fractal and fractional

MDPI

Editorial

Advances in Optimization and Nonlinear Analysis

Savin Treanţă [1,2]

[1] Department of Applied Mathematics, University Politehnica of Bucharest, 060042 Bucharest, Romania; savin.treanta@upb.ro
[2] Academy of Romanian Scientists, 54 Splaiul Independentei, 050094 Bucharest, Romania

1. Introduction

There are many applications of optimization and nonlinear analysis in various fields of basic science, engineering, and natural phenomena. In this regard, we have provided the Special Issue "Advances in Optimization and Nonlinear Analysis" to cover the new advances in these mathematical areas. In this Special Issue, we have focused on publishing research studies on optimization and nonlinear analysis by investigating the well-posedness and optimal solutions in new classes of (multiobjective) variational (control) problems governed by multiple and/or path-independent curvilinear integral cost functionals and mixed and/or isoperimetric constraints involving first- and second-order partial differential equations. Additionally, some applications of fractional calculus or related subjects (variational inequalities, equilibrium problems, fixed point problems, evolutionary problems, and so on) have been considered in this Special Issue. In response to our invitation, we received 41 papers from 22 countries (Egypt, Saudi Arabia, Morocco, Pakistan, Mexico, Romania, China, Iran, Tunisia, South Africa, Yemen, Korea, Turkey, Bangladesh, Australia, Indonesia, Thailand, India, Ecuador, Albania, Spain, Malaysia), of which 15 were published and 26 rejected/withdrawn.

2. Brief Overview of the Contributions

In a review conducted by Omar et al. [1], the spiral dynamics optimization (SDO) algorithm was comprehensively reviewed. It is well-known that SDO algorithm is one of the most straightforward physics-based optimization algorithms and it is successfully applied in various broad fields. This review paper describes the recent advances of the SDO algorithm, including its adaptive, improved, and hybrid approaches. The growth of the SDO algorithm and its application in various areas, theoretical analysis, and comparison with its preceding and other algorithms are also described in detail. A detailed description of different spiral paths, their characteristics, and the application of these spiral approaches in developing and improving other optimization algorithms are comprehensively presented. The review concludes the current works on the SDO algorithm, highlighting its shortcomings and suggesting possible future research perspectives.

In [2], Treanţă studies the well posedness for a new class of optimization problems with variational inequality constraints involving second-order partial derivatives. More precisely, by using the notions of lower semicontinuity, pseudomonotonicity, hemicontinuity and monotonicity for a multiple integral functional, and by introducing the set of approximating solutions for the considered class of constrained optimization problems, he establishes some characterization results on well posedness. Furthermore, to illustrate the theoretical developments included in this paper, some examples are presented.

Thakur et al.'s [3] study in this Special Issue investigates the existence of positive solutions for a class of fractional differential equations of arbitrary order $\delta > 2$, subject to boundary conditions that include an integral operator of the fractional type. The consideration of this type of boundary conditions allows to consider heterogeneity on the dependence specified by the restriction added to the equation as a relevant issue for applications. An existence result is

check for updates

Citation: Treanţă, S. Advances in Optimization and Nonlinear Analysis. *Fractal Fract.* **2022**, *6*, 364. https://doi.org/10.3390/fractalfract6070364

Received: 28 June 2022
Accepted: 29 June 2022
Published: 30 June 2022

obtained for the sublinear and superlinear case by using the Guo–Krasnosel'skii fixed point theorem through the definition of adequate conical shells that allow to localize the solution. As additional tools in the considered procedure, Thakur et al. obtain the explicit expression of Green's function associated to an auxiliary linear fractional boundary value problem, and study some of its properties, such as the sign and some useful upper and lower estimates. Finally, an example is given to illustrate the results.

A parametric intuitionistic fuzzy multi-objective fractional transportation problem (PIF-MOFTP) is analyzed in El Sayed et al. [4]. The PIF-MOFTP includes a single-scalar parameter in the objective functions and an intuitionistic fuzzy supply and demand. Based on the (α, β)-cut concept, a parametric (α, β)-MOFTP is proposed. Then, a fuzzy goal programming (FGP) approach is utilized to obtain (α, β)-Pareto optimal solution. Moreover, the authors investigates the stability set associated with the first kind (SSFK) corresponding to the solution by extending the Kuhn-Tucker optimality conditions of multi-objective programming problems. Also, an algorithm to crystalize the progressing SSFK for PIF-MOFTP is presented.

