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A Four Step Feedback Iteration and Its Applications in Fractals

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Abstract: Fractals play a vital role in modeling the natural environment. The present aim is to investigate the escape criterion to generate specific fractals such as Julia sets, Mandelbrot sets and Multicorns via F-iteration using complex functions $h(z) = z^n + c$, $h(z) = \sin(z^n) + c$ and $h(z) = e^{z^n} + c$, $n \geq 2, c \in \mathbb{C}$. We observed some beautiful Julia sets, Mandelbrot sets and Multi-corns for $n = 2$, 3 and 4. We generalize the algorithms of the Julia set and Mandelbrot set to visualize some Julia sets, Mandelbrot sets and Multi-corns. Moreover, we calculate image generation time in seconds at different values of input parameters.

Keywords: imaging; complex function; Julia set; Mandelbrot set; multi-corn

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1. Introduction

Fractals occur frequently in the natural environment and their role is vital to measure the optimization of energy, quantification of CO_2, enhancement of the range of antennas, and performance of the stock market and musical compositions due to their natural fractal patterns. Such patterns can be generated by using various iterative algorithms in computers. This approach can be successfully applied in various fields to create future technologies by ensuring secure development (https://fractal-project.eu/) and fixed point theory plays a very vital role in the fractal theory for fractal generation via escape time criteria. Initially, the fractal was sketched by Benoit Mandelbrot, who is known as the father of fractal geometry, extending the work of Gaston Julia [1] who started this work in 1918 and successfully approximated the complex function $z \longrightarrow z^2 + b$—where z, $b \in \mathbb{C}$—but was unable to sketch it. Mandelbrot [2] drew the graph of $z \longrightarrow z^2 + b$, where $z, b \in \mathbb{C}$, in 1983 and, by changing the role of z and b in J-set, defined a new set known as M-set. J-set elaborates the role of iterates for every z and M-set explains the connected J-set for each b by elaborating those sets. M-set using $h : z \to z^p + c$, where $p \geq 2$, is explained in [3]. Later on, anti-fractals were defined by Crow et al. [4] for $\bar{z}^2 + c$, which are tricorns.

Researchers use fixed point iterative schemes to generate fractals. Higher dimensional fractals are discussed in [5,6]. Generalized Julia sets and Mandelbrot sets are generated by using different iterative schemes such as Mann iteration [7], Ishikawa iteration [8], S-iteration [9], Noor iteration [10], SP-iteration [11] and CR-iteration [12]. Fractals via modified Jungck-S orbit are discussed in [13]. M-set and J-sets by Jungck type scheme with s-convexity are elaborated in [14]. Fractals such as filled J-set by Jungck Mann scheme are discussed in [15]. Fractals via extended Jungck-SP orbit are discussed in [16]. Further, we can find biomorphs in literature, which are generated by different iterations [17–19]. J-set and M-set for complex trigonometric function via different iterations are discussed in [20]. Hybrid Picard–Mann iteration has been used to generate anti-fractals [21]. In this article, we use F-iteration [22] to generate the Julia set, Mandelbrot set and Multi-corns for the Complex function $h(z) = z^n + c$, where $n \geq 2, c \in \mathbb{C}$.

2. Preliminaries

In this part of the paper, we study some well-known fractals and iterative algorithms.

Definition 1 ([1]). *Assume that F_h is the set of points in $\mathbb{C}$ such that $h : \mathbb{C} \to \mathbb{C}$ is a complex polynomial of degree ≥ 2. The set F_h is called a filled Julia set when the orbit of $F_h \nrightarrow \infty$ as $i \to \infty$, i.e.,*

$$F_h = \{z \in \mathbb{C} : \{|h^i(z)|\}_{i=0}^{\infty} \text{ is bounded}\}. \tag{1}$$

The set of boundary points of F_h is known as a simple J-set.

Definition 2 ([23]). *A set consisting of all the connected J-sets is called a Mandelbrot set (M-set), i.e.,*

$$M = \{c \in \mathbb{C} : F_h \text{ is connected}\}, \tag{2}$$

correspondingly, we can define M-set as [24]

$$M = \{c \in \mathbb{C} : \{h^i(0)\} \nrightarrow \infty \text{ as } i \to \infty\}, \tag{3}$$

where 0 is the only critical point $\acute{h}(0) = 0$. So, we choose 0 as the initial point.

Definition 3 (Multi-corn [23]). *Let $h_b : \mathbb{C} \to \mathbb{C}$ be a mapping and $h_b(z) = \bar{z}^n + b$ with $b \in \mathbb{C}$ being a parameter. Then, the multi-corn J^* for h_b is defined as the set of all $b \in \mathbb{C}$ for which the orbit of 0 under the mapping of h_b is bounded, i.e.,*

$$J^* = \{b \in \mathbb{C} : \{h_b^n\} \nrightarrow \infty\}, \tag{4}$$

where h_b^n is the n^{th} iteration of function $h_b(z)$.

Definition 4 (Picard Iteration). *Suppose that $h : \mathbb{C} \to \mathbb{C}$ is a complex function. Then, for any $z_0 \in \mathbb{C}$, Picard's iteration is given as*

$$z_{k+1} = h(z_k), \tag{5}$$

where $k = 0, 1, 2, \ldots$.

Definition 5 (Mann Iterative Process [25]). *Assume that $h : \mathbb{C} \to \mathbb{C}$ is a complex mapping. For any $z_0 \in \mathbb{C}$, Mann's iterative scheme is given as*

$$z_{k+1} = (1 - \zeta)z_k + \zeta h(z_k), \tag{6}$$

where $\zeta \in (0, 1]$ and $k = 0, 1, 2, \ldots$.

Definition 6 (Ishikawa Iterative Process [26]). *Let $h : \mathbb{C} \to \mathbb{C}$ be a mapping. For any $z_0 \in \mathbb{C}$, the Ishikawa process is stated as*

$$\begin{cases} z_{k+1} = (1 - \alpha)z_k + \alpha h(y_k), \\ y_k = (1 - \beta)z_k + \beta h(z_k), \end{cases} \tag{7}$$

where $\alpha, \beta \in (0, 1]$ and $k = 0, 1, 2, \ldots$.

Definition 7 (F-iterative Scheme [22]). *Assume that $h : \mathbb{C} \longrightarrow \mathbb{C}$ with $z_0 \in \mathbb{C}$ is an initial guess; then, the F-iteration is*

$$\begin{cases} u_k = (1-\alpha)z_k + \alpha h(z_k), \\ x_k = h(u_k), \\ y_k = h(x_k), \\ z_{k+1} = h(y_k), \end{cases} \tag{8}$$

where $\alpha \in (0,1]$ and $k = 0,1,2,\dots$

3. Main Results

In this section, we will prove some escape criteria for different functions via F-iteration.

Case I. (When $h_c(z) = z^n + c$.)

Theorem 1. *Suppose that $h_c(z) = z^n + c$ with $n \geq 2$, $c \in \mathbb{C}$ is a complex polynomial with $|z| \geq |c| > 2^{\frac{1}{n-1}}$ and $|z| > |c| > \left(\frac{2}{\alpha}\right)^{\frac{1}{n-1}}$. Further, suppose that $\{z_{k\in\mathbb{N}}\}$ is the sequence of iterates defined in (8); then, $|z_k| \to \infty$ as $i \to \infty$.*

Proof. Since $h_c(z) = z^n + c$ and assuming that $z_0 = z, u_0 = u$, the initial step of F-iteration is

$$|u_k| = |(1-\alpha)z_k + \alpha h(z_k)|.$$

For $k = 0$, we have

$$\begin{aligned} |u| &= |(1-\alpha)z + \alpha h(z)| \\ |u| &= |(1-\alpha)z + \alpha(z^n + c)| \\ |u| &\geq \alpha|z^n| - \alpha|c| - |z| + \alpha|z| \\ |u| &\geq \alpha|z^n| - \alpha|z| - |z| + \alpha|z| && \because |z| > |c| \\ |u| &\geq \alpha|z^n| - |z| \\ |u| &\geq |z|\Big(\alpha|z^{n-1}| - 1\Big) \\ |u| &\geq |z|. && \because |z| > \left(\frac{2}{\alpha}\right) \end{aligned}$$

The second step of F-iteration is given as

$$|x_k| = |h(u_k)|.$$

For $i = 0$, we have

$$\begin{aligned} |x| &= |h(u)| \\ |x| &= |u^n + c| \\ |x| &\geq |u^n| - |c| \\ |x| &\geq |u^n| - |u| \\ |x| &\geq |u|\Big(|u^{n-1} - 1\Big)| \\ |x| &\geq |u| && |z| > 2^{\frac{1}{n-1}} \end{aligned}$$

and the third step of F-iteration is

$$|y_k| = |h(x_k)|.$$

For $k = 0$, we have

$$\begin{aligned}
|y| &= |h(x)| \\
|y| &= |x^n + c| \\
|y| &\geq |x^n + c| \\
|y| &\geq |x^n + c| \\
|y| &\geq |x^n| - |x| \\
|y| &\geq |x|\Big(|x^{n-1}| - 1\Big)
\end{aligned}$$

Since $|u| \geq |z| > 2^{\frac{1}{n-1}}$, this implies that $|x^{n-1}| - 1 > 1$; therefore,

$$|y| \geq |x|.$$

For the last step of F-iteration, we have

$$|z_{k+1}| = |h(y_k)|.$$

For $k = 0$, we have

$$\begin{aligned}
|z_1| &= |h(y)| \\
|z_1| &= |y^n + c| \\
|z_1| &\geq |y^n| - |c| \\
|z_1| &\geq |z^n| - |z| \quad \because |y| \geq |x| \geq |u| \geq |z| \geq |c| \\
|z_1| &\geq |z|\Big(z^{n-1} - 1\Big).
\end{aligned}$$

For $k = 1$,

$$|z_2| \geq |z|\Big(z^{n-1} - 1\Big)^2.$$

Iterating up to the k^{th} term, we have

$$\begin{aligned}
|z_3| &\geq |z|\Big(z^{n-1} - 1\Big)^3. \\
|z_4| &\geq |z|\Big(z^{n-1} - 1\Big)^4. \\
&\;\;. \\
&\;\;. \\
&\;\;. \\
|z_k| &\geq |z|\Big(z^{n-1} - 1\Big)^k.
\end{aligned}$$

Since $|z| > 2^{\frac{1}{n-1}} \implies |z^{n-1}| - 1 > 1$, $z_k \to \infty$ as $k \to \infty$. $\square$

Corollary 1. *Consider that*

$$\Big\{ z_m > \max\Big\{ |b|, 2^{\frac{1}{n-1}}, \big(\tfrac{2}{\alpha}\big)^{\frac{1}{n-1}} \Big\}$$

for $m \geq 0$. *Since* $|z| > \big(\frac{2}{\alpha}\big)^{\frac{1}{n-1}} \implies \alpha|z^{n-1}| - 1 > 1$, $|z_{m+k}| > |z|(\alpha|z^{n-1}| - 1)^{m+k}$. *Hence,* $|z_k| \to \infty$ *as* $k \to \infty$.

Case II. (When $h_b(z) = \sin(z^n) + b$.)

