



Article

On Z-Intuitionistic Fuzzy Fractional Valuations for Medical Diagnosis: An Intuitionistic Fuzzy Knowledge-Based Expert System

Nitesh Dhiman¹, Madan M. Gupta², Dhan Pal Singh³, Vandana⁴, Vishnu Narayan Mishra^{5,*} 
and Mukesh K. Sharma^{1,*} 

- ¹ Department of Mathematics, Chaudhary Charan Singh University, Meerut 250004, Uttar Pradesh, India; niteshdhiman91@gmail.com
- ² Department of Statistics, Meerut College, Meerut 250001, Uttar Pradesh, India; madangupta22@gmail.com
- ³ Department of Mathematics, Keshav Mahavidyalaya, University of Delhi, Delhi 110034, Uttar Pradesh, India; drdhanpalsingh@gmail.com
- ⁴ Department of Computer Applications, Sir Chhotu Ram Institute of Engineering and Technology–SCRIET, Chaudhary Charan Singh University, Meerut 250004, Uttar Pradesh, India; ranavd@gmail.com
- ⁵ Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak, Anuppur 484887, Madhya Pradesh, India
- * Correspondence: vishnunarayanmishra@gmail.com (V.N.M.); drmukeshsharma@gmail.com (M.K.S.)

Abstract: In an uncertain situation, data may present in continuous form or discrete form. We have various techniques to deal with continuous data in a realistic situation. However, when data are in discrete form, the existing techniques are inadequate to deal with these situations, and these techniques cannot provide the proper modulation for adequate analysis of the system. In order to provide the proper acceleration to discrete data, we need an appropriate modulation technique that can help us to handle unconditional boundedness on the technique and will operate like the techniques used for continuous data with fractional variables. In this work, we developed an intuitionistic fuzzy fractional knowledge-based expert system using unconditional and qualified fuzzy propositions based on the Z-intuitionistic fuzzy fractional valuation probability density function. In this proposed method, the discrete fractional variables will be converted into intuitionistic fuzzy fractional numbers and then be used in our algorithm. The proposed Z-intuitionistic fuzzy fractional valuation knowledge-based system can easily be applied in the medical field for the diagnosis of diseases in a vague environment due to the ordered-pair characteristics of the Z-intuitionistic fuzzy fractional valuation. In this study, we collected data of dengue patients, which included seven clinical findings: Temperature, sugar, Pulse Rate (PR), age, cough, and Blood Pressure (BP). A numerical example was also carried out to elaborate on the present technique. In addition, a comparative study is discussed in this work. We also provide the managerial implications of the data, with the limitations of the proposed technique presented at the end of this work.

Keywords: intuitionistic fuzzy set; Z-intuitionistic fuzzy fractional valuation; knowledge-based system; intuitionistic fuzzy propositions; intuitionistic unconditional and qualified fraction fuzzy propositions



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1. Introduction

In real life, information may often be imperfect due to its vague nature. The aspects of imperfect knowledge are uncertain and imprecise. These uncertainties are characterized by the degree of truth in the form of the membership grade. In 1965, L.A. Zadeh [1] introduced the concept of fuzzy set theory. Fuzzy set theory is a useful tool to deal with these types of imprecise factors in the modelling of systems. The fuzzy logic approach has previously been used in the medical field [2,3] in existing studies. Different industrial and medical problems characterized by such uncertainty and imprecision [4,5] can be solved using fuzzy

set theory. To solve this type of problem, we have to develop an inference system that will include all the input factors and provide a final output value. The fuzzy inference system used in decision making is essentially based on the experience, intuition, and knowledge of experts. Due to the complexity of real-life problems, sometimes these features cannot address all the uncertainties of the considered problem. The Z-number, introduced by Zadeh [6], is applicable for such types of uncertainties present in a problem. By using the concept of Z-numbers, an estimation of the input and output factors can easily be performed by the concept of fuzzy logic. The Z-number can be represented by an ordered pair (A, B) , where A and B are fuzzy sets. The concept of Z-numbers has been used in many real-life problems [7]. Kang et al. [8,9] applied the Z-number in the context of a decision-making problem. Kang et al. [10] also provided a method for converting Z-numbers into the theory of a classical fuzzy number. Xiao [11] also used the concept of the Z-number in multi-criteria decision making. Azadeh et al. [12] provided a mathematical technique based on the analytical hierarchy process with the help of the Z-number extension of fuzzy sets. Many extended techniques of fuzzy logic, such as intuitionistic fuzzy logic [13,14] and mediative fuzzy logic [15], have also been used in engineering and agricultural fields and the medical field [16,17]. Mediative fuzzy logic is also applied in the extension technique [18] for consistent measurement in decision making. The concept of the Z-number is very closely related to the idea of linguistic terms [19]. Yager [20] provided an ordered triple (V, A, B) to represent Z-valuation, which is equal to the statement V is (A, B) . The Z-number is used to provide information about variable V . Z-valuation indicates that V takes A with certainty B . Let us consider V is (A, B) ; here we are assuming V is as random variable. Zadeh [6] suggested that this Z-valuation can also be formed as a restriction on V described as

$$\text{Prob}(V \text{ is } A) \text{ is } B$$

Let P (some probability distribution) be the space of all probability distributions [21] on universal set U . The given information can be handled by using a possibility distribution function. Using Zadeh's definition of fuzzy probability, we can express the given information as

$$\text{Porb}_p(V \text{ is } A) = \int A(x)p(x)dx$$

and

$$\mu(p) = \mu(\text{Porb}_p(V \text{ is } A)) = \mu\left(\int A(x)p(x)dx\right)$$

that will give the degree of truth of $\text{prob}_p(V \text{ is } A)$ that satisfies B . In this manner, as a possibility distribution over the space P , we may easily express the information V as (A, B) , under the assumption that V is a random variable. In 1986, Atanassov [22] proposed an idea to deal with the current uncertainty present in a problem with the consideration of favorable, as well as unfavorable cases. The intuitionistic fuzzy set defines the membership grade and non-membership grade of an element, regarding its belongings and non-belongings, with a hesitation margin, where the sum of these three values will always be equal to 1. In an intuitionistic environment, we may express the given information as

$$\mu(p) = \mu(\text{Porb}_p(V \text{ is } A)) = \mu\left(\int A(x)p(x)dx\right)$$

and

$$\vartheta(p) = \vartheta(\text{Porb}_p(V \text{ is } A)) = \vartheta\left(\int A(x)p(x)dx\right)$$

