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Remarks on the Quantum Effects of Screw Dislocation Topology and Missing Magnetic Flux

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Abstract: We revisit the interaction between a point charge and an inhomogeneous magnetic field that yields the magnetic quantum dot system. This magnetic field is defined by filling the whole space, except for a region of radius r_0. Then, we assume that there is an impenetrable potential wall located at r_0 and discuss the quantum effects of screw dislocation topology and the missing magnetic flux. We first show that Landau levels can be achieved even though there is the presence of an impenetrable potential wall. We go further by discussing the confinement of a point charge to a cylindrical wire. In both cases, we show Aharonov–Bohm-type effects for bound states can be obtained from the influence of the screw dislocation topology and the missing magnetic flux. Later, we discuss the influence of the screw dislocation topology and the missing magnetic flux on the magnetization and the persistent currents.

Keywords: missing magnetic flux; Aharonov–Bohm effect; screw dislocation; inhomogeneous magnetic field; cylindrical wire; magnetization; persistent currents

1. Introduction

According to [1–3], a magnetic quantum dot can be built from the interaction of a point charge with an inhomogeneous magnetic field. This magnetic field is null inside a cylinder and uniform outside it. An interesting aspect of the energy levels in the region where the magnetic field is uniform is the influence of the missing magnetic flux, which stems from the region where the magnetic field is null on them. This quantum effect associated with the missing magnetic flux can be viewed as the inverse of the Aharonov–Bohm proposal [4,5]. In their proposal, Aharonov and Bohm [4] showed that the wave function of a point charge acquires a quantum phase when it moves around a long, thin solenoid. This quantum phase is called the Aharonov–Bohm quantum phase [4]. In another perspective, when the point charge is confined to a one-dimensional quantum ring, and the energy levels are influenced by the Aharonov–Bohm quantum phase, this quantum effect is known as the Aharonov–Bohm effect for bound states [5]. In mesoscopic systems, this quantum phenomenon was experimentally observed [6–12].

The Aharonov–Bohm effect [4] has also been studied from the perspective of the geometric theory of defects [13–15]. In this line of research, topological defects in solids can be described through differential geometry. Disclinations, screw dislocations, and spiral dislocation are examples of topological defects in solids [13,16,17]. From this description of topological defects in solids, Berry-type quantum phases [18–20], Aharonov–Bohm-type effect for bound states [21–25], and quantum holonomies [26–28] have been studied. Recently, the topological effects of a screw dislocation on the interaction of a point charge with an inhomogeneous magnetic field proposed in Refs. [1–3] have been studied in Ref. [29].

In this work, we revisit the interaction of a point charge with an inhomogeneous magnetic field under the influence of the screw dislocation topology. We consider the inhomogeneous magnetic field proposed in Refs. [1–3], where we assume that there is an impenetrable potential wall located at the radius of the cylinder. We bring two different discussions about the quantum effects of screw dislocation topology and the missing

magnetic flux. In the first one, we show that the Landau levels [30] can be achieved, even though there is the presence of an impenetrable potential wall. In addition, we show that Aharonov–Bohm-type effects for bound states can be obtained from the influence of the screw dislocation topology and the missing magnetic flux on the Landau levels [30]. Our second discussion is about the confinement of a point charge to a cylindrical wire in the presence of the screw dislocation and the inhomogeneous magnetic field of Refs. [1–3]. Further, we discuss the magnetization [31–33] and persistent currents [34,35] at zero temperature with regard to previous systems.

The structure of this paper is as follows: we start by introducing the inhomogeneous magnetic field of Refs. [1–3]. Then, we introduce the screw dislocation description through the geometric theory of defects [13–15]. Thereby, we study the interaction of a point charge with an inhomogeneous magnetic field in the presence of an impenetrable potential wall located at the radius of the cylinder. In Section 3, we study the point charge confined to a cylindrical wire in the presence of the screw dislocation and the inhomogeneous magnetic field. In Section 4, we study the magnetization at zero temperature with regard to the previous systems; in Section 5, we discuss the arising of persistent currents at zero temperature with respect to the previous systems. In Section 6, we present our conclusions.

