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Review

A Review of Numerical Models for Slab-Asphalt Track Railways

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Abstract: Higher train speeds and heavier axle loads trigger elevated stresses and vibrations in the track, potentially increasing track deterioration rates and maintenance costs. Alternative track forms made of combinations of reinforced concrete and asphalt layers have been developed. A thorough understanding of the slab and asphalt tracks is needed to investigate track performance. Thus, analytical and numerical models have been developed and validated by many researchers. This paper reviews numerical models developed to investigate railway track performance. The synthesis of major finite element models is described in detail, highlighting the main components and their outputs. For slab track models, the use of a structural asphalt layer within the railway track remains an active research topic and firm conclusions on its efficacy are not yet available. It can be expected that slab track structures will also be affected by train-induced ground vibrations. There is thus a gap in the literature regarding the measurement of dynamic effects on high-speed railway lines, and further research is needed to investigate the dynamic behaviour of slab–asphalt track systems. In this review, novel solutions for mitigating the vibrations in high-speed rail are discussed and compared. The use of asphalt material in railways appears to have beneficial effects, such as increasing the bearing capacity and stiffness of the structure and improving its dynamic performance and responses, particularly under high-speed train loads.

Keywords: high-speed railway; slab track; critical speed; asphalt layer; numerical modelling; finite element analysis; soil nonlinearity

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1. Introduction

The railway is an important transportation system, preferred over other means for nearly two centuries. It also plays a crucial role in the economy of developed and developing countries alike. In recent years, the attention given to both speed and axle weight has increased, as higher train speeds and heavier axle loads induce higher pressures within tracks as trains pass. The more elevated the stresses imposed on the track, the larger the subsequent deterioration. The geometry of tracks must be almost uniform to enable trains to reach high speeds since deformations can significantly influence train behaviour.

The behaviour of railway tracks and foundations under high-speed trains and repetitive axle loading is a complex soil-structure interaction issue requiring a concerted research effort. As train speeds increase, the dynamic responses of the railway track and ground along the railway line become more pronounced [1]. For a high-speed train running on soft soil, resonance may occur, and consequently the dynamic responses of the track and ground are dramatically amplified. The speed at which an extraordinarily large dynamic response occurs is termed the 'critical speed' [2]. At the critical speed, train loads induce strong vibrations in the track structure, increasing the risk of derailment and the occurrence of damage to the track structure [3]. A situation similar to this has been reported to have occurred at Ledsgård in Sweden as a result of very soft clayey soil [4], and several researchers have proposed analytical and numerical approaches to assess the critical speed and nature of railway-induced ground vibrations [1,5–7].

The use of slab tracks is still being investigated as a means to reduce vibration levels for high-speed rail traffic. For instance, 120 km of ballastless track was laid between Amsterdam Schiphol Airport and the Belgian border for the high-speed HSL Zuid line in the Netherlands between 2002 and 2006 [8]. In the same period, the use of slab tracks came to the fore in Spain for the first time for sections of its extensive high-speed rail network. The first high-speed track section from Seville to Madrid was built almost wholly using slab track. Furthermore, the slab track was laid in many tunnel segments on the Madrid–Valladolid line running north-west from Madrid, for example, in the 27 km long Guadarrama Tunnel [8]. Slab track railways exhibit dynamic characteristics that differ from traditional ballasted railways in terms of transient responses and permanent deformations [3], so it is of interest to assess how they respond under train-induced ground vibrations. There is a gap in the literature regarding the measurement of dynamic effects associated with high-speed railway lines, and further research is needed to investigate the dynamic behaviour of slab track railways. This paper focuses on the literature with regard to the dynamics of high-speed railways as well as modelling approaches that take into account the nonlinearities of soils in railway tracks while considering high-speed effects.

2. Mechanism of Vibration Propagation in Railways

Vibration in railways occurs through two basic mechanisms. First, quasi-static excitation occurs due to slowly applied loading where the structure deforms at a very slow rate (low strain rate); thus, inertial contributions are small and can be ignored. It is mainly axle loads (as illustrated in Figure 1) that govern the magnitude of quasi-static excitation, in tandem with the geometry and stiffness of the surrounding soil and rocks (geomechanical properties). The resulting track deflection (deflection around the wheel) is called 'static' or 'quasi-static' as it corresponds to a static load on an elastically supported beam. As the wheel passes over the track, this deflection moves and generates periodic waves, which are affected by factors such as the characteristics of the train and sleeper spacing, among other effects. Quasi-static excitation typically occurs in the low frequency range (0–20 Hz). When trains run at conventional speeds, quasi-static excitation dominates up to a quarter wavelength away from the track [9]. For increasing train speeds, quasi-static excitations begin expanding to greater distances from the track.

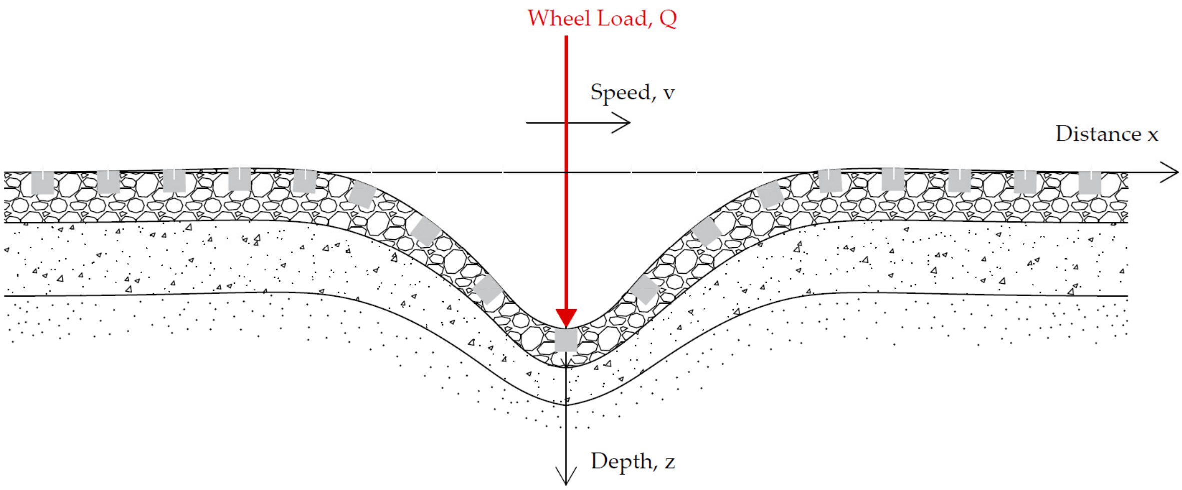

Figure 1. Deflection of the track structure response from one wheel load modified after [10].

The second vibration mechanism is dynamic excitation, which is caused by a vehicle's dynamic behaviour and its interaction with a rail through wheel/rail contacts. Furthermore, wheels and tracks can generate fluctuations, which result in dynamic deflection in addition to quasi-static deflection. In comparison to quasi-static excitation, this form of excitation tends to propagate at higher frequencies up to about 5 kHz [11]. It is mainly divided into 'wheel/rail unevenness excitation' and 'parametric excitation'. Wheel/rail unevenness excitation occurs when the wheels or rails are rough or irregular in some way, which can occur when there are changes in stiffness, such as at bridge abutments or over transition zones. Parametric excitation, on the other hand, occurs when the track stiffness changes periodically, such as at rail joints or due to sleeper spacing. Due to the increase in the use of continuously welded rails and improved track maintenance, this form of excitation has become less of an issue for the generation of dynamic excitation [12].

Quasi-static and dynamic excitation mechanisms generate forces that travel through the track and soil as seismic waves. These waves are classified as surface waves or body waves. Surface waves propagate along the soil surface and decay progressively with depth [13]. The extent and magnitude of these waves depend on both the geometry and stiffness of the surrounding soil and rocks. Body waves, on the other hand, propagate beneath the surface of the soil.

Compressional/dilatational seismic waves (known as P-waves or primary waves) travel longitudinally and at faster speeds than other waves. Shear waves (known as S-waves or secondary waves) travel in transverse directions and are always slower than P-waves. Rayleigh waves (R-waves) are the most important surface waves and result from the interaction of shear waves with the earth's surface. Among the types of seismic waves, Rayleigh waves are the slowest [14]. The speed of R-waves is between 87% and 95% of the shear wave speed (depending on the material's Poisson ratio). Rayleigh waves have varying propagation speeds as well as particle motions and will dissipate differently with increasing distance from the source. While other types of surface waves also exist (e.g., Love waves), P-waves, S-waves, and R-waves are the most common and will be the focus of this review.

The effect of different types of trains on vibration propagation was investigated by Kouroussis et al. [15] and the differences between quasi-static and dynamic features were explored, as well as defects such as irregularities in the wheel/rail. Kouroussis et al. [15] indicated that a variety of track quality classifications are available; however, it is difficult to compare their accuracy. They advised that vehicle models need to be as accurate as possible, and they also highlighted that should non-linear behaviour need to be included in a model, then time domain simulations are preferable. Lombaert et al. [16] used a numerical model for the prediction of railway-induced vibrations in order to determine whether quasi-static excitations and dynamic excitations are caused by random track unevenness. They effectively applied this approach for the calculation of both domestic and HSTs in Belgium. The main difficulty with the approach is differentiating between the unevenness of the track and the wheel.

3. The Damping of Vibrations

As vibrations propagate through materials, the intensity decreases with distance due to energy dissipation in the form of material and geometric damping. Geometric damping occurs as vibration energy is distributed over a larger surface area as the wavefront propagates away from the source. Material damping occurs due to friction or viscosity in a transmitting medium. The waves' energy is partly transformed into heat energy. These mechanisms are elaborated herein:

3.1. Geometrical Damping

Geometrical (or radiation) damping plays a significant role in the decay of both surface and body waves in elastic materials. Since the incident wave moves away from its source and propagates to infinity, vibration energy is spread over a larger area. The solution for the attenuation of waves was presented in a perfectly elastic half-space by Lamb [17]. Lamb's solution indicates that body waves undergo significant geometrical damping in every direction while propagating through a medium. However, Rayleigh waves propagate only along free surfaces; therefore, they are not exposed to the same radiation damping effects [10]. Rayleigh waves are therefore expected to maintain a higher amplitude (attenuate less) at corresponding points from the source.

3.2. Material Damping

Material damping can be simulated as a form of viscous (velocity-proportional) damping. For multi-degree-of-freedom modelling, a damping matrix can be formulated as a linear combination of mass and stiffness matrices by employing a Rayleigh damping approach [18], expressed as follows:

$$C = \alpha[M] + \beta[K] \tag{1}$$

where: $[M]$ is the mass matrix of the system, $[K]$ is the stiffness matrix of the system, α is the mass proportional damping coefficient, and β is the stiffness proportional damping coefficient. The determination of α and β is often done systematically using:

$$\alpha = \frac{2\xi\omega_1\omega_n}{\omega_1 + \omega_n} \tag{2}$$

$$\beta = \frac{2\xi}{\omega_1 + \omega_n} \tag{3}$$

where ω_1 is the first natural (circular) frequency and ω_n is a subsequent natural frequency (typically second) of the system. The damping ratio ξ is defined as the fraction of critical damping. In the case of a damping ratio of 1.0, the system is critically damped, meaning that it returns to equilibrium without any oscillations. An undamped system has a damping ratio of zero. Using the following formula enables the damping ratio to be expressed in terms of Rayleigh damping coefficients.

$$\xi = \frac{1}{2}\left(\frac{\alpha}{\omega_1} + \beta\omega_1\right) \tag{4}$$

4. Critical Speed and Resonance Effect

For trains travelling on railway tracks, numerical studies and field measurements have shown that vibrations and vertical displacements increase dramatically when a train reaches a specific speed, termed the critical speed [4–7,16,19–33]. This results as a consequence of trains moving at the same rate and in phase with Rayleigh waves travelling along the soil surface. This is a similar phenomenon to the "Mach" effect that results from supersonic jets breaking through the sound barrier [23].

