, then the unity of the syllogism's three determinations should be analogous to that of the three musical means in the *tetraktys*. Indeed, within the neo-Platonic tradition, this point had been explicitly made by Proclus, an author with whom Hegel was familiar. Thus, in his commentary on Plato's *Timaeus* Proclus says of the "three means" that Timaeus had earlier called them "bonds". "For the geometric [proportion] was said above to be the finest of bonds, and the other [proportions] are in them. But every bond is a sort of unification. [. . .] So surely, then, these means permeate all and make it a single whole from many parts, since they are allocated the power of connecting things that have various forms" [55] (p. 175). Proclus goes on to say that within such a structure the harmonic mean connects things in their "Samenesses", while the arithmetic mean connects them in their "various Differences" [55] (pp. 175–176), and this surely coincides with Hegel's distinction between singularity and particularity. Something is grasped in its singularity to the extent that it is different to other things, while grasped in its particularity—that is, as a particular instance of a universal—it is grasped by what it has in common with other instances of that universal. In Ploucquet's logic, as taught to Hegel in Tübingen, Ploucquet had distinguished between what he called "exclusive particulars" and "comprehensive" particulars in just this way [56] (§§ 14–15). An exclusive particular (Hegel's "singular") is cognized in a way that *excludes* other similar things from the scope of the referring term, while a comprehensive particular (Hegel's "particular") is cognized in a way that its universal subsumes or "comprehends" other similar things along with it. Singularity and particularity are unified together with universality in the structure of the syllogism in Book III of Hegel's *The Science of Logic*.

There Hegel treats the syllogism as generated by an expansion of a "judgment of the concept", the immediate form of which has a universal predicated of a singular, as in "this house is good" or "this house is bad". The judgment then develops from this immediate "assertoric" form through a "problematic" and into an "apodictic" judgment as in "the house, as so and so constituted, is *good*" [21] (p. 585). This is an implicit syllogism with a structure something like: this house is so and so constituted; a house so and so constituted is good; therefore, this house is good, which rendered in terms of the conceptual determinacies involved can be represented as *SP*, *PU*, and, therefore, *SU.*

Hegel reminds us that these structures are no longer abstract, as in the conception of judgments and syllogisms as found in the *logic of the understanding*. A singular, we must remember, is itself a *concrete thing*, a concept that has become "*external to itself*" and that has "stepped into actuality" [21] (p. 548). Moreover, having achieved its inherently syllogistic form, the apodictic judgment "is now *truly* objective" and is "the *truth* of the *judgment* in general. Subject and predicate correspond to each other, and have the same concept, and this *content* is itself posited *concrete universality*; that is to say, it contains the two moments, the objective universal or the genus and the *singularized universal*" [21] (pp. 585–586).

Hegel's syllogism is not a sequence of judgments in the usual sense but some concrete entity that manifests its universal—it is the sort of thing that "makes true" a true judgment (in the ordinary sense) made about it. But these concretizations of singularity and universality have become united only because of the mediating role of particularity. That is, as a *concretum*, the syllogism is nevertheless an element within a network of acts of expressed acts of judging and inferring made by concrete subjects reasoning about aspects of their world. Thus, there is a place for a formal consideration of logic in Hegel's account. His logic is not, as is often said, simply ontology [38].

It is, thus, important to grasp how Hegel's own *formal* syllogism cannot be restricted to the relatively indeterminate particularizations of universality as found in Aristotle's syllogism of the understanding. Aristotle, Hegel notes, "confined himself rather to the mere relation of inherence by defining the nature of the syllogism as follows: *When three terms are so related to each other that the one extreme is in the entire middle term, and this middle term is in the entire other extreme, then these two extremes are necessarily united in the conclusion*" [39]. Hegel goes on, "What is here expressed is the repetition of the *equal relation* of inherence of the one extreme to the middle term, and then again of this last to the other extreme, rather than the determinateness of the three terms to each other" [21] (p. 591). In this way, Aristotle's syllogism, without employing the distinction between singularity and particularity, was modelled on a continuous geometric sequence, *a*, *b*, *c*, in which *a*:*b* as *b*:*c.* In this sense, without singulars, logical space for Aristotle was homogeneous, as it would be for Kant. And just as the geometric mean cannot internally divide the octave, but needs supplementation by the arithmetic and harmonic means, the geometric structure of Aristotle's formal reasoning does not appropriately "divide" the constituents of the actual world: its particulars as determinations of universals must be further linked to instantiating singular terms.

6. Conclusions

I have argued that Hegel's version of the "Copernican revolution" in philosophy might best be understood as a Keplerian variant in contrast to Kant's Brunian one. Kant had thought of Euclid's geometry as providing *a priori* truths within which the laws of the physical world would eventually be articulated. For Kepler, however, for geometry to be applicable to the universe it had to adopt a form that differed from any ideal system—a form more appropriate to finite subjects, like us, who always observe and measure the world from locations within it, locations that limit observation and measurement to within some variable horizon beyond which our cognitions cannot (at least for the moment) stretch. In this way, geometry as applicable to the bounded world would be more projective than

Euclidean. For Hegel, similar considerations would apply to the forms of judgments used in coming to know the empirical world.