2. Topological Effects in an Inhomogeneous Magnetic Field

Let us begin with the interaction of a point charge with an inhomogeneous magnetic field proposed in Refs. [1–3]. The proposed inhomogeneous magnetic field is defined by filling the whole space except for a region of radius r_0, which can be viewed as the region inside a cylinder of radius r_0. Thereby, this inhomogeneous magnetic field is given by $B = 0$ for $r < r_0$ and $\vec{B} = B_0 \hat{z}$ for $r > r_0$, where $B_0 > 0$ is a constant. The vector potential associated with this inhomogeneous magnetic field, for $r > r_0$, is given by [1,2]:

$$\vec{A}(r) = B_0 \left[\frac{r}{2} - \frac{r_0^2}{2r} \right] \hat{\varphi}. \tag{1}$$

Thus, the vector potential (1) is null when $r < r_0$. Hence, this field configuration defines a magnetic quantum dot [1–3]. In particular, the term proportional to r_0^2 of the vector potential (1) provides information about the magnetic flux that is missing from the uniform magnetic field inside the region of radius r_0, i.e,

$$\varsigma = \oint \frac{B_0 r_0^2}{2r} \hat{\varphi} \cdot d\vec{r} = B_0 r_0^2 \pi. \tag{2}$$

Therefore, ς is called the missing magnetic flux quanta [3]. It represents the number of missing magnetic flux within the quantum dot. An interesting view of the interaction of the point charge with the inhomogeneous magnetic field described by the vector potential (1) is that this quantum system is the inverse of the Aharonov–Bohm proposal [4,5]. Thus, this interaction in the region $r > r_0$ can be influenced by the missing magnetic flux inside the region $r < r_0$. Recently, the influence of the missing magnetic flux ς on the interaction of the point charge with the inhomogeneous magnetic field in the region $r > r_0$ has been studied in the presence of a screw dislocation [29].

Let us also consider the presence of a screw dislocation, which consists in the distortion of a circular curve into a vertical spiral [16–20]. We can describe this linear topological defect through the Katanaev–Volovich approach [13], where this screw dislocation is represented by the line element:

$$ds^2 = dr^2 + r^2 d\varphi^2 + (dz + \beta \, d\varphi)^2. \tag{3}$$

In Equation (3), the screw dislocation is characterized by the parameter β, which is related to the Burgers vector $\vec{b}$ via $\beta = b/2\pi$. For the distortion of a circular curve into a vertical spiral, the Burgers vector is perpendicular to the plane $z = 0$. Note that the radius

r_0 in Equation (1) can also be viewed as the radius of the core of the topological defect, which characterizes a thick defect [29].

According to Refs. [15,36–38], the interaction of a point charge with a magnetic field in the presence of a screw dislocation can be studied through the Schrödinger equation (we will use the units $\hbar = 1$ and $c = 1$):

$$E\psi = -\frac{1}{2m}\frac{1}{\sqrt{g}}\left(\partial_l - iq\,A_l\right)\left[\sqrt{g}\,g^{lj}\left(\partial_j - iq\,A_j\right)\right]\psi. \tag{4}$$

Observe in Equation (4) that g_{ij} is the metric tensor. In addition, g^{ij} is the inverse of g_{ij} and $g = \det\left|g_{ij}\right|$. The parameter q represents the electric charge of the particle ($q > 0$), while A_j represents the covariant component of the electromagnetic four-vector potential: $A_\mu = \left(A_0,\,A_j\right)$.