where the first expression gives the degree of truth, and the second expression gives the false values of $\text{prob}_p(V \text{ is } A)$ that will satisfy B . In this work, the application of fuzzy logic in the measurement of disease diagnostic levels is considered. By using unconditional and qualified intuitionistic fuzzy propositions, we developed an intuitionistic fuzzy knowledge-based expert system based on the Z-intuitionistic fuzzy fractional valuation probability density function. We proposed a Z-intuitionistic fuzzy fractional valuation-based inference

system that is easily applicable in the medical diagnosis of patients with dengue in an intuitionistic fuzzy environment. Fuzzy set theory and intuitionistic fuzzy logic have many applications [23,24], including in the medical field [25,26]. The calculation of an intuitionistic fuzzy generator [27] is also used and has been applied in medical diagnoses. Dhiman and Sharma [28] provided a truth-value restriction method based on the inference of intuitionistic conditional and qualified fuzzy propositions. Szmidt and Kacprzyk proposed the concept of measuring distances between intuitionistic fuzzy sets [29,30] and provided a similarity measure for intuitionistic fuzzy sets in medical diagnostic [31] reasoning. The intuitionistic fuzzy set has also been applied [32] in the determination of students' performance. The aim of this proposed work is to generalize the concept of Z-valuation over Z-intuitionistic fuzzy fractional valuation and to apply this concept over an inference system based on unconditional and qualified intuitionistic fuzzy propositions. The proposed method can be applied in the diagnostic process of dengue disease to obtain the various severity levels of patients.

The basic objective of the study contains certain points, as follows:

- (1) We will generalize the concept of the Z-number to the Z-intuitionistic number to obtain knowledge about the valuation function.
- (2) We will introduce the concept of the Z-intuitionistic fuzzy fractional valuation based on these Z-intuitionistic numbers.
- (3) We will develop an intuitionistic fuzzy knowledge-based system based on the Z-intuitionistic fuzzy fractional valuation function by using unconditional and qualified intuitionistic fuzzy propositions in the context of the probability density function.
- (4) We will show the applicability of our proposed algorithm in the medical field and discuss the relationship between Z-intuitionistic fuzzy fractional valuation and symptomatic factors of dengue-infected patients.

The present research paper is divided into ten sections. In the Section 2, basic concepts related to our proposed work are defined. In the Section 3, we defined the structure of the proposed inference system. In the Section 4, we developed an algorithm for the proposed Z-intuitionistic fuzzy fractional valuation function. In the Section 5, we defined the mathematical formulation of our proposed work. In the Section 6, we defined the process of collecting dengue-patient data, and in this section, we present the data of fifteen infected patients, as well as their symptomatic structure. The numerical computations of this work are given in the Section 7. In the next section, we present the conclusions of the entire work, we also discussed a comparative study in this section. In Section 9, we give the results and discussions of the proposed work. In the last section, we present the future scope of this work.

2. Basic Concepts

2.1. Fuzzy Logic

The idea of fuzzy logic was first introduced by Lotfi A. Zadeh of the University of California at Berkeley in the 1960s. Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0) Boolean logic. It may help to view fuzzy logic as the way reasoning truly works, and binary or Boolean logic as simply a special case of it.

2.2. Fuzzy Sets

Fuzzy set A is defined based on universal set U , denoted as

$$A = \{(x, \mu_A(x)) : x \in U\} \quad (1)$$

where $\mu_A(x) : U \rightarrow [0, 1]$ represents the membership value of A given in universal set U .

2.3. Fuzzy Number

Fuzzy set A defined on the basis of universal set U is known as a fuzzy number if it satisfies the following three properties:

- Fuzzy set A should be “normal”, i.e., there exists a point $x \in U$ such that $\mu_A(x) = 1$.
- Fuzzy set A should be “convex”, i.e., $\forall x_1, x_2 \in U$ and $\alpha \in [0, 1]$

$$\mu_A(\alpha x_1 + (1 - \alpha)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

- Support of A, i.e., $\{x \in U: \mu_A(x) > 0\}$ is bounded.

2.4. Intuitionistic Fuzzy Set

Intuitionistic fuzzy set B is defined on the basis of universal set U, denoted as

$$B = \{(x, \mu_B(x), \vartheta_B(x)) : x \in U\} \quad (2)$$

where the functions $\mu_B(x)$ and $\vartheta_B(x) : U \rightarrow [0, 1]$ represent the membership and non-membership grades, respectively, and $\pi_B(x)$ represents the hesitation value of x in B such where

$$\mu_B(x) + \vartheta_B(x) + \pi_B(x) = 1$$

2.5. Intuitionistic Fuzzy Number

Intuitionistic fuzzy set B, defined on the basis of universal set U, is known as an intuitionistic fuzzy number if it satisfies the following three properties:

- The intuitionistic fuzzy set must satisfy the normal property, i.e., there exists a point $x \in U$ such that $\mu_B(x) = 1, \vartheta_B(x) = 0$.
- The intuitionistic fuzzy set must satisfy the convexity property, i.e., $\forall x_1, x_2 \in U$ and $\alpha \in [0, 1]$

$$\mu_B(\alpha x_1 + (1 - \alpha)x_2) \geq \min(\mu_B(x_1), \mu_B(x_2))$$

and

$$\vartheta_B(\alpha x_1 + (1 - \alpha)x_2) \leq \max(\mu_B(x_1), \mu_B(x_2))$$

- Support of B, i.e., $\{x \in U: \mu_B(x) > 0, \vartheta_B(x) < 1\}$ is bounded.

2.6. Z-Numbers

If A and B are two fuzzy numbers defined on the basis of universal set U, then the ordered pair (A, B) represents the Z-number. A Z-number can also be viewed as the fuzzy set of probability functions that arise from the malleable restriction of the probability of a fuzzy fact by a fuzzy probability. The representation of the Z-number requires complex calculations and does not reduce down to the original fuzzy fact when the probability is one.

2.7. Z-Intuitionistic Numbers

If A and B are two intuitionistic fuzzy numbers defined on universal set U, then the Z-intuitionistic number can be represented by an ordered pair (A, B). An intuitionistic Z-number can be viewed as the fuzzy set of probability functions that arise from the malleable restriction of the probability of an intuitionistic fuzzy fact by an intuitionistic fuzzy probability.

