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Fixed Point Results for Generalized F -Contractions in b -Metric-like Spaces

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Abstract: The purpose of this paper is to introduce several generalized F -contractions in b -metric-like spaces and establish some fixed point theorems for such contractions. Moreover, some nontrivial examples are given to illustrate the superiority of our results. In addition, as an application, we find the existence and uniqueness of a solution to a class of integral equations in the context of b -metric-like spaces.

Keywords: (s, q, F) -contraction; b -metric-like space; fixed point; integral equation

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1. Introduction

The Banach contraction principle (for short, BCP), the most celebrated theorem in metric fixed point theory, has undergone many extensions and generalizations due to its simple nature and wide range of applicability. Some significant generalizations of BCP are seen in [1–4]. Generally speaking, these generalizations usually contain two sides. On the one hand, BCP is extended from metric space to generalized metric space, such as b -metric space [1], partial metric space [5], ordered Banach space [6], b -metric-like space [7], etc. One of the most prevalent spaces is b -metric-like space, which was introduced by Alghamdi et al. [7] in 2013. In 2014, Hussain et al. [8] obtained fixed point results for Ćirić type contraction and ϕ -contraction in b -metric-like spaces. Afterwards, Joshi et al. [9] in 2017 presented fixed point theorems for generalized F -contractions in b -metric-like spaces. In the same year, Zoto et al. [10] offered some generalizations for $(\alpha-\psi, \phi)$ -contractions in b -metric-like spaces. Whereafter, Zoto et al. [11] in 2018 obtained fixed point theorems for (s, p, α) -contractive mappings in b -metric-like spaces. Subsequently, Zoto et al. [12] in 2019 investigated common fixed point theorems for a class of (s, q) -contractive mappings in b -metric-like spaces. In the meanwhile, De la Sen et al. [13] in 2019 gave fixed point results for (s, q) -graphic contraction in b -metric-like spaces. Later in 2020, Fabiano et al. [14] discussed fixed point theorems for (s, q) -Dass–Gupta–Jaggi type contraction in b -metric-like spaces. Recently in 2021, Mitrović et al. [15] established fixed point theorems for Jaggi- W -contraction in b -metric-like spaces.

On the other hand, BCP is extended for different contractive mappings. One of the most important contractions, is F -contraction on metric space, which was introduced by Wardowski [16] in 2012. Shukla et al. in 2014 established ordered F -contraction in [17] from metric spaces to partial metric spaces. In 2018, Kadelburg and Radenović in [18]

extended F -contraction in [16] from metric spaces to b -metric spaces. In 2019, Hammad and De la Sen [19] considered fixed point theorem for the generalized almost (s, q) -Jaggi F -contraction-type in b -metric-like spaces. As followed by them, Huang et al. in [20] introduced the notion of convex F -contraction and proved fixed point theorems for both continuous and discontinuous mappings.

Among these extensions cited above, throughout this paper, first and foremost, we initiate several generalized F -contractions, such as (s, q, F) -contraction, general (s, q, F) -contraction and r -order (s, q, F) -contraction. We give several fixed point theorems for such contractions in b -metric-like spaces. As compared with previous contractions from [8–10,12–14,19], our contractions mainly aim at generalized F -contractions, which are the sharp generalizations of F -contraction introduced by Wardowski [16]. As we know, F -contraction is one of the generalizations of Banach type contraction, whereas generalized F -contractions greatly extend F -contractions. As a result, our conclusions related to generalized F -contractions have strong theoretical significance and practical influence. It is worth mentioning that we demonstrate our assertions by much fewer conditions and more straightforward proofs than the counterpart from previous results. In addition, we illustrate the vitality of our conclusions by some supportive examples. As an application, we obtain the existence and uniqueness of solution to a class of integral equations. Regarding finding the solutions for such equations, there have emerged numerous versions in the existing literature; our method used in this paper is very easy to be understood since it contains simple conditions and comes straight to the point with short proof.

2. Preliminaries

It is customary for a paper to firstly list some useful definitions, lemmas and other contributed results.

In the following, unless otherwise specified, we always assume \mathbb{R} as the set of all real numbers, \mathbb{N} the set of all nonnegative integers, and \mathbb{N}^* the set of all positive integers.

Definition 1 ([7]). Let M be a nonempty set and $s \geq 1$ a constant. The mapping $b: M \times M \rightarrow [0, +\infty)$ is called a b -metric-like if for all $\xi, \eta, \zeta \in M$, the following conditions are satisfied:

- (b1) $b(\xi, \eta) = 0$ implies $\xi = \eta$;
- (b2) $b(\xi, \eta) = b(\eta, \xi)$;
- (b3) $b(\xi, \eta) \leq s[b(\xi, \zeta) + b(\zeta, \eta)]$.

The pair (M, b, s) is called a b -metric-like space with parameter $s \geq 1$.

In a b -metric-like space (M, b, s) , if $\xi, \eta \in M$ and $b(\xi, \eta) = 0$, then $\xi = \eta$; however, the converse need not be true, since $b(\eta, \eta)$ may be positive for some $\eta \in M$.

Example 1 ([11]). Let $M = [0, +\infty)$ and $p > 1$ be a constant. Define a function $b: M \times M \rightarrow [0, +\infty)$ by $b(\xi, \eta) = (\xi + \eta)^p$ or $b(\xi, \eta) = (\max\{\xi, \eta\})^p$. Then, (M, b, s) is a b -metric-like space with parameter $s = 2^{p-1}$. Clearly, (M, b, s) is neither a b -metric (see [1]), nor a metric-like space (see [5]), nor a partial b -metric space (see [21]).