In a recent work [29], by dealing with Equations (1)–(3), the solution to Equation (4) has been obtained in view of the cylindrical symmetry of the system. It is given by $\psi(r,\,\varphi,\,z) = e^{i\ell\varphi + ikz}\,u(r)$, where k is a constant and $\ell = 0, \pm 1, \pm 2, \pm 3, \pm 4 \ldots$. The function $u(r)$, in turn, is the solution to the radial equation. After defining $y = \frac{m\omega}{2}\,r^2$, the solution to the radial equation obtained in Ref. [29] was

$$u(y) = e^{-\frac{y}{2}} \times y^{|\gamma|/2} \times U\left(\frac{|\gamma|}{2} + \frac{1}{2} - \frac{\tau}{2m\omega},\,|\gamma| + 1;\,y\right), \tag{5}$$

where the function $U(a,\,b;\,y)$ is the confluent hypergeometric function of the second kind [39]. The solution (5) has been achieved by imposing that $u(y) \to 0$ when $y \to \infty$. Note that the function $U(a,\,b;\,y)$ is regular at $y \to \infty$ [39]. Moreover, we have that $a = \frac{|\gamma|}{2} + \frac{1}{2} - \frac{\tau}{2m\omega}$ and $b = |\gamma| + 1$, where parameters ω, γ, and τ have been defined as follows:

$$\begin{aligned}
\omega &= \frac{q\,B_0}{m}; \\
\gamma &= \ell - \beta k + \frac{q\,\varsigma}{2\pi}; \\
\tau &= 2mE + m\omega\,\gamma - k^2.
\end{aligned} \tag{6}$$

In this work, we also assume that there exists an impenetrable potential wall at $r = r_0$, as in Ref. [29]. Hence, the presence of this impenetrable potential wall establishes the following boundary condition:

$$u(y_0) = 0, \tag{7}$$

where $y_0 = \frac{m\omega\,r_0^2}{2}$ is defined when $r = r_0$. By substituting the wave function (5) into Equation (7), we obtain

$$u(y_0) = e^{-\frac{y_0}{2}} \times y_0^{|\gamma|/2} \times U\left(\frac{|\gamma|}{2} + \frac{1}{2} - \frac{\tau}{2m\omega},\,|\gamma| + 1;\,y_0\right) = 0. \tag{8}$$

However, in this work, we wish to explore the presence of this impenetrable potential wall at $r = r_0$ from a different approach. Our proposal is to deal with the case where $y_0 \ll 1$, which has not been studied in Ref. [29]. For $y_0 \ll 1$ and $b > 1$, the function $U(a,\,b;\,y_0)$ can be written in the form [39]:

$$U(a,\,b;\,y_0) \quad \propto \quad \frac{\Gamma(b - a)}{\Gamma(a)}\,y_0^{1-b}. \tag{9}$$

Hence, by substituting Equations (9) into Equation (8), thus, Equation (8) is satisfied only if $\Gamma(a) \to \infty$. This occurs when $a = -n$, where $n = 0, 1, 2, 3, \ldots$. With $a = \frac{|\gamma|}{2} + \frac{1}{2} - \frac{\tau}{2m\omega}$, thus, we obtain from $a = -n$:

$$E_{n,\ell} = \omega \left[n + \frac{1}{2} \left| \ell - \beta k + \frac{q\varsigma}{2\pi} \right| - \frac{1}{2} \left(\ell - \beta k + \frac{q\varsigma}{2\pi} \right) + \frac{1}{2} \right] + \frac{k^2}{2m}. \tag{10}$$

where n is called the radial quantum number.

Hence, although there exists an impenetrable potential wall at $r = r_0$ and the point charge interacts with an inhomogeneous magnetic field in the region $r > r_0$, the case $y_0 \ll 1$ allows us to achieve the Landau levels [30] in Equation (10). In this case, the Landau levels are influenced by the missing magnetic flux ς and the topology of the screw dislocation. Therefore, both the missing magnetic flux and the screw dislocation topology break the degeneracy of the Landau levels. In contrast to the energy levels obtained in Ref. [29], the radial quantum number has no upper limit. Note that even though the system is unconfined in the third dimension, we have bound states in two-dimensions, in agreement with Refs. [30,36,40,41]. The last term of Equation (10) corresponds to the translational kinetic energy of the free motion of the particle in the z-direction.