In 1998, in Ledsgård, the Swedish State Railway performed extensive measurements at a location with soft soil ground conditions [4]. When train speeds increased to 200 km/h (near the wave speed in the ground), the rail, embankment, and ground vibration magnitude substantially increased. A maximum displacement of 1.5~2.0 cm was recorded in the track [4,34]. This amount of displacement can lead to derailments. For this reason, the evaluation of critical speed has become a significant issue in railway engineering to enable designers to mitigate against this resonance [32].

In earlier studies, the ground was usually either ignored or included as part of the supporting layer of the track model to create continuous damped elastic Winkler foundations. In these types of models, the critical speed (V_{cr}) can be obtained through [35–37]:

$$V_{cr} = \sqrt[4]{\frac{4sEI}{(\rho A)^2}} \tag{5}$$

where s is the foundation stiffness per unit length, EI is the bending stiffness of the rail, and ρA is the rail mass per unit length. However, based on commonly assumed properties of subgrade stiffness, the estimated speed resulting from this model is approximately 500 m/s [38], much higher than the train speeds expected. Using a half-space ground model provides an alternative method for calculating the critical speed. There are two fundamental wave speeds in an elastic medium, compressional wave (P-wave, C_p) and shear wave (S-wave, C_s), as given by:

$$C_p = \sqrt{\frac{\lambda + 2G}{\rho}} \quad (6)$$

$$C_s = \sqrt{\frac{G}{\rho}} \quad (7)$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad (8)$$

where ν is the Poisson ratio, λ and G are the two Lamé constants; G is also called the shear modulus. Young's modulus can be derived as $E = 2G(1+\nu)$. The Rayleigh wave speed (C_R) for a half-space is slightly lower than the shear wave speed [10]:

$$C_R = \frac{0.87 + 1.12\nu}{1+\nu} C_S \quad (9)$$

Costa et al. [5] developed a track model on a layered ground profile which showed that two critical speeds are reached when the stiffness of the upper ground layer is greater than the lower ground layer. In the lower layer, the critical speed is approximately equal to the shear wave speed, and in the upper layer, the critical speed is approximately equal to the Rayleigh wave speed. For this reason, determining the value of the Rayleigh wave velocity in the soil is essential for the evaluation of critical train speeds (to avoid resonance). According to Krylov [39], Rayleigh waves typically travel in the soil at speeds of 250 to 500 m/s at small strains; however, this can be less than 100 m/s (360 km/h) for very soft soils (as seen in Table 1). As an example, peat, organic clays, and soft marine clays can have a Rayleigh wave velocity that is as low as 40–50 m/s, which is significantly slower than high-speed train's operational speeds [4].

Table 1. Range of (P), (S), and (R) wave velocities at small strains for different soil types [40].

Soil Type	P-Wave Velocity (m/s)	S-Wave Velocity (m/s)	Rayleigh-Wave Velocity (m/s) ($C_R \approx 0.9\ C_S$)
Water	1450	0	0
Glacial till	600–1800	300–600	270–540
Dry gravel	500–1000	250–400	225–360
Saturated gravel	1450	300–400	270–360
Dry sand	300–600	150–200	135–180
Saturated sand	1450	150–250	135–225
Silts and stiff clays	1450	100–200	90–180
Plastic clay	1450	50–100	45–90
Organic soils	1450	30–50	27–45

As seen in Table 1, specific train speeds are more likely to result in resonance effects in certain soils when these coincide with the wave speeds propagating through the soil media [10]. Due to the fact that waves begin propagating at the cut-off frequency in the soil, this means waves cannot propagate outward below this frequency, resulting in no radiation damping. According to the literature, this frequency is related to the layer depth and the velocity of the P-wave in the top layer [15,30,41].

5. Background to Railway Modelling

Both the speed and weight per axle of trains have increased year on year, which has caused new problems for railway geotechnics and train dynamics. The main issue relates to increasing dynamic track-ground amplification occurring as a result of train speeds exceeding approximately half of the small-strain natural wave propagation velocity of the track-ground model, thereby inducing resonance [42]. Moreover, it has become understood that for lower speed lines, it is essential to take into account the static stiffness of the track-ground model, whereas, for high-speed lines, dynamic stiffness must be taken into consideration in addition to static stiffness [42].

The analysis and simulation of railway track/foundation coupling and dynamic behaviour have been popular research topics for many researchers [31,43–48]. Significant research has been undertaken with a focus on using multi-degree of freedom (MDoF) and FEM techniques. Models have advanced from simple moving load models to coupled vehicle–track interaction, and various papers have been published about vehicle–track-ground interaction in the frequency domain [11,49,50]. Some solutions focus on the dynamic issues of moving loads on infinite elastic foundations using Fourier transform techniques with moving coordinate systems [51,52]. Converting track structures into infinite Euler–Bernoulli beam-on-elastic-foundation models was made more explicit by Jezequel [53], who modelled train loads as concentrated forces with uniform motion with a consideration of rotation and transverse shear forces. Solving differential equations for moving loads on simply supported beams in the time domain was undertaken in [54]. Through analytical methods, Warburton [55] demonstrated that at a certain speed, the deflection of beams would reach a maximum under moving loads. The dynamic response of infinite beams over periodic bearings was examined using modal superposition in [56] under moving loads.

As summarised above, many researchers have considered the track beam as part of a continuum and solved the differential equations through analytical methods. While these approaches are simple, they are not appropriate for multi-DoF vehicle–track models and provide limited insight into track dynamics [57]. In recent years, the finite element method (FEM) has been widely used in many engineering studies as an alternative to analytical modelling and is a popular solution to model train–track dynamics. The following subsections provide information related to analytical and numerical approaches for railway modelling and briefly cover the history of these modelling concepts in railways.

5.1. Analytical Models

Earlier studies on track vibration caused by moving loads neglected the ground or simply combined it with the track model to create a continuous damped elastic Winkler foundation. In addition, vehicles were modelled as multiple sets of moving loads, which omits any contribution from vehicle dynamic behaviour. Analytical models in railway modelling are based on some simplifications, such as the use of beams, springs, and dashpots to represent the track structure and foundation. Due to these simplifications, analytical methods are not suitable for detailed analysis of track under different conditions such as temperature change and dimensional effects (for instance, springs and dashpots are not able to take account of layer depth, width, and height). In many studies, Euler–Bernoulli beams have been used as rail models, which neglects any contribution from shear deformations (as can be incorporated using Timoshenko beams). Track substructures are generally considered elastic in simplified analytical approaches.

One of the most popular methods of track dynamic analysis employs beam on elastic foundation models, as shown in Figure 2, which were initially proposed by Winkler in 1867 and developed later by Zimmermann in 1887 [38]. While beam on elastic foundation models are straightforward to implement, they also have several drawbacks: (i) A ballast layer and supporting rail model are not taken into account in these models; (ii) Winkler formulations do not consider the effects of track geometry, including sleeper distance (cross ties) and ballast depth, as well as characteristics of subgrade materials (subgrade stiffness is the only parameter in the Winkler model to describe the subgrade's physical behaviour); (iii) it is not possible to consider longitudinal forces due to thermal stresses; and iv) the model does not account for subgrade inertia and damping forces.

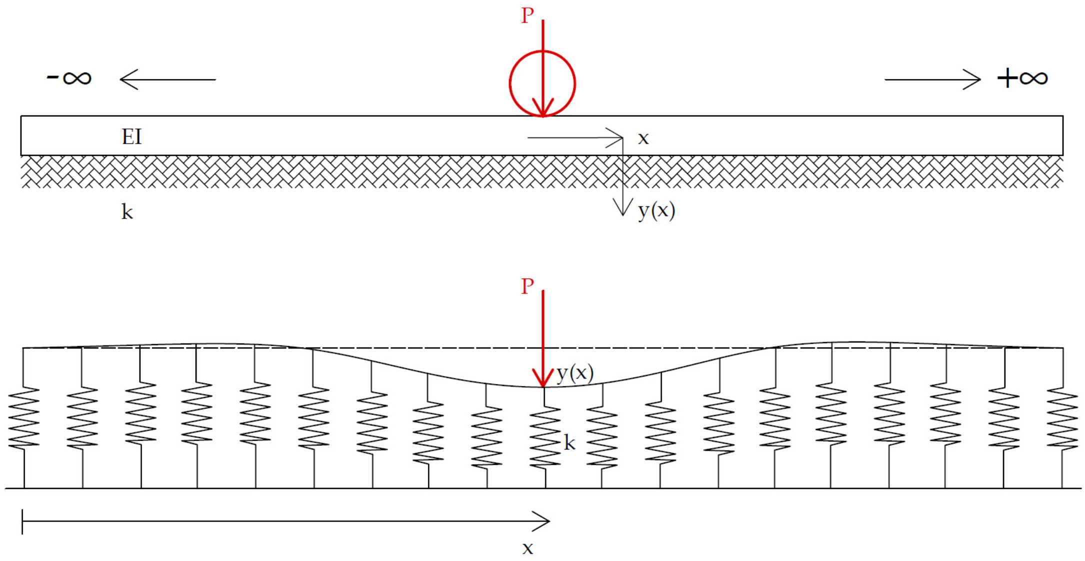

Figure 2. Beam on elastic foundation [38].

The trackbed support is assumed to be continuous in a Winkler formulation, but the rail in ballasted tracks is supported at discrete distances. This is referred to as a discretely supported beam model. The rail is supported by the track structure at discrete points with sleepers evenly spaced along the rail. In this discrete model, rail support is only taken into account at a sleeper's position. Each spring's stiffness is determined according to its transverse length on the rail footing to which it is attached. A solution to this problem was provided by [58] based on the Castigliano theorem. This solution was compared to that from the Winkler model, resulting in similar results [59]. In 2003, Kerr [59] also assumed a rigid rail–tie connection and a continuous moment at the rail support.

Since the development of beam on elastic foundation models, which provided a reasonable approximation for train–track analysis, more detailed models have been developed to more accurately represent track responses to train loads. The double beam model is well-known for its application to slab track modelling. As illustrated in Figure 3, this model contains two beams: the upper beam models the rails, and the lower beam represents track substructures such as bridges or slabs. A number of researchers have examined slab track behaviour using models of this nature, for example, Hussein and Hunt [60], who used the dispersion equation directly to determine the critical load velocity due to the effect of floating slabs on vibration attenuation. The vehicle–track interaction problem under dynamic loads was analysed by [61] using a double beam model. According to their findings, rail deflections increase with train speed. Another example of double beam modelling was presented by [62], who investigated the effects of double beam track on far-field vibrations, and their results showed that the track–soil model is longitudinally invariant, allowing for an efficient solution in the frequency–wavenumber domain.

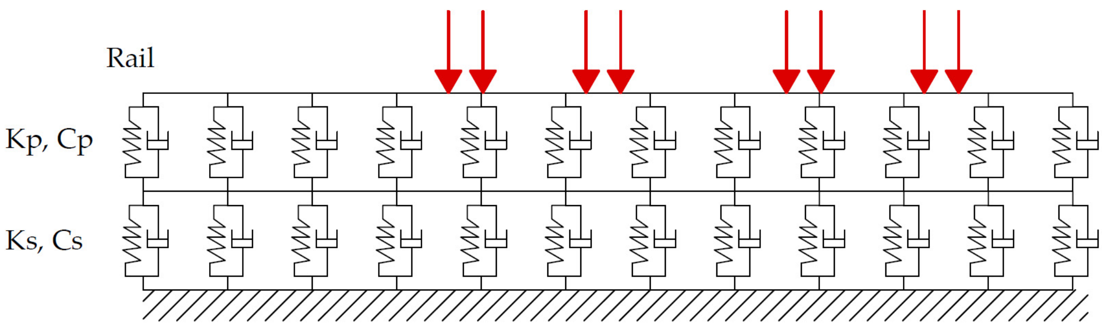

Figure 3. Double beam model [60].

5.2. Numerical Models

Since analytical methods do not facilitate in-depth analysis of railway track/foundation coupling under varying conditions, numerical models have also been developed in addition to the analytical models. Numerical modelling allows for a more detailed simulation of different aspects of the interaction process as well as an integrated model of the various layers. Additionally, moving loads and nonlinear material constitutive laws can be studied in three-dimensional models. Within certain spatial and frequency limits, numerical modelling provides more accurate results in analysing dynamic problems compared to analytical models [10]. To simulate the dynamic response of railway tracks, numerical methods such as the finite element method (FEM), boundary element method (BEM), finite difference method (FDM), and discrete element method (DEM) are usually used. Of these, FEM is the most common numerical approach used to simulate railway track responses [10].