Definition 2 ([7]). Let (M, b, s) be a b -metric-like space with parameter $s \geq 1$, $\{\eta_n\}$ a sequence in M and $\eta \in M$. We say:

- (i) $\{\eta_n\}$ is said to be a b -convergent sequence if $\lim_{n \rightarrow \infty} b(\eta_n, \eta) = b(\eta, \eta)$;
- (ii) $\{\eta_n\}$ is said to be a b -Cauchy sequence if $\lim_{n, m \rightarrow \infty} b(\eta_n, \eta_m)$ exists and is finite;
- (iii) (M, b, s) is called b -complete, if, for every b -Cauchy sequence $\{\eta_n\}$ in M , there exists $\eta \in M$ such that $\lim_{n, m \rightarrow \infty} b(\eta_n, \eta_m) = \lim_{n \rightarrow \infty} b(\eta_n, \eta) = b(\eta, \eta)$.

Definition 3 ([7]). Let (M, b, s) be a b -metric-like space with parameter $s \geq 1$ and $f: M \rightarrow M$ a function. We say that f is b -continuous if for each sequence $\{\eta_n\}$ in M with $b(\eta_n, \eta) \rightarrow b(\eta, \eta)$ as $n \rightarrow \infty$, then $b(f\eta_n, f\eta) \rightarrow b(f\eta, f\eta)$ as $n \rightarrow \infty$.

Remark 1 ([14]). In a b -metric-like space, if $\lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = 0$ and the limit of $\{\eta_n\}$ exists, then the limit is unique.

Lemma 1 ([22]). Let (M, b, s) be a b -metric-like space with parameter $s \geq 1$ and $\{\eta_n\}$ a sequence in M such that

$$b(\eta_{n+1}, \eta_{n+2}) \leq \lambda b(\eta_n, \eta_{n+1})$$

for some $\lambda \in [0, 1)$ and each $n \in \mathbb{N}^*$. Then, $\{\eta_n\}$ is a b -Cauchy sequence with $\lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = 0$.

In 2012, Wardowski [16] defined the F -contraction in metric spaces as follows:

Definition 4 ([16]). Let (M, d) be a metric space. The mapping $f : M \rightarrow M$ is called an F -contraction if there exists a function $F : (0, +\infty) \rightarrow \mathbb{R}$ such that

(F1) F is strictly increasing on $(0, +\infty)$;

(F2) for each sequence $\{\alpha_n\}$ of positive numbers, $\lim_{n \rightarrow \infty} \alpha_n = 0 \Leftrightarrow \lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$;

(F3) there exists $c \in (0, 1)$ such that $\lim_{\alpha \rightarrow 0^+} \alpha^c F(\alpha) = 0$;

(F4) there exists $\tau > 0$ such that

$$\tau + F(d(f\xi, f\eta)) \leq F(d(\xi, \eta)),$$

for all $\xi, \eta \in M$ with $f\xi \neq f\eta$.

Huang et al. [20] modified Definition 4 and defined the notion of convex F -contraction in the framework of b -metric spaces.

Definition 5 ([20]). Let $(M, d, s \geq 1)$ be a b -metric space and f a self-mapping on M . We say that f is a convex F -contraction if there exists a function $F : (0, +\infty) \rightarrow \mathbb{R}$ such that (F1) holds and:

(i) for each sequence $\{\alpha_n\}$ of positive numbers, if $\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$, then $\lim_{n \rightarrow \infty} \alpha_n = 0$;

(ii) there exists $c \in \left(0, \frac{1}{1 + \log_2 s}\right)$ such that $\lim_{\alpha \rightarrow 0^+} \alpha^c F(\alpha) = 0$;

(iii) there exist $\tau > 0$ and $\lambda \in [0, 1)$ such that

$$\tau + F(d_n) \leq F(\lambda d_n + (1 - \lambda)d_{n-1}),$$

for all $d_n > 0$ where $n \in \mathbb{N}^*$.

Remark 2. Condition (iii) yields that $d_n < d_{n-1}$ for all $n \in \mathbb{N}$. Hence, the sequence $\{d_n\}$ is a decreasing sequence.

Definition 6 ([23]). Let (M, b, s) be a b -metric-like space, f a self-mapping on M and $\{\eta_n\}$ a sequence in M . We say $\{\eta_n\}$ is a Picard iterative sequence generated by f if for any $\eta_0 \in M$, $\eta_{n+1} = f\eta_n$ holds for all $n \in \mathbb{N}$.

Inspired by the above notions, we provide some new definitions and theorems in the sequel.

3. Main Results

In this section, we introduce the notion of (s, q, F) -contraction, general (s, q, F) -contraction and r -order (s, q, F) -contraction and give some fixed point theorems based on them. We also provide three examples to support our conclusions.

First of all, motivated by Definitions 4 and 5, we present the following concept.

Definition 7. Let f be a self-mapping on b -metric-like space (M, b, s) with parameter $s \geq 1$, $\{\eta_n\}$ be a Picard iterative sequence generated by f and $F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing function. We say that f is an (s, q, F) -contraction if:

$$\tau + F(s^q b_n) \leq F(\alpha b_n + \beta b_{n-1}), \tag{1}$$

for all $b_n := b(\eta_n, \eta_{n+1}) > 0$, where $\tau, q > 0$, $\alpha, \beta \geq 0$ are constants with $0 < \alpha + \beta < 1$.

Remark 3. Definition 7 improves the corresponding definitions given in [13,20,24]. It contains a fewer conditions compared with the previous ones. It covers many contractive conditions since the set of all increasing functions F is a very broad set.

Remark 4. If $s = 1$, then we get the case in metric spaces. If $\alpha = 0$, then we get the definition of (s, q, λ) -contraction in [11].

Remark 5. Although the conditions of Definition 7 look strong since it involves the Picard iteration, to the best of our knowledge, the Picard iteration is one of the most frequently used iterations in fixed point theory. Hence, our object is more targeted for the convenience of applications. Otherwise, since $\tau > 0$ and F is increasing, then by (1), we speculate:

$$F(s^q b_n) < \tau + F(s^q b_n) \leq F(\alpha b_n + \beta b_{n-1}).$$

Then by the monotonicity of F , we get:

$$s^q b_n < \alpha b_n + \beta b_{n-1},$$

which implies that

$$b_n < \frac{\beta}{s^q - \alpha} b_{n-1}. \tag{2}$$

Clearly, $\frac{\beta}{s^q - \alpha} \in [0, 1)$. As a consequence, (s, q, F) -contraction generalizes usual contractions in general.