2.8. Z-Valuations

If A and B are two fuzzy numbers defined on the basis of universal set U, and V is a valuation function, then the ordered triple (V, A, B) will represent the Z-valuation, which deals with the statement “V is (A, B)”. The Z-valuation indicates that V takes A with certainty B. The Z-valuation can also be viewed as a restriction on V, described by previous authors [20] as Prob (V is A) is B.

2.9. Z-Intuitionistic Valuations

If A and B are two intuitionistic fuzzy numbers defined on the basis of universal set U, and V is a valuation function, then the ordered triple (V, A, B) will represent the Z-intuitionistic valuation, which deals with the statement “V is (A, B)”. The Z-intuitionistic valuation indicates that V takes A with certainty B (considering both membership and non-membership values).

2.10. Z-Intuitionistic Fuzzy Fractional Valuations

The Z-intuitionistic fuzzy fractional valuation deals with the statement “V is (A, B)”, and the Z-intuitionistic fuzzy fractional valuation deals with the uncertainty present in the fraction form of the intuitionistic fuzzy probability. We express the given information as

$$Porb_p(V \text{ is } A) = \int A(x)dx \tag{3}$$

and

$$\mu(p) = \mu(Porb_p(V \text{ is } A)) = \mu\left(\int \frac{M^u(x)}{M^l(x)}dx\right) \tag{4}$$

where $A(x) = M^u(x)/M^l(x)$, $M^u(x)$ and $M^l(x)$ represent the numerator and denominator parts of membership values, respectively, and

$$\vartheta(p) = \mu(Porb_p(V \text{ is } A)) = \vartheta\left(\int \frac{NM^u(x)}{NM^l(x)}dx\right) \tag{5}$$

where $A(x) = NM^u(x)/NM^l(x)$, $NM^u(x)$ and $NM^l(x)$ represent the numerator and denominator parts of non-membership values, respectively.

3. Proposed Z-Intuitionistic Fuzzy Fractional Valuation Knowledge-Based System

In this section, we developed an inference system based on the Z-intuitionistic valuation called the “Z-intuitionistic fuzzy fractional valuation knowledge-based system”. The architecture of the proposed system is presented in Figure 1, which consists of eight components.

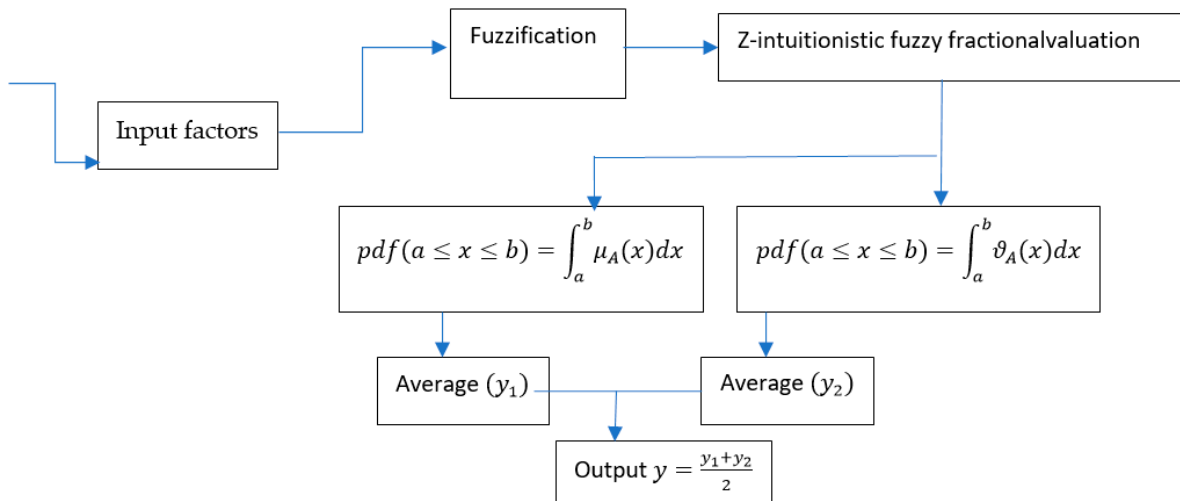


Figure 1. Proposed inference system using Z-intuitionistic fuzzy fractional valuation.

4. Algorithm of the Proposed Z-Intuitionistic Fuzzy Fractional Valuation-Based Inference System

The proposed algorithm of the system contains seven steps, and the process of each step is given below:

Step 1: Let us consider a set of input factors $I = \{\text{input-I, input-II, } \dots \text{input N}\}$ and an output factor, namely “Out”, which represents the output of the system.

Step 2: Categorize each input and output factor into three linguistic categories, viz low, medium, and high.

Step 3: Fuzzify each input and output factor using triangular- as well as trapezoidal-shaped intuitionistic fuzzy numbers; the selection of a triangular or trapezoidal shape is dependent upon the structure of the data.

Step 4: Now consider a Z-intuitionistic fuzzy fractional valuation for the uncertain variable V to express the knowledge in a probabilistic form, that is “ V is input-N is Out”.

Step 5: Now consider intuitionistic fuzzy rules with the help of unconditional and qualified propositions as mentioned above in step 4.

Step 6: To obtain a conventional and numeric value of the intuitionistic fuzzy rules produce probability density function P , as shown in Figure 1.

Step 7: Take the average of all outputs obtained from step 6 (for both membership and non-membership values) to obtain the final crisp output of the proposed system, which shows the specific categories of the output amongst the three linguistic categories.

5. Mathematical Formulation

In this section, we formulate the mathematical model of the symptoms included in our work. Seven symptoms, namely temperature, sugar, pulse rate, age, cough, and blood pressure are categorized into three linguistic categories, in the form of trapezoidal or triangular membership functions, the shape of which is dependent upon the trend of the data:

$$\mu_{\text{trimf}}(\tau) = \begin{cases} \frac{\tau - a_{11}}{a_{22} - a_{11}} & a_{11} \leq \tau \leq a_{22} \\ \frac{a_{33} - \tau}{a_{33} - a_{22}} & a_{22} \leq \tau \leq a_{33} \\ 0 & \text{else} \end{cases}$$

$$\nu_{\text{trimf}}(\tau) = \begin{cases} \frac{a_{22} - \tau}{a_{22} - \bar{a}_{11}} & \bar{a}_{11} \leq \tau \leq a_{22} \\ \frac{\tau - a_{22}}{a_{33} - a_{22}} & a_{22} \leq \tau \leq \bar{a}_{33} \\ 1 & \text{else} \end{cases}$$

where $\bar{a}_{11} < a_{11} < a_{22} < a_{33} < \bar{a}_{33}$ on the real line.