Vivas-Cortez et al. [5] use integral inequalities involving many fractional integral operators in order to solve various fractional differential equations. More precisely, the authors generalize the Hermite–Jensen–Mercer-type inequalities for an h-convex function via a Caputo–Fabrizio fractional integral. They develop some novel Caputo–Fabrizio fractional integral inequalities. Also, they establish Caputo–Fabrizio fractional integral identities for differentiable mapping, and these will be used to give estimates for some fractional Hermite–Jensen–Mercer-type inequalities. Some familiar results are recaptured as special cases of these results.

In Lai et al. [6], the authors establish Fritz John stationary conditions for nonsmooth, nonlinear, semidefinite, multiobjective programs with vanishing constraints in terms of convexificator. Also, they introduce generalized Cottle type and generalized Guignard type constraints qualification to achieve strong S—stationary conditions from Fritz John stationary conditions. Further, the authors establish strong S—stationary necessary and sufficient conditions, independently from Fritz John conditions. Some examples are provided to validate the established results.

The purpose of the next paper Khan et al. [7] published in this Special Issue is to introduce a new class of Hermite–Hadamard inequalities for LR-convex interval-valued functions, by means of a pseudo-order relation. This order relation is defined on interval space. Moreover, the interval Hermite–Hadamard–Fejér inequality is also derived for LR-convex interval-valued functions. These inequalities also generalize some new and known results. Useful examples that verify the applicability of the theory developed in this study are presented.

The Lieb concavity theorem, successfully solved in the Wigner–Yanase–Dyson conjecture, is an important application of matrix concave functions. Recently, the Thompson–Golden theorem, a corollary of the Lieb concavity theorem, was extended to deformed exponentials. Hence, it is worthwhile to study the Lieb concavity theorem for deformed exponentials. In Yang [8], the Pick function is used to obtain a generalization of the Lieb concavity theorem for deformed exponentials, and some corollaries associated with exterior algebra are obtained.

Nowadays, more and more consumers consider environmentally friendly products in their purchasing decisions. Companies need to adapt to these changes while paying attention to standard business systems such as payment terms. The purpose of the study realized by Sultana et al. [9] is to optimize the entire profit function of a retailer and to find the optimal selling price and replenishment cycle when the demand rate depends on the price and carbon emission reduction level. This study investigates an economic order quantity model that has a demand function with a positive impact of carbon emission reduction besides the selling price. In this model, the supplier requests payment in advance on the purchased cost while offering a discount according to the payment in the advanced decision. Three different types of payment-in-advance cases are applied: (1) payment

in advance with equal numbers of instalments, (2) payment in advance with a single instalment, and (3) the absence of payment in advance. Numerical examples and sensitivity analysis illustrate the proposed model. Here, the total profit increases for all three cases with higher values of carbon emission reduction level. Further, the study finds that the profit becomes maximum for case 2, whereas the selling price and cycle length become minimum. This study considers the sustainable inventory model with payment-in-advance settings when the demand rate depends on the price and carbon emission reduction level.

Convexity is crucial in obtaining many forms of inequalities. As a result, there is a significant link between convexity and integral inequality. Due to the significance of these concepts, the purpose of Khan et al.'s [10] study is to introduce a new class of generalized convex interval-valued functions called (p, s)-convex fuzzy interval-valued functions (for short, (p, s)-convex F-I-V-Fs) in the second sense and to establish Hermite–Hadamard (for short, H–H) type inequalities for (p, s)-convex F-I-V-Fs using fuzzy order relation. In addition, the authors demonstrate that the derived results include a large class of new and known inequalities for (p, s)-convex F-I-V-Fs and their variant forms as special instances. Furthermore, useful examples are given to demonstrate usefulness of the theory produced in this study. These findings and diverse approaches may pave the way for future research in fuzzy optimization, modeling, and interval-valued functions.