Kepler's critique of Euclidean geometry as appropriate for his bounded measurable universe had been expressed in his support for the "music of the spheres", creating an association for which Hegel had been condemned. But while Hegel's embrace of Kepler might indeed have extended to those more mystical elements of Kepler's thought, we should also see it as having made available for Hegel's logical use the unity of the Greek "musical" ratios at the heart of Kepler's implicit projective geometry. In short, rather than attesting to any melodies made by the movements of the planets, these ratios had provided a way of translating ideal magnitudes into ones applicable to the world by the finite beings living within it. It is appropriate, that Plato, or one of his followers, had described the device of the musical *tetraktys* as having been "granted to the human race by the blessed choir of the Muses" [57] (p. 991b)—a divine gift for the benefit of human beings and so adapted to the finitude of their worldly circumstance [40].

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Notes

1 For example, Klaus Brinkmann would write, "As I see it, Hegel came to the conclusion that Kant had pursued the Copernican revolution in philosophy only halfheartedly and had left it unfinished. On the other, Hegel saw no alternative to the completion of the Kantian project as he understood it" [1] (p. ix).

2 Copernicus recieves no more than a couple of passing references in Hegel's *Lectures on the Philosophy of Nature* [3] (pp. 269, 275), and while Bruno was a popular figure among some of Hegel's contemporaries, he is not mentioned at all in that work. While Bruno is discussed in detail in the *Lectures on the History of Philosophy* [4] (vol. 3, pp. 61–72), there is only passing mention of his cosmological ideas (p. 72).

3 See editor's comments in [5] (p. 167).

4 In terms popularized by Wilfrid Sellars, a new "scientific image" would come to challenge the traditional "manifest image" of "man-in-the-world" [7]. In the twentieth century, many philosophers on both sides of the "analytic–continental divide", would call for the reintegration of the scientific with the human, as with Sellars with his plea for a type of "stereoscopic vision" [7] (p. 372). Hegel's brand of idealism, I suggest, should be seen as an attempt to create such a vision.

5 As far as I can tell, Hegel nowhere invoked this popular image of Kant's "Copernican turn".

6 Hegel had commented on Bruno, "generally speaking, his philosophy is Spinozism" [4] (pp. 62–63).

7 This would be crucial for Newton who, following Henry More, treated the infinity of space as reflecting the fact of space being an attribute of God himself [18] (pp. 244–245).

8 "A new light broke upon the first person who demonstrated the isosceles triangle (whether he was called 'Thales' or had some other name). For he found that what he had to do was not to trace what he saw in this figure, or even trace its mere concept, and read off, as it were, from the properties of the figure; but rather that he had to produce the latter from what he himself thought into the object and presented (through construction) according to *a priori* concepts, and that in order to know something securely *a priori* he had to ascribe to the thing nothing except what followed necessarily from what he himself had put into it in accordance with its concept" [6] (p. Bxi–xii).

9 Later, certain neo-Kantians would attempt to augment Kant's *Erfahrung* with a conception of such a "lived" form of experience, "*Erlebnis*".

10 Here, Kant's metaphor of reason compelling nature to answer *its own* questions seems particularly apt, as the experience found in the types of experiment appealed to is being portrayed as resulting from a type of specific "yes–no" question being addressed to the experimental outcome by a reasoning inquirer. Here, a "yes" answer will be equivalent to an assertion of the whole proposition asked about.

11 In *On Interpretation*, ch. 7, Aristotle seems to discuss judgments about singulars (*ta de kath ekaston*) as of a distinct class from judgments about particulars [22] (17a40), although interpreters differ as to Aristotle's actual classification here. In the syllogistic

logic of *Prior Analytics* [22], however, there is no real place for singular judgments as the subject term of one premise may need to play the role of predicate in the other, and Aristotle does not allow singular terms as predicates.

12 In the equivalent passages of the *Encyclopedia Logic* Hegel tends to use the demonstrative "this rose" (e.g., 11, §§ 166 add., 172 add., 174 add., 178, rem).

13 That negation is logical or "external" is also shown by the fact that such sentences can be rephrased with the negation moved to the front of the sentence as in "it is *not the case that* the rose is an elephant".

14 Sometimes Prior suggested the stronger thesis of the *priority* of perspectival or modal judgments. Thus, Kit Fine describes Prior's major concerns as that of showing that "modal and tense logic could stand on their own, that talk of possible worlds or instants was to be reduced to them rather than the other way around [28] (p. 7).

15 Prior had been influenced by the Hegelian philosopher with an interest in modern logic, John N. Findlay, although Prior apparently did not recognize any Hegelian features of his own approach.

16 Not that we could ever make it the entire way, however, as the ladder was conceived as an infinite one. A version of Leibniz's ascent, subject to Kant's greater restrictions, is apparent in Kant's ascendable "prosyllogistic" chain of inferences in the *Critique of Pure Reason* [6] (pp. A331-332/B387-389).