Example 2. Let $M = [0, +\infty)$ and define a mapping on $M \times M$ by $b(\eta, \xi) = (\eta + \xi)^2$. Then, (M, b, s) is a b -complete b -metric-like space with parameter $s = 2$. Suppose that $F(t) = \ln t$ is a function on $(0, +\infty)$ and $f : M \rightarrow M$ is a mapping by $f\eta = \frac{1}{3}\eta$. It is easy to see that f is an (s, q, F) -contraction. Indeed, it is easy to show that (1) is satisfied for all $b_n = b(\eta_n, \eta_{n+1}) > 0$, where $\tau = \ln 2$, $q = 1$, $\alpha = 0$, $\beta = \frac{4}{5}$ and $\{\eta_n\}$ is a Picard iterative sequence generated by f .

The following lemma will be used in our main results.

Lemma 2. Let (M, b, s) be a b -metric-like space with parameter $s \geq 1$, f an (s, q, F) -contraction on M and $\{\eta_n\}$ a Picard iterative sequence generated by f . Then, $\{\eta_n\}$ is a b -Cauchy sequence with $\lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = 0$.

Proof. The proof is clear if there exists $n_0 \in \mathbb{N}$ such that $\eta_{n_0+1} = \eta_{n_0}$. Without loss of generality, we assume that $\eta_{n+1} \neq \eta_n$ for all $n \in \mathbb{N}$. Thus, $b_n := b(\eta_n, \eta_{n+1}) > 0$ for all $n \in \mathbb{N}$. Notice (2) and $\frac{\beta}{s^q - \alpha} \in [0, 1)$, via Lemma 1, the sequence $\{\eta_n\}$ is a b -Cauchy sequence in M with $\lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = 0$. \square

Now, our first theorem becomes valid for presentation, which generalizes many recent results.

Theorem 1. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$ and f a b -continuous (s, q, F) -contraction on M . Then, f has a fixed point in M provided that $b(f\eta, f\eta) \leq b(\eta, \eta)$ for all $\eta \in M$.

Proof. For any $\eta_0 \in M$, by Lemma 2, we can obtain that the Picard iterative sequence $\{\eta_n\}$ generated by f is a b -Cauchy sequence with $\lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = 0$. Since (M, b, s) is b -complete, then there exists $\eta^* \in M$ such that:

$$0 = \lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = \lim_{n \rightarrow \infty} b(\eta_n, \eta^*) = b(\eta^*, \eta^*).$$

Now that f is b -continuous, one has:

$$\lim_{n \rightarrow \infty} b(f\eta_n, f\eta^*) = b(f\eta^*, f\eta^*) \leq b(\eta^*, \eta^*) = 0.$$

By virtue of

$$b(\eta^*, f\eta^*) \leq s[b(\eta^*, \eta_{n+1}) + b(f\eta_n, f\eta^*)],$$

letting $n \rightarrow \infty$ in the above inequality, we obtain $b(\eta^*, f\eta^*) = 0$. Therefore, $f\eta^* = \eta^*$. That is to say, η^* is a fixed point of f . \square

Example 3. Under the hypotheses of Example 2, it is not hard to verify that $b(f\eta, f\eta) \leq b(\eta, \eta)$ for all $\eta \in M$. Therefore, all the conditions of Theorem 1 are satisfied and hence f has a fixed point $\eta^* = 0$ in M .

The following definition is the extension of (s, q) -Jaggi F -contractions related to [8,11,13,14,19,20,25,26].

Definition 8. Let (M, b, s) be a b -metric-like space with parameter $s \geq 1$, f be a self-mapping on M , $F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing mapping and $\Gamma : [0, +\infty) \times (0, +\infty) \rightarrow (0, +\infty)$ be a function such that $\Gamma(t, t) \leq 1$ for all $t \in (0, +\infty)$. Then, the mapping f is said to be a general (s, q, F) -contraction if:

$$\tau + F(s^q b(f\xi, f\eta)) \leq F(\alpha b(\eta, f\eta)\Gamma(b(\xi, f\xi), b(\xi, \eta)) + \beta b(\xi, \eta) + \gamma b(\eta, f\xi)), \quad (3)$$

for all $\xi, \eta \in M$ and $b(f\xi, f\eta) > 0$, where $\tau, q > 0$ and $\alpha, \beta, \gamma \geq 0$ are constants with $0 < \alpha + \beta + 2\gamma s < 1$.

Theorem 2. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$ and f a b -continuous general (s, q, F) -contraction on M . Then, f has a unique fixed point provided that $b(f\eta, f\eta) \leq b(\eta, \eta)$ for all $\eta \in M$.

Proof. Let $\eta_0 \in M$ and define the Picard iterative sequence $\{\eta_n\}$ as $\eta_{n+1} = f\eta_n$. If $\eta_{n_0+1} = \eta_{n_0}$ for some $n_0 \in \mathbb{N}$, then η_{n_0} is a fixed point of f because of $f\eta_{n_0} = \eta_{n_0}$. So we always assume that $\eta_{n+1} \neq \eta_n$, i.e., $b_n := b(f\eta_{n-1}, f\eta_n) = b(\eta_n, \eta_{n+1}) > 0$ for all $n \in \mathbb{N}^*$. By using (3) and the monotonicity of F , we have:

$$\begin{aligned} \tau + F(s^q b_n) &= \tau + F(s^q b(\eta_n, \eta_{n+1})) = \tau + F(s^q b(f\eta_{n-1}, f\eta_n)) \\ &\leq F(\alpha b(\eta_n, f\eta_n)\Gamma(b(\eta_{n-1}, f\eta_{n-1}), b(\eta_{n-1}, \eta_n)) + \beta b(\eta_{n-1}, \eta_n) + \gamma b(\eta_n, f\eta_{n-1})) \\ &= F(\alpha b(\eta_n, \eta_{n+1})\Gamma(b(\eta_{n-1}, \eta_n), b(\eta_{n-1}, \eta_n)) + \beta b(\eta_{n-1}, \eta_n) + \gamma b(\eta_n, \eta_n)) \\ &\leq F(\alpha b_n \Gamma(b_{n-1}, b_{n-1}) + \beta b_{n-1} + 2\gamma s b_{n-1}) \\ &\leq F(\alpha b_n + (\beta + 2\gamma s) b_{n-1}), \end{aligned}$$

which shows that (1) holds. Accordingly, f is an (s, q, F) -contraction. Thus, by virtue of Theorem 1, f has a fixed point.