In the paper Sajjadmanesh et al. [11], the authors are interested in an inverse geometric problem for the three-dimensional Laplace equation to recover an inner boundary of an annular domain. This work is based on the method of fundamental solutions (MFS) by imposing the boundary Cauchy data in a least-square sense and minimisation of the objective function. This approach can also be considered with noisy boundary Cauchy data. The simplicity and efficiency of this method is illustrated in several numerical examples.

Multiple attractors and their fractal basins of attraction can lead to the loss of global stability and integrity of Micro Electro Mechanical Systems (MEMS). In the paper of Zhu et al. [12], multistability of a class of electrostatic bilateral capacitive micro-resonator is researched in detail. First, the dynamical model is established and made dimensionless. Second, via the perturbating method and the numerical description of basins of attraction, the multiple periodic motions under primary resonance are discussed. It is found that the variation of AC voltage can induce safe jump of the micro resonator. In addition, with the increase of the amplitude of AC voltage, hidden attractors and chaos appear. The results may have some potential value in the design of MEMS devices.

The purpose of the study Khan et al. [13] is to define a new class of harmonically convex functions, which is known as left and right harmonically convex interval-valued functions (for short, LR-H-convex IV-F), and to establish novel inclusions for a newly defined class of interval-valued functions (for short, IV-Fs) linked to Hermite–Hadamard (for short, H-H) and Hermite–Hadamard–Fejér (H-H-Fejér) type inequalities via interval-valued Riemann–Liouville fractional (for short, IV-RL-fractional) integrals. These findings enable the authors to identify a new class of inclusions that may be seen as significant generalizations. Some examples are included in the considered findings that may be used to determine the validity of the results.

The study developed in Daqaq et al. [14] describes a novel manta ray foraging optimization approach based non-dominated sorting strategy, namely (NSMRFO), for solving the multi-objective optimization problems (MOPs). The proposed powerful optimizer can efficiently achieve good convergence and distribution in both the search and objective spaces. In the NSMRFO algorithm, the elitist non-dominated sorting mechanism is followed. Afterwards, a crowding distance with a non-dominated ranking method is integrated for the purpose of archiving the Pareto front and improving the optimal solutions coverage. To judge the NSMRFO performances, a bunch of test functions are carried out including classical unconstrained and constrained functions, a recent benchmark suite known as the completions on evolutionary computation 2020 (CEC2020) that contains twenty-four multimodal optimization problems (MMOPs), some engineering design problems, and also the modified real-world issue known as IEEE 30-bus optimal power flow involving

the wind/solar/small-hydro power generations. Comparison findings with multimodal multi-objective evolutionary algorithms (MMMOEAs) and other existing multi-objective approaches with respect to performance indicators reveal the NSMRFO ability to balance between the coverage and convergence towards the true Pareto front (PF) and Pareto optimal sets (PSs). Thus, the competing algorithms fail in providing better solutions while the proposed NSMRFO optimizer is able to attain almost all the Pareto optimal solutions.

The last paper published in the considered Special Issue (see Elkasem et al. [15]) presents an innovative strategy for load frequency control (LFC) using a combination structure of tilt-derivative and tilt-integral gains to form a TD-TI controller. Furthermore, a new improved optimization technique, namely the quantum chaos game optimizer (QCGO) is applied to tune the gains of the proposed combination TD-TI controller in two-area interconnected hybrid power systems, while the effectiveness of the proposed QCGO is validated via a comparison of its performance with the traditional CGO and other optimizers when considering 23 bench functions. Correspondingly, the effectiveness of the proposed controller is validated by comparing its performance with other controllers, such as the proportional-integral-derivative (PID) controller based on different optimizers, the tilt-integral-derivative (TID) controller based on a CGO algorithm, and the TID controller based on a QCGO algorithm, where the effectiveness of the proposed TD-TI controller based on the QCGO algorithm is ensured using different load patterns (i.e., step load perturbation (SLP), series SLP, and random load variation (RLV)). Furthermore, the challenges of renewable energy penetration and communication time delay are considered to test the robustness of the proposed controller in achieving more system stability. In addition, the integration of electric vehicles as dispersed energy storage units in both areas has been considered to test their effectiveness in achieving power grid stability. The simulation results elucidate that the proposed TD-TI controller based on the QCGO controller can achieve more system stability under the different aforementioned challenges.

Funding: This research received no external funding.