Let $h(z) = \sin(z^n) + b$ with $n \geq 2, b \in \mathbb{C}$ be a sine function. Then, the Maclaurin expansion is

$$\begin{aligned} |\sin(z^n)| &= \left| z^n - \frac{z^{3n}}{3!} + \frac{z^{5n}}{5!} - \frac{z^{7n}}{7!} + \dots \right| \\ &\geq |z^n| \left| 1 - \frac{z^{2n}}{3!} + \frac{z^{4n}}{5!} - \frac{z^{6n}}{7!} + \dots \right| \end{aligned}$$

Assume that

$$|\eta_4| \leq \left| 1 - \frac{z^{2n}}{3!} + \frac{z^{4n}}{5!} - \frac{z^{6n}}{7!} + \dots \right|$$

so, we have

$$|\sin(z^n)| \geq |\eta_4||z^n|.$$

Similarly,

$$\begin{aligned} |\sin(y^n)| &\geq |\eta_3||y^n|. \\ |\sin(x^n)| &\geq |\eta_2||x^n|. \\ |\sin(u^n)| &\geq |\eta_1||u^n|. \end{aligned}$$

Theorem 2. *Suppose $h_b(z) = \sin(z)^n + b$ with $n \geq 2$, $b \in \mathbb{C}$ is a trigonometric function with $|z| \geq |b| > \left(\frac{2}{\alpha|\eta_4|}\right)^{\frac{1}{n-1}}, |z| \geq |b| > \left(\frac{2}{|\eta_1|}\right)^{\frac{1}{n-1}}, |z| \geq |b| > \left(\frac{2}{|\eta_2|}\right)^{\frac{1}{n-1}}$ and $|z| \geq |b| > \left(\frac{2}{|\eta_3|}\right)^{\frac{1}{n-1}}$. Further, suppose that $\{z_{k \in \mathbb{N}}\}$ is the sequence of iterates defined in (8); then, $|z_k| \to \infty$ as $k \to \infty$.*

Proof. As $h_b(z) = \sin(z)^n + b$, the initial step of F-iterative process is given as

$$|u_k| = |(1-\alpha)z_k + \alpha h(z_k)|.$$

for $k = 0$,

$$\begin{aligned} |u_0| &= |(1-\alpha)z_0 + \alpha h(z_0)| \\ |u| &= |\alpha h(z) + (1-\alpha)z| \\ |u| &\geq |\alpha h(z)| - |(1-\alpha)z| \\ |u| &\geq \alpha|\sin(z^n) + b| - (1-\alpha)|z| \\ |u| &\geq \alpha|z^n||\eta_4| - \alpha|b| - (1-\alpha)|z| \\ |u| &\geq \alpha|z^n||\eta_4| - |z| \qquad \because |z| \geq |b| \\ |u| &\geq |z|(\alpha|z^{n-1}||\eta_4| - 1) \\ |u| &\geq |z|. \qquad \because |z| > \left(\frac{2}{\alpha|\eta_4|}\right)^{n-1} \end{aligned}$$

The second step of the F-iterative scheme is

$$|x_k = h(u_k)|.$$

for $k = 0$,

$$\begin{aligned}
|x_0| &= |h(u_0)| \\
|x| &= |\sin(u^n) + b| \\
|x| &\geq |\sin(u^n)| - |b| \\
|x| &\geq |u^n||\eta_1| - |b| \\
|x| &\geq |u^n||\eta_1| - |u| \qquad \because |u| \geq |z| \geq |b| \\
|x| &\geq |u|(|u^{n-1}||\eta_1| - 1) \\
|x| &\geq |u| \qquad \because |z| > \left(\frac{2}{\eta_1}\right)^{n-1}.
\end{aligned}$$

The third step of F-iteration is

$$|x_k| = |h(u_k)|$$

for $k = 0$,

$$\begin{aligned}
|y_0| &= |h(x_0)| \\
|y| &= |\sin(x^n) + b| \\
|y| &\geq |\sin(x^n)| - |b| \\
|y| &\geq |x^n||\eta_2| - |b| \\
|y| &\geq |x^n||\eta_2| - |x| \qquad \because |z| \geq |b| \\
|y| &\geq |x|\Big(|x^{n-1}||\eta_2| - 1\Big) \\
|y| &\geq |x|. \qquad \because |z| > \left(\frac{2}{\eta_2}\right)^{n-1}
\end{aligned}$$

The final step of F-iteration is as follows:

$$|z_{k+1}| = |h(y_k)|$$

for $k = 0$,

$$\begin{aligned}
|z_1| &= |h(y_0)| \\
|z_1| &= |\sin(y^n) + b| \\
|z_1| &\geq |\sin(y^n)| - |b| \\
|z_1| &\geq |y^n||\eta_3| - |b| \\
|z_1| &\geq |z^n||\eta_3| - |z| \qquad \because |z| \geq |b| \\
|z_1| &\geq |z|(|z^{n-1}||\eta_3| - 1)
\end{aligned}$$

Iterating this up to the k^{th} terms,

$$\begin{aligned}
|z_2| &\geq |z|(|z^{n-1}||\eta_3| - 1)^2 \\
|z_3| &\geq |z|(|z^{n-1}||\eta_3| - 1)^3 \\
&. \\
&. \\
&. \\
|z_k| &\geq |z|(|z^{n-1}||\eta_3| - 1)^k
\end{aligned}$$

Since $|z| > \left(\frac{2}{\eta_3}\right)^{\frac{1}{n-1}} \implies |\eta_3||z^{n-1}| - 1 > 1$, $z_k \to \infty$ as $k \to \infty$. $\square$

Corollary 2. *Suppose*

$$\left\{z_m > \max\left\{|b|, \left(\frac{2}{|\eta_1|}\right)^{\frac{1}{n-1}}, \left(\frac{2}{|\eta_2|}\right)^{\frac{1}{n-1}}, \left(\frac{2}{|\eta_3|}\right)^{\frac{1}{n-1}}, \left(\frac{2}{\alpha|\eta_4|}\right)\right\}\right.$$

for some $m \geq 0$. *Since* $|z| > \left(\frac{2}{|\eta_1||\eta_2||\eta_3||\eta_4|\alpha}\right)^{\frac{1}{n-1}} \implies (|\eta_1||\eta_2||\eta_3||\eta_4|\alpha|z^{n-1}| - 1) > 1$, $|z_{m+k}| > |z|(|\eta_1||\eta_2||\eta_3||\eta_4|\alpha|z^{n-1}| - 1)^{m+k}$. *Hence,* $|z_k| \to \infty$ *as* $k \to \infty$.

Case III. (When $h_b(z) = e^{z^n} + b$.)

Let $h_b(z) = e^{z^n} + b$ with $n \geq 2$, $b \in \mathbb{C}$ be a exponential function. Then, the Maclaurin expansion is

$$\begin{aligned} |e^{z^n}| &= \left|1 + z^n + \frac{z^{2n}}{2!} + \frac{z^{3n}}{3!} + \frac{z^{4n}}{4!} + \dots\right| \\ &\geq \left|z^n + \frac{z^{2n}}{2!} + \frac{z^{3n}}{3!} + \frac{z^{4n}}{4!} + \dots\right| \\ &\geq |z^n|\left|1 + \frac{z^n}{2!} + \frac{z^{2n}}{3!} + \frac{z^{3n}}{4!} + \dots\right|. \end{aligned}$$

Assume that

$$|\eta_4| < \left|1 + \frac{z^n}{2!} + \frac{z^{2n}}{3!} + \frac{z^{3n}}{4!} + \dots\right|$$

so, we have

$$|e^{z^n}| \geq |\eta_4||z^n|.$$

Similarly

$$\begin{aligned} |e^{y^n}| &\geq |\eta_3||y^n|. \\ |e^{x^n}| &\geq |\eta_2||x^n|. \\ |e^{u^n}| &\geq |\eta_1||u^n|. \end{aligned}$$

Theorem 3. *Suppose that* $h_b(z) = e^{z^n} + b$ *with* $n \geq 2$, $b \in \mathbb{C}$ *is a exponential function with* $|z| \geq |b| > \left(\frac{2}{\alpha|\eta_4|}\right)^{\frac{1}{n-1}}$, $|z| \geq |b| > \left(\frac{2}{|\eta_1|}\right)^{\frac{1}{n-1}}$, $|z| \geq |b| > \left(\frac{2}{|\eta_2|}\right)^{\frac{1}{n-1}}$, $|z| \geq |b| > \left(\frac{2}{|\eta_3|}\right)^{\frac{1}{n-1}}$. *Further, suppose that* $\{z_{k\in\mathbb{N}}\}$ *is the sequence of iterates defined in* (8)*; then,* $|z_k| \to \infty$ *as* $k \to \infty$.

Proof. As $h_b(z) = e^{z^n} + b$, the initial step of F-iteration is given as

$$|u_k| = |(1-\alpha)z_k + \alpha h(z_k)|$$

for $k = 0$,

$$\begin{aligned}
|u_0| &= |(1-\alpha)z_0 + \alpha h(z_0)| \\
|u| &= |\alpha h(z) + (1-\alpha)z| \\
|u| &\geq |\alpha h(z)| - |(1-\alpha)z| \\
|u| &\geq \alpha |e^{z^n} + b| - (1-\alpha)|z| \\
|u| &\geq \alpha |z^n||\eta_4| - \alpha|b| - (1-\alpha)|z| \\
|u| &\geq \alpha |z^n||\eta_4| - |z| \qquad \because |z| \geq |b| \\
|u| &\geq |z|(\alpha |z^{n-1}||\eta_4| - 1) \\
|u| &\geq |z|. \qquad\qquad \because |z| > \left(\frac{2}{\alpha|\eta_4|}\right)^{n-1}
\end{aligned}$$

The second step of F-iteration is given as

$$|x_k| = |h(u_k)|.$$

for $k = 0$,

$$\begin{aligned}
|x_0| &= |h(u_0)| \\
|x| &= |e^{u^n} + b| \\
|x| &\geq |e^{u^n}| - |b| \\
|x| &\geq |u^n||\eta_1| - |b| \\
|x| &\geq |u^n||\eta_1| - |u| \qquad \because |z| \geq |b| \\
|x| &\geq |u|((|u^{n-1}||\eta_1| - 1) \\
|x| &\geq |u| \qquad \because |z| > \left(\frac{2}{\eta_1}\right)^{n-1}.
\end{aligned}$$

The third step of F-iteration is

$$|x_k| = |h(u_k)|$$

for $k = 0$,

$$\begin{aligned}
|y_0| &= |h(x_0)| \\
|y| &= |e^{x^n} + b| \\
|y| &\geq |e^{x^n}| - |b| \\
|y| &\geq |x^n||\eta_2| - |b| \\
|y| &\geq |x^n||\eta_2| - |x| \qquad \because |z| \geq |b| \\
|y| &\geq |x|\Big(|x^{n-1}||\eta_2| - 1\Big) \\
|y| &\geq |x|. \qquad \because |z| > \left(\frac{2}{\eta_2}\right)^{n-1}
\end{aligned}$$