We show that f has a unique fixed point in M . Indeed, first of all, we prove that $b(\eta^*, \eta^*) = 0$ if η^* is a fixed point of f . On the contrary, assume $b(\eta^*, \eta^*) > 0$, i.e., $b(f\eta^*, f\eta^*) > 0$, then by (3) and the monotonicity of F , we get:

$$\begin{aligned} F(s^q b(\eta^*, \eta^*)) &< \tau + F(s^q b(f\eta^*, f\eta^*)) \\ &\leq F(\alpha b(\eta^*, f\eta^*)\Gamma(b(\eta^*, f\eta^*), b(\eta^*, \eta^*)) + \beta b(\eta^*, \eta^*) + \gamma b(\eta^*, f\eta^*)) \\ &= F(\alpha b(\eta^*, \eta^*)\Gamma(b(\eta^*, \eta^*), b(\eta^*, \eta^*)) + \beta b(\eta^*, \eta^*) + \gamma b(\eta^*, \eta^*)) \\ &\leq F((\alpha + \beta + \gamma)b(\eta^*, \eta^*)) \\ &\leq F(b(\eta^*, \eta^*)), \end{aligned}$$

which implies that:

$$s^q b(\eta^*, \eta^*) < b(\eta^*, \eta^*).$$

As a consequence of $b(\eta^*, \eta^*) > 0$, then $s^q < 1$. This is a contradiction with $s \geq 1$ and $q > 0$. Accordingly, $b(\eta^*, \eta^*) = 0$.

Let ζ^* and η^* be two distinct fixed points of f . By the above statement, we have $b(\zeta^*, \zeta^*) = 0$ and $b(\eta^*, \eta^*) = 0$. In view of $\zeta^* \neq \eta^*$, that is, $b(\zeta^*, \eta^*) > 0$, i.e., $b(f\zeta^*, f\eta^*) > 0$, then from (3) and the monotonicity of F , we speculate:

$$\begin{aligned} F(s^q b(\zeta^*, \eta^*)) &< \tau + F(s^q b(f\zeta^*, f\eta^*)) \\ &\leq F(\alpha b(\eta^*, f\eta^*)\Gamma(b(\zeta^*, f\zeta^*), b(\zeta^*, \eta^*)) + \beta b(\zeta^*, \eta^*) + \gamma b(\eta^*, f\zeta^*)) \\ &= F(\alpha b(\eta^*, \eta^*)\Gamma(b(\zeta^*, \zeta^*), b(\zeta^*, \eta^*)) + \beta b(\zeta^*, \eta^*) + \gamma b(\eta^*, \zeta^*)) \\ &= F((\beta + \gamma)b(\zeta^*, \eta^*)) \leq F(b(\zeta^*, \eta^*)), \end{aligned}$$

which establishes that

$$s^q b(\zeta^*, \eta^*) < b(\zeta^*, \eta^*).$$

In view of $b(\zeta^*, \eta^*) > 0$, then $s^q < 1$. This is a contradiction with $s \geq 1$ and $q > 0$. Therefore, $b(\zeta^*, \eta^*) = 0$, i.e., $\zeta^* = \eta^*$. In other words, the fixed point is unique. \square

Remark 6. Theorem 2 generalizes the previous theorems from [9,10,12,15,19,20,24–26] and some of them used rational expressions under the contractive conditions. If we take the function $\Gamma : (0, +\infty) \times (0, +\infty) \rightarrow (0, +\infty)$ as $\Gamma(p, t) = \frac{p}{t}$, we can obtain from the above results in case of Jaggi and Gupta contractions.

Corollary 1. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$, f be a b -continuous self-mapping on M , $F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing mapping. If

$$\tau + F(s^q b(f\zeta, f\eta)) \leq F\left(\alpha b(\eta, f\eta) \frac{b(\zeta, f\zeta)}{b(\zeta, f\zeta) + b(\zeta, \eta)} + \beta f(\zeta, \eta) + \gamma b(\eta, f\zeta)\right),$$

for all $\zeta, \eta \in M$ and $b(f\zeta, f\eta) > 0$, where $\tau, q > 0$ and $\alpha, \beta, \gamma \geq 0$ are constants with $0 < \alpha + \beta + 2\gamma s < 1$. Then, f has a unique fixed point in M .

Proof. Use the function $\Gamma(x, y) = \frac{x}{x+y}$ on $[0, +\infty) \times (0, +\infty)$ in (3). By Theorem 2, we obtain the proof. \square

Corollary 2. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$, f be a b -continuous self-mapping on M , and $F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing mapping. If

$$\tau + F(s^q b(f\zeta, f\eta)) \leq F\left(\alpha b(\eta, f\eta) \frac{\sqrt{b(\zeta, f\zeta)b(\zeta, \eta)}}{1 + b(\zeta, \eta)} + \beta b(\zeta, \eta) + \gamma b(\eta, f\zeta)\right),$$

for all $\zeta, \eta \in M$ and $b(f\zeta, f\eta) > 0$, where $\tau, q > 0$ and $\alpha, \beta, \gamma \geq 0$ are constants with $0 < \alpha + \beta + 2\gamma s < 1$. Then, f has a unique fixed point in M .