FEM is used for a variety of modelling applications, primarily due to its advantages over other numerical methods. The use of finite element (FE) modelling, for example, provides the capability to define a track's geometry in detail as well as considering advanced constitutive models (such as Drucker–Prager or Mohr-Coulomb) to define material behaviour. It can therefore accurately simulate the response of railway tracks and the propagation of waves induced by trains moving over the ground. It is important to note that if model boundary conditions are not adequately introduced, this can lead to misleading results. For example, when the mesh boundaries of a model are constrained or fixed, the waves generated by dynamic loads will reflect from the boundaries rather than propagating continuously [63,64]. For this reason, dynamic models require large model domains (far boundaries), significantly larger than those used in static models, or they require the implementation of absorbing boundaries capable of reducing reflections. It is suggested that for an accurate representation of the wave transmission through the model, the size of elements should be less than or equal to one-eighth of the wavelength corresponding to the maximum frequency of interest [65].

The number of calculations required in two-dimensional FEM is much lower than in three-dimensional FEM, significantly reducing the computational costs. Computing requirements for large scale dynamic train–track models are a significant challenge; thus, a concerted research effort has been put into increasing modelling efficiencies. 2D numerical models have been developed that can simulate the dynamic interaction between a lumped mass vehicle and a discretely supported continuous rail track at the wheel/rail interface [66]. Models of this nature have been used in several studies [11,38]. A 2D dynamic FEM using ABAQUS was developed in [67] to examine stress changes with the effect of train speeds, acceleration/braking, rail surface shape, and unsupported sleepers during the passage of a train. The results indicated that dynamic effects cannot be discounted, and it was observed that the stresses due to dynamic factors increased significantly when the train speed was greater than 10% of the Rayleigh wave speed.

The 2D simulation of ground vibration from high-speed trains was conducted in [34], where the track embankment was modelled as a viscoelastic beam and the ground was modelled as a viscoelastic half-space separated into layers. The numerical solution demonstrated the effectiveness of strengthening the embankment for mitigating ground vibration. The disadvantage of 2D railway modelling is that one dimension, generally the perpendicular axis to the direction of the train passage, is disregarded. To facilitate more accurate modelling of vibration along railways, 2.5D models have been developed. In these models, the considered domain is 2D, while the considered excitation is 3D. The advantage of this is that the computational requirements to run a 2.5D model are lower than those for running a full 3D model if reduced accuracy is acceptable.

A 2.5D finite element method capable of handling complex geometry and materials was proposed by [68], and they proved that it is possible to simulate exactly the behaviour of waves generated by moving loads at all speeds by using finite/infinite elements. They additionally studied [41] the transmission of vibration induced by trains for sub-critical and supercritical speeds to estimate the effects of each parameter (such as the shear wave velocity, damping ratio, and depth of the soil layers) on the ground response induced by moving trains. Yang et al. [41] found that by increasing the shear wave speed of soils, the responses can be reduced for both speeds; however, by increasing the damping ratio, the response is only reduced for the supercritical speed. Furthermore, a model was developed in Ref. [69] for determining the ground vibration induced by trains moving either on the ground surface or in tunnels, and the authors demonstrated that the wavenumber-based modelling approach (2.5D) can be applied to surface vibration and tunnel vibration analyses. Experimental works (homologation tests of the new high speed track on the line L2 between Brussels and Köln) were validated in Ref. [70] using a 2.5D FEM model, which accounts for the dynamic interaction between the train, track, and soil, and relatively good agreement was shown. This numerical model was also used to determine the quasi-static excitations resulting from railway-induced vibrations as well as the dynamic vibrations caused by random track irregularities [16]. The results revealed that the track response is dominated by quasi-static contributions, while the free-field response is dominated by dynamic contributions.

High-speed train induced ground vibrations using 2.5D models were simulated by Bian et al. [71], and the results suggested that the ground vibrations of a moving train are limited to a local area when it moves at a low speed. When the moving speed approaches or exceeds the critical speed of the specific ground–track model, the amplitude of ground vibration and the area of impact increases significantly. A 2.5D model was presented in Ref. [72] to determine vibrations induced by trains, including the train/track dynamic interaction. The numerical model was calibrated with experimental measurements implemented on a section of the Portuguese railway network. It was found that wave propagation mechanisms are strongly influenced by factors such as local inhomogeneities in stiffness and damping of the ground.

While 2.5D models can achieve similar levels of accuracy to full 3D models, they are highly dependent on wavenumber sampling, which involves determining the wavenumber range and the number of the sampling points. In order to obtain high levels of accuracy, wavenumber sampling must be high, which means that the computational requirements become similar to those required for 3D modelling [12]. The 2.5D modelling approach is too computationally intensive to be practical for preliminary studies (as compared to 2D models). Also, it is not sufficiently flexible to be used in detailed investigations. By explicitly modelling the third dimension, 3D models avoid many pitfalls related to 2.5D models. This enables the modelling of almost any track and its surroundings in a geometrically realistic manner. Even though they face major problems in terms of modelling efficiency, 3D models do not suffer the same computational issues that 2.5D models experience [42].

Full 3D models (as schematically shown in Figure 4) are advantageous for analysis in time or frequency domains when track properties and/or geometries are complicated [73]. For frequency domain analysis, the implementation of absorbing boundaries is less complicated than in time domain simulations [74]. In contrast, time-domain analyses [75–77] are more suited to implementing nonlinear material models and wheel–rail contact algorithms [7,48].

A 3D model where track geometry is analysed as a periodic elastically coupled beam model on a Winkler foundation was introduced in Ref. [78] using spring and dashpot elements, representing the mechanism of rail pads, ties, and the ballast bed. A theoretical study was conducted regarding ground vibrations created by superfast trains close to or greater than 300 km/h, taking into account the contribution of each track sleeper subjected to the action of the carriage wheel axles [23]. Site measurements for a high-speed train on a soft soil site in Sweden [4] were carried out, and the resulting dynamic response was analysed in detail. Through measurements, it was discovered that track responses amplify when trains pass at speeds that approach an apparent critical speed, in agreement with Refs. [1,3,27]. Furthermore, it is suggested that the critical speed is controlled by the minimum phase velocity of the first Rayleigh mode of the soil and embankment profile at the site. A full 3D analysis and a 2.5D model were conducted using the coupled FE/BE method for high-speed train–track–soil–structure dynamic interactions in the time domain in Ref. [74], and the dynamic behaviour of a transition zone between ballast and slab tracks was investigated. It was found that to obtain accurate results for the problems presented by the transition zone, a three-dimensional model should be used instead of a 2.5D model. The authors also pointed out that for predicting track and soil responses at low frequencies, the suspended mass should be considered.

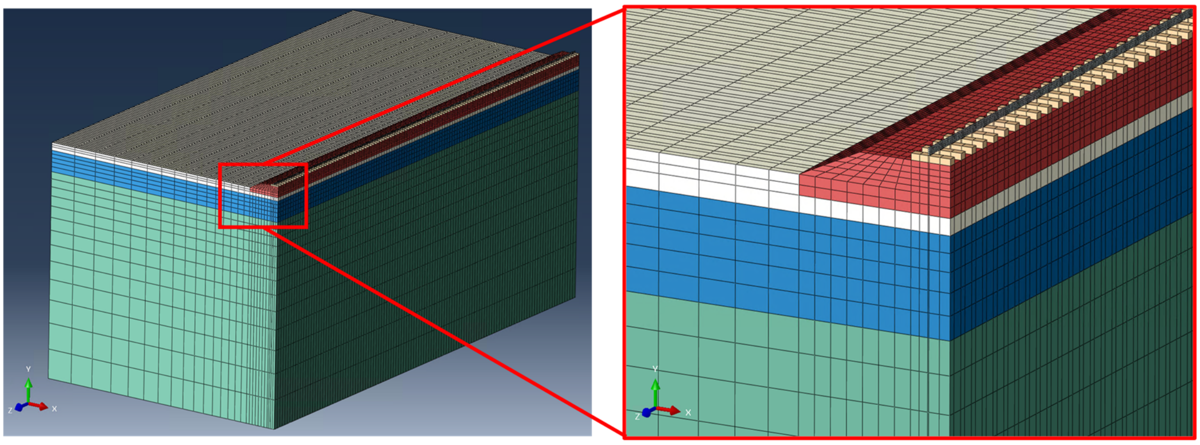

Figure 4. Example of a 3D model on ABAQUS.

An uncoupled approach for the 3D vehicle–track–soil model was used in Ref. [79]. The vehicle–track subsystem was first simulated to estimate the ground forces, which in turn were applied to the model of the soil. In another study [80], a 3D model was developed whereby a time-domain explicit dynamic finite element model capable of simulating nonlinear excitation mechanisms was implemented. The soil structure was modelled using an elongated, spherical geometry to increase boundary absorption performance. A 3D FEM for the three-slab track structure was used in [81] to find the fatigue life design of concrete slab tracks and provide the principal vibration modes. A train–track dynamic interaction model with component damping was investigated computationally using 3D FEM by Ref. [30] to estimate the vibrations induced from trains at high-speeds, and it was found that an increase in track deflection occurs, especially when the train exceeds the Rayleigh ground speed. This is in agreement with Refs. [1,3,6,27,31,48].

Some applications require the modelling of train-induced dynamic ground effects on a large scale in terms of the depth and width of the soil domain [64,77,82]. According to Ref. [42], soil modelling methods which use shallow soil dimensions can be accurate at low speeds, but as the speed increases and the dynamic effects become more prominent, the soil dimensions used in the analysis need to be larger to avoid the generation of errors.

6. The Dynamics of Slab Track Railways

Slab track railways have been the dominant infrastructure for high-speed rail, and as train speeds increase, there has been a growing need for further research into problems such as the wheel–rail interaction, vibration, and noise. Many authors have therefore devoted their studies to highlighting these problems to support the rail industry in their developments [83]. Over forty years, slab tracks have used Japanese Shinkansen systems due to the negative experiences associated with the use of ballasted tracks during the operation of the first high-speed line. Slab lines have demonstrated significantly reduced track maintenance costs and maintained good track geometry [84]. Moreover, it has been shown that maintenance costs for the Sanyo Shinkansen line reduced by 25% between 1975 and 1998.

Slab track performance was compared to a ballasted track using 2D FEM analysis in Ref. [85], and the results showed that the slab track reduced train-induced vibrations and track level displacements by almost 50% when compared to the ballasted track. The performance of the slab track was studied with a prototype fatigue test considering cycling load, temperature variation, and water erosion in Ref. [86]. The dynamic response of different track components and conditions showed that while fatigue damage for the slab structure is lower, the cement asphalt (CA) mortar used between the precast upper slab and an in-situ concrete base deteriorated quite rapidly from the edge and corner of the precast slabs. A 3D FEM was developed in Ref. [87] that covers all infrastructure components such as the fastening system, pad, concrete slab, frost protection, and hydraulically bound layers, and the numerical model was calibrated dynamically using a full-scale test rig. Good agreement was achieved between the numerical and experimental results for the seven load scenarios considered. Experiments were carried out in Ref. [88] using a 55.17 m long test rig where the dynamic performance of four typical China Railway Track Systems (CRTS) with operating wheel-drop loads and four different slab track variations were compared and discussed. Four different 3D slab track cross-sections and track–soil interfaces were modelled in Ref. [89] by estimating the vertical free-field response to trains moving along the track at a fixed speed. It was discovered that in-plane track–soil coupling significantly influences the vertical vibration in the free field. Considerable differences were obtained for a thin slab vs a thick slab, where a thick slab provided more satisfactory responses. Two interface interactions were considered in Ref. [90] for a slab track: the longitudinal resistance between the rail and rail pad, and the cohesive longitudinal force between the slab and CA mortar layers. A 2D vehicle-slab track vertical-longitudinal coupled dynamic model was established by applying the longitudinal motions and interface interactions. The established model could capture the longitudinal interactions between the track structures and was capable of considering the evolution of damage at the interface between the track slab and CA mortar subjected to complex train dynamic loads.