The last step of F-iteration is as follows:

$$|z_{k+1}|| = |h(y_k)|$$

for $k = 0$,

$$
\begin{aligned}
|z_1| &= |h(y_0)| \\
|z_1| &= |e^{y^n} + b| \\
|z_1| &\geq |e^{y^n}| - |b| \\
|z_1| &\geq |y^n||\eta_3| - |b| \\
|z_1| &\geq |z^n||\eta_3| - |z| \qquad \because |z| > |b| \\
|z_1| &\geq |z|(|z^{n-1}||\eta_3| - 1)
\end{aligned}
$$

Iterating this up to k^{th} terms,

$$
\begin{aligned}
|z_2| &\geq |z|(|z^{n-1}||\eta_3| - 1)^2 \\
|z_3| &\geq |z|(|z^{n-1}||\eta_3| - 1)^3 \\
&. \\
&. \\
&. \\
|z_k| &\geq |z|(|z^{n-1}||\eta_3| - 1)^k
\end{aligned}
$$

Since $|z| > \left(\frac{2}{\eta_3}\right)^{\frac{1}{n-1}} \implies |\eta_3||z^{n-1}| - 1 > 1$, $z_k \to \infty$ as $k \to \infty$. $\square$

Corollary 3. *Consider*

$$
\left\{|z_m| > \max\left\{|b|, \left(\frac{2}{\eta_1}\right)^{\frac{1}{n-1}}, \left(\frac{2}{\eta_2}\right)^{\frac{1}{n-1}}, \left(\frac{2}{\eta_3}\right)^{\frac{1}{n-1}}, \left(\frac{2}{\alpha\eta_4}\right)\right\}\right.
$$

for some $m \geq 0$. *Since* $|z| > \left(\frac{2}{|\eta_1||\eta_2||\eta_3||\eta_4|\alpha}\right)^{\frac{1}{n-1}} \implies (|\eta_1||\eta_2||\eta_3||\eta_4|\alpha|z^{n-1}| - 1) > 1$, $|z_{m+k}| > |z|(|\eta_1||\eta_2||\eta_3||\eta_4|\alpha|z^{n-1}| - 1)^{m+k}$. *Hence,* $|z_k| \to \infty$ *as* $k \to \infty$.

4. Applications

This section presents some anti-Julia sets via proposed iteration. To generate anti-Julia sets, a criteria to execute image is needed to run the algorithm. In the generation of fractals, some popular algorithms are used (i.e., Distance Estimator [27], Potential Function Algorithm [28] and escape criteria [29,30]).

In this paper, we use escape criteria in Algorithms 1–3 to generate the J-sets, M-sets and Multi-corns. The graphs are generated on an "Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz 2.90GHz " computer using Mathematica 9.0 at Sub-Campus Depalpur, University of Agriculture, Faisalabad Pakistan.

4.1. Julia Set

In this part, we explain some J-sets for the polynomials $h(z) = z^n + c$, $h(z) = \sin(z^n) + c$ and $h(z) = e^{z^n} + c$ at different n in the orbit of F-iteration. We generated Julia sets for $n = 2$, $n = 3$ and $n = 3$ via F-iterative scheme. For each graph, we set $I = 100$ (i.e., the highest number of iterates) in Algorithm 1.

Example 1. *In this example, we present Julia sets for the function* $h(z) = z^n + c$ *at n = 2, 3 and 4. In Figures 1–3, we fix* $\alpha = 0.9$ *and change the values of c. In Figures 4–6, we fix the parameter* $\alpha = 0.8$ *and change the values of c. In Figures 7–9, we fix the value of* $\alpha = 0.7$ *for different values of c. In Figures 10, we fix the value of* $\alpha = 0.6$ *for different values of* $c = 0.56 + 0.9I$. *We set the occupied area* $A = [-1.5, 1.5]^2$. *We noted the generation time of all images in seconds and noticed that while increasing the value of n and decreasing the occupied area, images take less time for generation. Quadratic Julia sets take more generation time than Cubic and Bi-quadratic.*

Algorithm 1: Geometry of Julia Set

Input: $h(z)$—proposed complex polynomial, $A \subset \mathbb{C}$—area, κ—fixed number of iterates, $\alpha \in (0,1]$—input parameters, $c \in \mathbb{C}$—a complex constant, Coloursmap $[0 \ldots C-1]$.

Output: J-set.

for $z_0 \in$ A **do**
  R= Escape threshold for F-iteration from developed Corollary
  $k = 0$
  while $k \leq \kappa$ **do**
    $u_k = (1-\alpha)z_k + \alpha h(z_k)$,
    $x_k = h(u_k)$,
    $y_k = h(x_k)$,
    $z_{k+1} = h(y_k)$,
    if $|z_{k+1}| > R$ **then**
      break
    $k = k + 1$
  $k = \lfloor (C-1)k/\kappa \rfloor$
  colour z_0 with colurmap[i]

Algorithm 2: Geometry of M-Set

Input: $h(z)$—proposed complex polynomial, $A \subset \mathbb{C}$—area, κ—fixed number of iterates, $\alpha \in (0,1]$—input parameters, $c \in \mathbb{C}$—a complex constant, Coloursmap $[0 \ldots C-1]$.

Output: Mandelbrot Set

for $c \in$ A **do**
  R= escape threshold for F-iteration from Corollary
  $k = 0$
  z_0—any critical point of $h(z)$
  while $k \leq \kappa$ **do**
    $u_k = (1-\alpha)z_k + \alpha h(z_k)$,
    $x_k = h(u_k)$,
    $y_k = h(x_k)$,
    $z_{k+1} = h(y_k)$,
    if $|z_{k+1}| > R$ **then**
      break
    $k = k + 1$
  $k = \lfloor (C-1)k/\kappa \rfloor$
  colour z_0 with colurmap[i]

Example 2. *In this example, we present Julia sets for the function $h(z) = \sin(z^n) + c$ at $n = 2, 3$ and 4. In Figures 11 and 12, we fix $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.8$ and the values of $c = 1.09$. In Figures 13 and 14, we fix $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.95$ and the values of $c = 0.3 - 0.7i$ and $c = 0.55$, respectively. In Figures 15–17, we fix $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.5$ and the values of $c = 0.5 + 0.75i$, $c = 0.51$ and $c = 0.55$, respectively. In Figures 18 and 19, we fix $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.85$ and the values of $c = 0.51$ and $c = 0.55$, respectively. For $n = 2$ set, we set area $A = [-2, 5, 2.5]^2$, and for $n = 3, 4$ J-sets, area $A = [-2, 2]^2$. In Figures 20–25, we present J-sets for the function $h(z) = e^{z^n} + c$ at $n = 2$, 3 and 4. In Figures 20–22, we fix $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.009$, and in Figures 23–25, we fix $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.001$. We set occupied area $A = [-2.5, 2.5]^2$ and noted the generation time of all images in seconds.*

Algorithm 3: Geometry of Multi-corn

Input: $h(\bar{z})$—proposed complex polynomial, $A \subset \mathbb{C}$—area, κ—fixed number of iterates, $\alpha \in (0,1]$—input parameters, $c \in \mathbb{C}$—a complex constant, Coloursmap $[0 \ldots C-1]$.
Output: Multi-corn

for $c \in$ A **do**
  R=escape threshold for f-iteration from established Corollary
  $k = 0$
  z_0—any CP of $h(\bar{z})$
  while $k \leq \kappa$ **do**
    $u_k = (1-\alpha)z_k + \alpha h(z_k),$
    $x_k = h(u_k),$
    $y_k = h(x_k),$
    $z_{k+1} = h(y_k),$
    if $|z_{k+1}| > R$ **then**
      break
    $k = k + 1$
  $k = \lfloor (C-1)k/\kappa \rfloor$
  colour z_0 with colurmap[i]

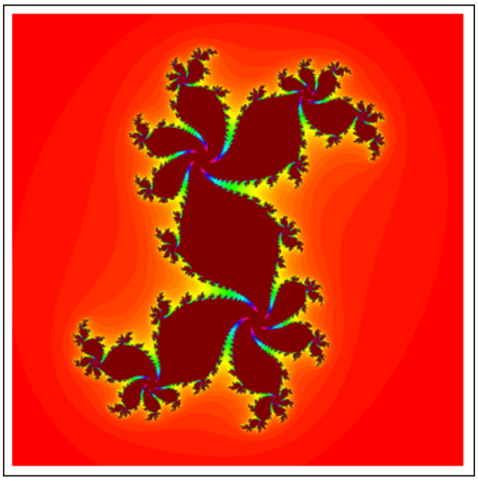

Figure 1. Second degree J-set via F-iteration with visualization period $= 76.6$ s.

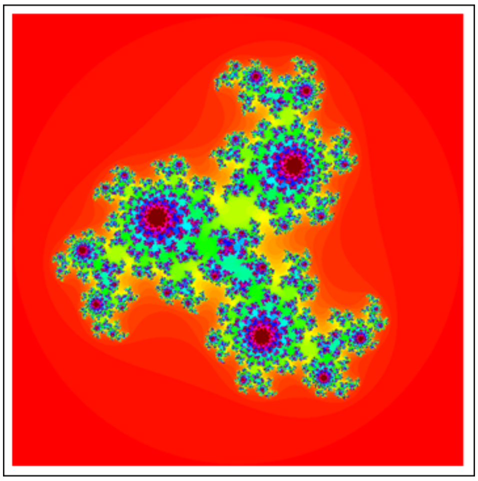

Figure 2. Third degree J-set via F-iteration with visualization period $= 53.8$ s.

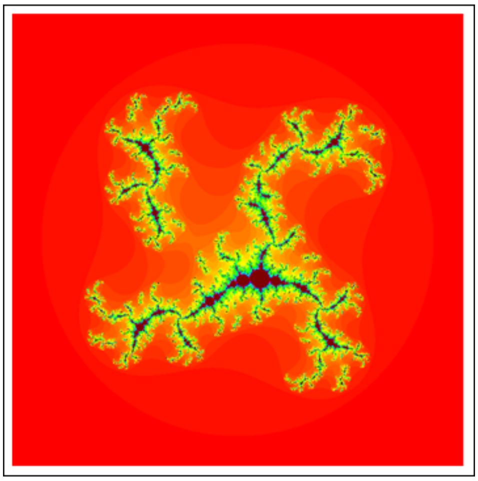

Figure 3. Fourth degree J-set via F-iterative scheme with image visual time = 31.18 s.

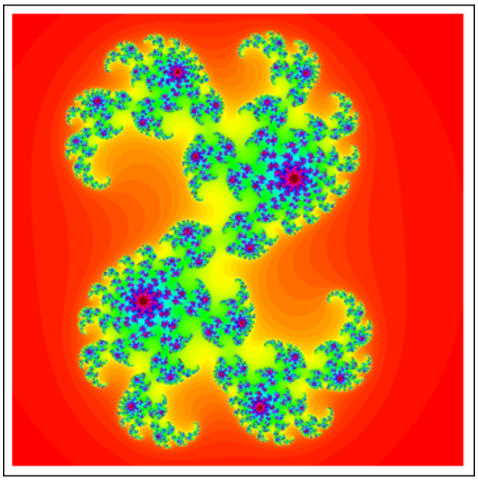

Figure 4. Second degree J-set via F-iteration with image visual time = 54.45 s.