Proof. Use the function $\Gamma(x, y) = \frac{\sqrt{xy}}{1+y}$ on $[0, +\infty) \times (0, +\infty)$ in (3). Via Theorem 2, we get the desired result. \square

Example 4. Let $M = [0, +\infty)$ and $b(\xi, \eta) = \xi^2 + \eta^2 + (\xi - \eta)^2$ for all $\xi, \eta \in M$. It is not hard to verify that (M, b, s) is a b -complete b -metric-like space with parameter $s = 2$. Suppose that $F(t) = \ln t$ is a function on $(0, +\infty)$ and $f : M \rightarrow M$ is a mapping by $f\xi = \frac{1}{4} \ln(1 + \xi)$ for all $\xi \in M$. Then f is a b -continuous general (s, q, F) -contraction on M . Indeed, it is easy to show that (3) is satisfied for all $\xi, \eta \in M$ and $b(f\xi, f\eta) > 0$, where $\tau = \ln 2, q = 2, \alpha = \gamma = 0, \beta = \frac{1}{2}, \Gamma : [0, +\infty) \times (0, +\infty) \rightarrow (0, +\infty)$ is a function such that $\Gamma(t, t) \leq 1$ for all $t \in (0, +\infty)$. Therefore, f has a unique fixed point $0 \in M$.

Now we show that (3) is satisfied for all $\xi, \eta \in M$ and $b(f\xi, f\eta) > 0$. As a matter of fact, by $\ln(1 + t) \leq t$ for all $t \in [0, +\infty)$, using the mean value theorem of differentials, we have:

$$\begin{aligned} & \tau + F(s^q b(f\xi, f\eta)) \\ &= \ln 2 + \ln \left\{ 4 \left[f^2 \xi + f^2 \eta + (f\xi - f\eta)^2 \right] \right\} \\ &= \ln 2 + \ln \left\{ 4 \left[\left(\frac{\ln(1 + \xi)}{4} \right)^2 + \left(\frac{\ln(1 + \eta)}{4} \right)^2 + \left(\frac{\ln(1 + \xi)}{4} - \frac{\ln(1 + \eta)}{4} \right)^2 \right] \right\} \\ &\leq \ln 2 + \ln \left\{ 4 \left[\frac{1}{16} \xi^2 + \frac{1}{16} \eta^2 + \frac{1}{16} (\xi - \eta)^2 \right] \right\} \\ &= \ln 2 + \ln \left\{ \frac{1}{4} \left[\xi^2 + \eta^2 + (\xi - \eta)^2 \right] \right\} \\ &= \ln \left\{ \frac{1}{2} \left[\xi^2 + \eta^2 + (\xi - \eta)^2 \right] \right\} \\ &\leq F(\alpha b(\eta, f\eta) \Gamma(b(\xi, f\xi), b(\xi, \eta)) + \beta b(\xi, \eta) + \gamma b(\eta, f\xi)), \end{aligned}$$

for all $\xi, \eta \in M$ and $b(f\xi, f\eta) > 0$.

Theorem 3. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$, f be a b -continuous self-mapping on $M, F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing mapping. If

$$\tau + F(s^q b(f\xi, f\eta)) \leq F\left(\alpha b(\xi, \eta) + \beta \frac{b(\xi, f\eta)}{2s} + \gamma \frac{b(\eta, f\xi)}{2s}\right), \tag{4}$$

for all $\xi, \eta \in M$ with $b(f\xi, f\eta) > 0$, where $\tau, q > 0$ and $\alpha, \beta, \gamma \geq 0$ are constants with $0 < \alpha + \beta + \gamma < 1$. Then, f has a unique fixed point provided that $b(f\eta, f\eta) \leq b(\eta, \eta)$ for all $\eta \in M$.

Proof. Let $\{\eta_n\}$ be a Picard iterative sequence as $\eta_{n+1} = f\eta_n$ initiated on each point $\eta_0 \in M$. Assume the general case that $\eta_{n+1} \neq \eta_n$, i.e., $b_n := b(f\eta_{n-1}, f\eta_n) = b(\eta_n, \eta_{n+1}) > 0$ for all $n \in \mathbb{N}^*$. Considering (4) and the monotonicity of F , we have:

$$\begin{aligned} \tau + F(s^q b_n) &= \tau + F(s^q b(f\eta_{n-1}, f\eta_n)) \\ &\leq F\left(\alpha b(\eta_{n-1}, \eta_n) + \beta \frac{b(\eta_{n-1}, f\eta_n)}{2s} + \gamma \frac{b(\eta_n, f\eta_{n-1})}{2s}\right) \\ &= F\left(\alpha b(\eta_{n-1}, \eta_n) + \beta \frac{b(\eta_{n-1}, \eta_{n+1})}{2s} + \gamma \frac{b(\eta_n, \eta_n)}{2s}\right) \\ &\leq F\left(\alpha b_{n-1} + \beta \frac{b_{n-1} + b_n}{2} + \gamma b_{n-1}\right) \\ &= F\left(\frac{\beta}{2} b_n + \left(\alpha + \gamma + \frac{\beta}{2}\right) b_{n-1}\right). \end{aligned}$$

Hence, f is an (s, q, F) -contraction. By Theorem 1, f has a fixed point.