A dynamic model for a slab track model was presented based on uniform vibration excitation using MATLAB in Ref. [91]. The effects of track parameters, vibration intensity, and running speed on the dynamic responses and vehicle running safety were analysed, and the findings indicated that track parameters have a significant effect on the dynamic responses. It has also been observed that the vibration intensity has a significant effect on the derailment coefficient, which is defined as the ratio of the lateral force to the vertical force. A dynamic shakedown solution, which is a technique used to assess the safety factor for structures subjected to cyclic loading against instantaneous plastic collapse, fatigue, and excessive accumulated strains, was found for slab track under train loads at different speeds, and the critical speeds were obtained by plotting the shakedown limits versus train speeds [92].

A dynamic model based on track, embankment, and ground was created in Ref. [3] using 2.5D FEM, and the critical speed for slab track was found to be higher than the Rayleigh wave velocity of soil, in contrast to the findings of [5]. The underlying soil stiffness is the main factor in controlling the track vibration amplitude. The dynamic performance of a low vibration slab track on a high-speed passenger and freight railway was studied in Ref. [93], where an optimal modulus of the isolation layer was implemented (rubber pad to cover the adaptability of the track system to the dynamic actions of both high-speed passenger and heavy axle-load freight trains). 3D FEM models were developed in Ref. [94] for vehicle–slab–track interactions with an asphalt support layer (ASL), and the dynamic behaviour of the ASL was investigated using both modelling and in-situ measurements. The results suggested that the zone of influence for the dynamic responses in the ASL reached almost 8 m in the longitudinal direction under the moving loads and a thicker ASL is more appropriate for riding comfort and structural stability in high-speed railways. After this study, the effects of temperature on ASL in slab track railways (see [95]) were investigated both numerically and via in-situ measurements. This study showed that temperature is distributed nonuniformly within the ASL under meteorological factors including solar radiation intensity, air temperature, and wind speed, and that the temperature of the ASL is more sensitive to sunlight and wind speed during hot months.

The effect of dynamic material properties on train–track vibration interactions was highlighted, and a nonlinear 3D FEM vehicle slab track model was developed based on multi-body simulation [96]. Three types of material properties were used: static stiffness for rail pads and static moduli of elasticity for concrete and CA mortar, dynamic stiffness for rail pads and static moduli of elasticity for concrete and CA mortar, and dynamic stiffness for rail pads and strain-rate-dependent moduli for concrete and CA mortar. In this study, the authors recommended adopting suitable and realistic material properties of high-speed slab tracks in practice. The performance of a vibration isolation mat for a slab track model was studied by [97] using 3D FEM with a moving load at a particular speed, and the isolation mat was modelled as a continuous viscoelastic layer between the track slab and the supporting plate. In another study [98], rubber mats were implemented with the aim of improving the dynamic performance of track transition zones, and these offered a solution for the transition zones where severe vibration and deterioration in track structures gradually changed the stiffness.

Furthermore, transition zones between three different track typologies: ballasted track, asphalt slab track, and concrete slab track were studied by [99]. 3D FEM dynamic analysis was used in this study to estimate the transition behaviour between the typologies and the accelerations induced along the transition zones. They found that the stiffness gradient can be improved in the transition area either by reducing the elastomer stiffness in the concrete slab track or by using resilient mats under the concrete slab track.

A vehicle–rail–slab track coupled model [100] was developed using a periodic structure to mitigate the vibration responses of track structures, and the results showed that the proposed slab track design can adequately decrease vibration levels.

Mud pumping, which occurs as the fast-upward movement of fine subsoil particles from a saturated foundation through the voids [101], is a typical issue in railway infrastructure. Mud pumping only arises in expansion joints in the slab track structure. This issue was investigated in Ref. [102], and its influence on the dynamic behaviour of the slab track–subgrade was studied via a field investigation and a full-scale experimental model. The authors realised that mud pumping occurred only at expansion joints in the concrete base and that subgrade settlement increased continuously over time.

Many researchers have examined the critical speed phenomenon for ballasted railways. For slab tracks, the critical speed was investigated using 2.5D FEM [3] to estimate the vibrations produced by a high-speed train. The results showed that the critical speed of the slab track is higher than the Rayleigh wave velocity of the underlying soil and quite close to the Rayleigh wave velocity of the subgrade. The underlying soil stiffness is the crucial factor in specifying the track vibration amplitude. Hu et al. [3] also found that the embankment plays a vital role in reducing the heterogeneous lateral stress propagation and the amplitude of vertical stress in slab tracks. The dynamic response of slab track was also studied with different train speeds and subsoil stiffnesses, and the critical velocity was modelled using 2D FEM in Ref. [103]. According to this study, the critical velocity at which maximum dynamic amplification occurs for subsoil is completely different from that observed for concrete slab tracks, and a similar finding was also described by Krylov [23].

7. Soil Modelling in Train–Track Interaction

With increasing train speeds, elastodynamic waves propagate deeper into the soil, and this wave energy creates large strain levels, leading to reduced soil stiffness due to nonlinear behaviour. The situation results in an increase in track displacement [29]. Laboratory tests have shown that soil stiffness is related to various parameters, including cyclic strain amplitude, void ratio, mean principal effective stress, plasticity index (PI), overconsolidation ratio, and the number of loading cycles [13]. The shear modulus is the most important parameter for modelling nonlinear soil behaviour, especially at shear strains greater than ~10^{-5} to 10^{-4} [104]. It has been found that the shear modulus will reduce with increasing shear strain over this level [105]. This is sometimes referred to as shear modulus degradation (or stiffness degradation). Compared to the frequency of the loading and number of loading cycles, the over-consolidation ratio, void ratio, degree of saturation, and grain characteristics, Ref. [105] showed that shear strain, mean effective confining stress, soil type, and PI were the principal factors affecting shear modulus degradation.

Increasing shear strain results in an increase in the damping ratio. Besides evaluating the shear strain level, confining stress, soil type, and PI, it is also essential to examine how frequently and how many times a load is applied [105,106]. As a result, linear elastic models are usually used to model ground vibration caused by railways. However, when the train speed is close to the critical speed, the soil behaves nonlinearly, especially in the region near the track. The degradation of soil cannot be accounted for by linear models. Thus, soil nonlinearity needs to be considered to improve predictions.

7.1. Equivalent Linear Model

As an alternative to fully nonlinear calculations, equivalent linear models provide a simple way to approximate nonlinear behaviour. In several studies, equivalent linear models have been used to include nonlinear soil effects in the study of passing train loads [4,7,34,77,82,107,108]. In order to achieve convergence, it is necessary to apply an iterative procedure based on the effective shear strain. Convergence is achieved when the difference between the shear modulus and damping ratio obtained in successive iterations is less than about 10% [13]. 'Effective' shear strain amplitudes are determined by proportional values of the maximum shear strains from the time-varying strains derived from initial calculations based on the smallest damping ratio and maximum shear modulus. Based on the measured curves, an updated value of the shear modulus and damping ratio is obtained, as shown in Figure 5. This cycle is repeated until the corresponding modulus is close to that of the previous iteration.

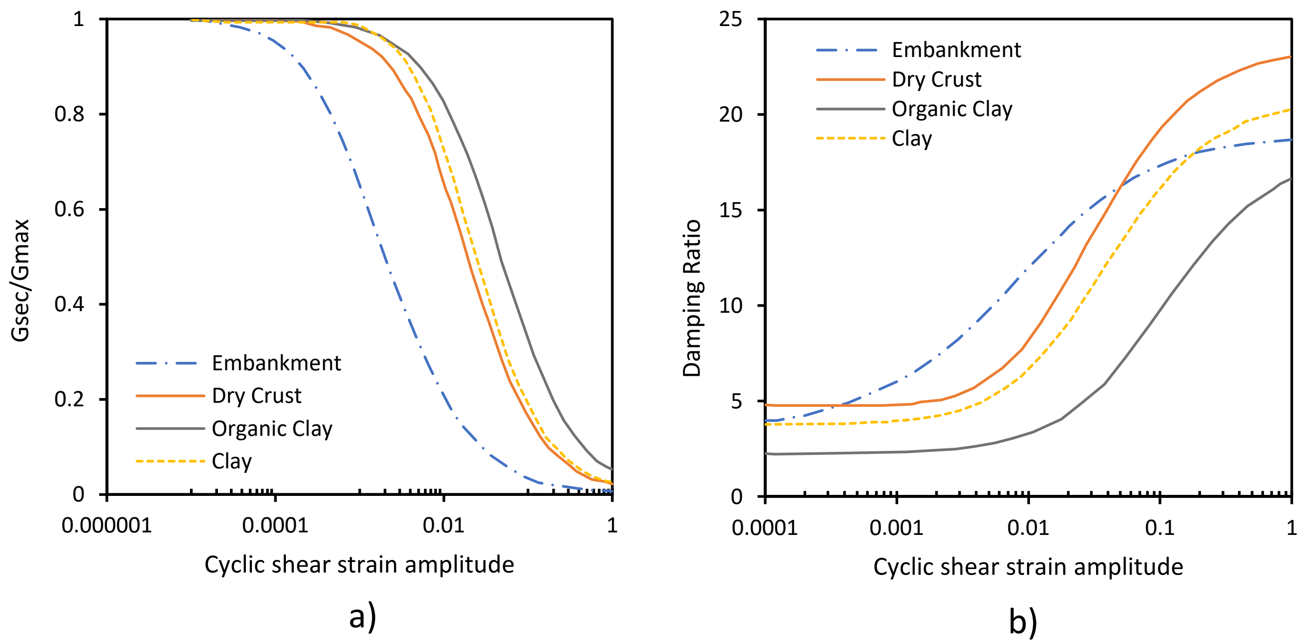

Figure 5. Example of (**a**) soil shear modulus and (**b**) damping ratio versus shear strain for certain soil types [82].

7.2. Constitutive Models

A constitutive model typically requires a yield surface, which describes the condition at which elastic behaviour appears. The hardening rule specifies how material size and shape at yield surfaces change as plastic deformation occurs, and a flow rule relates plastic strain increments to stresses. Constitutive models provide flexibility and generality in modelling soil responses to cyclic loading for general initial stress conditions. Other factors are cyclic or monotonic loading, rotating principal stress axes, high or low strain rates, a wide variety of stress paths, and drained and undrained conditions.

A 2D nonlinear model with plane-strain was developed by Celebi and Kirtel [109] and Celebi and Goktepe [110] with an elastoplastic Mohr-Coulomb constitutive law, and they conducted an analysis of the performance of different vibration reduction strategies, i.e., waves, trenches, and wave impeding barriers (WIBs). They found that mitigating structural vibrations is highly dependent on the location of the installation of reduction strategies. The influence of soil constitutive models (hardening soil model with small-strain stiffness (HSsmall) and Mohr–Coulomb (MC)) on the study of railway vibrations in tunnels is discussed by [111], who showed that the HSsmall model is more accurately suited to real data than the MC model with stiffness E_{50} (Young's modulus at 50% failure load). However, when considering the Mohr–Coulomb model with stiffness E_0, the use of HSsmall does not seem to be advantageous. A FE approach coupled with a Drucker–Prager isotropic hardening model was presented by [112] to assess the properties of capping layer soil contained in a semi-confined mould subject to monotonic or cyclic penetration. It is possible to reduce the stresses developed on the top of weaker layers below by using a thicker capping layer; however, according to large-scale experiments, the thicker the capping layer, the more prone it is to showing large deformation.

8. Using Asphalt on Railways

There have been various materials applied to railway tracks that consider the changes in train speeds, annual ton-miles (a metric used in quantifying traffic intensity), and axle loads associated with heavy axle trains and high-speed passenger lines. Also, to improve trackbed stability and mitigate ballast degradation, certain authors have proposed innovative solutions and technologies. In this regard, it has been discussed in the literature that the use of elastic elements such as under-sleeper pads, ballast mats, or crumb rubber mixed with ballast aggregate can contribute a significant reduction in track deterioration and vibration [113–115]. Moreover, these solutions enable the adjustment of the vertical stiffness of a track to a required level for each particular point. However, it is important to note that elastic elements within (in high amounts) or beneath the ballast layer might decrease the bearing capacity and consequently increase the overall settlement [116].

