

Supplementary Information

1. SI – Theory

1.1 Pyramid

Assuming the pyramid base has a unity width of 1, the total length from A to C or \overleftrightarrow{AC} is $\sqrt{1^2 + 1^2} = \sqrt{2}$. From this it follows that the height of the pyramid \overleftrightarrow{EM} is equal to half that distance $\frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}$.

The corner F or $\angle EFG$ is then found by taking the tangent of the purple-grey triangle formed by

$\triangle FEM$, $F = \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\right) = \tan^{-1}\left(\frac{2}{\sqrt{2}}\right) = \tan^{-1}(\sqrt{2}) = 54.74^\circ$. The corner E or $\angle GEF$ can then

simply be found by taking twice $\angle FEM$: $\angle FEG = 2 \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\right) = 2 \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) = 2 \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) = 2 \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 70.53^\circ$.

1.2 First Corner: Edge A-E, 109.47°

When looking at the grey triangle $\triangle EQJ$ (Figure S1b), the GF is given by the diagonal \overleftrightarrow{KQ} (yellow dotted line in Figure S1b), perpendicular to the outside edge. This is again the edge of a right-angled triangle, that is given by $\sqrt{\frac{1}{2}(\sqrt{3})^2} t = \sqrt{\frac{3}{2}} t = 1.22 t$.

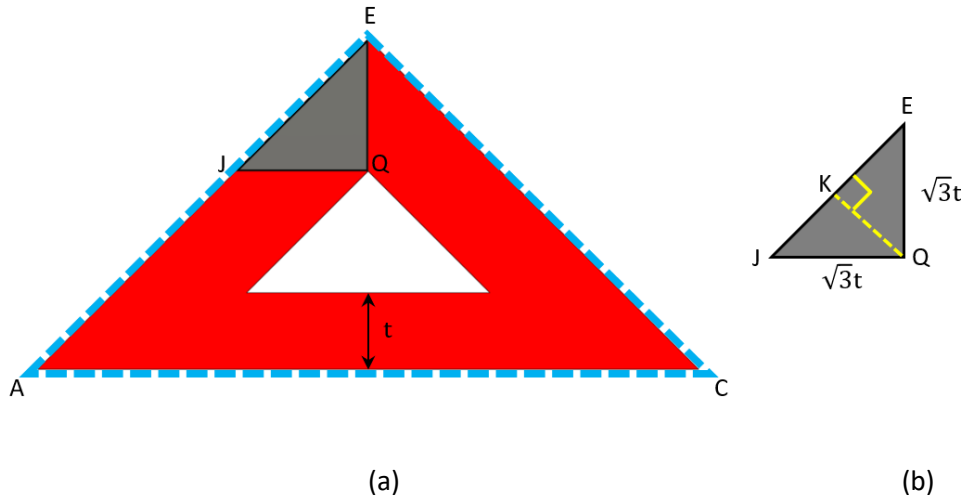


Figure S1: (a) Cross-section of the pyramid over the blue dashed line or through A, C and E. (b) The grey triangle EQJ of (a) in detail.

1.3 Second Corner: Vertex E, 70.53°

Figure S2 corresponds to the triangle $\triangle EFG$. The GF \overleftrightarrow{EQ} at the centre vertex or apex of the pyramid is calculated from the grey triangle $\triangle EQR$ Figure S2b. From this triangle, one can get the ratio between the sides from the dimensions of the original pyramid, so a base of $\frac{1}{2}$, a height of $\frac{1}{\sqrt{2}}$ and

thus the diagonal side \overleftrightarrow{FE} with a length of $\overleftrightarrow{FE} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{2}\sqrt{3}$.

Also the length of yellow dashed diagonal \overleftrightarrow{QS} is known since this is the deposited thickness and this line forms another 2 similar triangles. From the ratios of the grey triangle (Figure S2b), the length

between the apex of the pyramid and the inner tip of the deposited layer \overrightarrow{EQ} can be calculated. With a diagonal of length t , the length \overrightarrow{QR} at the bottom of the grey triangle is given by $\overrightarrow{QR} = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{\sqrt{2}}} t =$

$\sqrt{\frac{3}{2}} t$. Combining this with the ratio between height and bottom gives:

$$\overrightarrow{EQ} = \frac{1}{\sqrt{2}} / \sqrt{\frac{3}{2}} t = \sqrt{3} t = 1.73 t.$$