Regarding the influence of the screw dislocation topology, observe that the interaction between the point charge and the defect does not exist. Thereby, the topological defect influence on the energy levels (10) yields an Aharonov–Bohm-type effect for bound states [5,15,38]. By taking $\beta = 0$, we thus obtain the energy levels in the absence of screw dislocation. Besides, with $\beta = 0$, the energy levels (10) remain influenced by the missing magnetic flux. This dependence of the Landau levels on the missing magnetic flux gives rise to an Aharonov–Bohm-type effect. As pointed out in Ref. [1], the missing magnetic flux (2) is the inverse of the Aharonov–Bohm proposal [4]. Then, by dealing with the missing magnetic flux influence on the Landau levels (10), we have the inverse of the proposal of the Aharonov–Bohm effect for bound states [5], which we call the Aharonov–Bohm effect for missing magnetic flux [29].

3. Topological Effects in a Cylindrical Wire

In this section, our aim is to study the confinement of a point charge to a cylindrical wire in the presence of the screw dislocation (3) and the inhomogeneous magnetic field established by Equation (1). By following Refs. [25,36,42,43], the cylindrical wire is described by considering a fixed radius R ($R > r_0$). Note that, for a fixed radius R, the line element (3) becomes [36]

$$ds^2 = R^2 d\varphi^2 + (dz + \beta\, d\varphi)^2. \tag{11}$$

In this way, the Schödinger-Pauli Equation (4) is written in the form:

$$\begin{aligned} E\psi &= -\frac{1}{2mR^2} \left[\frac{\partial}{\partial\varphi} - \beta \frac{\partial}{\partial z} \right]^2 \psi - \frac{1}{2m} \frac{\partial^2\psi}{\partial z^2} + i\frac{qB_0}{2m} \left(1 - \frac{r_0^2}{R^2} \right) \left[\frac{\partial}{\partial\varphi} - \beta \frac{\partial}{\partial z} \right] \psi \\ &\quad + \frac{q^2 B_0^2}{8mR^2} \left(R^2 - r_0^2 \right)^2 \psi. \end{aligned} \tag{12}$$

As $\hat{p}_z = -i\partial_z$ commutes with the Hamiltonian operator given in the left-hand side of Equation (12), we can achieve a solution to Equation (12) in the form:

$$\psi(\varphi, z) = e^{ikz} g(\varphi), \tag{13}$$

where k is a constant. By substituting the wave function (13) into Equation (12), we find

$$\frac{d^2 g}{d\varphi^2} - 2i\,\delta\,\frac{dg}{d\varphi} + \tau\, g = 0, \tag{14}$$

where we have defined

$$
\begin{aligned}
\delta &= \beta k - \frac{q\,\varsigma}{2\pi} + \frac{1}{2}\,m\omega R^2; \\
\tau &= 2m\,R^2\,E - \delta^2 - R^2\,k^2.
\end{aligned}
\tag{15}
$$

Let us go further by writing the solution to Equation (14) as follows:

$$
g(\varphi) = a_0\,e^{\eta\,\varphi},
\tag{16}
$$

where a_0 is a constant and η is a parameter to be determined. After substituting (16) into Equation (14), we find $\eta = i\left(\delta \pm \sqrt{\delta^2 + \tau}\right)$. Then, the wave function (16) becomes

$$
g(\varphi) = a_0\,e^{i\left(\delta \pm \sqrt{\delta^2 + \tau}\right)\varphi}.
\tag{17}
$$

For a spinless particle, we have the boundary condition:

$$
g(\varphi + 2\pi) = g(\varphi),
\tag{18}
$$

because $0 \leq \varphi \leq 2\pi$. Next, by substituting the wave function (17) into Equation (18), we find the relation:

$$
\delta \pm \sqrt{\delta^2 + \tau} = \ell,
\tag{19}
$$

where $\ell = 0, \pm 1, \pm 2, \pm 3, \dots$ and with δ and τ being defined in Equation (15). Hence, from Equation (19), we obtain

$$
E_\ell = \frac{1}{2m\,R^2}\left[\ell - \beta k + \frac{q\varsigma}{2\pi} - \frac{1}{2}\,m\,\omega\,R^2\right]^2 - \frac{k^2}{2m}.
\tag{20}
$$