An asphalt layer within the railway substructure is another solution that is used in many countries to enhance performance, optimise stress distribution, dampen dynamic loads, and reduce vibrations [117–121]. A number of early works focused on applying asphalt layers to the railway structure [117,122–126]. These studies concluded that the asphalt layer contributed to the performance, especially for the vibration attenuation of ballasted track. Fang et al. proved that the asphalt layer is able to reduce the maximum vertical deformation of the subgrade [120]. According to Yu et al., the pressure on the subgrade was 40.6 kPa for ballasted track, but this was reduced to 11.7 kPa in their hybrid asphalt-ballast track, which is a 72% reduction [127]. It has been also demonstrated with full-scale testing that asphalt trackbeds are effective at reducing noise and vibration due to their viscoelastic properties [128]. On the other hand, studies on slab track incorporating an asphalt layer are sparse.

Asphalt layers in railway substructures were analysed using FEM [120,129,130] in Ref. [129], and researchers found that the asphalt increases the resilient modulus and stress distribution while decreasing vibrations. A full-scale static test was reported by [131,132] to evaluate the performance of an asphalt trackbed that can support a railway track without experiencing major cracks. It was observed that after the static loading was applied, any permanent deformations were less than 2 mm. The finding is directly in line with another study [118], where it was reported that asphalt mixtures successfully resisted fatigue cracking and permanent deformation.

Many new tracks are being constructed as slab tracks in order to increase the durability of the tracks (against high speed and heavy-haul train loads) and enhance maintenance efficiency. Slab tracks, however, have considerably higher noise and vibration levels than traditional ballasted trackbeds [133]. Based on research studies [122,134], there has been no damage or cracking reported in viscous asphalt layers used in ballasted high-speed railway substructures after many years of service under widely varying conditions. Also, according to [123], the rutting issue, which refers to the gradual accumulation of permanent deformation in asphalt over time, does not cause a significant problem on asphalt tracks because the stress from train loads is distributed over a sufficiently wide area. German authorities approved the Getrac system, which is effectively an asphalt slab track, as a ballastless track for high speeds, and research has shown that Getrac has similar advantages to other ballastless tracks while being quick and easy to install [135].

9. Discussion and Recommendations

In this paper, the authors have reviewed research that deals with the prediction of dynamic effects on high-speed railway lines by implementing analytical methods, numerical modelling, and full-scale testing. This section discusses and summarises some of the reasons for the results obtained. Based on the insights gained, some research recommendations are offered.

As summarised above in Section 5.1, there have been many researchers who have examined track dynamics using analytical methods. Although these methods are simple, they provide limited in-depth insight into railway track/foundation coupling under varying conditions [10].

The numerical modelling in Section 5.2 is shown to better simulate railway dynamic responses. However, even though better results are achieved when using numerical approaches, they face major problems in terms of computing efficiency [42]. Unlike static models, dynamic models either require large model domains (away from boundaries), significantly larger than those used in static models, or they require absorbing boundaries capable of reducing reflections [63,64], as used in Refs. [94,136,137]. Additionally, the mesh size should be less than one-eighth of the shortest wavelength to ensure accuracy [65]. On the other hand, finer meshes exponentially increase computational costs. For this reason, the mesh size needs to be chosen with caution.

The use of finite element (FE) modelling also provides the capability to define track nonlinearity by considering advanced constitutive models (such as Drucker–Prager or Mohr–Coulomb). Based on the literature, it would generally appear that in the presence of elastodynamic waves in the soil, large strain levels lead to a reduction in soil stiffness, referred to as nonlinear behaviour. Stress and strain in the soil are approximately proportional under small strains, so the soil response can be considered linear and elastic at these strain levels, as many researchers have assumed. This, however, is not always the case, and it is important to consider soil nonlinearity to enhance predictions. Nonlinearity has been considered in several studies [4,7,34,77,82,107–110,138], as discussed above, and relatively good agreement with field measurements has been found. However, all these studies were applied on ballasted tracks in terms of both equivalent linear and constitutive approaches. As a result, there is a shortage of models that consider soil nonlinearity when simulating dynamic effects on high-speed slab track lines, as seen in the summary in Table 2. This is therefore an area where further study is recommended.

Table 2. Summary of research findings.

Ref.	Ref.	Method Info.	Track Layers	Nonlinear Behaviour	Boundary Conditions	Focus	Model Length	Train Speed (km/h)	Parameters Studied
[3]	(Hu et al., 2019)	2.5D FEM	R + F + S + CAM +BC + RB + Sub + G	-	-	Critical speed	-	360	VerAcc, VerDis, VerStress
[85]	(Kece et al., 2019)	2D FEM	S + CF + E + G	-	IB	The effects of subsoil stiffness, track speed, and track type	-	150 to 400	VerDis
[87]	(Sainz-Aja et al., 2020)	3D FEM	R + EVAP + SP + EPDMP + TBS + S + GM + HBL + FPL + Sub	-	FB	Calibration of the slab track model	2.2	160–360	VerDis, VerAcc
[88]	(Wang et al., 2017)	FST	R + RP + S + CAM + SL + Sub	-	-	Dynamic characteristics of different slab tracks	55.17	-	VerAcc, VerAcc, TF, TDS, DC
[91]	(Lou et al., 2019)	3D BSDM	R + F + S + SCCL + CL + BL	-	-	Track parameters, seismic intensity, running speed	-	200 to 350	VerAcc, VerDis, TStress, DC
[93]	(He et al., 2018)	3D BSDM	R + F + S + GM + RuP + SL + G	-	FB	Optimal modulus of the rubber pad	5.6	160–400	VerDis, CStress, TStress, VerAcc,
[94]	(Liu et al., 2019)	3D FEM	R + S + SCCL + BP + US + LS + Sub + G	-	VB	In-depth study of asphalt layer	16.8	350	VerStress, LatStress, LongStress, VerDis, VerAcc
[95]	(Liu et al., 2020)	3D FEM	R + F + S + SCCL + BP + ASL + Sub	-	AB	Temperature features of asphalt layer	5.6	-	T, SR, WS
[133]	(Lee et al., 2021)	FST	R + RP + S + GT + SB + US + LS	-	-	Effect of different thickness of asphalt layer	20	140	SP, VerDis, TS, SL, TStrain, Cstrain
[136]	(Feng et al., 2017)	3D FEM	R + S + CAM + BC + SU + LS + G + BB	-	VB	The effects of subgrade treatment and ground vibration	70	300–360–667	VerDis, LongDis, VerAcc
[137]	(Bian et al., 2015)	2.5D FEM	R + F + S + CAM + BC + RB + Sub + E+ + SCM +SC + Si	-	FB	Track irregularities	-	100–700	VerDis, VerAcc

Table 2. *Cont.*

Ref.	Ref.	Method Info.	Track Layers	Nonlinear Behaviour	Boundary Conditions	Focus	Model Length	Train Speed (km/h)	Parameters Studied
[139]	(Chen and Zhou, 2018)	3D FEM	R + RP + S + CAM + SL + US + LS + E + G	-	VB	Dynamic responses of different train speed and line patterns	-	250–300–360	Vertical stress, VerDis, VerAcc, VerAcc
[140]	(Chen and Zhou, 2020)	3D FEM	R + F + S + CAM + SL + US + LS + E + G	-	FB	Effect of subgrade, foundation modulus, and fastener stiffness	10	360	VerDis
[141]	(Tang, Xiao and Yang, 2019)	3D FEM	R + F + S + CAM + BC + RB + US + LS + Sub + GC	ELM	IB	Geosynthetic-reinforced pile foundation	58	200 to 550	VerDis, DS, VerAcc
[142]	(Yang et al., 2015)	3D FEM	S + BC + ASL + US + LS + Sub	-	FB	Material composition and mechanical response of asphalt layer	-	-	TransStress, VerStress
[143]	(Thölken et al., 2021)	3D FEM	R + RP + S + GM + HBL + FPL + Sub	-	-	Validation of experimental results	2.2	-	VerDis, VerStress
[144]	(Ramos et al., 2021)	3D FEM	R + RP + S + GM + HBL + FPL + Sub	-	FB	Validation of experimental results	6.2	-	VerDis, VerAcc
[145]	(Shi, Yu and Shi, 2016)	2D BSDM	R + RP + F + S + CAM + TP + SL+ Sub	-	-	CAM deterioration and vibration responses	-	300	VerDis, VerAcc
[146]	(Ntotsios, Thompson and Hussein, 2019)	3D SAM	R + RP + S + HBL + G	-	-	Comparison of ground vibration due to ballasted and slab tracks	-	120–300	VerDis, VerVel
[147]	(Marolt Čebašek et al., 2018)	FST	R + RP + S + GM + HBL + FPL + Sub	-	-	Long-term settlement performance	6.2	-	VerDis, CS
[148]	(Esen et al., 2021)	FST	R + RP + S + GM + HBL + FPL + Sub with geogrid	-	-	Geosynthetically reinforced soil performance	6.2	-	VerDis, CS

Table 2. *Cont.*

Ref.	Ref.	Method Info.	Track Layers	Nonlinear Behaviour	Boundary Conditions	Focus	Model Length	Train Speed (km/h)	Parameters Studied
[149]	(Yao et al., 2016)	SAM	R + RP + S + US + Sub + G	-	-	Ground vibration	-	70 to 432	VerDis, VerAcc, VerStress
[150]	(Yusupov et al., 2020)	3D FEM	R + RP + S + CAM + PCC + US + LS + Sub	-	IB	Effects of different temperatures	80	350	VerDis, LongDis, TStrain

Method Info.: FEM (finite element method), FST (full scale testing), SAM (semi-analytical model), BSDM (beams/springs/dashpots model). **Track Layers:** ACWL (asphalt concrete waterproofing layer), ASL (asphalt support layer), BB (bedrock base), BC (base concrete), BL (base layer), BP (base plate), CAM (cement asphalt mortar), CF (concrete foundation), CL (cushion layer), E (embankment), EPDMP (ethylene propylene diene monomer pad), EVAP (extreme vibration attenuation pad), F (fastener), FPL (frost protection layer), G (ground), GC (gravel cushion), GM (grout mass), GT (geotextile), HBL (hydraulically bonded layer), LS (lower subgrade), PCC (Portland cement concrete), R (rail), RB (road bed), RP (rail pad), RuP (rubber pad), S (slab), SB (sub-ballast), SC (silt clay), SCCL (self-compacting concrete layer), SCM (silty clay with mud), Si (silt), SL (supporting layer), SP (steel plate), SS (subsoil), Sub (subgrade), TBS (twin block sleepers) TP (track plate), US (upper subgrade). **Nonlinear behaviour:** ELM (equivalent linear model). **Boundary Conditions:** VB (viscous boundary), FB (fixed boundary), AB (adiabatic boundary), IB (infinite boundary) **Parameters Studied:** CPUT (CPU Time), CS (cumulative settlement), CStrain (compressive strain), CStress (compression stress), DC (damping coefficient), DC (derailment coefficient), DS (dynamic stress), LatStrain (lateral strain), LatStress (lateral stress), LongDis (longitudinal displacement), LongStrain (longitudinal strain), LongStress (longitudinal stress), MU (memory ssage), SL (strain level), SP (subgrade pressure), SR (solar radiation), T (temperature), TDS (track dynamic stiffness), TF (transfer function), TS (track stiffness), TransStress (transverse stress), TStrain (tensile strain), TStress (tensile stress), TStress (tension stress), VerAcc (vertical acceleration), VerDis (vertical displacement), VerStrain (vertical strain), VerStress (vertical stress), VerVel (vertical velocity), WS (wind speed).

It is also suggested to evaluate the application possibilities for an asphalt layer within a slab track model by taking into account a variety of factors including the local economy, implementation cost, technical requirement, advantages, challenges, and environmental impact.

An overview of recent studies is tabulated in Table 2, classified according to the computational method, nonlinear soil modelling, boundary conditions, research focus, model length, train speed, and the parameters studied.