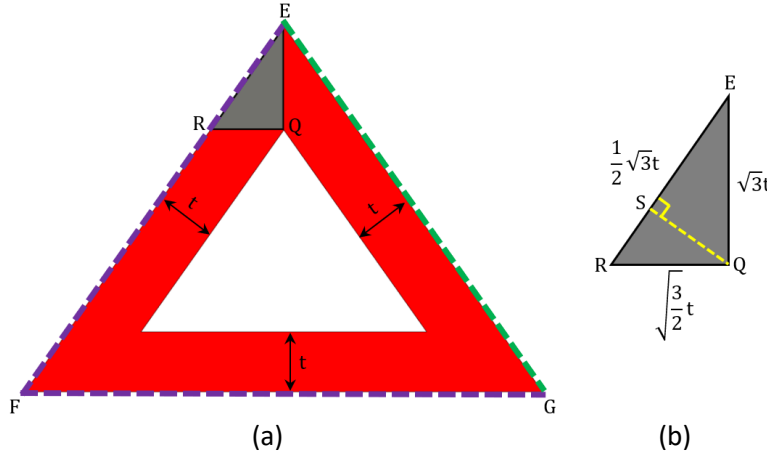


Figure S2: (a) Cross-section of the pyramid over the green and purple dashed line or through F, E and G. (b) The grey triangle of EQR in detail with both the calculated lengths and ratios between the edges.

1.4 Third Corner: Edge A-B, 54.74°

Figure S3 illustrates the next sharpest corner which is the edge at the base of the pyramid with the angle at \overrightarrow{AB} given by F or $\angle EFG$. The etching distance for this edge is indicated by the hypotenuse of the yellow right-angled triangle ΔTUF in Figure S3b. For calculating the length, the height is given by t , but for calculating the base side the previously calculated triangle ΔQER and an equally shaped smaller ΔVUT triangle are used. It follows that, $\overrightarrow{QR} = \overrightarrow{SF} = \sqrt{\frac{3}{2}} t$, $\overrightarrow{ER} = \overrightarrow{TF} = \frac{1}{2}\sqrt{3} t$ and $\overrightarrow{EQ} = \overrightarrow{TS} = \sqrt{3} t$. The length of SU is given by the ratio between \overrightarrow{EQ} and \overrightarrow{QR} in combination with the known thickness resulting in $\overrightarrow{SU} = \frac{1}{\sqrt{2}} t$.

This means that the total etching length of the diagonal is given by:

$$\sqrt{\left(\sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}}\right)^2 + 1^2} t = \sqrt{3 + \sqrt{3}} t = 2.18 t$$

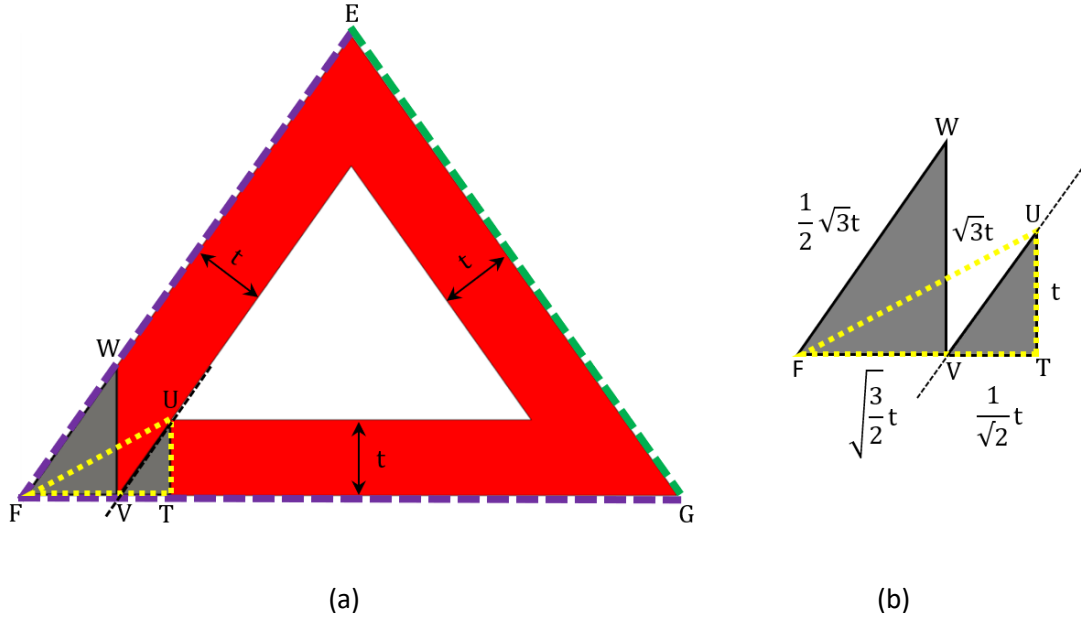


Figure S3 Cross-section of the pyramid over the green and purple dashed line or through F, E and G where ΔVWF is ΔQER .