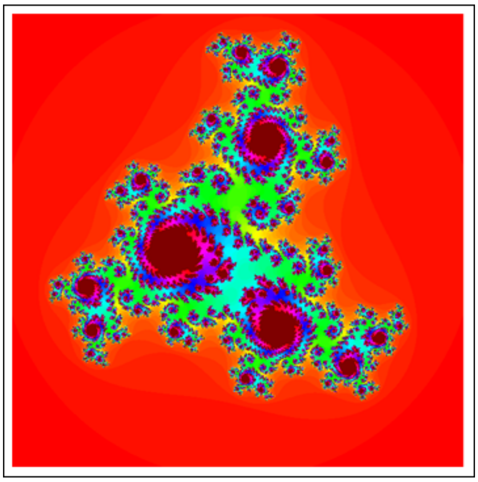

Figure 5. J-set of degree three via F-iterative process with image visual time = 75.12 s.

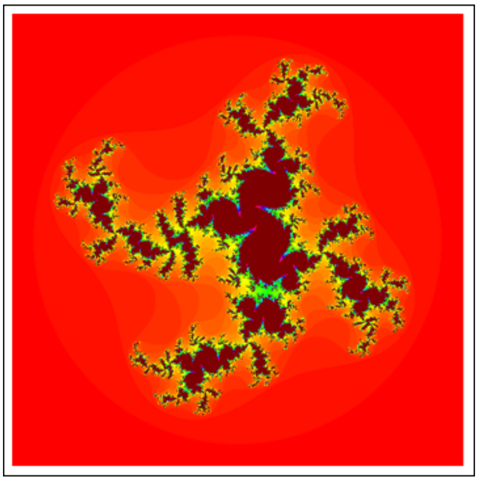

Figure 6. J-set of degree 4 via F-iterative scheme with image visual time = 69.85 s.

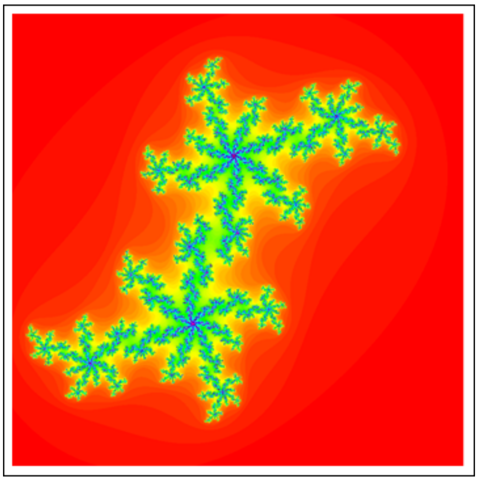

Figure 7. J-set via F-iteration with image visual period = 57 s.

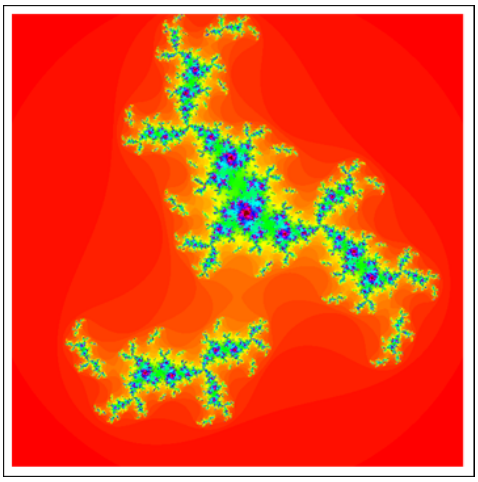

Figure 8. Cubic J-set via F-iteration with image visual time = 33.48 s.

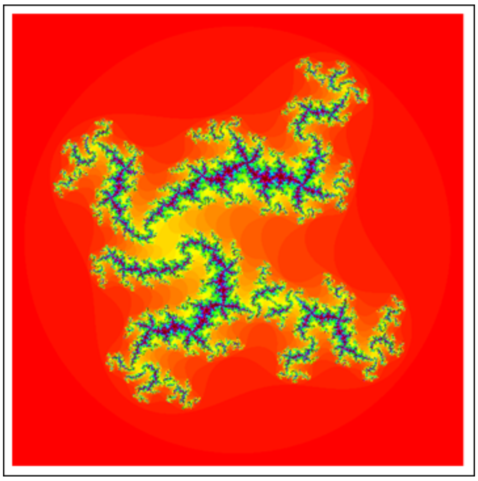

Figure 9. J-set via F-iteration with image visual time = 51.28 s.

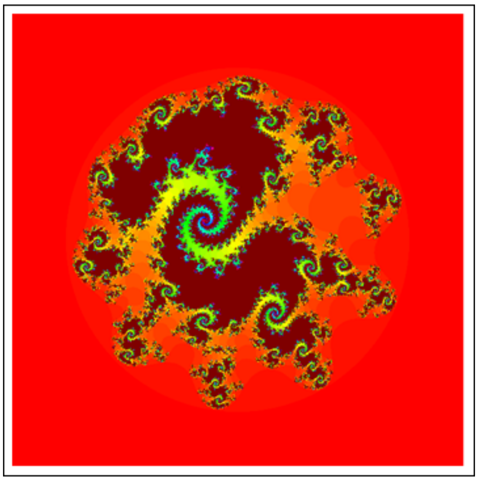

Figure 10. Decade Julia set for the function $h(z) = z^{10} + c$ via F-iteration.

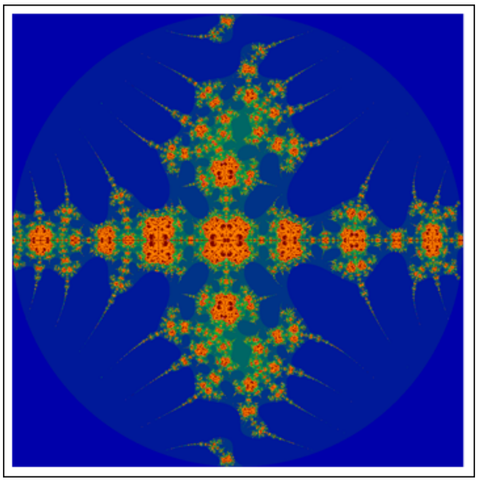

Figure 11. J-set via F-iteration for $\sin(z^2) + 1.09$ with visualization period = 119.03 s.

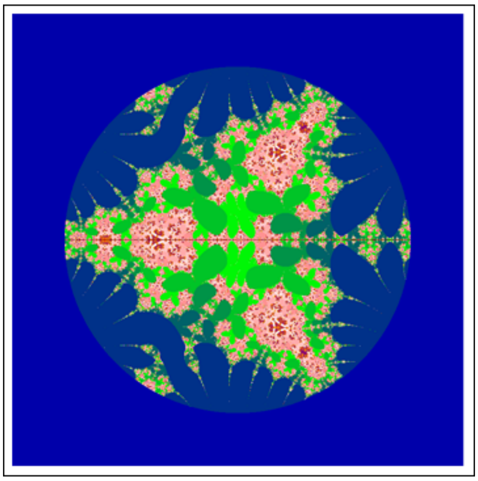

Figure 12. J-set via F-iterative scheme for $\sin(z^3) + 1.09$ with image visual time $= 119.03$ s.

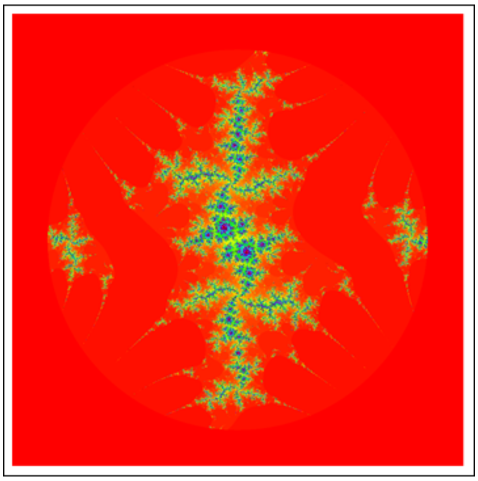

Figure 13. J-set via F-iteration for $\sin(z^2) + (0.3 - 0.7i)$ with image visual time $= 83.5$ s.

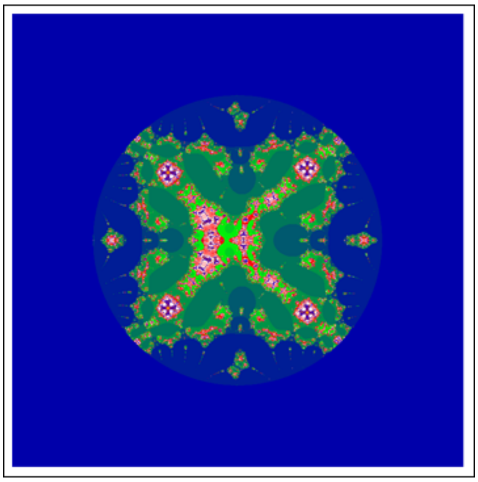

Figure 14. J-set via F-iteration $\sin(z^4) + 0.55$ with visualization time $= 62.5$ s.

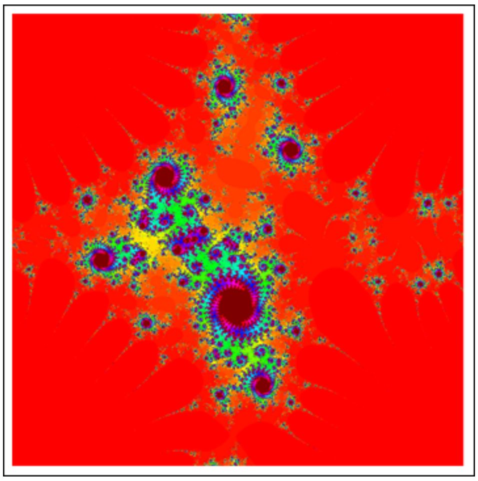

Figure 15. J-set via F-iteration $\sin(z^2) + (0.5 + 0.75i)$ with image visual period $= 203.5$ s.

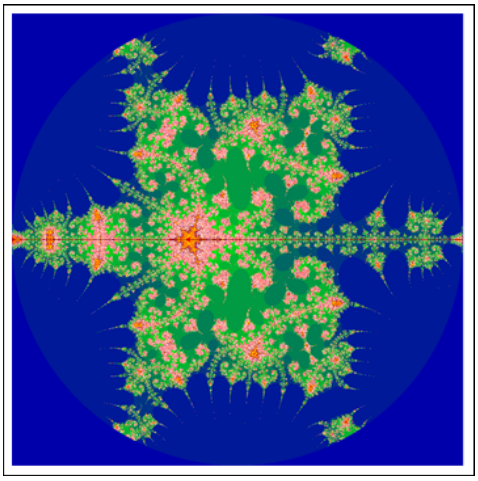

Figure 16. J-set via F-iteration for $\sin(z^3) + 0.51$ with image visual time $= 62.37$ s.