Now we show the fixed point of f is unique. To this end, assume that there exist two distinct fixed points ζ^* and η^* , then $b(\zeta^*, \eta^*) > 0$, i.e., $b(f\zeta^*, f\eta^*) > 0$. By (4) and the monotonicity of F , we have:

$$\begin{aligned} F(s^q b(\zeta^*, \eta^*)) &< \tau + F(s^q b(\zeta^*, \eta^*)) = \tau + F(s^q b(f\zeta^*, f\eta^*)) \\ &\leq F\left(\alpha b(\zeta^*, \eta^*) + \beta \frac{b(\zeta^*, f\eta^*)}{2s} + \gamma \frac{b(\eta^*, f\zeta^*)}{2s}\right) \\ &= F\left(\alpha b(\zeta^*, \eta^*) + \beta \frac{b(\zeta^*, \eta^*)}{2s} + \gamma \frac{b(\eta^*, \zeta^*)}{2s}\right) \\ &\leq F((\alpha + \beta + \gamma)b(\zeta^*, \eta^*)) \leq F(b(\zeta^*, \eta^*)), \end{aligned}$$

which establishes that:

$$s^q b(\zeta^*, \eta^*) < b(\zeta^*, \eta^*).$$

In view of $b(\zeta^*, \eta^*) > 0$, then $s^q < 1$. This is a contradiction with $s \geq 1$ and $q > 0$. Therefore, $b(\zeta^*, \eta^*) = 0$. It leads to $\zeta^* = \eta^*$. That is to say, the mapping f has a unique fixed point in M . \square

Definition 9. Let f be a self-mapping on b -metric-like space (M, b, s) with parameter $s \geq 1$, and let $\{\eta_n\}$ be a Picard iterative sequence generated by f , $F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing function. We say that f is a r -order (s, q, F) -contraction if for all $b_n := b(\eta_n, \eta_{n+1}) > 0$, it satisfies

$$\tau + F(s^q b_n) \leq F\left((\alpha b_n^r + \beta b_{n-1}^r)^{\frac{1}{r}}\right), \tag{5}$$

where $\tau, q, r > 0$ and $\alpha, \beta \geq 0$ are constants with $0 < \alpha + \beta < 1$.

Remark 7. Clearly, (s, q, F) -contraction is 1-order (s, q, F) -contraction. Hence, (s, q, F) -contraction is the special case of r -order (s, q, F) -contraction. In other words, r -order (s, q, F) -contraction greatly generalizes (s, q, F) -contraction. In addition, by replacing $s = 1$, we obtain the notion of r -order F -contraction in the setting of metric spaces.

Theorem 4. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$ and f a b -continuous r -order (s, q, F) -contraction. Then, f has a fixed point provided that $b(f\eta, f\eta) \leq b(\eta, \eta)$ for all $\eta \in M$.

Proof. Let $\{\eta_n\}$ be a Picard iterative sequence as $\eta_{n+1} = f\eta_n$ initiated on each point $\eta_0 \in M$. Without loss of generality, we assume that $\eta_{n+1} \neq \eta_n$, i.e., $b_n := b(f\eta_{n-1}, f\eta_n) = b(\eta_n, \eta_{n+1}) > 0$ for all $n \in \mathbb{N}^*$. Taking advantage of (5), we obtain:

$$F(s^q b_n) < \tau + F(s^q b_n) \leq F\left((\alpha b_n^r + \beta b_{n-1}^r)^{\frac{1}{r}}\right).$$

By the monotonicity of F , we have:

$$s^q b_n < (\alpha b_n^r + \beta b_{n-1}^r)^{\frac{1}{r}},$$

which follows that

$$s^{qr} b_n^r < \alpha b_n^r + \beta b_{n-1}^r.$$

This leads to

$$b_n < \left(\frac{\beta}{s^{qr} - \alpha}\right)^{\frac{1}{r}} b_{n-1}.$$

Note that $\left(\frac{\beta}{s^{qr}-\alpha}\right)^{\frac{1}{r}} < 1$, then by Lemma 1, $\{\eta_n\}$ is a b -Cauchy sequence in M such that $\lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = 0$. Since (M, b, s) is b -complete, then there exists some $\eta^* \in M$ such that

$$0 = \lim_{n,m \rightarrow \infty} b(\eta_n, \eta_m) = \lim_{n \rightarrow \infty} b(\eta_n, \eta^*) = b(\eta^*, \eta^*).$$

Following the same argument as in Theorem 1, we claim that f has a fixed point. \square

Theorem 5. Let (M, b, s) be a b -complete b -metric-like space with parameter $s \geq 1$, f be a b -continuous self-mapping on M , $F : (0, +\infty) \rightarrow \mathbb{R}$ be an increasing mapping and $\Gamma : [0, +\infty) \times (0, +\infty) \rightarrow (0, +\infty)$ be a function such that $\Gamma(t, t) \leq 1$ for all $t \in (0, +\infty)$. If

$$\begin{aligned} & \tau + F(s^q b(f\xi, f\eta)) \\ & \leq F\left(\left(\alpha(b(\eta, f\eta)\Gamma(b(\xi, f\xi), b(\xi, \eta)))\right)^r + \beta(b(\xi, \eta))^r + \gamma(b(\eta, f\xi))^r\right)^{\frac{1}{r}}, \end{aligned} \tag{6}$$

for all $\xi, \eta \in M$ and $b(f\xi, f\eta) > 0$, where $\tau, q, r > 0$ and $\alpha, \beta, \gamma \geq 0$ are constants with $0 < \alpha + \beta + 2^r s^r \gamma < 1$. Then, the mapping f has a unique fixed point provided that $b(f\eta, f\eta) \leq b(\eta, \eta)$ for all $\eta \in M$.

Proof. Let $\{\eta_n\}$ be a Picard iterative sequence as $\eta_{n+1} = f\eta_n$ initiated on each point $\eta_0 \in M$. Assume the general case that $\eta_{n+1} \neq \eta_n$, i.e., $b_n := b(f\eta_{n-1}, f\eta_n) = b(\eta_n, \eta_{n+1}) > 0$ for all $n \in \mathbb{N}^*$. By using (6), we have

$$\begin{aligned} \tau + F(s^q b_n) &= \tau + F(s^q b(f\eta_{n-1}, f\eta_n)) \\ &\leq F\left(\left(\alpha(b(\eta_n, f\eta_n)\Gamma(b(\eta_{n-1}, f\eta_{n-1}), b(\eta_{n-1}, \eta_n)))\right)^r \right. \\ &\quad \left. + \beta(b(\eta_{n-1}, \eta_n))^r + \gamma(b(\eta_n, f\eta_{n-1}))^r\right)^{\frac{1}{r}} \\ &= F\left(\left(\alpha(b(\eta_n, \eta_{n+1})\Gamma(b(\eta_{n-1}, \eta_n), b(\eta_{n-1}, \eta_n)))\right)^r \right. \\ &\quad \left. + \beta(b(\eta_{n-1}, \eta_n))^r + \gamma(b(\eta_n, \eta_n))^r\right)^{\frac{1}{r}} \\ &\leq F\left(\left(\alpha b_n^r + (\beta + 2^r s^r \gamma) b_{n-1}^r\right)^{\frac{1}{r}}\right), \end{aligned}$$

which shows that (5) holds, and hence f is an r -order (s, q, F) -contraction. Thus, by Theorem 4, f has a fixed point.