1.5 Fourth Corner: Vertex A, 45°

Figure S4 shows the fourth corner which is the sharpest corners and is formed by the vertices at the base of the pyramid $\angle FEG$. For calculating, a similar approach is taken as before as shown in Figure S3b, but now we use the hypotenuse of the left smaller grey triangle ΔOPA to find the length \overline{AN} of the base of the yellow triangle ΔALN . This is given by $\sqrt{2 \cdot \frac{3}{2}t} = \sqrt{3}t$, leading to a total etching length given by: $\sqrt{(\sqrt{3} + 1)^2 + 1^2}t = \sqrt{5 + 2\sqrt{3}}t = 2.91t$

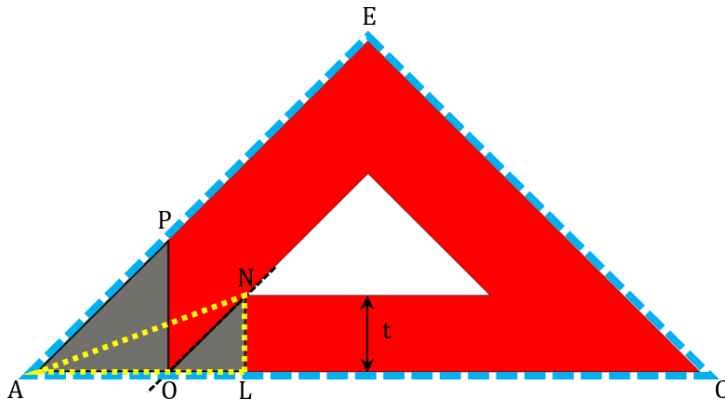


Figure S4: Cross-section of the pyramid over the blue dashed line through A, C and E.

2. SI – Extra Etching Corner

As mentioned in the Theory section; when etching not through a circular, but an elongated aperture in the SiO_2 hard-mask, a rectangular structure can be obtained in the Si as can be seen in **Figure S5**. The extra vertex corner obtained at the bottom of this structure has an etching factor of 1.2707, only 0.046 more than the edge \overline{AE} at the thickest spot. However, this means that the etching window in which one has control is less than 5% of the deposited silicon nitride thickness. That means that for

any deposition significantly less than 100 nm your etching window is only several atoms thick and most of the mask is even thinner and thus not practically applicable as a hard-mask against etching.

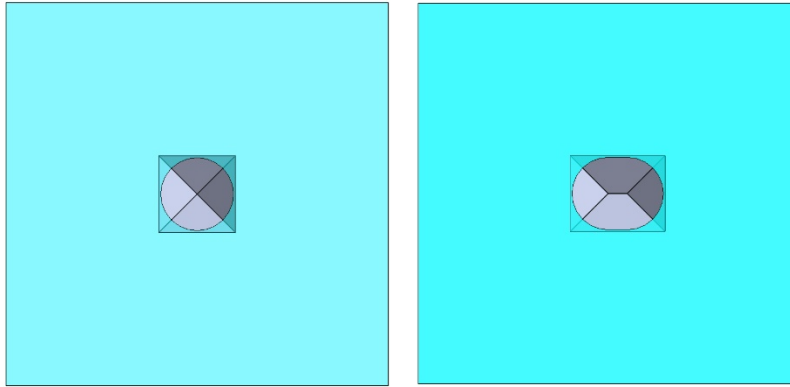


Figure S5: Left: Inverted pyramid etched in Si through a circular hole in the SiO_2 hard-mask. Right: Inverted hip roof structure etched in Si under an elongated hole in the SiO_2 hard-mask.

3. SI – Undercut Mask, Cornered Edges

3.1 Undercut Mask

As described in the text, the planar top mask will be undercut when the SiO_2 of the G1 previous fractal generation pyramid is etched. This needs to be etched since otherwise the small hole in the apices left by the Si_3N_4 nanodots of the previous generation limit the thickness of the Si_3N_4 that can be deposited in the G2 or following generation by growing shut. A way to limit the undercut, is by not fully removing the SiO_2 on the $\{111\}$ planes, but only the free-standing part. This can be achieved by etching only 50% of the thickness since it will be etched from both sides as is shown in Figure S6.

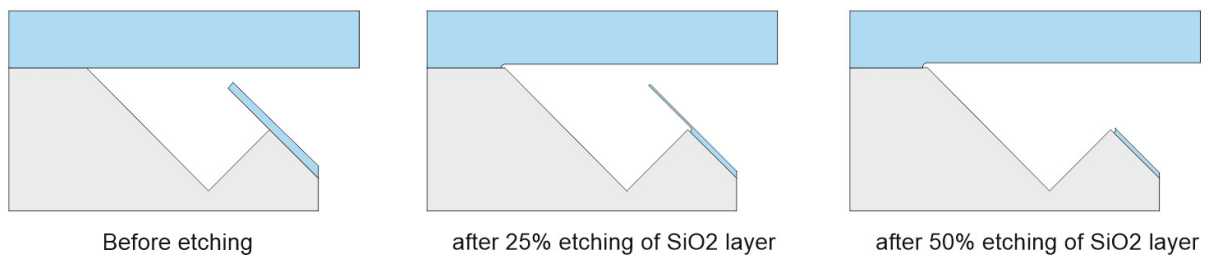


Figure S6: Undercutting effect of the planar mask by removing the SiO_2 walls of the pyramid and showing only 50% of the thickness needs to be removed.