10. Conclusions

High-speed trains have been widely investigated to understand how waves are generated and propagate in railway tracks, especially in terms of dynamic effects. This review provides an overview of the methods that have been developed for predicting track responses. Data collected in the field after condition monitoring proved to be crucial for numerical model validation. For this reason, a detailed examination of innovative solutions to investigate track performance was carried out. The use of a structural asphalt layer within the slab track is still under investigation, and a consensus on the advantages and disadvantages related to dynamic effects is yet to be reached. However, the slab–asphalt combination is worth examining as a possible effective solution to provide good superstructure bearing capacity and stiffness while improving its dynamic performance and responses, especially for high-speed trains. From a numerical point of view, some notable methodologies were explained that provide a suitably accurate prediction of track and soil dynamic behaviours.

The most recent FEM models were discussed. Some important parameters such as boundary conditions and domain sizes were identified as well as rarely considered concepts such as soil non-linearity. Parameters that were more worth investigating in order to understand the performance of asphalt-slab track systems were highlighted for future numerical models.

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References

1. Hu, J.; Bian, X.; Jiang, J. Critical Velocity of High-speed Train Running on Soft Soil and Induced Dynamic Soil Response. *Procedia Eng.* **2016**, *143*, 1034–1042. [CrossRef]
2. Krylov, V.V. *Noise and Vibration from High-Speed Trains*; ICE Publishing: London, UK, 2001. [CrossRef]
3. Hu, J.; Bian, X.; Xu, W.; Thompson, D. Investigation into the critical speed of ballastless track. *Transp. Geotech.* **2019**, *18*, 142–148. [CrossRef]
4. Madshus, C.; Kaynia, A.M. High-speed railway lines on soft ground: Dynamic behaviour at critical train speed. *J. Sound Vib.* **2000**, *231*, 689–701. [CrossRef]
5. Costa, P.A.; Colaço, A.; Calçada, R.; Cardoso, A.S. Critical speed of railway tracks. Detailed and simplified approaches. *Transp. Geotech.* **2015**, *2*, 30–46. [CrossRef]
6. Sayeed, M.A.; Shahin, M.A. Three-dimensional numerical modelling of ballasted railway track foundations for high-speed trains with special reference to critical speed. *Transp. Geotech.* **2016**, *6*, 55–65. [CrossRef]
7. Shih, J.Y.; Thompson, D.J.; Zervos, A. The influence of soil nonlinear properties on the track/ground vibration induced by trains running on soft ground. *Transp. Geotech.* **2017**, *11*, 1–16. [CrossRef]
8. Freudenstein, S.; Geisler, K.; Mölter, T.; Mißler, M.; Stolz, C. *Ballastless Tracks*; John Wiley and Sons: Hoboken, NJ, USA, 2018; ISBN 9783433030738.

9. Thompson, D. *Railway Noise and Vibration: Mechanisms, Modelling and Means of Control*; Elsevier: Amsterdam, The Netherlands, 2008.
10. Hall, L. Simulations and Analyses of Train-induced Ground Vibrations: A Comparative Study of Two-and Three-dimensional Calculations with Actual Measurements. Ph.D. Thesis, KTH Royal Institute of Technology, Stockholm, Sweden, 2000.
11. Knothe, K.L.; Grassie, S.L. Modelling of Railway Track and Vehicle/Track Interaction at High Frequencies. *Int. J. Veh. Mech. Mobil.* **1993**, *22*, 209–262. [CrossRef]
12. Connolly, D.P. Ground Borne Vibrations from High Speed Trains. Ph.D. Thesis, The University of Edinburgh, Edinburgh, UK, 2013.
13. Kramer, S.L. *Geotechnical Earthquake Engineering*; Prentice Hall: Upper Saddle River, NJ, USA, 1996; ISBN 0133749436.
14. Thompson, D. *Railway Noise and Vibration*; Elsevier: Amsterdam, The Netherlands, 2009; ISBN 9780080451473. [CrossRef]
15. Kouroussis, G.; Connolly, D.P.; Verlinden, O. Railway-induced ground vibrations—A review of vehicle effects. *Int. J. Rail Transp.* **2014**, *2*, 69–110. [CrossRef]
16. Lombaert, G.; Degrande, G. Ground-borne vibration due to static and dynamic axle loads of InterCity and high-speed trains. *J. Sound Vib.* **2009**, *319*, 1036–1066. [CrossRef]
17. Lamb, H. On Group-Velocity. *Proc. Lond. Math. Soc.* **1904**, *2*, 473–479. [CrossRef]
18. Caughey, T.K. Classical normal modes in damped linear dynamic systems. *J. Appl. Mech. Trans. ASME* **1960**, *27*, 269–271. [CrossRef]
19. Fernández-Ruiz, J.; Miranda, M.; Castro, J.; Medina Rodríguez, L. Improvement of the critical speed in high-speed ballasted railway tracks with stone columns: A numerical study on critical length. *Transp. Geotech.* **2021**, *30*, 100628. [CrossRef]
20. Sunaga, M.; Sekine, E.; Ito, T. Vibration behaviors of roadbed on soft grounds under train load. *Q. Rep. Railw. Tech. Res. Inst.* **1990**, *31*, 29–35.
21. Huang, H.; Chrismer, S. Discrete element modeling of ballast settlement under trains moving at "critical Speeds". *Constr. Build. Mater.* **2013**, *38*, 994–1000. [CrossRef]
22. Picoux, B.; Le Houédec, D. Diagnosis and prediction of vibration from railway trains. *Soil Dyn. Earthq. Eng.* **2005**, *25*, 905–921. [CrossRef]
23. Krylov, V.V. Generation of ground vibrations by superfast trains. *Appl. Acoust.* **1995**, *44*, 149–164. [CrossRef]
24. Dimitrovová, Z.; Varandas, J.N. Critical velocity of a load moving on a beam with a sudden change of foundation stiffness: Applications to high-speed trains. *Comput. Struct.* **2009**, *87*, 1224–1232. [CrossRef]
25. Auersch, L. The effect of critically moving loads on the vibrations of soft soils and isolated railway tracks. *J. Sound Vib.* **2008**, *310*, 587–607. [CrossRef]
26. Costa, P.A.; Lopes, P.; Cardoso, A.S. Soil shakedown analysis of slab railway tracks: Numerical approach and parametric study. *Transp. Geotech.* **2018**, *16*, 85–96. [CrossRef]
27. Costa, P.A.; Soares, P.; Colaço, A.; Lopes, P.; Connolly, D. Railway critical speed assessment: A simple experimental-analytical approach. *Soil Dyn. Earthq. Eng.* **2020**, *134*, 106156. [CrossRef]
28. Dong, K.; Connolly, D.P.; Laghrouche, O.; Woodward, P.K.; Alves Costa, P. The stiffening of soft soils on railway lines. *Transp. Geotech.* **2018**, *17*, 178–191. [CrossRef]
29. Dong, K.; Connolly, D.P.; Laghrouche, O.; Woodward, P.K.; Alves Costa, P. Non-linear soil behaviour on high speed rail lines. *Comput. Geotech.* **2019**, *112*, 302–318. [CrossRef]
30. El Kacimi, A.; Woodward, P.K.; Laghrouche, O.; Medero, G. Time domain 3D finite element modelling of train-induced vibration at high speed. *Comput. Struct.* **2013**, *118*, 66–73. [CrossRef]
31. Mezher, S.B.; Connolly, D.P.; Woodward, P.K.; Laghrouche, O.; Pombo, J.; Costa, P.A. Railway critical velocity—Analytical prediction and analysis. *Transp. Geotech.* **2016**, *6*, 84–96. [CrossRef]
32. Sheng, X.; Jones, C.J.C.; Thompson, D.J. A theoretical study on the influence of the track on train-induced ground vibration. *J. Sound Vib.* **2004**, *272*, 909–936. [CrossRef]
33. Takemiya, H. Simulation of track-ground vibrations due to a high-speed train: The case of X-2000 at Ledsgard. *J. Sound Vib.* **2003**, *261*, 503–526. [CrossRef]
34. Kaynia, A.M.; Madshus, C.; Zackrisson, P. Ground Vibration from High-Speed Trains: Prediction and Countermeasure. *J. Geotech. Geoenviron. Eng.* **2000**, *126*, 531–537. [CrossRef]
35. Frýba, L. *Vibration of Solids and Structures under Moving Loads*; ICE Publishing: London, UK, 1999. [CrossRef]
36. Timoshenko, S. Method of Analysis of Statical and Dynamical Stresses in Rail. In Proceedings of the 2nd International Congress for Applied Mechanics, Zurich, Switzerland, 12–17 September 1926.
37. Kenney, J.T. Steady-state vibrations of beam on elastic foundation for moving load. *J. Appl. Mech. ASME* **1954**, *21*, 359–364. [CrossRef]
38. EsveldR, C. *Modern Railway Track*; MRT-Productions: Zaltbommel, The Netherlands, 2001; ISBN 9080032433.
39. Krylov, V.V. Effects of Track Properties on Ground Vibrations Generated by High-Speed Trains. *Acustica* **1998**, *84*, 78–90.
40. Massarsch, K.R.; Fellenius, B.H. Ground vibrations induced by impact pile driving. In Proceedings of the Sixth International Conference in Case Histories in Geotechnical Engineering, Arlington, VA, USA, 11–16 August 2008.
41. Yang, Y.B.; Hung, H.H.; Chang, D.W. Train-induced wave propagation in layered soils using finite/infinite element simulation. *Soil Dyn. Earthq. Eng.* **2003**, *23*, 263–278. [CrossRef]