$$\mu_{\text{trapmf}}(\tau) = \begin{cases} \frac{\tau - a_{11}}{a_{22} - a_{11}} & a_{11} \leq \tau \leq a_{22} \\ 1 & a_{22} \leq \tau \leq a_{33} \\ \frac{a_{44} - \tau}{a_{44} - a_{33}} & a_{33} \leq \tau \leq a_{44} \\ 0 & \text{else} \end{cases}$$

$$\nu_{\text{trapmf}}(\tau) = \begin{cases} \frac{a_{22} - \tau}{a_{22} - \bar{a}_{11}} & \bar{a}_{11} \leq \tau \leq a_{22} \\ 0 & a_{22} \leq \tau \leq a_{33} \\ \frac{\tau - a_{33}}{a_{44} - a_{33}} & a_{33} \leq \tau \leq \bar{a}_{44} \\ 1 & \text{else} \end{cases}$$

where $\bar{a}_{11} < a_{11} < a_{22} < a_{33} < a_{44} < \bar{a}_{44}$ on the real line.

Let us consider probability distribution function P , which shows the space of all probability distributions on universal set U . The possibility distribution function is used to handle the given information. Using Zadeh’s definition of the fuzzy probability, we express the given information as

$$\text{Porb}_p(V \text{ is } A) = \int A(x) dx$$

that provides the degree of truth of $\text{Porb}_p(V \text{ is } A)$, which satisfies

$$\mu(p) = \mu(\text{Porb}_p(V \text{ is } A)) = \mu\left(\int A(x) dx\right) = \mu\left(\int \frac{M^u(x)}{M^l(x)} dx\right)$$

and

$$\vartheta(p) = \vartheta(\text{Porb}_p(V \text{ is } A)) = \vartheta\left(\int A(x)dx\right) = \vartheta\left(\int \frac{NM^\mu(x)}{NM^l(x)}dx\right)$$

that provides the degree of true and false values of $\text{Porb}_p(V \text{ is } A)$, which satisfy B. In this work, the application of intuitionistic fuzzy logic to measure disease diagnostic levels is considered.

For a particular patient with various input factors (symptoms), we must consider an intuitionistic fuzzy rule based on unconditional and qualified intuitionistic fuzzy propositions, in which the antecedent part of input factor 'I' is represented by an intuitionistic fuzzy number (which can be trapezoidal or triangular) and the consequent part 'O' is represented by the probability distribution function with the help of the Z-intuitionistic fuzzy fractional valuation, given as follows:

A_i is an intuitionistic fuzzy set, $\mu_{A_i}(x)$ is the membership function of A_i , and $\vartheta_{A_i}(x)$ is the non-membership function of A_i over universal sets U_i , $i = 1, 2, \dots, n$.

A continues this version of the intuitionistic fuzzy rule, while the function with 'n' arguments is

$$\begin{aligned} R(I) &: A_1 \text{ is } I_1 \text{ and } A_2 \text{ is } I_2 \text{ and } A_3 \text{ is } I_3 \dots A_n \text{ is } I_n \\ R(O) &= \text{Average} \{ \mu_{A_i}(I_i) \} = \sum_{i=1}^n \mu\left(\int A_i(I_i)dx\right) \text{ and } \text{Average} \{ \vartheta_{A_i}(I_i) \} \\ &= \sum_{i=1}^n \vartheta\left(\int A_i(I_i)dx\right) \text{ and} \\ R(O) &: \frac{\sum_{i=1}^n \mu\left(\int A_i(I_i)dx\right) + \sum_{i=1}^n \vartheta\left(\int A_i(I_i)dx\right)}{2} \end{aligned}$$

6. Data Collection of Dengue Infected Patients

We applied our approach to the data [25], and we collected seven clinical factors: Temperature, sugar, Pulse Rate (PR), age, cough, and Blood Pressure (BP), as shown in Table 1.

Table 1. Data of dengue-infected patients.

	Temperature (Degree F)	Sugar (mg/dL)	PR (beats/min.)	Age (Years)	BP (mm Hg)	Cough	Chills
P-I	97.5	100	108	35	120	No	No
P-II	99.5	98	70	40	140	No	Yes
P-III	98.6	110	65	25	110	Yes	No
P-IV	98	126	80	55	120	No	No
P-V	102	99	75	23	130	No	Yes
P-VI	97.5	105	98	28	98	No	Yes
P-VII	98	115	88	30	110	No	No
P-VIII	102	120	104	47	98	No	No
P-IX	98.8	98	77	58	112	Yes	Yes
P-X	99	137	88	65	115	No	No
P-XI	98	125	110	77	98	No	No
P-XII	103	88	90	24	110	Yes	No
P-XIII	97.5	97	110	49	120	No	No
P-XIV	99	142	85	70	140	No	Yes

7. Numerical Computation

In this section, we conducted a study based on the data of dengue-infected patients. For the applicability process, we estimated the severity level of dengue-infected patients. In this study, we considered five (out of seven) input factors, which are categorized into three linguistic categories, and their ranges are given in Figures 2–6. Furthermore, we also categorized the output factor ‘Out’ into three linguistic categories with the following ranges: Less severe ($Out \leq 25$), normal ($25 < Out \leq 40$), and severe ($40 \leq Out$).

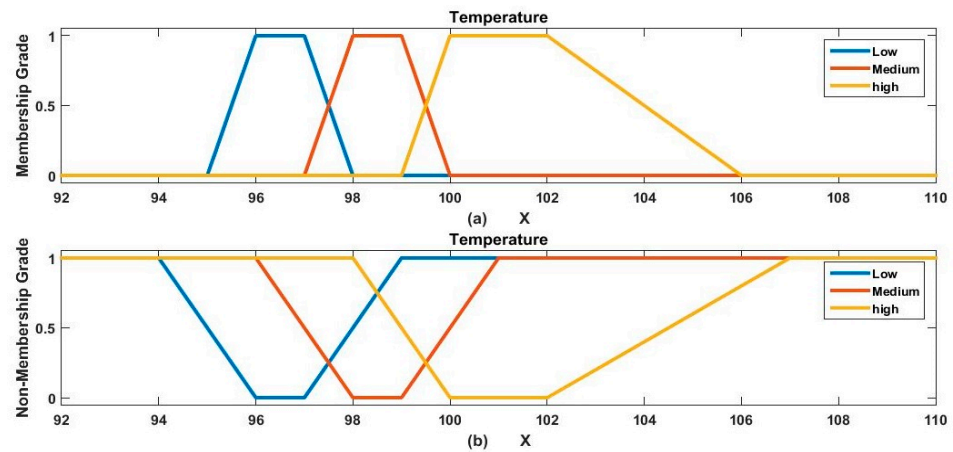


Figure 2. Three linguistic categories of temperature with their trapezoidal membership (a) and non-membership grades (b).