However, this still causes etching of the planar SiO_2 mask on top creating a gap or extra edge at the base of the inverted G2 pyramids, meaning the original edges and vertices are not the longest distances that will be etched, which might influence the corner lithography. Figure S9 shows the effect on the etching of the vertices of the G2 by taking the cross-section as is illustrated by the red dashed line in Figure S7. Figure S10 shows the effect on the etching of the edges of both the G1 and G2 by taking the cross-section as is illustrated by the green dashed line in Figure S7.

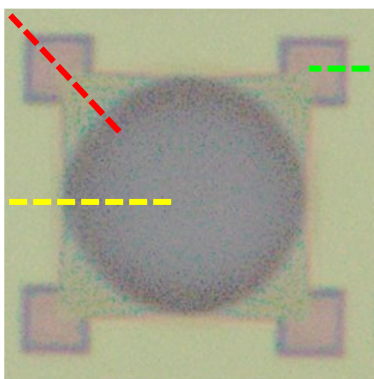


Figure S7: Locations of cross-sections for illustrating mask-etching effect. Yellow dashed line: Cross-section for Figure S8. Red dashed line: Cross-section for Figure S9. Green dashed line: Cross-section for Figure S10.

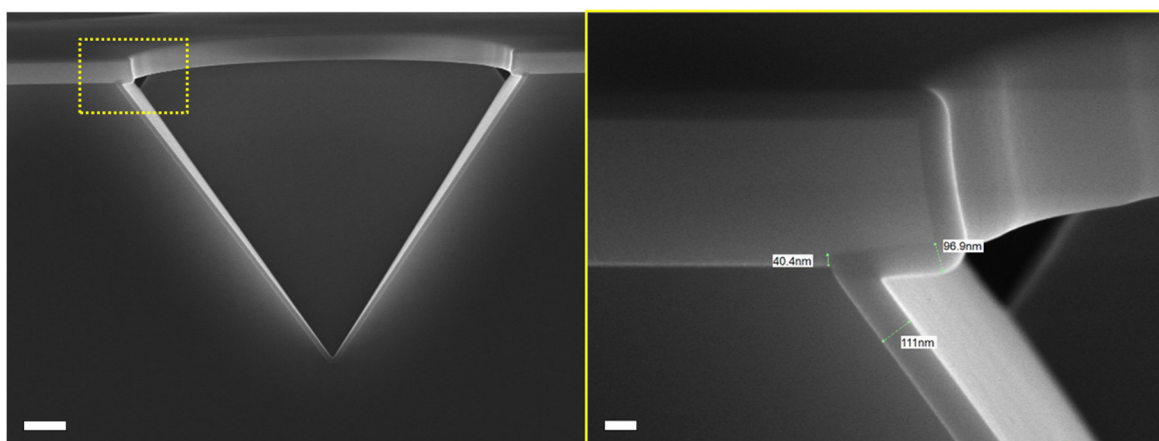


Figure S8: SEM images of the cross-section as indicated in Figure S7b showing the undercutting of the planar mask after which Si_3N_4 has been deposited. The dashed rectangle in the image on the left indicates the location of the zoomed in image on the right. Scalebars: left image is $1\ \mu\text{m}$ and right image 100 nm.