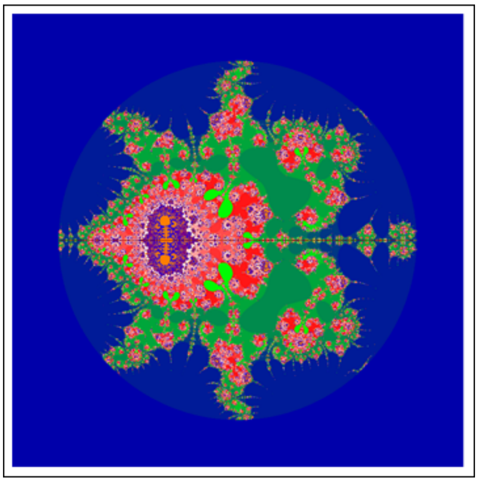

Figure 17. J-set via F-iteration for $\sin(z^4) + 0.55$ with image visual time $= 91.7$ s.

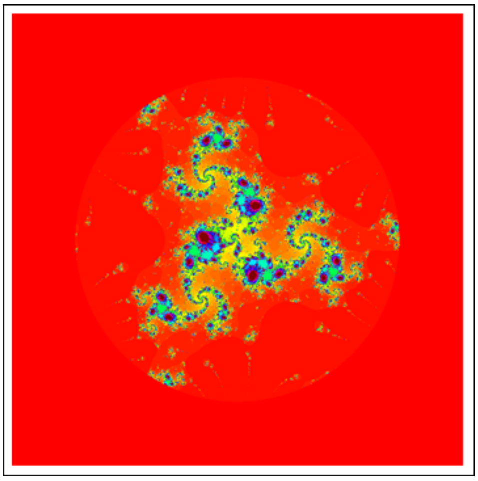

Figure 18. J-set via F-iterative process for $\sin(z^3) + 0.51$ with image visual time $= 46.5$ s.

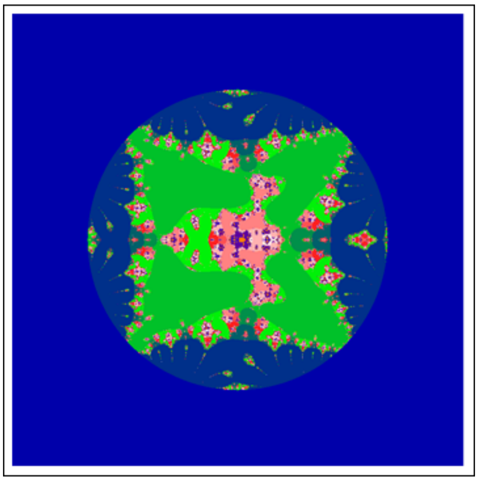

Figure 19. J-set via F-iteration for $\sin(z^4) + 0.55$ with image visual time $= 61.23$ s.

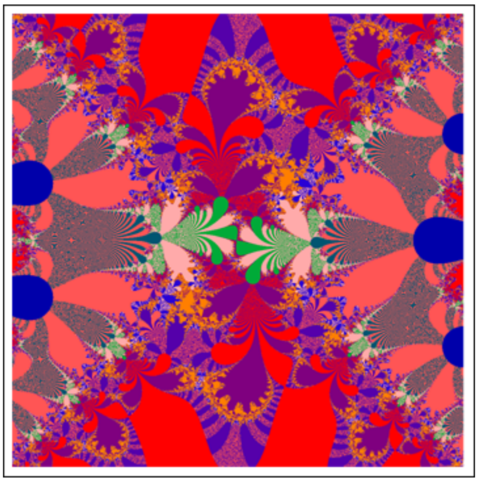

Figure 20. J-set via F-iterative scheme for $e^{z^2} + (-0.1i)$ with image visual time $= 69.3$ s.

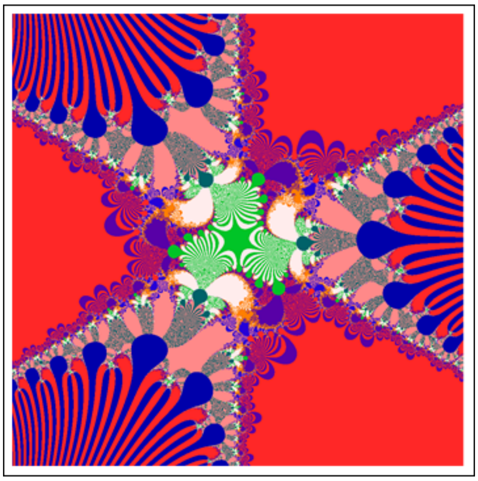

Figure 21. J-set via F-iterative process for $e^{z^3} + (-0.1i)$ with image visual time $= 103$ s.

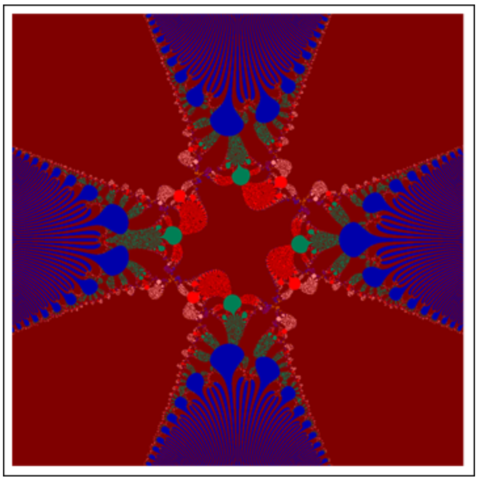

Figure 22. J-set via F-iterative scheme for $e^{z^4} + (-0.1i)$ with image visual time $= 1270$ s.

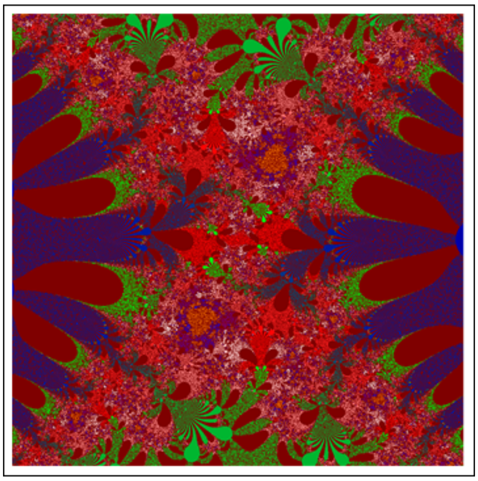

Figure 23. J-set via F-iterative scheme for $e^{z^2} + (-0.5i)$ with image visual time $= 1209$ s.

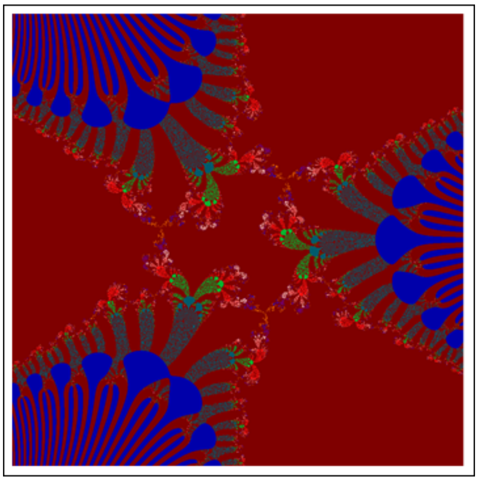

Figure 24. J-set via F-iterative process for $e^{z^3} + (-0.5i)$ with image visual time = 1206 s.

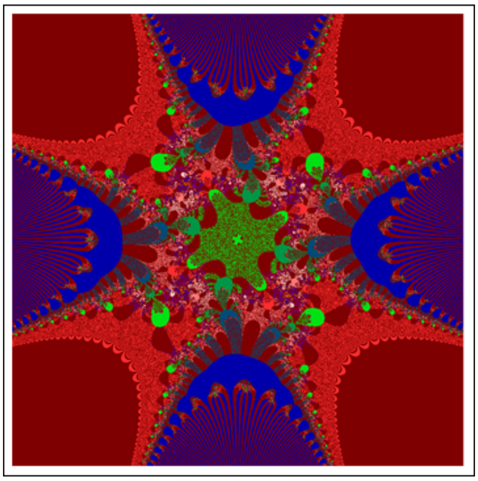

Figure 25. J-set via F-iteration for for $e^{z^4} + (-0.5i)$ with image visual time = 1111 s.

4.2. Mandelbrot Set

Here, we discuss some Mandelbrot sets for the functions $h(z) = z^n + c, h(z) = \sin(z^n) + c$ and $h(z) = e^{z^n} + c$ at different n in the orbit of proposed iteration. We have generated Mandelbrot sets for $n = 2, 3, 4$ via F-iteration. In all graphs, we set $I = 100$ (i.e., the highest number of iterates) in Algorithm 2.

Example 3. *Here, we explain M-sets for the function $h(z) = z^n + c$ at n = 2, 3 and 4. In Figures 26–28, we fix the parameter $\alpha = 0.9$. In Figures 29–31, we fix the parameter $\alpha = 0.8$. In Figures 32 and 33, we fix the parameter $\alpha = 0.99$. In Figures 34, we fix the parameters $\alpha = 0.7$. In Figures 35, we fix the parameters $\alpha = 0.1$. We set the occupied area $A = [-1.5, 1.5]^2$. We noted the generation time of all images in seconds and noticed that while increasing the value n and decreasing the occupied area, images take less time for generation. Quadratic Mandelbrot sets take more generation time than Cubic and Bi-quadratic.*

Example 4. *In this example, we explain M-sets for the function $h(z) = \sin(z^n) + c$ at n = 2, 3 and 4. In Figures 36, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.8$. In Figures 37 and 38, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.7$. In Figures 39 and 40, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.85$. In Figures 41–43, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.07$. In Figures 44, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.95$. In Figures 45, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.009$. In Figures 46, we fix the parameters $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.001$. We set the occupied area $A = [-2.5, 2.5]^2$. We noted the generation time of all images* *in seconds and noticed that while increasing the value n and decreasing the occupied area, images take less time for generation.*

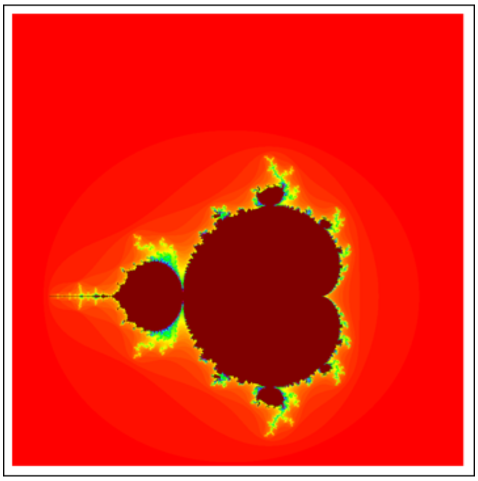

Figure 26. M-set of degree 2 via F-iterative scheme with image visual time = 87.8 s.

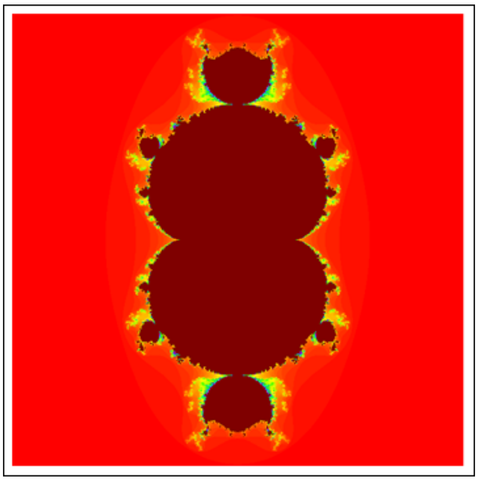

Figure 27. M-set of third degree via F-iteration with image visual time = 95.9 s.