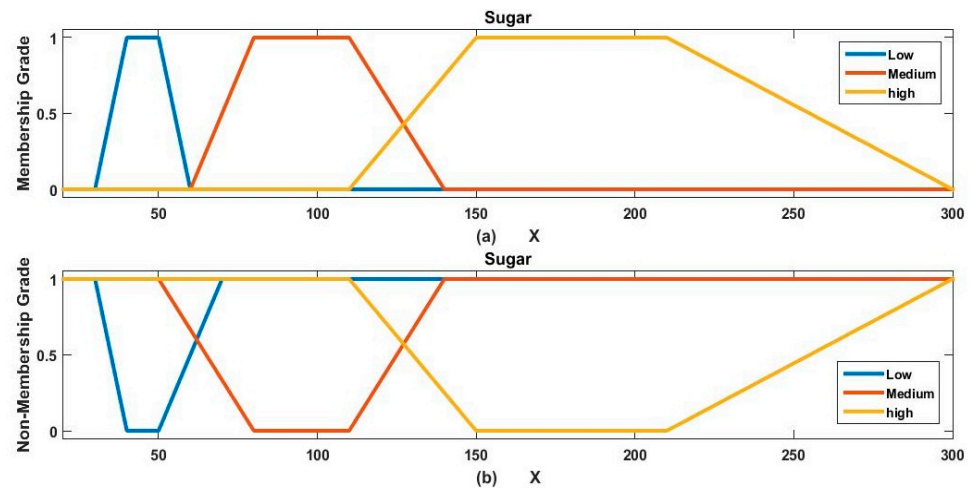


Figure 3. Three linguistic categories of sugar with their trapezoidal membership (a) and non-membership grades (b).

We considered patients with the identities P-XII and P-XIV for the numerical computation as follows:

For patient P-XII, the value of temperature is high

$$\int_{99}^{106} \mu(x)dx = \int_{99}^{100} \frac{x-99}{1} dx + \int_{100}^{102} 1 dx + \int_{102}^{106} \frac{106-x}{4} dx = 4.5 \text{ and}$$

$$\int_{98}^{107} \vartheta(x)dx = \int_{98}^{100} \frac{100-x}{2} dx + \int_{100}^{102} 0 dx + \int_{102}^{107} \frac{x-102}{4} dx = 4.125$$

sugar is medium

$$\int_{60}^{140} \mu(x)dx = \int_{60}^{80} \frac{x-50}{20} dx + \int_{80}^{110} 1 dx + \int_{110}^{140} \frac{140-x}{30} dx = 65 \text{ and}$$

$$\int_{58}^{140} \vartheta(x)dx = \int_{58}^{80} \frac{80-x}{22} dx + \int_{80}^{110} 0 dx + \int_{110}^{140} \frac{x-110}{40} dx = 31$$

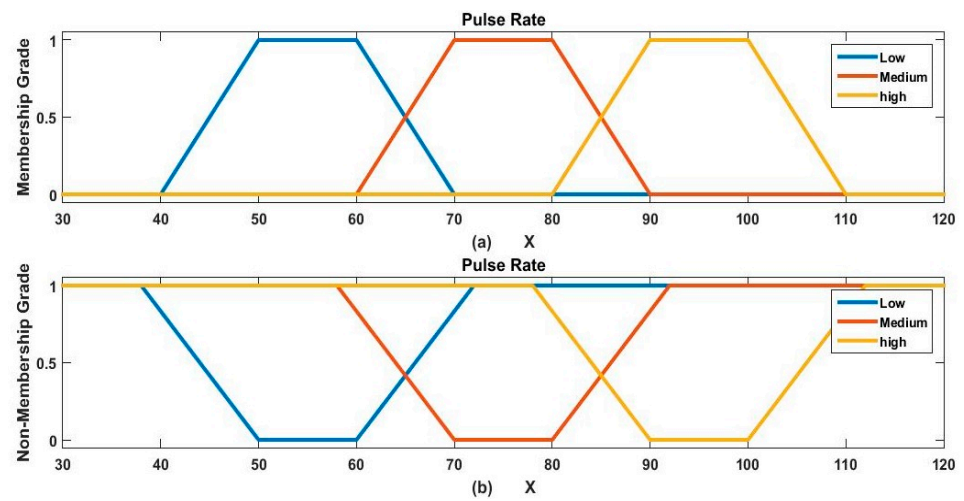


Figure 4. Three linguistic categories of pulse rate with their trapezoidal membership (a) and non-membership grades (b).

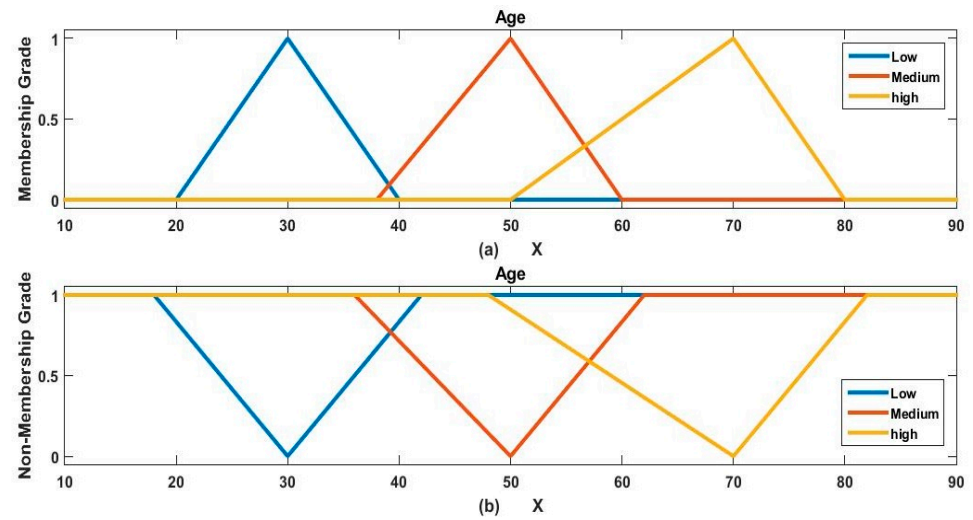


Figure 5. Three linguistic categories of age with their triangular membership (a) and non-membership grades (b).