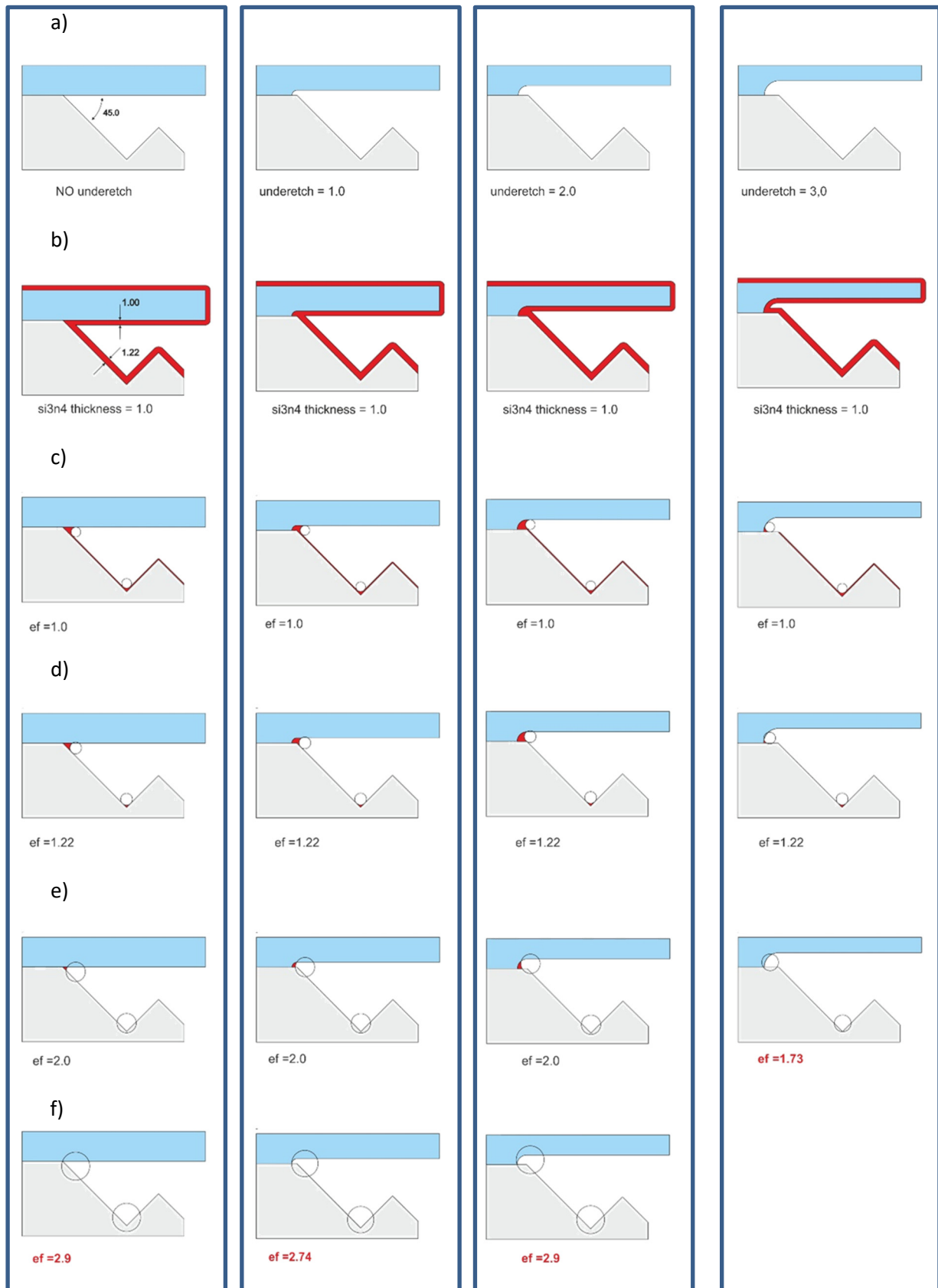


Figure S9: The effect on the etching factor (EF) of the underetching of the planar mask on corner lithography in the vertex of the G2 or latest generation. The underetching factor here is relative to the thickness of the deposited Si_3N_4 .

