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Editorial

Editorial for Special Issue "Fractional Dynamics: Theory and Applications"

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The investigation of fluctuations and random processes in complex systems and random environments has been attracting much attention for years. The theoretical modelling of diffusion in heterogeneous and disordered media, and determining the governing equations require interdisciplinary collaboration between scientists working in physics, mathematics, computer science, biology, chemistry, economy, etc. The diffusion and relaxation processes in heterogeneous and disordered materials and random environments, such as materials with defects, multi-scale amorphous composites, fractals, comb-like structures, weighted graphs, and networks, are often anomalous. This means that the mean squared displacement shows a power-law dependence on time, while the relaxation law is non-exponential or non-Debye.

Many of the aforementioned processes can be described by various random walk models, generalised and fractional diffusion equations, and Fokker–Planck and Langevin equations, and many of them can be described by means of fractional and generalised derivatives and integrals.

The purpose of this Special Issue on "Fractional Dynamics: Theory and Applications" was to reflect the current situation in fractional dynamics theory, and to collect various models for the description of anomalous diffusion and random walks in complex systems, a purpose that was successfully addressed by the accepted papers.

A brief summary of all the published papers in the Special Issue follows.

In the paper by Tawfik and Hefny, "Subdiffusive Reaction Model of Molecular Species in Liquid Layers: Fractional Reaction-Telegraph Approach" [1], the authors considered a time-fractional reaction–telegraph equation model for the description of molecular species movement in liquid layers and analysed the corresponding equation with the help of the Fourier–Laplace transform method. The mean squared displacement was calculated using the Mittag-Leffler functions and their asymptotic behaviour to obtain the short and long time limits for the different molecular species.

In the paper by Bazhlekova, "An Inverse Source Problem for the Generalized Subdiffusion Equation with Nonclassical Boundary Conditions" [2], the author analysed generalised diffusion and relaxation equations with a memory kernel in the presence of a source term. The elegant subordination approach was used to find the conditions under which the solution of the generalized diffusion equation is non-negative. Representative memory kernels for the description of anomalous diffusion were considered, and a special case of fractional Jeffreys-type equation was also recovered.

Iomin, in his paper "Quantum Walks in Hilbert Space of Lévy Matrices: Recurrences and Revivals" [3], considered the quantum evolution of wave functions, which is controlled by the spectrum of Lévy random matrices. By the analytical treatment of quantum recurrences and revivals in the Hilbert space, the author showed that, for chaotic systems with a uniform mixing property, the distribution of the return probability is exponential, while in systems with nonuniform mixing, the distribution of the return probability is algebraic in large recurrence times.

Koltun et al., in their paper "Diffusion in Heterogenous Media and Sorption—Desorption Processes" [4], considered an adsorption–desorption process of a system described by a diffusion equation in a heterogeneous medium, characterised by a spatially dependent diffusion coefficient. The solution of the equation was obtained in terms of a stretched exponential, which is opposite to the Gaussian distribution for the free diffusion process. The authors considered a fractional kinetic equation with a memory-dependent adsorption term for the processes on the surface, which recovers various scenarios of non-Debye relaxation.

Gao, Peng and Tang, in the paper "Optimizing the First-Passage Process on a Class of Fractal Scale-Free Trees" [5], investigated the first-passage process for a class of fractal scale-free trees. The authors also constructed a biased random walk strategy by introducing weight to each edge of the fractal scale-free trees, thus obtaining the weighted fractal scale-free trees. The global mean first-passage times for unbiased and biased random walks were obtained, and the point at which the minimum for a particular value of the weight was achieved was found.

In the paper by Górska, Horzela and Penson, "Non-Debye Relaxations: The Ups and Downs of the Stretched Exponential vs. Mittag–Leffler's Matchings" [6], the authors elaborated the stochastic description of non-exponential relaxation via a subordination approach by using different characteristic Lévy exponents. The Mittag-Leffler functions were used to analyse the relaxation functions, showing that, with them, it is possible to capture the stretched exponential behaviour of the relaxation function in the short time limit and the power-law decay in the long time limit.

In the paper "A Novel Analytical Formula for the Discounted Moments of the ECIR Process and Interest Rate Swaps Pricing" [7] by Boonklurb et al., the extended Cox–Ingersoll–Ross (ECIR) process, which is an economic model used to describe the dynamics of interest rates, was considered by means of a stochastic differential equation. A numerical method for numerically evaluating the analytical formulas and numerical validations for the formulas was introduced, and a fractional generalisation of the ECIR process was analysed analytically and numerically.

Petreska et al., in the paper "Tuning of the Dielectric Relaxation and Complex Susceptibility in a System of Polar Molecules: A Generalised Model Based on Rotational Diffusion with Resetting" [8], considered the generalised rotational diffusion equation in the context of non-exponential dielectric relaxation in a medium of polar molecules. A stochastic resetting in the equation was also introduced, and it was shown that the resetting leads to a saturation of the autocorrelation function to a constant value, and that the dielectric relaxation dynamics can be tuned by changing the resetting rate.

In the paper by Sibatov et al., "Modelling of Electron and Thermal Transport in Quasi-Fractal Carbon Nitride Nanoribbons" [9], the electronic and phonon transport in a device based on quasi-fractal carbon nitride nanoribbons with Sierpinski triangle blocks was studied. The hopping electron transport was analysed by introducing energetic disorder, and a transient anomalous diffusion was observed by the calculation of the mean squared displacement. Such a hopping in quasi-fractal nanoribbons can be considered as an explicit physical implementation of the generalised comb model.

In the paper by Padash et al., "Asymmetric Lévy Flights Are More Efficient in Random Search" [10], the first arrival (hitting) dynamics and efficiency of the asymmetric Lévy flights were studied using the Fokker–Planck equation with a δ-sink and an asymmetric space-fractional derivative operator. An optimal search strategy, depending on the initial distance and location of the target with respect to the initial searcher position, was developed. It was shown that the asymmetry in jumps can lead to essentially higher efficiency of the Lévy search compared to symmetric Lévy flights at both short and long distances.

As a Guest Editor, I would like to thank all the authors for their contributions with original research papers in the field of fractional dynamics. My gratitude also goes to all the peer reviewers for their help in the reviewing process. Finally, I particularly thank the Section Managing Editor Ms. Dale Du for her diligent work and assistance throughout the process of managing this Special Issue.

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