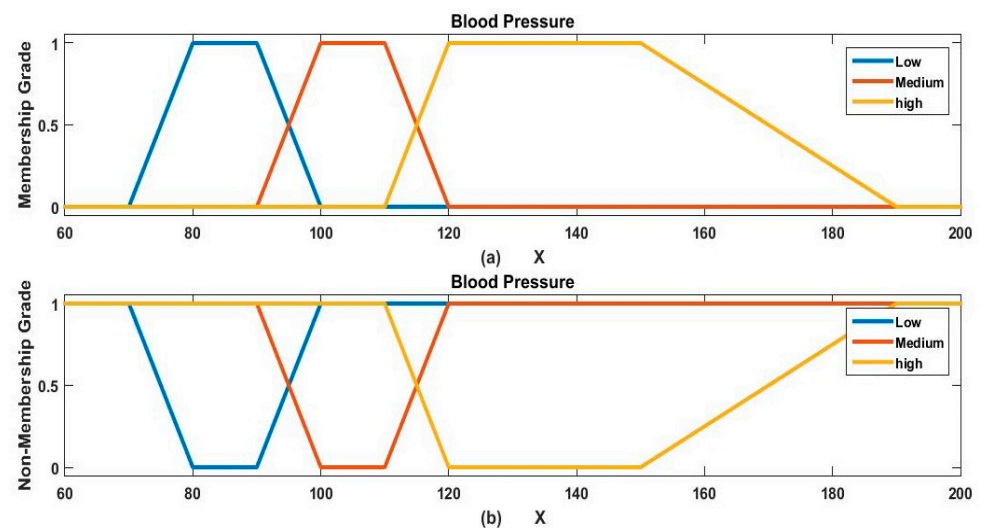


Figure 6. Three linguistic categories of blood pressure with their trapezoidal membership (a) and non-membership grade (b).

PR is high

$$\int_{80}^{110} \mu(x) dx = \int_{80}^{90} \frac{x-80}{10} dx + \int_{90}^{100} 1 dx + \int_{100}^{110} \frac{110-x}{10} dx = 20 \text{ and}$$

$$\int_{78}^{115} \vartheta(x) dx = \int_{78}^{90} \frac{90-x}{12} dx + \int_{90}^{100} 0 dx + \int_{100}^{115} \frac{x-100}{15} dx = 13.50$$

age is low

$$\int_{20}^{40} \mu(x) dx = \int_{20}^{30} \frac{x-20}{10} dx + \int_{30}^{40} \frac{40-x}{10} dx = 10 \text{ and}$$

$$\int_{18}^{42} \vartheta(x) dx = \int_{18}^{30} \frac{30-x}{12} dx + \int_{30}^{42} \frac{x-30}{12} dx = 12$$

and BP is high

$$\int_{90}^{120} \mu(x) dx = \int_{90}^{100} \frac{x-90}{10} dx + \int_{100}^{110} 0 dx + \int_{110}^{120} \frac{120-x}{10} dx = 10 \text{ and}$$

$$\int_{90}^{120} \vartheta(x) dx = \int_{90}^{100} \frac{100-x}{10} dx + \int_{100}^{110} 0 dx + \int_{110}^{120} \frac{x-110}{10} dx = 10$$

$$\text{Average (For membership values)} = \frac{4.5 + 65 + 20 + 10 + 10}{5} = 21.90$$

$$\text{Average (For non – membership values)} = \frac{4.125 + 31 + 13.50 + 12 + 10}{5} = 14.125$$

$$\text{Final output} = \frac{21.9 + 14.125}{2} = 18.0125.$$

For patient P-XIV, the value for temperature is medium

$$\int_{97}^{100} \mu(x) dx = \int_{97}^{98} \frac{x-97}{1} dx + \int_{98}^{99} 1 dx + \int_{99}^{100} \frac{100-x}{1} dx = 2 \text{ and}$$

$$\int_{96}^{101} \vartheta(x) dx = \int_{96}^{98} \frac{98-x}{2} dx + \int_{98}^{99} 0 dx + \int_{99}^{101} \frac{x-99}{2} dx = 3$$

sugar is high

$$\int_{110}^{300} \mu(x) dx = \int_{110}^{150} \frac{x-110}{40} dx + \int_{150}^{210} 1 dx + \int_{210}^{300} \frac{300-x}{90} dx = 125 \text{ and}$$

$$\int_{105}^{300} \vartheta(x) dx = \int_{105}^{150} \frac{150-x}{45} dx + \int_{150}^{210} 0 dx + \int_{210}^{300} \frac{x-210}{90} dx = 80.97$$

PR is medium

$$\int_{60}^{90} \mu(x) dx = \int_{60}^{70} \frac{x-60}{10} dx + \int_{70}^{80} 1 dx + \int_{80}^{90} \frac{90-x}{10} dx = 20 \text{ and}$$

$$\int_{58}^{95} \vartheta(x) dx = \int_{58}^{70} \frac{70-x}{12} dx + \int_{70}^{80} 0 dx + \int_{80}^{95} \frac{x-80}{15} dx = 13.5$$

age is high

$$\int_{50}^{80} \mu(x) dx = \int_{50}^{70} \frac{x-50}{20} dx + \int_{70}^{80} \frac{80-x}{10} dx = 10 \text{ and } \int_{48}^{85} \vartheta(x) dx = \int_{48}^{70} \frac{70-x}{22} dx + \int_{70}^{80} \frac{x-70}{10} dx = 16$$

and BP is high

$$\int_{110}^{190} \mu(x) dx = \int_{110}^{120} \frac{x-110}{10} dx + \int_{120}^{150} 1 dx + \int_{150}^{190} \frac{190-x}{40} dx = 55 \text{ and}$$

$$\int_{110}^{190} \vartheta(x) dx = \int_{110}^{120} \frac{120-x}{10} dx + \int_{120}^{150} 0 dx + \int_{150}^{190} \frac{x-180}{40} dx = 15$$

$$\text{Average (For membership values)} = \frac{2 + 125 + 20 + 10 + 55}{5} = 42.4$$

$$\text{Average (For non – membership values)} = \frac{3+80.97+13.5+16+15}{5} = 49.294$$

$$\text{Final crispoutput} = \frac{42.40 + 49.294}{2} = 45.847.$$

8. Conclusions

Decisions involved in any problem are based upon the available information. The concept of a Z-intuitionistic-number relates to the issue of the reliability of the information in two ways: First, if it considers the membership function of the information, and second, if it considers the non-membership function of the information. We generalized the concept of the Z-number with the computation of Z-intuitionistic fuzzy fractional numbers. The concept of a Z-intuitionistic fuzzy fractional number can be applied in many areas, especially in decision making and risk assessment in the medical field and in many prediction-making problems.