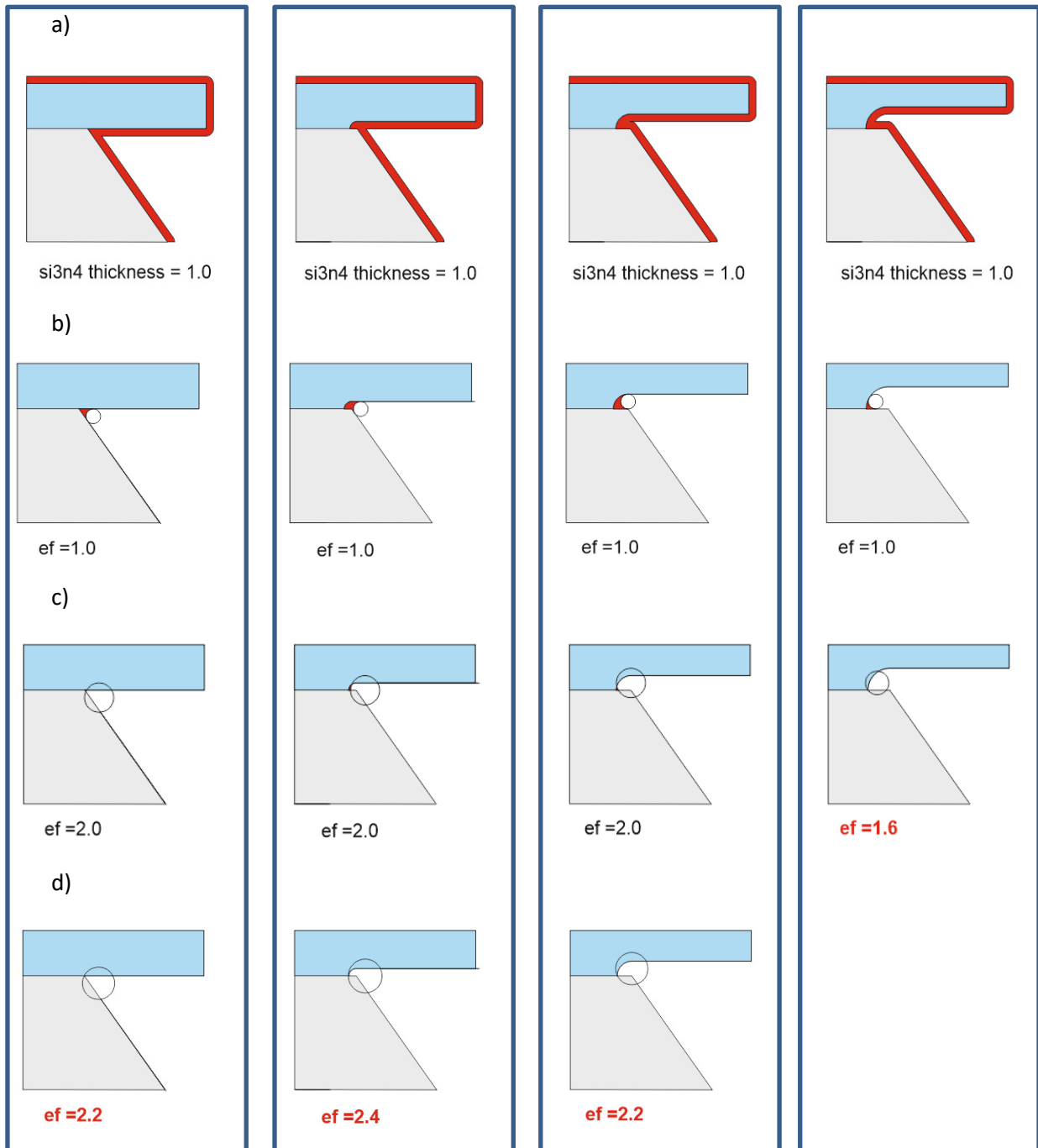


Figure S10: The effect of the underetching of the planar mask on corner lithography in the edge of the G2, from left to right the columns represent an underetching factor of 0, 1, 2 and 3 respectively. The underetching factor here is relative to the thickness of the deposited Si_3N_4 .

Looking at Table 1 it can be seen that the last edge has an etching window between an EF of 1.73 and 2.18 while the vertex has an etching window between an EF of 2.18 and 2.91. From Figure S10 it can be seen that at an underetching factor of 2, only an EF of ~ 0.02 still remains to selectively only etch these edges. This means that as long as the underetching factor stays significantly below 2, this edge can still selectively be used for masking. When looking at Figure S9 it shows that with an underetching factor of 3, the newly shaped edge is clearly dominating the etching, removing both these vertices and edges before the previous vertex (γ) is removed. However, at an underetching

factor of 2 or lower, these vertices can still be selectively etched and used and in all cases corner lithography as demonstrated in this paper for creating more fractals, can still be applied.

3.2 Cornered Edges

As described, there is not only an undercutting of the mask, but due to the use of TMAH, the base of the pyramid has an extra corner instead of a sharp edge. The combined result of these two effects can be seen in on the SEM image of a cross-section in Figure S11.

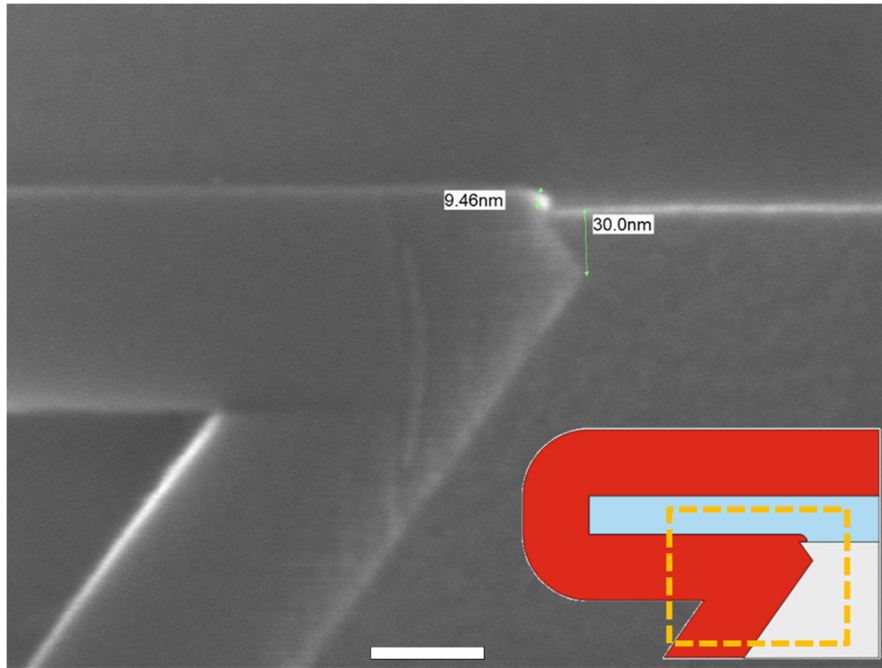


Figure S11: SEM image of the undercut and extra corner with Si_3N_4 deposited on it. The sketch in the bottom right shows the situation with a dashed rectangle of which part is shown in the SEM image. Scalebar is 50 nm.