17 It would, for the same reasons, I suggest, be in tension with Leibniz's imagery of an ascent to a "God's-eye view", but pursuing this is beyond the scope of this essay.

18 In relation to this, Kepler had apparently thought of Plato's *Timaeus* as having provided a commentary on the Book of Genesis [36] (p. 1).

19 Bruno had, of course, been burnt at the stake in Rome, some six years earlier, for heresies including his views on cosmology.

20 In a subsequent work [2], Koyré was somewhat more sympathetic to this aspect of Kepler's position.

21 As is well known, Hegel rejected the usual contrast of finitude with what he called the "bad infinite". On the role of mathematics in Hegel thinking here, see [38].

22 From a later perspective which includes the finitude of the speed of light, for any observer located outside the "light cone" radiating from some event in the universe, that event will be in principle unobservable.

23 Judith Field makes a similar point: "If the Universe is immense or infinite, in the sense that we must believe it to contain objects we cannot observe, such as stars that are sometimes too far away for us to see them ... then we cannot know how to construct theories to explain what we observe." "[I]n modern terms" she adds, that for Kepler "the word 'Universe' must be taken to mean 'observable Universe'" [36] (p. 18).

24 "It is therefore evident that there is also no place or void or time outside the heaven. For in every place body can be present; and void is said to be that in which the presence of body, though not actual, is possible ... and outside heaven, as we have shown, body neither exists nor can come to exist" [40] (pp. 279a12-16).

25 While Kepler does not tend to speculate on what may be beyond the sphere of the fixed stars, a comment in *Epitome of Copernican Astronomy* suggests the Stoic void rather than Aristotle's nothing. The sphere of the fixed stars, he says, works like a wall or a garment to keep "the heat from flowing out" [41] (p. 15). Presumably this heat from the sun would flow *somewhere.*

26 In April 1610 Kepler would read Galileo's *Siderius Nuncius*, published only in the previous month, containing his telescopic observations of the moons of Jupiter [42] (pp. 77–78). Kepler quickly designed his own version of the telescope and put it into use a year later, and attempted to explain the operations of lenses in *Dioptrice* in the same year.

27 C.f., "In Euclidean geometry, constructions are made with the rule and compass. Projective geometry is simpler: its constructions require only the ruler. We consider the straight line join two points, and the point of intersection of two lines, with the further simplification that two lines never fail to meet!" [43] (p. v).

28 Sectioning the cone by a plane at right-angles to the cone's axis produced a circle, while altering the angle produced at first an ellipse, then a parabola, and then a hyperbola. Strictly, two parabolas and hyperbolas are produced because the cone is considered to continue in a second cone standing tip-to-tip with the first.

29 For an account of the history of what would be called the "principle of continuity" after Kepler see [47] (ch. 9).

30 This in turn would imply a complete symmetry between points and lines within projective geometry: just as every pair of points define some particular line that runs through them, *every* pair of lines define some point as the point at which they meet. This "duality" of points and lines would mean that in projective geometry, every theorem concerning relations holding among points will be matched by an equivalent theorem concerning analogous relations holding among lines.

31 Analogous to this, an equivalent double ratio can be considered as holding among the angles of a "pencil" of four rays.

32 This is how it was often understood in the nineteenth century.

33 These are the "complete quadrilateral", formed by four intersecting lines on a plane, and the "complete quadrangle", formed by lines joining four points on a plane [43] (pp. 6–8).

34 Strictly, the value is -1, as in the nineteenth century the idea of assigning line-segments a value of + or $-$ on the basis of their direction or "orientation" had been introduced. Thus, if the segment *AB* is deemed positive, that of *BA* is negative.

35 Desargues's work had been largely forgotten but was revived in the nineteenth century in France by the military engineers Gaspard Monge (1746–1818) and Lazare Carnot (1753–1823). The use of the cross-ratio device had proved invaluable in the field during the Revolutionary Wars for assessing distances between objects in a landscape observed from a particular point of view [50].

36 As can be seen in Figure 2, the geometric mean falls approximately midway between the harmonic and arithmetic means, but now taking the harmonic and arithmetic means of *the interval between the first harmonic and arithmetic means* will produce an even closer approximation. The operation can, of course, be repeated indefinitely, giving a series of closer and closer approximations. The musical *tetraktys* thus provides the basis of an algorithm for approximations of a number that otherwise could not be represented directly.

37 Because he took the soundless "harmonizing" of the movements of planets quite literally, Kepler, drawing on the musical theory of Ptolemy, expanded the range of whole number ratios that would be employed, considering the musical *tetraktys* itself as providing too narrow an array of consonant intervals for the type of modern polyphonic music that he thought most pleasing to God. These further ratios, however, were still based on the model of the musical *tetraktys*.

38 As for those "objective" aspects of Hegel's logic, I suggest it is less misleading to think of them as presenting a type of *semantics* for his "subjective" or formal or "uninterpreted" logic

39 This is effectively a quotation from *Prior Analytics* [22] (25b32–35).

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