We prove that $b(\eta^*, \eta^*) = 0$ if η^* is a fixed point of f . Indeed, by supposing the contrary, that is, $b(\eta^*, \eta^*) > 0$, i.e., $b(f\eta^*, f\eta^*) > 0$. It follows immediately from (6) that:

$$\begin{aligned} & F(s^q b(\eta^*, \eta^*)) < \tau + F(s^q b(f\eta^*, f\eta^*)) \\ & \leq F\left(\left(\alpha(b(\eta^*, f\eta^*)\Gamma(b(\eta^*, f\eta^*), b(\eta^*, \eta^*)))\right)^r + \beta(b(\eta^*, \eta^*))^r + \gamma(b(\eta^*, f\eta^*))^r\right)^{\frac{1}{r}} \\ & \leq F\left(\left(\alpha(b(\eta^*, \eta^*))^r + \beta(b(\eta^*, \eta^*))^r + \gamma(b(\eta^*, \eta^*))^r\right)^{\frac{1}{r}}\right) \\ & \leq F\left(\left((b(\eta^*, \eta^*))^r\right)^{\frac{1}{r}}\right) = F(b(\eta^*, \eta^*)). \end{aligned}$$

Making full use of the monotonicity of F , we claim that

$$s^q b(\eta^*, \eta^*) < b(\eta^*, \eta^*).$$

As a consequence of $b(\eta^*, \eta^*) > 0$, then $s^q < 1$. This is a contradiction with $s \geq 1$ and $q > 0$. Accordingly, $b(\eta^*, \eta^*) = 0$.

Let ζ^* and η^* be two distinct fixed points of f . By the above statement, we have $b(\zeta^*, \zeta^*) = 0$ and $b(\eta^*, \eta^*) = 0$. In view of $\zeta^* \neq \eta^*$, that is, $b(\zeta^*, \eta^*) > 0$, i.e., $b(f\zeta^*, f\eta^*) > 0$. then from (6) and the monotonicity of F , we speculate:

$$\begin{aligned} & F(s^q b(\zeta^*, \eta^*)) < \tau + F(s^q b(\zeta^*, \eta^*)) < \tau + F(s^q b(f\zeta^*, f\eta^*)) \\ & \leq F\left(\left(\alpha(b(\eta^*, f\eta^*)\Gamma(b(\zeta^*, f\zeta^*), b(\zeta^*, \eta^*)))^r + \beta(b(\zeta^*, \eta^*))^r + \gamma(b(\eta^*, f\eta^*))^r\right)^{\frac{1}{r}}\right) \\ & = F\left(\left(\alpha(b(\eta^*, \eta^*)\Gamma(b(\zeta^*, \zeta^*), b(\zeta^*, \eta^*)))^r + \beta(b(\zeta^*, \eta^*))^r + \gamma(b(\eta^*, \eta^*))^r\right)^{\frac{1}{r}}\right) \\ & = F\left(\left(\beta(b(\zeta^*, \eta^*))^r\right)^{\frac{1}{r}}\right) \leq F\left(\left((b(\zeta^*, \eta^*))^r\right)^{\frac{1}{r}}\right) = F(b(\zeta^*, \eta^*)), \end{aligned}$$

which follows from the monotonicity of F that

$$s^q b(\zeta^*, \eta^*) < b(\zeta^*, \eta^*).$$

Notice that $b(\zeta^*, \eta^*) > 0$ implies $s^q < 1$. This is a contradiction with $s \geq 1$ and $q > 0$. Thus, $b(\zeta^*, \eta^*) = 0$. Therefore, $\zeta^* = \eta^*$. That is to say, the fixed point of f is unique. \square

Remark 8. It can be easily shown that our new approach of r -order (s, q, F) -contraction covers many classical types of contractions such as Kannan, Reich, Chatteria, Hardy, Ćirić, etc. Consequently, it could be developed as a prospective work in the future. Kindly see the reference from [27].

4. Application

Stimulated by the work in [6,7,28,29], we investigate the existence of solution to a class of nonlinear integral equations utilizing the results proved in the previous section.

Consider the integral equation:

$$\zeta(t) = \int_0^T G(t, s)f(s, \zeta(s)) ds, \quad \text{for all } t \in [0, T], \tag{7}$$

where $T > 0$ is a constant, $\zeta : [0, T] \rightarrow \mathbb{R}$, $f : [0, T] \times \mathbb{R} \rightarrow \mathbb{R}$ and $G : [0, T] \times [0, T] \rightarrow [0, +\infty)$ are continuous functions.

Let $C([0, T])$ denote the set of real continuous functions on $[0, T]$.

Theorem 6. Suppose that:

$$\sup_{t \in [0, T]} \left(\int_0^T G^2(t, s) ds \right) \leq L, \tag{8}$$

and

$$\left(|f(t, \zeta(t))| + |f(t, \eta(t))| \right)^2 \leq \left(|\zeta(t)| + |\eta(t)| \right)^2, \tag{9}$$

for all $\zeta(t), \eta(t) \in C([0, T])$ and $t \in [0, T]$, where $L \in (0, \frac{1}{T})$ is a constant. Then the integral Equation (7) has a unique solution in $C([0, T])$.