42. Connolly, D.P.; Dong, K.; Alves Costa, P.; Soares, P.; Woodward, P.K. High speed railway ground dynamics: A multi-model analysis. *Int. J. Rail Transp.* **2020**, *8*, 324–346. [CrossRef]
43. Nguyen, K.; Goicolea, J.M.; Galbadon, F. Dynamic effect of high speed railway traffic loads on the ballast track settlement. In Proceedings of the Conference of Numerical Methods in Engineering, Coimbra, Portugal, 14–17 June 2011.
44. Bosso, N.; Gugliotta, A.; Zampieri, N. A Comprehensive Strategy to Estimate Track Condition and its Evolution. *Int. J. Railw. Technol.* **2012**, *1*, 1–19. [CrossRef]
45. Indraratna, B.; Nimbalkar, S.; Rujikiatkamjorn, C. Modernisation of Rail Tracks for Higher Speeds and Greater Freight. *Int. J. Railw. Technol.* **2013**, *2*, 1–20. [CrossRef]
46. Fortunato, E.; Paixão, A.; Calçada, R. Railway Track Transition Zones: Design, Construction, Monitoring and Numerical Modelling. *Int. J. Railw. Technol.* **2013**, *2*, 33–58. [CrossRef]
47. Momoya, Y.; Nakamura, T.; Fuchigami, S.; Takahashi, T. Improvement of Degraded Ballasted Track to Reduce Maintenance Work. *Int. J. Railw. Technol.* **2016**, *5*, 31–54. [CrossRef]
48. Woodward, P.K.; Laghrouche, O.; Mezher, S.B.; Connolly, D.P. Application of Coupled Train-Track Modelling of Critical Speeds for High-Speed Trains using Three-Dimensional Non-Linear Finite Elements. *Int. J. Railw. Technol.* **2015**, *4*, 1–35. [CrossRef]
49. Grassie, S.; Gregory, R.; Johnson, K. The dynamic response of railway track to high frequency lateral excitation. *J. Mech. Eng. Sci.* **1982**, *24*, 91–95. [CrossRef]
50. Grassie, S.; Gregory, R.; Johnson, K. The dynamic response of railway track to high frequency longitudinal excitation. *J. Mech. Eng. Sci.* **1982**, *24*, 97–102. [CrossRef]
51. Mathews, P.M. Vibrations of a beam on elastic foundation. *Z. Angew. Math. Mech.* **1959**, *39*, 13–19. [CrossRef]
52. Mathews, P. Vibrations of a beam on elastic foundation II. *Z. Angew. Math. Mech.* **1958**, *38*, 105–115. [CrossRef]
53. Jezequel, L. Response of periodic systems to a moving load. *J. Appl. Mech. Trans. ASME* **1981**, *48*, 613–618. [CrossRef]
54. Timoshenko, S.; Young, D.; Weaver, J. *Vibration Problems in Engineering*, 5th ed.; John Wiley & Sons: Hoboken, NJ, USA, 1990.
55. Warburton, G. *The Dynamical Behavior of Structures*, 2nd ed.; Elsevier: Amsterdam, The Netherlands, 1976; ISBN 0080203647. [CrossRef]
56. Cai, C.W.; Cheung, Y.K.; Chan, H.C. Dynamic response of infinite continuous beams subjected to a moving force—An exact method. *J. Sound Vib.* **1988**, *123*, 461–472. [CrossRef]
57. Lei, X. *High Speed Railway Track Dynamics: Models, Algorithms and Applications*; Springer: Cham, Switzerland, 2016; ISBN 9789811020391. [CrossRef]
58. Boresi, A.P.; Schmidt, R.J. *Advanced Mechanics of Materials*, 6th ed.; John Wiley & Sons, Inc.: Hoboken, NJ, USA, 2003; ISBN 9780471438816.
59. Kerr, A.D. *Fundamentals of Railway Track Engineering*; Simmons Boardman Publishing Co.: New York, NY, USA, 2003; ISBN 9780911382402.
60. Hussein, M.F.M.; Hunt, H.E.M. Modelling of floating-slab tracks with continuous slabs under oscillating moving loads. *J. Sound Vib.* **2006**, *297*, 37–54. [CrossRef]
61. Kuo, C.M.; Huang, C.H.; Chen, Y.Y. Vibration characteristics of floating slab track. *J. Sound Vib.* **2008**, *317*, 1017–1034. [CrossRef]
62. Lombaert, G.; Degrande, G.; Vanhauwere, B.; Vandeborght, B.; François, S. The control of ground-borne vibrations from railway traffic by means of continuous floating slabs. *J. Sound Vib.* **2006**, *297*, 946–961. [CrossRef]
63. Kouroussis, G.; Verlinden, O.; Conti, C. Finite-Dynamic Model for Infinite Media: Corrected Solution of Viscous Boundary Efficiency. *J. Eng. Mech.* **2011**, *137*, 509–511. [CrossRef]
64. Shih, J.Y.; Thompson, D.J.; Zervos, A. The effect of boundary conditions, model size and damping models in the finite element modelling of a moving load on a track/ground system. *Soil Dyn. Earthq. Eng.* **2016**, *89*, 12–27. [CrossRef]
65. Lysmer, J.; Kuhlemeyer, R.L. Finite Dynamic Model for Infinite Media. *J. Eng. Mech. Div.* **1969**, *95*, 859–877. [CrossRef]
66. Zhai, W.; Cai, Z. Dynamic interaction between a lumped mass vehicle and a discretely supported continuous rail track. *Comput. Struct.* **1997**, *63*, 987–997. [CrossRef]
67. Yang, L.A.; Powrie, W.; Priest, J.A. Dynamic stress analysis of a ballasted railway track bed during train passage. *J. Geotech. Geoenviron. Eng.* **2009**, *135*, 680–689. [CrossRef]
68. Yang, Y.-B.; Hung, H.-H. A 2.5D finite/infinite element approach for modelling visco-elastic bodies subjected to moving loads. *Int. J. Numer. Methods Eng.* **2001**, *51*, 1317–1336. [CrossRef]
69. Sheng, X.; Jones, C.J.C.; Thompson, D.J. Prediction of ground vibration from trains using the wavenumber finite and boundary element methods. *J. Sound Vib.* **2006**, *293*, 575–586. [CrossRef]
70. Lombaert, G.; Degrande, G.; Kogut, J.; François, S. The experimental validation of a numerical model for the prediction of railway induced vibrations. *J. Sound Vib.* **2006**, *297*, 512–535. [CrossRef]
71. Bian, X.C.; Chen, Y.M.; Hu, T. Numerical simulation of high-speed train induced ground vibrations using 2.5D finite element approach. *Sci. China Ser. G Phys. Mech. Astron.* **2008**, *51*, 632. [CrossRef]
72. Costa, P.A.; Calçada, R.; Cardoso, A.S. Track-ground vibrations induced by railway traffic: In-situ measurements and validation of a 2.5D FEM-BEM model. *Soil Dyn. Earthq. Eng.* **2012**, *32*, 111–128. [CrossRef]
73. De Abreu Corrêa, L.; Quezada, J.C.; Cottereau, R.; d'Aguiar, S.C.; Voivret, C. Randomly-fluctuating heterogeneous continuum model of a ballasted railway track. *Comput. Mech.* **2017**, *60*, 845–861. [CrossRef]

74. Galvín, P.; Romero, A.; Domínguez, J. Fully three-dimensional analysis of high-speed traintracksoil-structure dynamic interaction. *J. Sound Vib.* **2010**, *329*, 5147–5163. [CrossRef]
75. Varandas, J.N.; Hölscher, P.; Silva, M.A.G. Dynamic behaviour of railway tracks on transitions zones. *Comput. Struct.* **2011**, *89*, 1468–1479. [CrossRef]
76. Olivier, B.; Connolly, D.P.; Alves Costa, P.; Kouroussis, G. The effect of embankment on high speed rail ground vibrations. *Int. J. Rail Transp.* **2016**, *4*, 229–246. [CrossRef]
77. Hall, L. Simulations and analyses of train-induced ground vibrations in finite element models. *Soil Dyn. Earthq. Eng.* **2003**, *23*, 403–413. [CrossRef]
78. Cai, Z.; Raymond, G.P. Modelling the dynamic response of railway track to wheel/rail impact loading. *Struct. Eng. Mech.* **1994**, *2*, 95–112. [CrossRef]
79. Kouroussis, G.; Verlinden, O.; Conti, C. Ground propagation of vibrations from railway vehicles using a finite/infinite-element model of the soil. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2009**, *223*, 405–413. [CrossRef]
80. Connolly, D.; Giannopoulos, A.; Forde, M.C. Numerical modelling of ground borne vibrations from high speed rail lines on embankments. *Soil Dyn. Earthq. Eng.* **2013**, *46*, 13–19. [CrossRef]
81. Poveda, E.; Yu, R.C.; Lancha, J.C.; Ruiz, G. A numerical study on the fatigue life design of concrete slabs for railway tracks. *Eng. Struct.* **2015**, *100*, 455–467. [CrossRef]
82. Costa, P.A.; Calçada, R.; Cardoso, A.S.; Bodare, A. Influence of soil non-linearity on the dynamic response of high-speed railway tracks. *Soil Dyn. Earthq. Eng.* **2010**, *30*, 221–235. [CrossRef]
83. Di Mascio, P.; Loprencipe, G.; Moretti, L. Competition in rail transport: Methodology to evaluate economic impact of new trains on track. In Proceedings of the 3rd International Conference on Tranportation Infrastructure, ICTI 2014—Sustainability, Eco-Efficiency and Conservation in Transportation Infrastructure Asset Management, Pisa, Italy, 22–25 April 2014.
84. Ando, K.; Sunaga, M.; Aoki, H.; Haga, O. Development of slab tracks for Hokuriku Shinkansen line. *Q. Rep. Railw. Tech. Res. Inst.* **2001**, *42*, 35–41. [CrossRef]
85. Kece, E.; Reikalas, V.; DeBold, R.; Ho, C.L.; Robertson, I.; Forde, M.C. Evaluating ground vibrations induced by high-speed trains. *Transp. Geotech.* **2019**, *20*, 100236. [CrossRef]
86. Zhao, P.; Ding, C.; Guo, L.; Zhang, Y.; Liu, X. A prototype fatigue test for slab track subjected to the coupling action of wheel load, temperature variation, and water erosion. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2019**, *233*, 566–579. [CrossRef]
87. Sainz-Aja, J.; Pombo, J.; Tholken, D.; Carrascal, I.; Polanco, J.; Ferreño, D.; Casado, J.; Diego, S.; Perez, A.; Filho, J.E.A.; et al. Dynamic calibration of slab track models for railway applications using full-scale testing. *Comput. Struct.* **2020**, *228*, 106180. [CrossRef]
88. Wang, M.; Cai, C.; Zhu, S.; Zhai, W. Experimental study on dynamic performance of typical nonballasted track systems using a full-scale test rig. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2017**, *231*, 470–481. [CrossRef]
89. Malmborg, J.; Persson, P.; Persson, K. Effects of modeling strategies for a slab track on predicted ground vibrations. *Soil Dyn. Earthq. Eng.* **2020**, *136*, 106254. [CrossRef]
90. Luo, J.; Zhu, S.; Zhai, W. Theoretical modelling of a vehicle-slab track coupled dynamics system considering longitudinal vibrations and interface interactions. *Veh. Syst. Dyn.* **2020**, *59*, 1313–1334. [CrossRef]
91. Lou, P.; Gong, K.; Zhao, C.; Xu, Q.; Luo, R.K. Dynamic Responses of Vehicle-CRTS III Slab Track System and Vehicle Running Safety Subjected to Uniform Seismic Excitation. *Shock Vib.* **2019**, *2019*, 5308209. [CrossRef]
92. Wang, J.; Liu, S.; Yang, W. Dynamics shakedown analysis of slab track substructures with reference to critical speed. *Soil Dyn. Earthq. Eng.* **2018**, *106*, 1–13. [CrossRef]
93. He, Q.; Cai, C.; Zhu, S.; Zhang, J.; Zhai, W. Dynamic performance of low vibration slab track on shared high-speed passenger and freight railway. *Transport* **2018**, *33*, 669–678. [CrossRef]
94. Liu, S.; Chen, X.; Ma, Y.; Yang, J.; Cai, D.; Yang, G. Modelling and in-situ measurement of dynamic behavior of asphalt supporting layer in slab track system. *Constr. Build. Mater.* **2019**, *228*, 116776. [CrossRef]
95. Liu, S.; Chen, X.; Yang, J.; Cai, D.; Yang, G. Numerical study and in-situ measurement of temperature features of asphalt supporting layer in slab track system. *Constr. Build. Mater.* **2020**, *233*, 117343. [CrossRef]
96. Li, T.; Su, Q.; Kaewunruen, S. Influences of piles on the ground vibration considering the train-track-soil dynamic interactions. *Comput. Geotech.* **2020**, *120*, 103455. [CrossRef]
97. Malmborg, J.; Persson, K.; Persson, P. Evaluating the effect of vibration isolation mats on train-induced ground vibrations. In Proceedings of the 7th International Conference on Structural Engineering, Mechanics and Computation—Advances in Engineering Materials, Structures and Systems: Innovations, Mechanics and Applications, Cape Town, South Africa, 2–4 September 2019. [CrossRef]
98. Xin, T.; Ding, Y.; Wang, P.; Gao, L. Application of rubber mats in transition zone between two different slab tracks in high-speed railway. *Constr. Build. Mater.* **2020**, *243*, 118219. [CrossRef]
99. Real, T.; Zamorano, C.; Hernández, C.; García, J.A.; Real, J.I. Static and dynamic behavior of transitions between different railway track typologies. *KSCE J. Civ. Eng.* **2016**, *20*, 1356–1364. [CrossRef]
100. Hu, Y.; Cheng, Z.; Shi, Z. Vibration Reduction Performance of a Periodic Layered Slab Track. In *Lecture Notes in Civil Engineering*; Springer: Cham, Switzerland, 2020. [CrossRef]