This again also can have an impact on the corner lithography if the extra corner is spaced at significantly large distance from the planar hard-mask with respect to the deposited Si_3N_4 layer thickness. However, when the distance is in a smaller order than the layer thickness the situation arises as is sketched in Figure S12. This shows that the edge is fully etched at an EF of 1.8, meaning there is only an EF of 0.7 to selectively etch only these edges. For the vertex the maximum etching has been reduced to 2.4, so it is still possible to selectively etch these vertices. Meaning that generation of fractals using corner lithography can still be performed as is also demonstrated.

When desired extra corners can also be removed using some extra steps as long as all the SiO_2 masks are thick enough. The same situation will be reached as shown on the right in Figure S6, leaving the SiO_2 on the $\{111\}$ Si sidewalls of all the previous generations. However, in this case the SiO_2 of the planar mask needs to be etched back far enough to reach beyond the corner caused by the TMAH etching. Meaning that it needs to align with the preferred $\langle 111 \rangle$ plane of the pyramid or go even beyond that. That means the LOCOS oxide after the last corner lithography step needs to be thick enough for this. Then everything can be etched shortly in TMAH again where the exposed $\langle 100 \rangle$ plane at the top will etch quickly, making pyramids of the latest generation slightly bigger while the still present SiO_2 on the previous generations prevent etching of those pyramids.

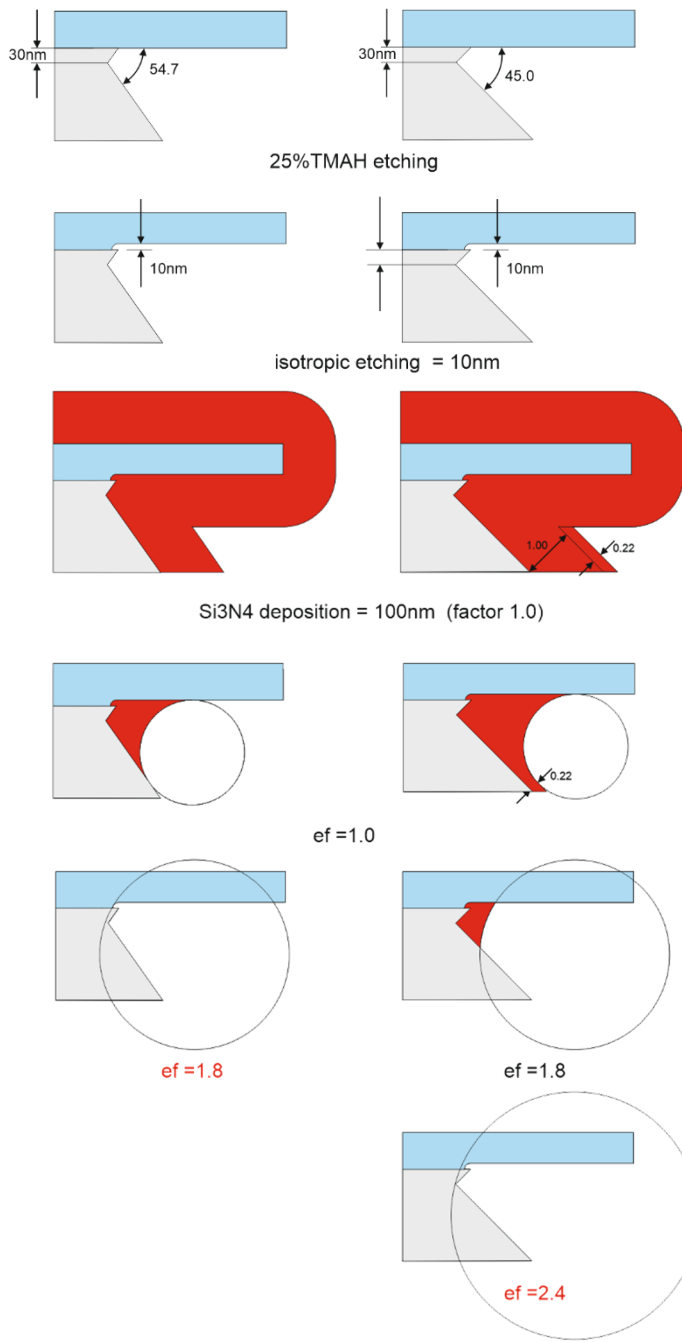


Figure S12: The effect of the extra corner and undercut on corner lithography. On the left the cross-section of the edge as indicated in Figure S2B. On the right the cross-section of the apex as indicated in Figure S7a.