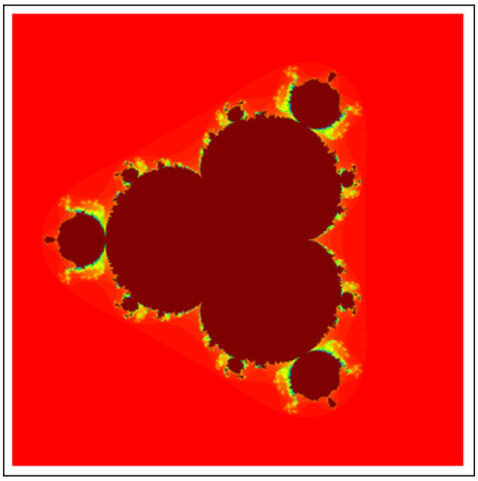

Figure 28. Bi-quadratic M-set via F-iteration with visual time = 127.07 s.

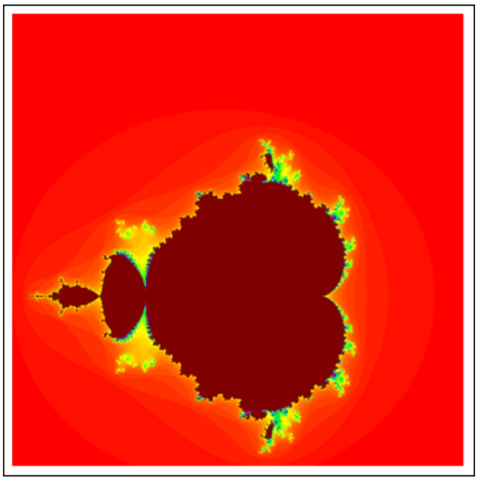

Figure 29. Quadratic M-set via F-iteration with image visual time = 118.7 s.

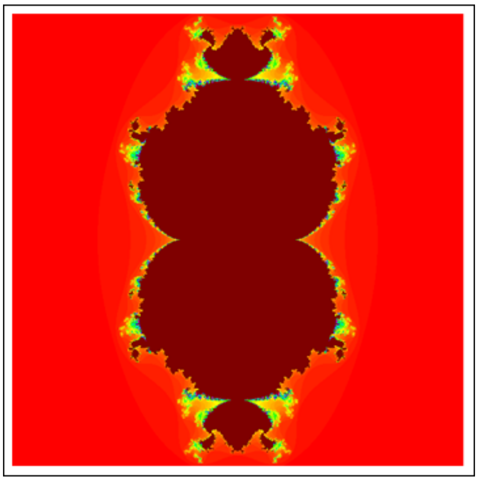

Figure 30. Third degree M-set via F-iteration with image visual time = 109.1 s.

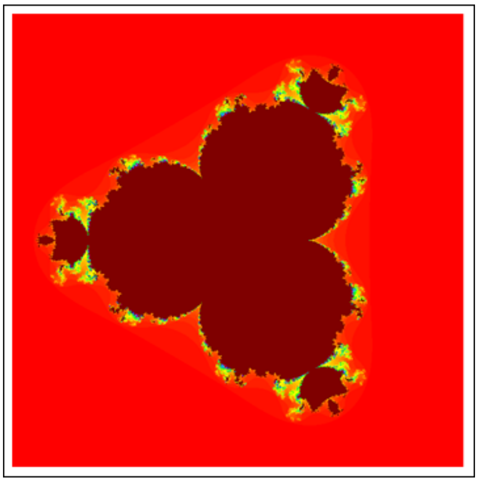

Figure 31. Bi-quadratic M-set via F-iteration with visual time = 139.7 s.

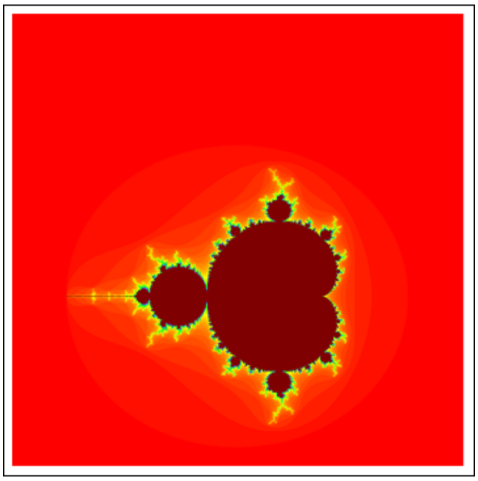

Figure 32. Quadratic M-set via F-iteration with image visual time = 69.3 s.

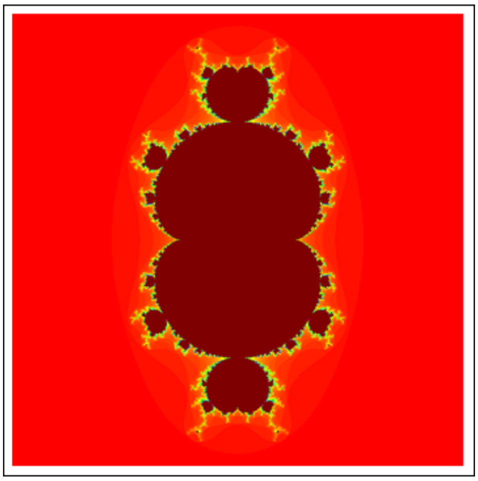

Figure 33. Third degree M-set via F-iteration with image visual time = 81.39 s.

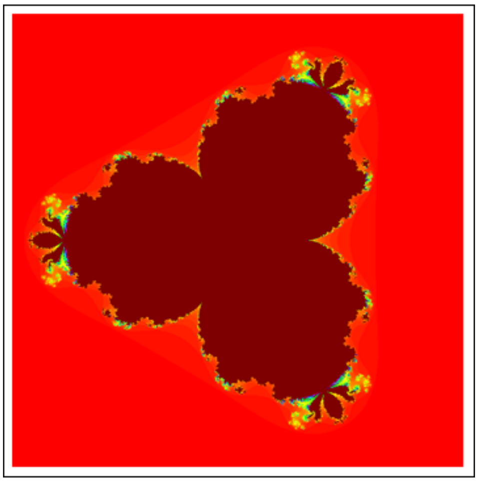

Figure 34. Bi-quadratic M-set with image visual time = 148.6 s.

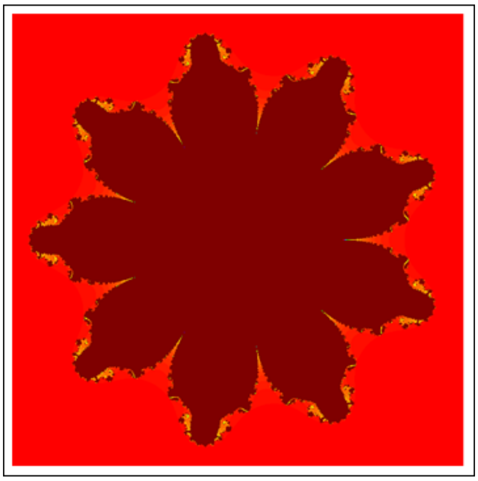

Figure 35. Decade Mandelbrot set.

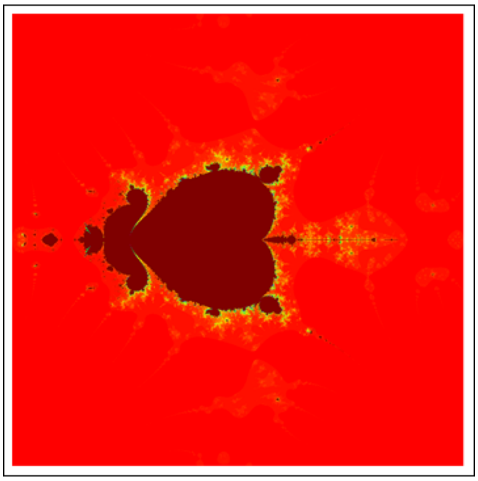

Figure 36. M-set via F-iterative scheme for $\sin(z^2) + c$ with image visual time = 176.9.6 s.

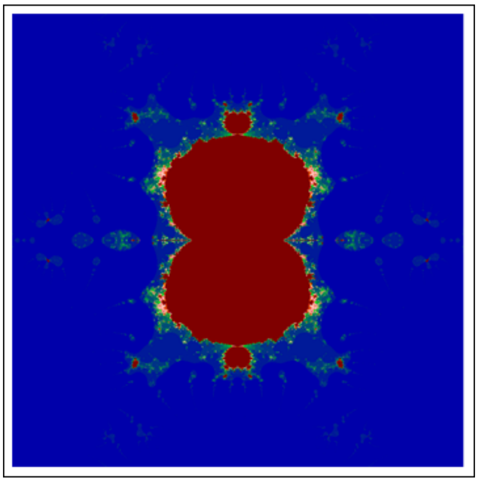

Figure 37. M-set in F-orbit for $\sin(z^3) + c$ with image visualization time = 155.4.

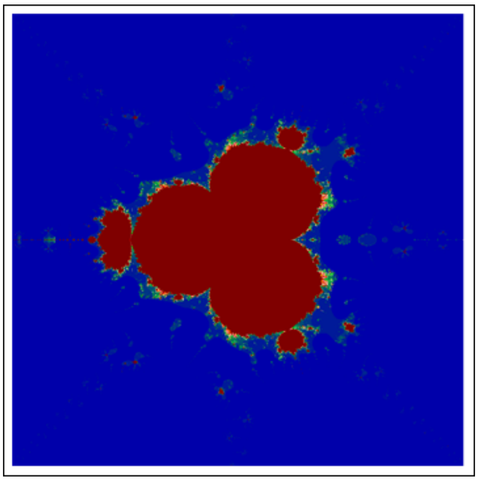

Figure 38. M-set via F-iteration for $\sin(z^4) + c$ with image visual time = 175.2 s.

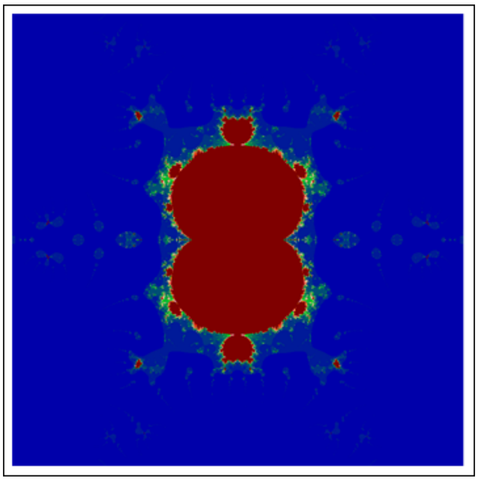

Figure 39. M-set via F-iteration for $\sin(z^3) + c$ with image visual time = 140.4 s.