The entire work is illustrated in the following points:

- (1) We used the concept of Zadeh’s Z-numbers and generalized the concept of a Z-number to a Z-intuitionistic number to obtain knowledge about a valuation function (uncertain variable) in the context of forming Z-intuitionistic fuzzy fractional valuations (V, A, B).
- (2) Z-intuitionistic fuzzy fractional valuation describes the uncertain knowledge in probabilistic form, which indicates the fact that “V is A is equal to B”. We interpreted this as Z-intuitionistic fuzzy fractional valuation demonstrating the knowledge.
- (3) We developed an intuitionistic fuzzy knowledge-based system based on the Z-intuitionistic fuzzy fractional valuation function. In this proposed work, we used unconditional and qualified fuzzy propositions in the form of the probability density function.
- (4) A comparative study between Z-valuation, Z-intuitionistic valuation, and Z-intuitionistic fuzzy fractional valuation is also discussed in this study (see Table 2).
- (5) The utility of the proposed algorithm based on the intuitionistic fuzzy Z-valuation method is established in the medical field. This concept will provide a remarkable landmark in decision making regarding the severity levels of dengue patients. This proposed method shows the relationship between Z-intuitionistic fuzzy fractional valuation and the symptomatic factors of dengue-infected patients.
- (6) A numeric computation was carried out to represent the severity level of dengue-infected patients. The numerical computation allowed us to observe that patient P-XII with an output value of 18.0125 belongs to the less-severe category in terms of dengue risk. However, in the case of patient P-XIV, the output value was 45.847, which indicates the patient is affected by dengue disease at a high severity level.

Table 2. Comparative study of existing techniques with proposed technique.

Z-Valuations	Z-Intuitionistic Valuation	Z-Intuitionistic Fuzzy Fractional Valuation
It considers only membership value or truth value.	It considers both membership value as well as non-membership value.	It considers both membership value as well as non-membership value in fractional form.
It deals with the uncertainty present in the form of fuzzy probability.	It deals with the uncertainty present in the form of intuitionistic fuzzy probability.	It deals with the uncertainty present in the fraction form of intuitionistic fuzzy probability.
Quite reliable without computation.	Reliable and complex in computation.	More reliable without computation.
May be applied in the medical field or various decision-making problems.	Can be applied in the medical field or various decision-making problems in a more enhanced manner than the Z-valuation.	Can be applied in the medical field or various decision-making problems in a more appropriate manner than the Z-valuation and Z-intuitionistic valuation.

The proposed technique is applicable to various datasets, whether it is provided in an indiscrete or continuous form. The managerial implication of this study is not restricted to the medical field, and it may be applied in engineering, agriculture, economic fields, etc. However, the limitations of this model depend upon the nature, accuracy level, and sample size of the experimental data.

The proposed technique is easily applicable in the diagnostic process of diseases, requiring less computation. Existing intuitionistic fuzzy inference technique-based models may lead to high levels of computation due to the defuzzification and aggregation processes. Throughout this entire work, there was no requirement for any complex computation techniques such as defuzzification in order to obtain the required output of the proposed system.

9. Results and Discussion

In this work, we determined the objective severity levels of dengue-infected patients. In this work, we provide an organized structure, the objective of the research, with conclusive outcomes given in the conclusion section. To make it easy for the reader to understand the whole concept, the mathematical structure, and the methodology, Figure 1 provides a graphical representation of the proposed Z-intuitionistic fuzzy fractional valuation-based inference system with all eight components. During this study, we obtained seven symptoms of the patients, including temperature, sugar, pulse rate, age, cough, and blood pressure, as shown in Table 1, and the various linguistic categories of these input variables are shown in Figures 2–6 with the help of triangular and trapezoidal intuitionistic fuzzy numbers. The output factors reflect the various categories of severity of dengue patients, with some fixed values in the form of ranges (as discussed earlier in Section 7). The interpretation of the results is important for the proper treatment of infected patients. As a result, physicians and doctors can recommend the exact treatment and dose quantity to an infected patient by determining the exact severity level of dengue disease.

10. Future Aspects of the Work

From a future perspective, we may extend the proposed Z-intuitionistic fuzzy fractional valuation to generalized concepts of fuzzy set theory, such as mediative fuzzy logic or intuitionistic type-II fuzzy logic, to create an inference system that will manage the uncertainty in a more appropriate manner. We may use the inference system based on Sugeno's approach [33] or Tsukamoto's approach [34] by taking a certain linear or monotonic function for the consequent part of the knowledge-based fuzzy rules.