4. Formulae for Determining the Corner Width

4.1 Width a

$$\alpha_a = 2 \tan^{-1}(\sqrt{2}) \quad \text{Original corner } 109.47^\circ$$

$$\beta_a = \frac{1}{2} \alpha_a = \tan^{-1}(\sqrt{2})$$

$$width_a/t = 2 \sin \left(\sin^{-1} \left(GF_a \frac{\sin(\beta_a)}{EF} \right) - \beta_a \right) EF$$

For EF from 0 to $GF_a = \sqrt{\frac{3}{2}}$

4.2 Width b

$$\alpha_b = 2 \tan^{-1} \left(\frac{1}{\sqrt{2}} \right) \quad \text{Original corner } 70.53^\circ$$

$$\beta_b = \frac{1}{2} \alpha_b = \tan^{-1} (1/\sqrt{2})$$

$$width_b/t = 2 \sin \left(\sin^{-1} \left(GF_b \frac{\sin(\beta_b)}{EF} \right) - \beta_b \right) EF$$

For EF from 0 to $GF_b = \sqrt{3}$.

4.3 Width c

$$\alpha_c = 2 \beta_c \quad \text{Original corner } 54.74^\circ$$

$$\beta_c = \tan^{-1} \left(\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{2}} \right)$$

$$width_c/t = \sin \left(\sin^{-1} \left(GF_c \frac{\sin(\beta_c)}{EF} \right) - \beta_c \right) EF$$

For EF from 0 to $GF_d = \sqrt{3 + \sqrt{3}}$.

4.4 Width d

$$\alpha_{d1} + \alpha_{d2} = 45^\circ \quad \text{Original corner } 45^\circ$$

$$\alpha_{d1} = \tan^{-1} \left(\frac{1}{\sqrt{3}} + 1 \right)$$

$$\alpha_{d2} = 45^\circ - \alpha_{d1}$$

$$width_d/t = \frac{\sin \left(\sin^{-1} \left(\frac{GF_d \sin(\alpha_{d1})}{EF} \right) + \sin^{-1} \left(\frac{GF_d \sin(\alpha_{d2})}{EF} \right) - (\alpha_{d1} + \alpha_{d2}) \right) EF}{\sin \left(\frac{180 - \sin^{-1} \left(\frac{GF_d \sin(\alpha_{d1})}{EF} \right) - \sin^{-1} \left(\frac{GF_d \sin(\alpha_{d2})}{EF} \right) + (\alpha_{d1} + \alpha_{d2})}{2} \right)}$$

For EF from $\sqrt{\frac{3}{2}}$ to $GF_d = \sqrt{5 + 2\sqrt{3}}$ where the lower limits comes from the fact that the first corner over edge \overleftrightarrow{AE} needs to be fully etched before you can speak of a limited corner width in the direction for d.

5. SI – SEM Images Information

Table S1: Overview of the used acceleration voltages, magnification and SEM used for taking the SEM images. The numbering in the magnification column refers to numbering in the corresponding figures or tables.

Table/Figure	SEM Model	kV	Magnification
Table 3	FEI Sirion HR-SEM	3	(1.1) 100.000 (1.4) 100.000 (1.8) 100.000
Table 3 2.2	FEI Sirion HR-SEM	5	(2.2) 100.000
Figure 11	FEI Sirion HR-SEM	5	(a) 1500 (b) 100.000 100.000
Figure 12	FEI Sirion HR-SEM	5	(a) 2679 (b) 85.738
Figure 13	FEI Sirion HR-SEM	5	(b) 8032 (c) 8004 (d) 8001 (e) 8088
Figure 14	JEOL JSM 7610FPlus FEG SEM	5	(a) 3300 (b) 5000 (c) 30.000 100.000
Figure 15	JEOL JSM 7610FPlus FEG SEM	5	(a) 10.000 (b) 27.000 200.000
Figure 16	JEOL JSM 7610FPlus FEG SEM	5	(a) 2500 (b) 7000 (c) 65.000 200.000
Figure 17	JEOL JSM 7610FPlus FEG SEM	2	(a) 2500 (b) 4000 (c) 70.000 (d) 3000 (e) 7000 22.000
Figure 19	JEOL JSM 7610FPlus FEG SEM	15	(a) 12.000 (b) 300.000 200.000
Figure 20	JEOL JSM 7610FPlus FEG SEM	5	(a) 50.000 200.000