According to Refs. [36,40,41], the quantum wire has bound states in two-dimensions, although the system is unconfined in the third dimension. Therefore, Equation (20) is the energy levels of the point charge confined to a cylindrical wire. The last term of Equation (20) is the translational kinetic energy of the free motion of the particle in the z-direction. From the first term of Equation (20), we observe the influence of the missing magnetic flux ς, as well as the screw dislocation topology. The topological effects of the screw dislocation also yield an Aharonov–Bohm-type effect for bound states [36]. With respect to the influence of the missing magnetic flux on the energy levels (20), we also obtain the Aharonov–Bohm effect for missing magnetic flux [29] when the point charge is confined to the cylindrical wire.

Moreover, the energy levels (20) are a periodic function of the missing magnetic flux ς. As example, for the periodicity $\phi_0 = \frac{2\pi}{q}$, we have $\mathcal{E}_\ell\left(\varsigma + \frac{2\pi}{q}\right) = \mathcal{E}_{\ell+1}(\varsigma)$. This means that the quantum states determined by $\{\ell\}$ and $\{\ell + 1\}$ are degenerate. If we consider the periodicity $\phi_0 = -\frac{2\pi}{q}$, we have $\mathcal{E}_\ell\left(\varsigma - \frac{2\pi}{q}\right) = \mathcal{E}_{\ell-1}(\varsigma)$. Then, the quantum states $\{\ell\}$ and $\{\ell - 1\}$ are degenerate.

4. Magnetization at Zero Temperature

In this section, our focus is on the magnetization of the systems at zero temperature. According to Refs. [31–33], the magnetization at zero temperature is defined as $\mathcal{M} = \sum_{n,\ell} \mathcal{M}_{n\ell}$, where $\mathcal{M}_{n\ell}$ is the magnetic moment of the $\{n, \ell\}$-th state. Thereby, the magnetic moment in a state determined by the quantum numbers $\{n, \ell\}$ is given by [31–33]

$$
\mathcal{M}_{n\ell} = -\frac{\partial E_{n,\ell}}{\partial B_0}.
\tag{21}
$$

In the following, we discuss the magnetic moment (21) with regard to the systems studied in the previous sections.

4.1. Landau Levels

In Equation (6), we have defined the angular frequency or cyclotron frequency: $\omega = q\,B_0/m$ [30]. Then, by dealing with Equation (10), the magnetic moment (21) is

$$\mathcal{M}_{n\ell} = -\frac{q}{m}\left[n + \frac{1}{2}\left|\ell - \beta k + \frac{q\varsigma}{2\pi}\right| - \frac{1}{2}\left(\ell - \beta k + \frac{q\varsigma}{2\pi}\right) + \frac{1}{2}\right]. \tag{22}$$

Hence, Equation (22) shows that the magnetic moment is influenced by the topology of the screw dislocation, as well as the missing magnetic flux. By taking $\beta = 0$, we obtain magnetization in the absence of the defect. Even though, in the absence of the defect, the magnetization is influenced by the missing magnetic flux.

4.2. Cylindrical Wire

With regard to the energy levels (20), we can see a contribution to the energy levels related to the cyclotron frequency $\omega = q\,B_0/m$. In this way, the magnetic moment (21) is given by

$$\mathcal{M}_{n\ell} = \frac{q}{2m}\sum_{\ell}\left[\ell - \beta k + \frac{q\varsigma}{2\pi} - \frac{1}{2}\,m\,\omega\,R^2\right]. \tag{23}$$

Thereby, both screw dislocation topology and the missing magnetic flux influence the magnetic moment (23). If we take $\beta = 0$ in Equation (23), we obtain the magnetic moment and the magnetization in the cylindrical wire in the absence of the topological defect. Therefore, the magnetization depends on the missing magnetic flux even in the absence of the screw dislocation.