101. Duong, T.V.; Cui, Y.J.; Tang, A.M.; Dupla, J.C.; Canou, J.; Calon, N.; Robinet, A. Investigating the mud pumping and interlayer creation phenomena in railway sub-structure. *Eng. Geol.* **2014**, *171*, 45–58. [CrossRef]
102. Huang, J.; Su, Q.; Wang, W.; Phong, P.D.; Liu, K. Field investigation and full-scale model testing of mud pumping and its effect on the dynamic properties of the slab track–subgrade interface. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2019**, *233*, 802–816. [CrossRef]
103. Nsabimana, E.; Jung, Y.H. Dynamic subsoil responses of a stiff concrete slab track subjected to various train speeds: A critical velocity perspective. *Comput. Geotech.* **2015**, *69*, 7–21. [CrossRef]
104. Beresnev, I.A.; Wen, K.-L. Nonlinear soil response—A reality? *Bull. Seismol. Soc. Am.* **1996**, *86*, 964–1978.
105. Zhang, J.; Andrus, R.D.; Juang, C.H. Normalized Shear Modulus and Material Damping Ratio Relationships. *J. Geotech. Geoenviron. Eng.* **2005**, *131*, 453–464. [CrossRef]
106. Shankar Kumar, S.; Murali Krishna, A.; Dey, A. Parameters Influencing Dynamic Soil Properties: A Review Treatise. *Int. J. Innov. Res. Sci.* **2014**, *3*, 47–60.
107. Thach, P.N.; Liu, H.L.; Kong, G.Q. Vibration analysis of pile-supported embankments under high-speed train passage. *Soil Dyn. Earthq. Eng.* **2013**, *55*, 92–99. [CrossRef]
108. Freitas da Cunha, J.P. Modelling of Ballasted Railway Tracks for High-Speed Trains. Ph.D. Thesis, University of Minho, Braga, Portugal, 2013.
109. Çelebi, E.; Kirtel, O. Non-linear 2-D FE modeling for prediction of screening performance of thin-walled trench barriers in mitigation of train-induced ground vibrations. *Constr. Build. Mater.* **2013**, *42*, 122–131. [CrossRef]
110. Çelebi, E.; Göktepe, F. Non-linear 2-D FE analysis for the assessment of isolation performance of wave impeding barrier in reduction of railway-induced surface waves. *Constr. Build. Mater.* **2012**, *36*, 1–13. [CrossRef]
111. Ruiz, J.F.; Rodríguez, L.M. Application of an advanced soil constitutive model to the study of railway vibrations in tunnels through 2D numerical models: A real case in Madrid (Spain). *Rev. Constr.* **2015**, *14*, 55–63. [CrossRef]
112. Radampola, S.S.; Gurung, N.; McSweeney, T.; Dhanasekar, M. Evaluation of the properties of railway capping layer soil. *Comput. Geotech.* **2008**, *35*, 719–728. [CrossRef]
113. Sol-Sánchez, M.; Thom, N.H.; Moreno-Navarro, F.; Rubio-Gámez, M.C.; Airey, G.D. A study into the use of crumb rubber in railway ballast. *Constr. Build. Mater.* **2015**, *75*, 19–24. [CrossRef]
114. Indraratna, B.; Salim, W.; Rujikiatkamjorn, C. *Advanced Rail Geotechnology—Ballasted Track*; CRC Press: Boca Raton, FL, USA, 2011; ISBN 9780203815779. [CrossRef]
115. Sol-Sánchez, M.; Moreno-Navarro, F.; Rubio-Gámez, M.C. The use of deconstructed tires as elastic elements in railway tracks. *Materials* **2014**, *7*, 5903–5919. [CrossRef] [PubMed]
116. Sol-Sánchez, M.; Pirozzolo, L.; Moreno-Navarro, F.; Rubio-Gámez, M.C. A study into the mechanical performance of different configurations for the railway track section: A laboratory approach. *Eng. Struct.* **2016**, *119*, 13–23. [CrossRef]
117. Rose, J.G. Selected in-track applications and performances of hot-mix asphalt trackbeds. In Proceedings of the 2013 Joint Rail Conference, JRC 2013, Knoxville, TN, USA, 15–18 April 2013. [CrossRef]
118. Lee, S.H.; Lee, J.W.; Park, D.W.; Vo, H.V. Evaluation of asphalt concrete mixtures for railway track. *Constr. Build. Mater.* **2014**, *73*, 13–18. [CrossRef]
119. Di Mino, G.; Di Liberto, M.; Maggiore, C.; Noto, S. A Dynamic Model of Ballasted Rail Track with Bituminous Sub-Ballast Layer. *Procedia Soc. Behav. Sci.* **2012**, *53*, 366–378. [CrossRef]
120. Fang, M.; Cerdas, S.F.; Qiu, Y. Numerical determination for optimal location of sub-track asphalt layer in high-speed rails. *J. Mod. Transp.* **2013**, *21*, 103–110. [CrossRef]
121. Rose, J.G.; Souleyrette, R. Hot-mix asphalt (bituminous) railway trackbeds: In-track tests, evaluations, and performances—A global perspective: Part I—Introduction to asphalt trackbeds and international applications and practices. In Proceedings of the 3rd International Conference on Tranportation Infrastructure, ICTI 2014—Sustainability, Eco-Efficiency and Conservation in Transportation Infrastructure Asset Management, Pisa, Italy, 22–25 April 2014.
122. Rose, J.G.; Bryson, L.S. Hot mix asphalt railway trackbeds: Trackbed materials, performance evaluations ans significant implications. In Proceedings of the International Conference on Perpetual Pavements, Columbus, OH, USA, 30 September–2 October 2009.
123. Rose, J.; Teixeira, P.; Ridgway, N. Utilization of Asphalt/Bituminous Layers and Coatings in Railway Trackbeds: A Compendium of International Applications. In Proceedings of the 2010 Joint Rail Conference, Urbana, IL, USA, 27–29 April 2010; Volume 1. [CrossRef]
124. Rose, J.G.; Teixeira, P.F.; Veit, P. International design practices, applications, amd performances pf asphalt/bituminous railway trackbeds. In Proceedings of the International Symposium Railway Geotechnical Engineering (GeoRail 2011), Paris, France, 19–20 May 2011.
125. Huang, Y.H.; Rose, J.G.; Khoury, C.J. Hot-mix asphalt railroad trackbeds. *Transp. Res. Rec.* **1986**, *1095*, 102–110.
126. Hensley, M.; Rose, J.G. Design, construction and performance of Hot Mix Asphalt for railway trackbeds. In Proceedings of the 1st World of Asphalt Pavements International Conference, Sydney, Australia, 20–24 February 2000.
127. Yu, Z.; Connolly, D.P.; Woodward, P.K.; Laghrouche, O. Settlement behaviour of hybrid asphalt-ballast railway tracks. *Constr. Build. Mater.* **2019**, *208*, 808–817. [CrossRef]

128. Lee, S.H.; Vo, H.V.; Park, D.W. Investigation of asphalt track behavior under cyclic loading: Full-scale testing and numerical simulation. *J. Test. Eval.* **2018**, *46*, 20160554. [CrossRef]
129. Fang, M.; Qiu, Y.; Rose, J.G.; West, R.C.; Ai, C. Comparative analysis on dynamic behavior of two HMA railway substructures. *J. Mod. Transp.* **2011**, *19*, 26–34. [CrossRef]
130. Fang, M.; Cerdas, S.F. Theoretical analysis on ground vibration attenuation using sub-track asphalt layer in high-speed rails. *J. Mod. Transp.* **2015**, *23*, 214–219. [CrossRef]
131. Lee, S.H.; Park, D.W.; Vo, H.V.; Dessouky, S. Asphalt mixture for the first asphalt concrete directly fastened track in Korea. *Adv. Mater. Sci. Eng.* **2015**, *2015*, 701940. [CrossRef]
132. Lee, S.H.; Choi, Y.T.; Lee, H.M.; Park, D.W. Performance evaluation of directly fastened asphalt track using a full-scale test. *Constr. Build. Mater.* **2016**, *113*, 404–414. [CrossRef]
133. Lee, S.H.; Eum, K.Y.; Ho Minh Le, T.; Park, D.W. Evaluation on mechanical behavior of asphalt concrete trackbed with slab panel using full-scale static and dynamic load test. *Constr. Build. Mater.* **2021**, *276*, 122207. [CrossRef]
134. Roberts, F.L.; Kandhal, P.S.; Braown, E.R. *Hot Mix Asphalt Materials, Mixture Design, and Constrution*; National Center for Asphalt Technology: Auburn, AL, USA, 1996.
135. Jadidi, K.; Esmaeili, M.; Kalantari, M.; Khalili, M.; Karakouzian, M. A review of different aspects of applying asphalt and bituminous mixes under a railway track. *Materials* **2021**, *14*, 169. [CrossRef] [PubMed]
136. Feng, S.J.; Zhang, X.L.; Zheng, Q.T.; Wang, L. Simulation and mitigation analysis of ground vibrations induced by high-speed train with three dimensional FEM. *Soil Dyn. Earthq. Eng.* **2017**, *94*, 204–214. [CrossRef]
137. Bian, X.; Jiang, H.; Chang, C.; Hu, J.; Chen, Y. Track and ground vibrations generated by high-speed train running on ballastless railway with excitation of vertical track irregularities. *Soil Dyn. Earthq. Eng.* **2015**, *76*, 29–43. [CrossRef]
138. Indraratna, B.; Ionescu, D.; Christie, H.D. Shear Behavior of Railway Ballast Based on Large-Scale Triaxial Tests. *J. Geotech. Geoenviron. Eng.* **1998**, *124*, 439–449. [CrossRef]
139. Chen, J.; Zhou, Y. Dynamic responses of subgrade under double-line high-speed railway. *Soil Dyn. Earthq. Eng.* **2018**, *110*, 1–12. [CrossRef]
140. Chen, J.; Zhou, Y. Dynamic vertical displacement for ballastless track-subgrade system under high-speed train moving loads. *Soil Dyn. Earthq. Eng.* **2020**, *129*, 105911. [CrossRef]
141. Tang, Y.; Xiao, S.; Yang, Q. Numerical study of dynamic stress developed in the high speed rail foundation under train loads. *Soil Dyn. Earthq. Eng.* **2019**, *123*, 36–47. [CrossRef]
142. Yang, E.; Wang, K.C.P.; Luo, Q.; Qiu, Y. Asphalt concrete layer to support track slab of high-speed railway. *Transp. Res. Rec.* **2015**, *2505*, 6–14. [CrossRef]
143. Thölken, D.; Abdalla Filho, J.E.; Pombo, J.; Sainz-Aja, J.; Carrascal, I.; Polanco, J.; Esen, A.; Laghrouche, O.; Woodward, P. Three-dimensional modelling of slab-track systems based on dynamic experimental tests. *Transp. Geotech.* **2021**, *31*, 100663. [CrossRef]
144. Ramos, A.; Gomes Correia, A.; Calçada, R.; Alves Costa, P.; Esen, A.; Woodward, P.K.; Connolly, D.P.; Laghrouche, O. Influence of track foundation on the performance of ballast and concrete slab tracks under cyclic loading: Physical modelling and numerical model calibration. *Constr. Build. Mater.* **2021**, *277*, 122245. [CrossRef]
145. Shi, H.; Yu, Z.; Shi, H. An improved method for dynamic modelling of a slab track on a high-speed railway. *WIT Trans. Built Environ.* **2016**, *162*, 225–237. [CrossRef]
146. Ntotsios, E.; Thompson, D.J.; Hussein, M.F.M. A comparison of ground vibration due to ballasted and slab tracks. *Transp. Geotech.* **2019**, *21*, 100256. [CrossRef]
147. Marolt Čebašek, T.; Esen, A.F.; Woodward, P.K.; Laghrouche, O.; Connolly, D.P. Full scale laboratory testing of ballast and concrete slab tracks under phased cyclic loading. *Transp. Geotech.* **2018**, *17*, 33–40. [CrossRef]
148. Esen, A.F.; Woodward, P.K.; Laghrouche, O.; Čebašek, T.M.; Brennan, A.J.; Robinson, S.; Connolly, D.P. Full-scale laboratory testing of a geosynthetically reinforced soil railway structure. *Transp. Geotech.* **2021**, *28*, 100526. [CrossRef]
149. Yao, H.L.; Hu, Z.; Lu, Z.; Zhan, Y.X.; Liu, J. Prediction of Ground Vibration from High Speed Trains Using a Vehicle-Track-Ground Coupling Model. *Int. J. Struct. Stab. Dyn.* **2016**, *16*, 1550051. [CrossRef]
150. Yusupov, B.; Qiu, Y.; Ding, H.; Rahman, A. Temperature and material behaviour effects on dynamic responses of asphalt concrete trackbed. *Int. J. Rail Transp.* **2020**, *8*, 66–79. [CrossRef]