Proof. Put $M = C([0, T])$. We endow M with the b -metric-like

$$b(\zeta, \eta) = \sup_{t \in [0, T]} \left(|\zeta(t)| + |\eta(t)| \right)^2, \quad \text{for all } \zeta, \eta \in M.$$

Clearly, (M, b, s) is a b -complete b -metric-like space with parameter $s = 2$. Assume that $F(u) = \ln u$ is a function on $(0, +\infty)$, $\Gamma(x, y) = \frac{xy}{1+y^2}$ is a function on $[0, +\infty) \times (0, +\infty)$. Define the mapping $T : M \rightarrow M$ by

$$T\zeta(t) = \int_0^T G(t, s)f(s, \zeta(s)) \, ds, \quad \text{for all } t \in [0, T],$$

then

$$\tau + F(s^q b(T\zeta, T\eta)) \leq F(\alpha b(\eta, T\eta)\Gamma(b(\zeta, T\zeta), b(\zeta, \eta)) + \beta b(\zeta, \eta) + \gamma b(\eta, f\zeta)), \quad (10)$$

for all $\zeta, \eta \in M$ and $b(T\zeta, T\eta) > 0$, where $\gamma = 0, \lambda > 1, \tau = \ln \lambda, q > 0, \alpha, \beta \geq 0$ are constants such that $0 < \alpha + \beta < 1$ and $LT < \frac{\beta}{2^q \lambda}$.

Indeed, (10) becomes

$$\ln \lambda + \ln(2^q b(T\zeta, T\eta)) \leq \ln \left(\alpha b(\eta, T\eta) \cdot \frac{b(\zeta, T\zeta)b(\zeta, \eta)}{1 + b^2(\zeta, \eta)} + \beta b(\zeta, \eta) \right),$$

which is equivalent with the following inequality:

$$2^q \lambda b(T\zeta, T\eta) \leq \alpha b(\eta, T\eta) \cdot \frac{b(\zeta, T\zeta)b(\zeta, \eta)}{1 + b^2(\zeta, \eta)} + \beta b(\zeta, \eta). \quad (11)$$

By the Cauchy–Schwarz inequality and (9), for all $\zeta, \eta \in M$, we have:

$$\begin{aligned} (|T\zeta| + |T\eta|)^2 &= \left(\left| \int_0^T G(t, s)f(s, \zeta(s)) \, ds \right| + \left| \int_0^T G(t, s)f(s, \eta(s)) \, ds \right| \right)^2 \\ &\leq \left(\int_0^T G(t, s)|f(s, \zeta(s))| \, ds + \int_0^T G(t, s)|f(s, \eta(s))| \, ds \right)^2 \\ &= \left(\int_0^T G(t, s)(|f(s, \zeta(s))| + |f(s, \eta(s))|) \, ds \right)^2 \\ &\leq \left(\int_0^T G^2(t, s) \, ds \right) \cdot \left(\int_0^T (|f(s, \zeta(s))| + |f(s, \eta(s))|)^2 \, ds \right) \\ &\leq T \left(\int_0^T G^2(t, s) \, ds \right) \cdot \sup_{t \in [0, T]} (|\zeta(t)| + |\eta(t)|)^2, \end{aligned}$$

which follows that:

$$\sup_{t \in [0, T]} (|T\zeta| + |T\eta|)^2 \leq T \sup_{t \in [0, T]} \left(\int_0^T G^2(t, s) \, ds \right) \cdot \sup_{t \in [0, T]} (|\zeta(t)| + |\eta(t)|)^2.$$

Taking advantage of (8), we arrive at:

$$b(T\zeta, T\eta) \leq \frac{\beta}{2^q \lambda} b(\zeta, \eta). \quad (12)$$

Thus, one gets:

$$2^q \lambda b(T\zeta, T\eta) \leq \beta b(\zeta, \eta) \leq \alpha b(\eta, T\eta) \cdot \frac{b(\zeta, T\zeta)b(\zeta, \eta)}{1 + b^2(\zeta, \eta)} + \beta b(\zeta, \eta).$$

Therefore, (11) holds. Accordingly, (10) is satisfied for all $\zeta, \eta \in M$ and $b(T\zeta, T\eta) > 0$. Hence, T is a general (s, q, F) -contraction. By (12), we obtain:

$$b(T\eta, T\eta) \leq \frac{\beta}{2^q \lambda} b(\eta, \eta) \leq b(\eta, \eta), \quad \text{for all } \eta \in M.$$

As a consequence, all the conditions of Theorem 2 are satisfied. Therefore, by Theorem 2, T has a unique fixed point in M . That is to say, the integral Equation (7) has a unique solution in $C([0, T])$. \square

Remark 9. As compared with Theorem 6.1 of [7], Theorem 6 has much fewer conditions and more straightforward proof. We consider the existence and uniqueness of the solution to the integral Equation (7) in the setting of b -metric-like spaces, which are different from other counterparts in the existing literature. Our conditions are not complicated and our proof is quite forthright.

Remark 10. It has been more widely used for the fixed point theory in b -metric-like spaces. It is not only for all kinds of integral equations (see [7,9–12]), but also for other types of equations. As an example, it has been applied to an electric circuit equation (see [19]), the conversion of solar energy to electrical energy (see [9]), impulsive differential equations (see [30]) and fractional differential equations (see [31,32]). As a result, our results may have wide applications in the future.

5. Conclusions

Nowadays, fixed point theory plays an important role in natural science and in solving different social problems. In this work, a technique is furnished, based on generalized F -contractions, such as the (s, q, F) -contraction, the general (s, q, F) -contraction, and the r -order (s, q, F) -contraction. We establish several fixed point theorems on such contractions with illustrative examples in the framework of b -metric-like spaces. As has been observed in studies, the class of b -metric-like spaces contains the other classes of generalized metric spaces (e.g., b -metric spaces, metric-like spaces, etc.), then our results in this paper generalize and improve many known results in the existing literature. Additionally, we have applied our results to obtain the existence of a solution for a class of integral equations. We believe that the idea of further elaborating our method, which is presented in the main result section, is very useful and can be applied to impulsive differential and nonlinear fractional differential equations in the future.

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