5. Persistent Currents

Persistent current is a phenomenon observed in quantum ring systems, which arises when the magnetic flux enclosed by the quantum ring induces an asymmetry between electrons with clockwise and counterclockwise momenta, and then leads to a thermodynamic state with a persistent current [44]. Persistent currents have been studied from theoretical and experimental points of view [45–55].

In a recent work [29], due to the fact that we can vary the missing magnetic flux inside the region $r < r_0$ by modifying the intensity of the magnetic field B_0 in the region $r > r_0$, the missing magnetic flux influence on the energy levels has raised the possibility of having an analogue of persistent currents [34]. There, the persistent currents (at temperature $T = 0$) with respect to the missing magnetic flux have been calculated via the Byers–Yang relation [34,35]:

$$\mathcal{I}|_{T=0} = -\sum_{n,l}\frac{\partial E_{n,l}}{\partial\varsigma}. \tag{24}$$

In the following, we are going to calculate the persistent currents in the Landau levels and the cylindrical wire studied previously.

5.1. Landau Levels

We have seen in Equation (10) that the Landau levels depend on the missing magnetic flux. Thereby, by using the eigenvalues of energy (10), the persistent currents associated with the missing magnetic flux (at temperature $T = 0$) are given by

$$\mathcal{I}|_{T=0} = \frac{q\omega}{4\pi} - \frac{q\omega}{4\pi}\sum_{\ell}\frac{\gamma}{|\gamma|}. \tag{25}$$

Hence, this analogue of the persistent currents [34] is influenced by the screw dislocation topology. This behaviour differs from that viewed in Ref. [29], where the corresponding persistent currents have no influence on the topological defect. Furthermore, Equation (25) shows that this analogue of the persistent currents is influenced by the missing magnetic flux.

5.2. Cylindrical Wire

We have seen that the energy levels (20) are also a periodic function of the missing magnetic flux. In this way, from the energy levels (20), we have

$$\mathcal{I}|_{T=0} = -\frac{q}{2\pi\,m\,R^2}\sum_\ell\left[\ell - \beta\,k + \frac{q\,\varsigma}{2\pi} - \frac{1}{2}\,m\,\omega\,R^2\right]. \tag{26}$$

Therefore, Equation (26) shows us that the influence of the screw dislocation topology and the missing magnetic flux on the persistent currents in the cylindrical wire exists.

6. Conclusions

We have revisited the interaction of a point charge with an inhomogeneous magnetic field under the influence of screw dislocation topology. The inhomogeneous magnetic field has been defined by filling the whole space except for a region of radius r_0 [1–3]. Then, we have assumed the existence of an impenetrable potential wall at $r = r_0$ [29]. We have shown a case where the Landau levels [30] can be achieved, even in the presence of the impenetrable potential wall at $r = r_0$, which differs from that of Ref. [29]. We have seen that the Landau levels are modified by the screw dislocation topology, as well as the missing magnetic flux. From this influence of the screw dislocation topology and the missing magnetic flux, we have obtained Aharonov–Bohm-type effects. In particular, the dependence of the Landau levels on the missing magnetic flux yields the inverse of the proposal of the Aharonov–Bohm effect for bound states, which we have called the Aharonov–Bohm effect for missing magnetic flux [29].

We have also studied the influence of screw dislocation topology and the missing magnetic flux on the confinement of the point charge to a cylindrical wire. We have seen that the energy levels are modified by the screw dislocation topology and the missing magnetic flux. Hence, we have also obtained Aharonov–Bohm-type effects. With regard to the dependence of the energy levels on the missing magnetic flux, we have also called the Aharonov–Bohm effect for missing magnetic flux. Further, we have seen that the energy levels are a periodic function of the missing magnetic flux.

We have gone further by searching for topological effects on magnetization. We have seen in both Landau levels and cylindrical wire that the magnetization is influenced by the screw dislocation topology and the missing magnetic flux. Finally, we have assumed that this missing magnetic flux can vary inside the region $r < r_0$ [29]. This has given rise to the appearance of persistent currents associated with the missing magnetic flux. In the Landau levels and the cylindrical wire, the persistent currents are influenced by the missing magnetic flux and the linear topological defect.

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