fractal and fractional

Editorial

Editorial for Special Issue "Fractional Calculus and Special Functions with Applications"

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The study of fractional integrals and fractional derivatives has a long history, and they have many real-world applications due to their properties of interpolation between operators of integer order. This field has covered classical fractional operators such as Riemann–Liouville, Weyl, Caputo, Grünwald–Letnikov, etc. Also, especially in the last two decades, many new operators have appeared, often defined using integrals with special functions in the kernel, such as Atangana–Baleanu, Prabhakar, Marichev–Saigo–Maeda, and tempered, as well as their extended or multivariable forms. These have been intensively studied because they can also be useful in modelling and analysing real-world processes because of their different properties and behaviours, which are comparable to those of the classical operators.

Special functions, such as the Mittag-Leffler functions, hypergeometric functions, Fox's H-functions, Wright functions, Bessel and hyper-Bessel functions, etc., also have some more classical and fundamental connections with fractional calculus. Some of them, such as the Mittag-Leffler function and its generalisations, appear naturally as solutions of fractional differential equations or fractional difference equations. Furthermore, many interesting relationships between different special functions may be discovered using the operators of fractional calculus. Certain special functions have also been applied to analyse the qualitative properties of fractional differential equations, such as the concept of Mittag-Leffler stability.

In early 2020, we opened a Special Issue in the journal *Fractal and Fractional*, with the aim of exploring and celebrating the diverse connections between fractional calculus and special functions, as well as their associated applications. The deadline was initially set as 31 December 2020, and was later extended to 31 March 2021 after the havoc caused by the COVID-19 pandemic. We received a total of 15 submissions for this Special Issue and, after a thorough peer-review process, nine of these were ultimately published, including several from experts in the field whom we had personally invited to contribute.

The published papers in our Special Issue are briefly summarised as follows.

In [1], Aljoudi et al. considered a nonlinear coupled system of Caputo–Hadamard fractional ordinary differential equations on a finite interval, with Hadamard integral boundary conditions and incommensurate fractional orders between 1 and 2. Under certain boundedness and Lipschitz-type assumptions, they proved the existence of solutions for this system, and the uniqueness of solutions under some extra boundedness assumptions.

In [2], Salem and Alghamdi considered a nonlinear sequential-type Caputo fractional ordinary differential equation on a finite interval, with nonlocal multi-point boundary conditions and an overall fractional order between 1 and 3. They proved the existence and/or uniqueness of solutions for this Langevin-type equation in three main results, under an array of possible conditions.

In [3], Zine and Torres introduced a new type of stochastic fractional operator, a way of applying fractional integrals and derivatives to stochastic processes. They proved

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many fundamental properties of these operators, including boundedness, semigroup and inversion properties, and an integration by parts rule, before posing stochastic fractional Euler–Lagrange equations to investigate the variational principles of the new operators.

In [4], Fernandez and Husain defined and investigated modified versions of the classical Mittag-Leffler functions of one, two, and three parameters. They found appropriate convergence conditions for the new series in each case, established complex integral representations of the new functions, and then used them to extend the definition of Atangana–Baleanu and Prabhakar fractional calculus, providing analytic continuations of the original definitions to wider domains for the parameter α.

In [5], Yılmaz et al. studied k-generalised Appell functions, based on the existing theory of k-fractional calculus and k-variants of special functions such as the gamma, beta, and hypergeometric functions (the k-variants being identical to the original versions up to some substitutions and constant multiples). They proved various functional equation relations and generating relations for the k-generalised Appell functions using k-fractional derivatives. Please kindly note that a corrigendum to this paper was also published [5].

In [6], Acay and Inç studied several variants of a differential equation used to model RC, LC, and RLC electric circuits under Kirchhoff's law. The function representing the source voltage was taken to be either constant, exponential, or a power function, and the differential operator was taken to be a so-called non-local fractional M-derivative, which is a constant times the usual Caputo derivative taken with respect to a power function. A comparative analysis was performed to compare the results achieved by using the M-derivative and by using the usual Caputo derivative with respect to t.

In [7], Uçar et al. considered a system of first-order ordinary differential equations, which is used to model the effect of computer worms, and replaced the first-order derivatives with fractional derivatives of Atangana–Baleanu type to obtain a different system, which they studied using fixed-point and Laplace transform techniques to prove existence, uniqueness, and stability properties.

In [8], Özarslan and Fernandez introduced a new five-parameter Mittag-Leffler function, defined by a single series but used to construct bivariate (double integral) fractional operators. They proved fundamental properties such as boundedness, Laplace transforms, semigroup and inversion properties, and series formulae. In the non-singular case, they derived a special case which is a mixed bivariate version of the Atangana–Baleanu operators.

In [9], Bargamadi et al. considered a coupled system of integro-differential equations involving Caputo fractional derivatives, with simple initial conditions and incommensurate fractional orders between 0 and 1. They used the Chebyshev wavelets method to estimate the Caputo derivatives and find approximate numerical solutions to the system, and performed error analysis on their approximations both analytically and numerically.

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