



Editorial

# New Challenges Arising in Engineering Problems with Fractional and Integer Order

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Mathematical models have been frequently studied in recent decades in order to obtain the deeper properties of real-world problems. In particular, if these problems, such as finance, soliton theory and health problems, as well as problems arising in applied science and so on, affect humans from all over the world, studying such problems is inevitable. In this sense, the first step in understanding such problems is the mathematical forms. Extracted results are generally in the form of numerical solutions, analytical solutions, approximate solutions and periodic properties. With the help of newly developed computational systems, experts have investigated and modeled such problems. Moreover, their graphical simulations also have been presented in the literature.

In response to the call for papers, 33 submissions were received. All submissions were reviewed by at least three experts in the field. Finally, 12 papers were accepted for publication in this special issue, all of which were of high quality and represented the areas covered by this Special Issue well. This corresponds to an acceptance rate of 36%.

This Special Issue is based on fractional differentiation and integration, both with respect to theoretical and numerical aspects including the integer order.

The published papers in this Special Issue were briefly studied as follows.

In [1], the solution of a fractional kinetic equation (FKE), associated with the incomplete I-function (IIF) by using the well-known integral transform (Laplace transform), was investigated. The FKE plays an important role in solving astrophysical problems. The solutions in terms of the IIF were represented. They presented some interesting corollaries by specializing the parameters of the IIF in the form of simpler special functions. The authors also mentioned a few known results that are very useful in solving physical or real-life problems. Finally, some graphical results were presented to demonstrate the influence of the order of the fractional integral operator on the reaction rate.

In [2], the Sumudu decomposition method (SDM), a way to find approximate solutions to two-dimensional fractional partial differential equations, was used. A numerical algorithm for solving a fractional Riccati equation was investigated. The authors formed a combination of the Sumudu transform method and the decomposition method. The fractional derivative was described in the Caputo sense.

According to [3], time scales have been the target of work for many mathematicians for more than a quarter century. They used the fractional maximal integrals to establish integral inequalities on time scales. Moreover, their findings showed that inequality is valid for discrete and continuous conditions.

In [4], obtaining approximate solutions for a fractional order Burgers' equation was presented in a reproducing kernel Hilbert space (RKHS). Some special reproducing kernel spaces were identified according to the inner products and norms. Then, an iterative approach was constructed by using kernel functions. The convergence of this



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approach and its error estimates was given. The numerical algorithm for the method was presented. In addition, the numerical outcomes were shown with tables and graphics for some examples.

In [5], the authors solved economic models based on market equilibrium with a constant proportional Caputo derivative by using the Laplace transform. They proved the accuracy and efficiency of the method. They constructed the relations between the solutions of the problems and bivariate Mittag–Leffler functions.

In [6], the authors applied an integral transform-based technique to solve a fractional order Volterra-type integro-differential equation (FVIDE) involving the generalized Lorenzo–Hartely function and generalized Lauricella confluent hypergeometric function with several complex variables in the kernel. They also investigated and introduced the Elzaki transform of the Hilfer derivative, generalized Lorenzo–Hartely function and generalized Lauricella confluent hypergeometric function. In this article, they established three results that were present in the form of lemmas, which give us new results for the above-mentioned three functions. By using these results, they derived results that were given in the form of theorems.

In [7], a fractional predator–prey model with a harvesting rate was considered. Besides the existence and uniqueness of the solution to the model, the local stability and global stability were examined. A novel discretization depending on the numerical discretization of the Riemann–Liouville integral was introduced, and the corresponding numerical discretization of the predator–prey fractional model was obtained. The net reproduction number  $R_0$  was obtained for the prediction and persistence of the disease. The dynamical behavior of the equilibria was examined by using the stability criteria. Furthermore, numerical simulations of the model were performed. Their graphical representations supported the numerical discretizations, emphasized the effectiveness of our theoretical results and monitored the effect of an arbitrary order derivative.

In [8], the application of the  $(m+1/G')$ -expansion method to the  $(2+1)$ -dimensional hyperbolic nonlinear Schrödinger equation was investigated. With the help of the proposed method, the periodic and singular complex wave solutions to the considered for the model were derived. Various figures such as 3D and 2D surfaces with suitable parameter values were plotted.

In [9], fractional order derivatives for the management and simulation of a fractional order disorderly finance system were investigated. In the developed system, the authors added the critical minimum interest rate parameter in order to develop a new stable financial model. The new emerging paradigm on the demand for innovation, which is the gateway to the knowledge economy, was surveyed. The derivatives were characterized in the Caputo fractional and the Atangana–Baleanu sense. They proved the existence and uniqueness of the solutions with fixed point theorems and an iterative scheme.

In [10], a new reproducing kernel approach was developed to obtain a numerical solution for multi-order fractional nonlinear three-point boundary value problems. This approach was based on a reproducing kernel, which was constructed by shifted Legendre polynomials (L-RKM). In the problem considered, fractional derivatives with respect to  $\alpha$  and  $\beta$  were defined in the Caputo sense. This method was applied to some examples that had exact solutions. In order to show the robustness of the proposed method, some examples were solved, and the numerical results were given in tabulated forms.

In [11], one of the special cases of an auxiliary method, named the Bernoulli sub-equation function method, was applied for the nonlinear modified alpha equation. The characteristic properties of these solutions, such as complex and soliton solutions, were extracted. Moreover, the strain conditions of the solutions were also reported in detail. Observing the figures plotted, by considering various values of the parameters of these solutions, the effectiveness of the approximation method used for the governing model was confirmed.

In [12], the authors proved the equivalence of the norm of the restricted centered fractional maximal diamond- $\alpha$  integral operator to the norm of the centered fractional

maximal diamond- $\alpha$  integral operator  $M_{\alpha}$  on the time scales in variable exponent Lebesgue spaces. This study considered problems such as the boundedness and compactness of the integral operators in relation to the time scales.

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