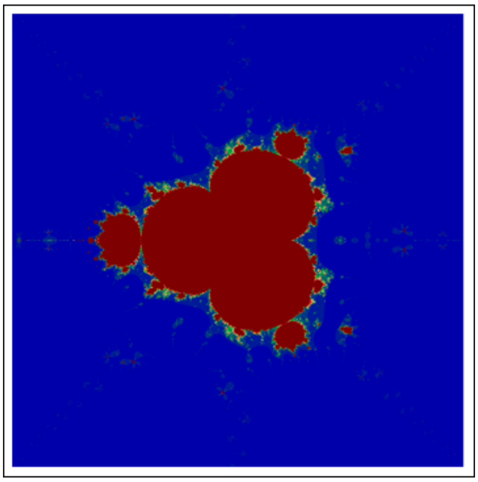

Figure 40. M-set via F-iterative scheme for $\sin(z^4) + c$ with image visual time = 294.9 s.

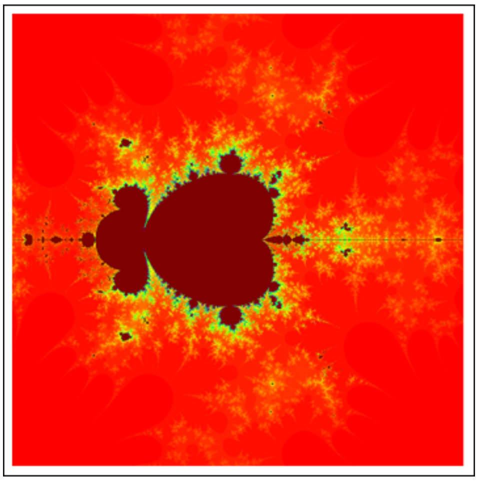

Figure 41. M-set via F-iteration for $\sin(z^2) + c$ with time of generation = 239.5.

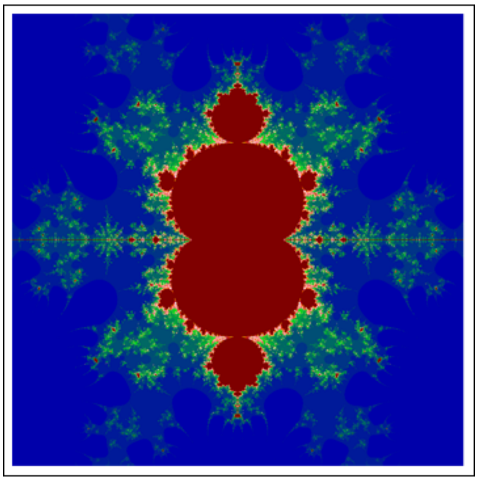

Figure 42. M-set via F-iteration $\sin(z^3) + c$ with image visual time = 155.4 s.

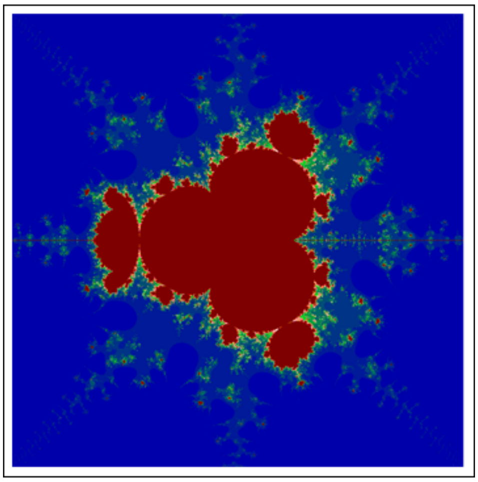

Figure 43. M-set via F-iteration $\sin(z^4) + c$ with image visual time = 202 s.

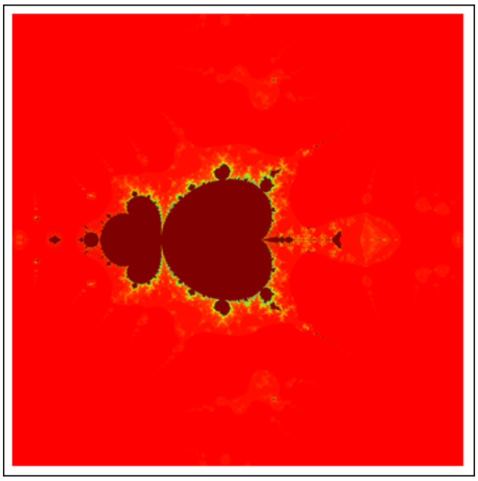

Figure 44. M-set via F-iteration for $\sin(z^2) + c$ with image visual time = 149.2 s.

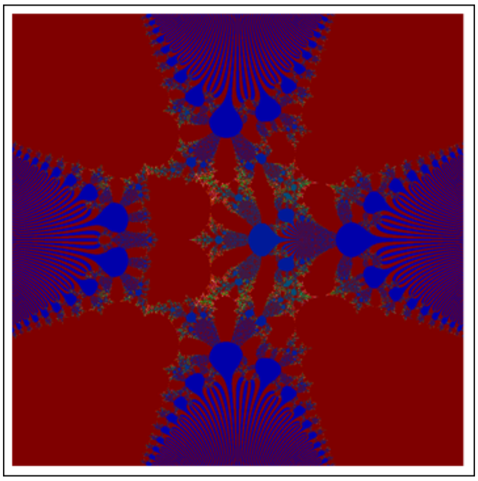

Figure 45. M-set via F-iteration for $e^{z^4} + c$ with image visual time = 1948 s.

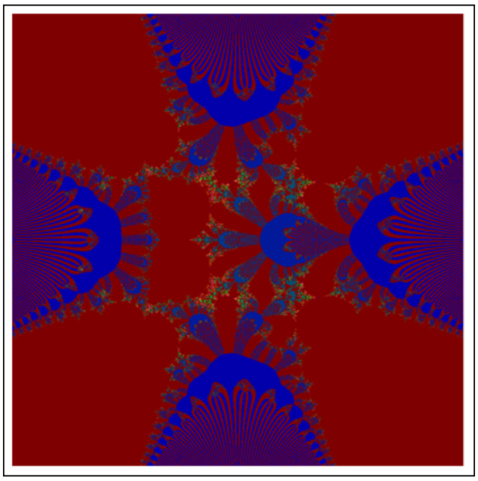

Figure 46. M-set via F-iteration for $e^{z^4} + c$ with image visual time = 1713 s.

4.3. Multi-Corn

Here, we discuss some multibrot set for for the polynomial $h(z) = \bar{z}^n + c$ at n = 2, 3 and 4. We noticed that for $n = 2$, Multi-corns become tricorns. We noted the generation time of each figure. We fixed the number of iteration up to 100 for each image in Algorithm 3.

Example 5. *Here, we explain Multi-corns for the function* $h(z) = \bar{z}^n + c$ *at* $n = 2$, *3 and 4. In Figures 47–49, we fix the parameter* $\alpha = 0.9$. *In Figures 50–52, we fix the parameter* $\alpha = 0.8$. *In Figures 53–55, we fix the parameter* $\alpha = 0.7$. *We noticed that the tricorn is three-cornered and the style of its self similarity is exactly the same as that of the Mandelbrot set. All Multi-corns have* $n + 1$ *lashes.*

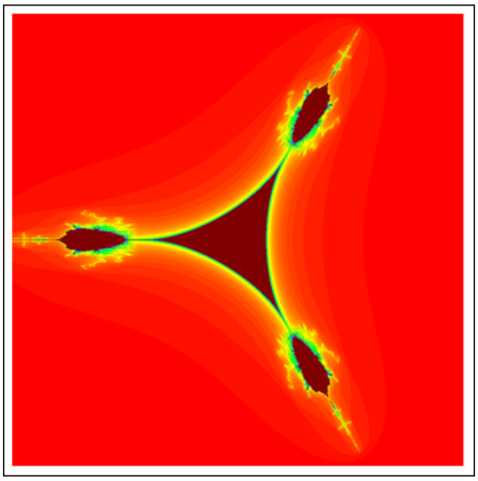

Figure 47. Tricorn via F-iteration with image visual time = 45.6 s.

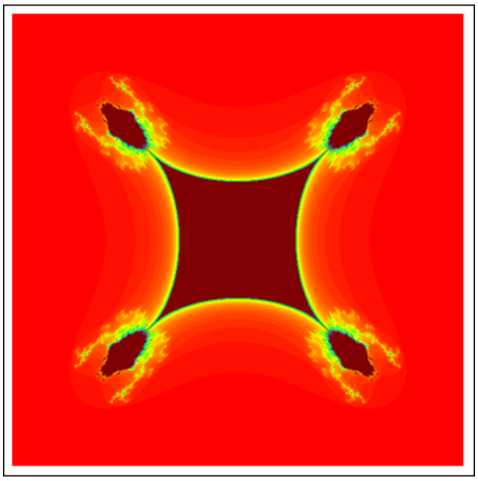

Figure 48. Multi-corn via F-iteration with image visual time = 66.3 s.

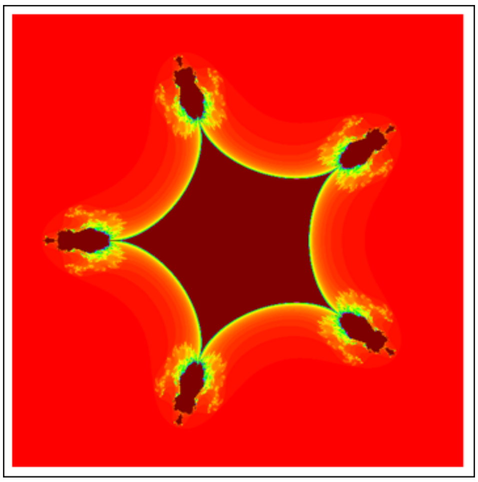

Figure 49. Multi-corn via F-iteration with image visual time = 74.23 s.

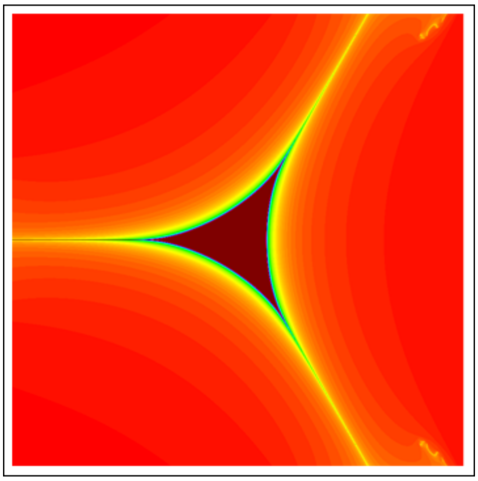

Figure 50. Tricorn via F-iteration with image visual time = 61.594 s.

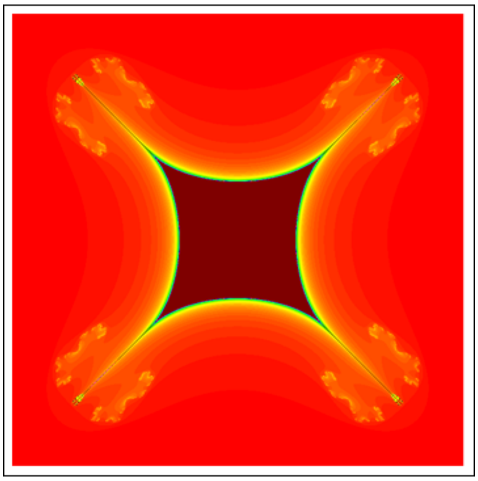

Figure 51. Multi-corn via F-iteration with image visual time = 45.7 s.