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References

- Zadeh, L.A. Fuzzy sets. *Inf. Control.* **1965**, *8*, 338–353. [[CrossRef](#)]
- Sharif, S.; Seno, S.A.; Rowhanimanesh, A. A fuzzy-logic-based fault detection system for medical Internet of Nano Things. *Nano Commun. Netw.* **2021**, *30*, 100366. [[CrossRef](#)]
- Bora, D.J. Contrast Improvement of Medical Images Using Advanced Fuzzy Logic-Based Technique. In *Advanced Machine Vision Paradigms for Medical Image Analysis*; Academic Press: Cambridge, MA, USA, 2021; pp. 229–257.
- Klir, G.J.; Yuan, B. *Fuzzy Sets and Fuzzy Logic: Theory and Applications*; Prentice Hall of India: New Delhi, India, 2003; ISSN 0-13-101171-5.
- Yen, J.; Langari, R.; Zadeh, L.A. *Industrial Applications of Fuzzy Logic and Intelligent Systems*; IEEE Press Piscataway: Hoboken, NJ, USA, 1995.
- Zadeh, L.A. A note on Z-numbers. *Inf. Sci.* **2011**, *181*, 2923–2932. [[CrossRef](#)]
- Aliev, R.A.; Huseynov, O.H.; Aliyev, R.R.; Alizadeh, A. *Arithmetic on Z-Numbers. The Theory and Application*; World Scientific Publishing Co.: Singapore, 2015; ISBN 9789814675291/9814675296.
- Aliev, R.A.; Zeinalova, L.M. Decision Making under Z-Information. In *Human-Centric Decision-Making Models for Social Sciences*; Studies in Computational Intelligence; Guo, P., Pedrycz, W., Eds.; Springer: Berlin/Heidelberg, Germany, 2014; Volume 502, pp. 233–252.
- Kang, B.; Wei, D.; Li, Y.; Deng, Y. Decision making using z-numbers under uncertain environment. *J. Comput. Inf. Syst.* **2012**, *8*, 2807–2814.
- Kang, B.; Wei, D.; Li, Y.; Deng, Y. A method of converting z-number to classical fuzzy number. *J. Comput. Inf. Syst.* **2012**, *9*, 703–709.
- Xiao, Z.Q. Application of Z-numbers in multi-criteria decision making. In Proceedings of the Informative and International Conference on Cybernetics for Computational Social Systems, Qingdao, China, 9–10 October 2014; pp. 91–95.
- Azadeh, A.; Saberi, M.; Atashbar, N.Z.; Chang, E.; Pazhoheshfar, P. Z-AHP: A Z-number extension of fuzzy analytical hierarchy process. In Proceedings of the International Conference on Digital Ecosystems and Technologies (DEST), Menlo Park, CA, USA, 24–26 July 2013; pp. 141–147.
- Atanassov, K.; Gargov, G. Elements of Intuitionistic fuzzy logic part 1. *Fuzzy Sets Syst.* **1989**, *95*, 39–52. [[CrossRef](#)]
- Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [[CrossRef](#)]
- Montiel, O.; Castillo, O.; Melin, P.; Rodríguez, D.A.; Sepúlveda, R. Reducing the cycling problem evolutionary algorithms. In Proceedings of the 2005 International Conference on Artificial Intelligence (ICAI-2005), Las Vegas, NV, USA, 27–30 June 2005; pp. 426–432.
- Kumar, S.; Sharma, M.K.; Dhiman, N.; Joshi, D. Meditative fuzzy logic and agricultural production planning: State level investigation of wheat crop in India. *Int. J. Agricult. Stat. Sci.* **2020**, *16*, 889–900.
- Sharma, M.K.; Dhiman, N.; Mishra, V.N. Meditative fuzzy logic mathematical model: A contradictory management prediction in COVID-19 pandemic. *Appl. Soft Comput.* **2021**, *105*, 107285. [[CrossRef](#)] [[PubMed](#)]
- Sharma, M.K.; Dhiman, N.; Mishra, L.N.; Mishra, V.N.; Sahani, S.K. Meditative Fuzzy Extension Technique and Its Consistent Measurement in the Decision Making of Medical Application. *Math. Probl. Eng.* **2021**, *2021*, 9. [[CrossRef](#)]
- Yager, R.R. On a View of Zadeh's Z-Numbers. In *Advances in Computational Intelligence. IPMU 2012. Communications in Computer and Information Science*; Greco, S., Bouchon-Meunier, B., Coletti, G., Fedrizzi, M., Matarazzo, B., Yager, R.R., Eds.; Springer: Berlin/Heidelberg, Germany, 2012; Volume 299, pp. 90–101.
- Yager, R.R. On Z-Valuations using Zadeh's z-number. *Int. J. Intell. Syst.* **2012**, *27*, 259–278. [[CrossRef](#)]
- Zadeh, L.A. Probability measures of fuzzy events. *J. Math. Anal. Appl.* **1968**, *10*, 421–427. [[CrossRef](#)]
- Atanassov, K.T. New operations defined over Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1994**, *61*, 137–142. [[CrossRef](#)]
- Chaira, T. *Fuzzy Set and Its Extension: The Intuitionistic Fuzzy Set*; John Wiley & Sons, Inc.: Hoboken, NJ, USA, 2019; ISBN 978-1-119-54422-7.
- Zimmermann, L.J. *Fuzzy Set Theory-and Its Applications*, 3rd ed.; Springer: Berlin/Heidelberg, Germany, 1996; ISBN 978-94-015-8702-0.
- Sharma, M.K.; Dhiman, N.; Verma, S.; Mishra, V.N. Meditative neuro fuzzy inference and meditative fuzzy expert system for the identification of severity diagnosis of the dengue patients. *AIP Conf. Proc.* **2021**, *2364*, 020029.
- Dhiman, N.; Sharma, M.K. Fuzzy Logic Inference System for Identification and Prevention of Coronavirus (COVID-19). *Int. J. Innov. Technol. Explor. Eng. (IJITEE)* **2020**, *9*, 1575–1580.
- Dhiman, N.; Sharma, M.K. Calculus of new intuitionistic fuzzy generator: In generated intuitionistic fuzzy sets and its applications in medical diagnosis. *Int. J. Adv. Appl. Sci.* **2020**, *7*, 125–130. [[CrossRef](#)]
- Dhiman, N.; Sharma, M.K. Diabetes Diagnostic Model Based on Truth-value Restrictions Method Using Inference of Intuitionistic Conditional and Qualified Fuzzy Propositions. *Int. J. Eng. Adv. Technol. (IJEAT)* **2019**, *9*, 5015–5021.
- Szmidt, E.; Kacprzyk, J. On measuring distances between intuitionistic fuzzy sets. *Notes Intuit. Fuzzy Sets* **1997**, *3*, 1–3.
- Szmidt, E.; Kacprzyk, J. Distances between intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **2000**, *114*, 505–518. [[CrossRef](#)]
- Szmidt, E.; Kacprzyk, J. Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets. In Proceedings of the 8th International Conference on Intuitionistic Fuzzy Sets, Varna, Bulgaria, 20–21 June 2004; Volume 10, pp. 61–69.
- Vasanti, G.; Viswanadham, T. Intuitionistic fuzzy sets and its application in student performance determination of a course via normalized Euclidean distance method. *Int. J. Multidiscip. Sci. Emerg. Res.* **2015**, *4*, 1053–1055.

33. Lohani, A.K.; Goel, N.K.; Bhatia, K.K. Takagi–Sugeno fuzzy inference system for modeling stage–discharge relationship. *J. Hydrol.* **2006**, *331*, 146–160. [[CrossRef](#)]
34. Wahyuni, I.; Mahmudy, W.F.; Iriany, A. Rainfall prediction in Tengger region Indonesia using Tsukamoto fuzzy inference system. In Proceedings of the 2016 1st International Conference on Information Technology, Information Systems and Electrical Engineering (ICITISEE), Yogyakarta, Indonesia, 23–24 August 2016; pp. 130–135.