Acknowledgments: I am thankful the editors and reviewers of the *Fractal and Fractional* journal for their help and support.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Omar, M.B.; Bingi, K.; Prusty, B.R.; Ibrahim, R. Recent Advances and Applications of Spiral Dynamics Optimization Algorithm: A Review. *Fractal Fract*. **2022**, *6*, 27. [CrossRef]
2. Treanţă, S. Well Posedness of New Optimization Problems with Variational Inequality Constraints. *Fractal Fract*. **2021**, *5*, 123. [CrossRef]
3. Thakur, A.; Ali, J.; Rodríguez-López, R. Existence of Solutions to a Class of Nonlinear Arbitrary Order Differential Equations Subject to Integral Boundary Conditions. *Fractal Fract*. **2021**, *5*, 220. [CrossRef]
4. El Sayed, M.A.; El-Shorbagy, M.A.; Farahat, F.A.; Fareed, A.F.; Elsisy, M.A. Stability of Parametric Intuitionistic Fuzzy Multi-Objective Fractional Transportation Problem. *Fractal Fract*. **2021**, *5*, 233. [CrossRef]
5. Vivas-Cortez, M.; Saleem, M.S.; Sajid, S.; Zahoor, M.S.; Kashuri, A. Hermite–Jensen–Mercer-Type Inequalities via Caputo–Fabrizio Fractional Integral for h-Convex Function. *Fractal Fract*. **2021**, *5*, 269. [CrossRef]
6. Lai, K.K.; Hassan, M.; Singh, S.K.; Maurya, J.K.; Mishra, S.K. Semidefinite Multiobjective Mathematical Programming Problems with Vanishing Constraints Using Convexificators. *Fractal Fract*. **2022**, *6*, 3. [CrossRef]
7. Khan, M.B.; Treanţă, S.; Soliman, M.S.; Nonlaopon, K.; Zaini, H.G. Some Hadamard–Fejér Type Inequalities for LR-Convex Interval-Valued Functions. *Fractal Fract*. **2022**, *6*, 6. [CrossRef]
8. Yang, G.; Li, Y.; Wang, J.; Sun, H. Application of the Pick Function in the Lieb Concavity Theorem for Deformed Exponentials. *Fractal Fract*. **2022**, *6*, 20. [CrossRef]
9. Sultana, S.; Mashud, A.H.M.; Daryanto, Y.; Miah, S.; Alrasheedi, A.; Hezam, I.M. The Role of the Discount Policy of Prepayment on Environmentally Friendly Inventory Management. *Fractal Fract*. **2022**, *6*, 26. [CrossRef]
10. Khan, M.B.; Treanţă, S.; Budak, H. Generalized p-Convex Fuzzy-Interval-Valued Functions and Inequalities Based upon the Fuzzy-Order Relation. *Fractal Fract*. **2022**, *6*, 63. [CrossRef]
11. Sajjadmanesh, M.; Aydi, H.; Ameer, E.; Park, C. The Method of Fundamental Solutions for the 3D Laplace Inverse Geometric Problem on an Annular Domain. *Fractal Fract*. **2022**, *6*, 66. [CrossRef]
12. Zhu, Y.; Shang, H. Multistability of the Vibrating System of a Micro Resonator. *Fractal Fract*. **2022**, *6*, 141. [CrossRef]

13. Khan, M.B.; Macías-Díaz, J.E.; Treanţă, S.; Soliman, M.S.; Zaini, H.G. Hermite-Hadamard Inequalities in Fractional Calculus for Left and Right Harmonically Convex Functions via Interval-Valued Settings. *Fractal Fract*. **2022**, *6*, 178. [CrossRef]
14. Daqaq, F.; Kamel, S.; Ouassaid, M.; Ellaia, R.; Agwa, A.M. Non-Dominated Sorting Manta Ray Foraging Optimization for Multi-Objective Optimal Power Flow with Wind/Solar/Small- Hydro Energy Sources. *Fractal Fract*. **2022**, *6*, 194. [CrossRef]
15. Elkasem, A.H.A.; Khamies, M.; Hassan, M.H.; Agwa, A.M.; Kamel, S. Optimal Design of TD-TI Controller for LFC Considering Renewables Penetration by an Improved Chaos Game Optimizer. *Fractal Fract*. **2022**, *6*, 220. [CrossRef]