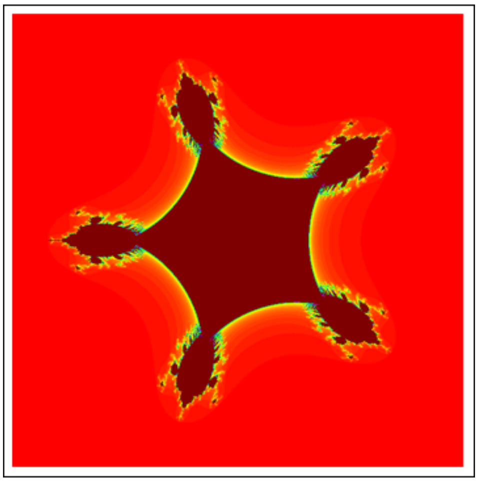

Figure 52. Multi-corn via F-iteration with image visual time = 84.25 s.

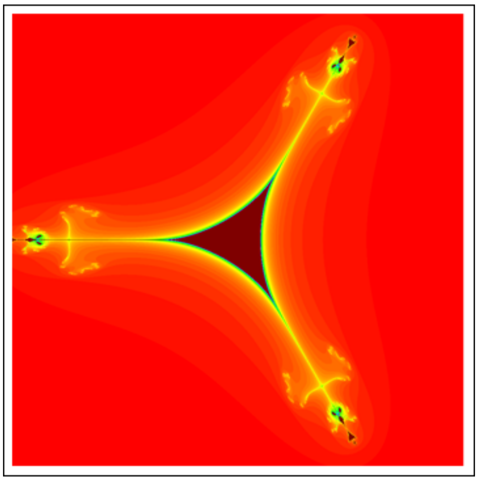

Figure 53. Tricorn via F-iteration with image visual time = 51.86 s.

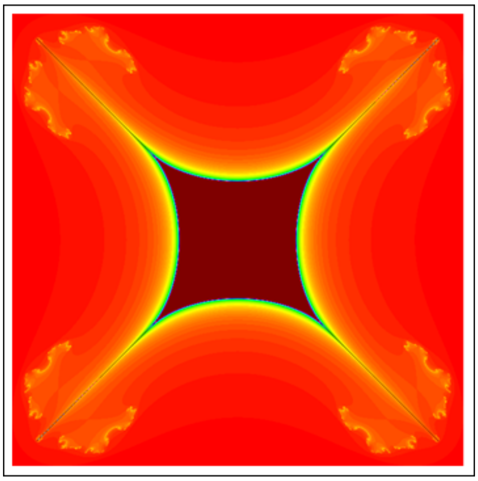

Figure 54. Multi-corn via F-iteration with image visual time = 50.1 s.

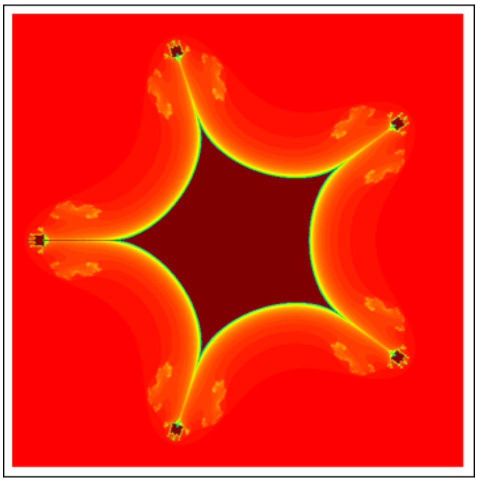

Figure 55. Multi-corn via F-iteration with image visual time = 66.1 s.

Example 6. *In this example, we present Multi-corns for the function* $h(z) = \sin \bar{z}^n + c$ *at* n *= 2, 3 and 4. In Figures 56–58, we fix the parameters* $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.85$*. In Figures 59 and 60, we fix the parameters* $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.0.07$*. In Figure 61, we fix the parameters* $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.0.7$*. In Figure 62, we fix the parameters* $\alpha, |\eta_1|, |\eta_2|, |\eta_3|, |\eta_4| = 0.95$*. For Tricorns, we set area* $A = [-2.5.2.5]^2$*; for Multi-corns, we set area* $A = [-2, 2]^2$*.*

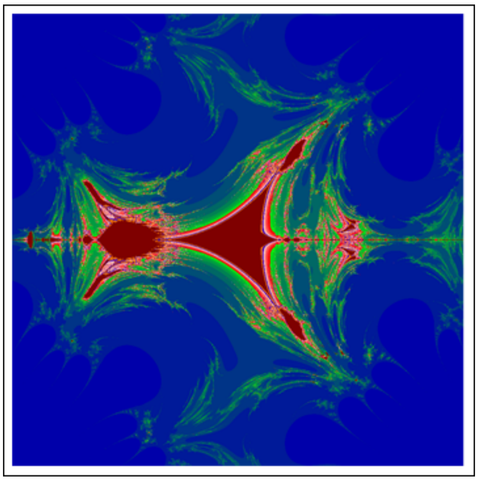

Figure 56. Tricorn via F-iteration with image visual time = 225.9 s.

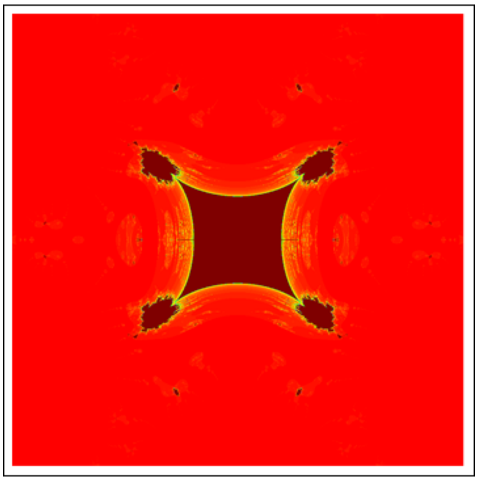

Figure 57. Multi-corn via F-iteration with image visual time = 145.9 s.

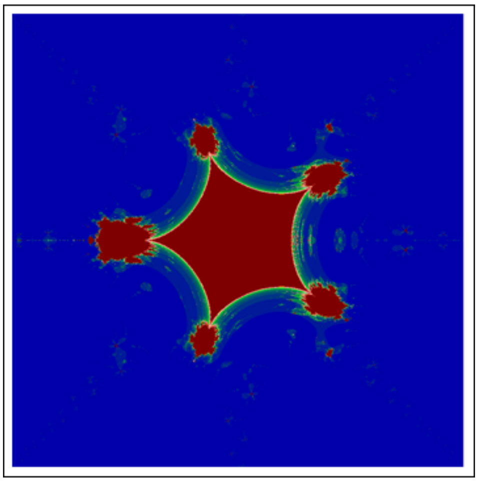

Figure 58. Multi-corn via F-iteration with image visual time = 201 s.

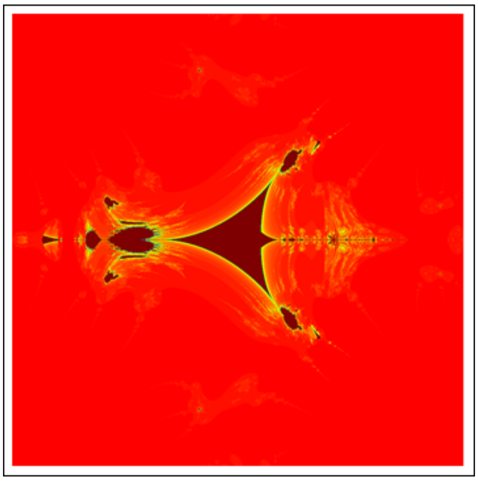

Figure 59. Tricorn via F-iteration with image visual time = 154.01 s.

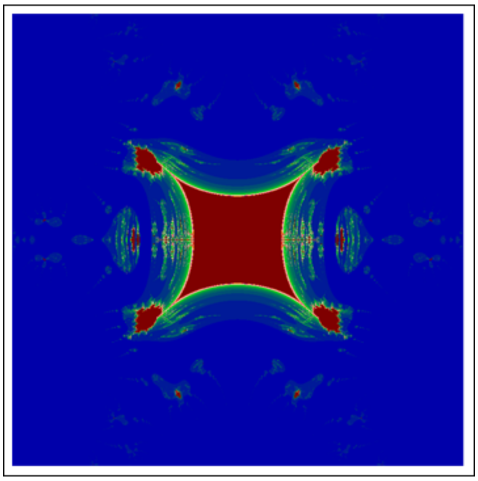

Figure 60. Multi-corn via F-iteration with image visual time = 193.3 s.

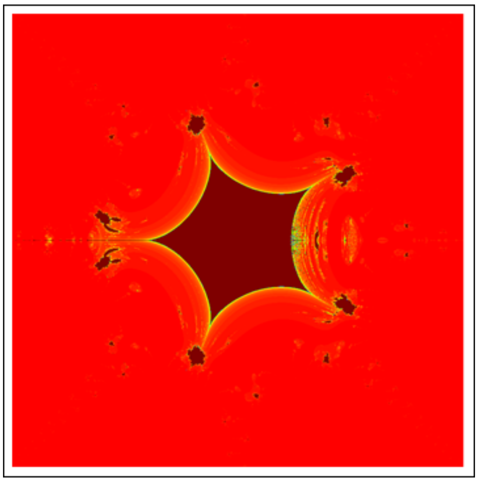

Figure 61. Multi-corn via F-iteration with image visual time = 155.1 s.

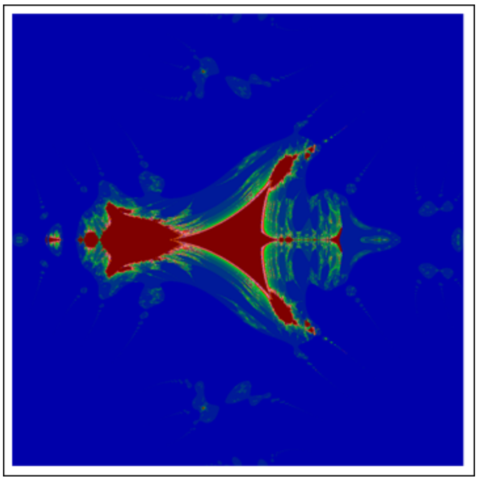

Figure 62. Tricorn via F-iteration with image visual time = 181.9 s.

5. Conclusions

We proved escape criterion for a complex function $h(z) = z^n + c$, complex trigonometric function $h(z) = \sin(z^n) + c$ and complex exponential function $h(z) = e^{z^n} + c$ with $n \geq 2$ and $c \in \mathbb{C}$ via F-iteration. We used the established results in Algorithms 1–3 for the Julia sets, Mandelbrot sets and Multi-corns in the orbit of our proposed iteration. We generated Quadratic, Cubic and Bi-quadratic Julia Mandelbrot sets, and some Tricorns and Multi-corns.

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