SUPPLEMENTARY MATERIALS

to

Comparative Evaluation of Two Analytical Functions for the Microdosimetry of Ions from ^{1}H to ^{238}U

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SM1. MCF MKM equations

In the MCF MKM [1, 2], the linear (α) and quadratic (β) terms of the linear quadratic model (LQM) of cell survival are calculated as

$$\alpha = \alpha_0 \int \left(1 + \frac{\beta_0}{\alpha_0} \frac{y}{\rho\, \pi\, r_d^2}\right) c(y)\; d(y)\; dy \qquad \text{(SM1)}$$

$$\beta = \beta_0 \left[\int c(y) d(y)\; dy\right]^2 \qquad \text{(SM2)}$$

$$c(y) = \frac{1 - \exp\left[-\alpha_0 \left(1 + \frac{\beta_0}{\alpha_0} \frac{y}{\rho\, \pi\, r_d^2}\right) \frac{y}{\rho\, \pi\, R_n^2} - \beta_0 \left(\frac{y}{\rho\, \pi\, R_n^2}\right)^2\right]}{\alpha_0 \left(1 + \frac{\beta_0}{\alpha_0} \frac{y}{\rho\, \pi\, r_d^2}\right) \frac{y}{\rho\, \pi\, R_n^2} + \beta_0 \left(\frac{y}{\rho\, \pi\, R_n^2}\right)^2} \qquad \text{(SM3)}$$

where *c(y)* is a correction factor accounting for the non-Poisson distribution of lethal lesions, *y* is the lineal energy and *d(y)* is the dose probability density of the lineal energy, α_0 and β_0 are the LQM terms in the limit of $y \rightarrow 0$, R_n is the mean radius of the cell nucleus, r_d is the mean radius of the subnuclear domains, and ρ is the density of the cell nucleus (set to 1 g/cm^3).

It can be demonstrated [3] that, since R_n is much larger than r_d,

$$\alpha_0 \left(1 + \frac{\beta_0}{\alpha_0} \frac{y}{\rho\, \pi\, r_d^2}\right) \frac{y}{\rho\, \pi\, R_n^2} + \beta_0 \left(\frac{y}{\rho\, \pi\, R_n^2}\right)^2 \cong \alpha_0 \left(1 + \frac{\beta_0}{\alpha_0} \frac{y}{\rho\, \pi\, r_d^2}\right) \frac{y}{\rho\, \pi\, R_n^2} \qquad \text{(SM 4)}$$

Therefore, Equations SM1 and SM2 become

$$\alpha = \int \frac{1-\exp\left[-\alpha_0\left(1+\frac{\beta_0}{\alpha_0}\frac{y}{\rho\,\pi\,r_d^2}\right)\frac{y}{\rho\,\pi\,R_n^2}\right]}{\frac{y}{\rho\,\pi\,R_n^2}}\ d(y)\ dy \tag{SM5}$$

$$\beta = \beta_0\left[\int \frac{1-\exp\left[-\alpha_0\left(1+\frac{\beta_0}{\alpha_0}\frac{y}{\rho\,\pi\,r_d^2}\right)\frac{y}{\rho\,\pi\,R_n^2}\right]}{\alpha_0\left(1+\frac{\beta_0}{\alpha_0}\frac{y}{\rho\,\pi\,r_d^2}\right)\frac{y}{\rho\,\pi\,R_n^2}}\ d(y)\ dy\right]^2 \tag{SM6}$$

For relatively sparsely ionizing radiation such as photons, *c(y)* is roughly 1 [1, 2].

Therefore, α_0 and β_0 can be derived from the known response of the cells to the reference photons:

$$\alpha_0 = \alpha_{ref} - \beta_0\frac{\bar{y}_{D,ref}}{\rho\,\pi\,r_d^2} \tag{SM7}$$

$$\beta_0 = \beta_{ref} \tag{SM8}$$

where $\bar{y}_{D,ref}$ is the dose-mean lineal energy for the reference photon exposure.

Finally, Equations 4 and 5 in the main body of the article were obtained by substituting the expressions for α_0 and β_0 (Equations SM7 and SM8) into Equations SM5 and SM6.

References

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