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Trajectory Tracking Control of Euler–Lagrange Systems Using a Fractional Fixed-Time Method

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Abstract: The results of this research provide fixed-time fractional-order control for Euler–Lagrange systems that are subject to external disturbances. The first step in the process of developing a new system involves the introduction of a method known as fractional-order fixed-time non-singular terminal sliding mode control (FoFtNTSM). The advantages of fractional-order calculus and NTSM are brought together in this system, which result in rapid convergence, fixed-time stability, and smooth control inputs. Lyapunov analysis reveals whether the closed-loop system is stable over the duration of the time period specified. The performance of the suggested method when applied to the dynamics of the Euler–Lagrange system is evaluated and demonstrated with the help of computer simulations.

Keywords: fixed-time convergence; sliding mode control; fractional control; Euler–Lagrange system

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1. Introduction

In the study and modelling of non-linear dynamical systems, the Euler–Lagrange formulation, which describes the behaviour of a large group of industrial applications, is utilized. Over the past few years, there has been a rise in the number of people interested in the investigation of Euler–Lagrange systems. In general, Euler–Lagrange systems represent a wide variety of real-world practical systems, such as mobile robot platforms [1], helicopters [2], aircraft [3], pneumatic muscles [4], robotic manipulators [5], exoskeleton or bipedal robots [6,7], cranes [8], spacecraft [9], military, automation sector, monitoring, and even space travel, are just some of the many potential applications for these technologies. The fact that this is a non-linear system with a large degree of mechanical instability means that a high level of control and stability is required for it to function properly. For uncertain Euler–Lagrange systems that are affected by external disturbances, several viable solutions have been given. Thus, a robust control such as H_∞/H_2 control and sliding mode control can be built to ensure that the system continues to fulfill its intended functions despite the presence of unknown dynamics. The approach behind robust schemes has several advantages, such as the control system provides a high performance of tracking and must be strongly robust to guarantee the required level of stability.

Control of Euler–Lagrange systems is currently receiving a significant amount of interest from researchers [1]. As a solution to the tracking problem, the academic community has devised a number of different approaches, including the observer-based disturbance estimation [8], the sliding mode control (SMC) [9], proportional–integral (PI) control scheme [10], and fuzzy control [11]. Additional features are incorporated to account for unknown parametric uncertainty [12]. When it comes to controlling uncertain non-linear systems, two of the most common control techniques are known as adaptive control and robust control approaches [6]. The robust method, in contrast to the adaptive strategy, minimizes the computational cost for complex processes while still requiring a fixed upper bound on the uncertainties.

A type of notable non-linear robust control approach is known as sliding mode control (SMC) [13–15]. It has the capability to successfully control uncertain non-linear dynamics with low susceptibility to changes in system parameters and disturbances. There have been a number of developments in the area of SMC, including terminal SMC (TSMC), non-singular SMC (NSMC), fast SMC (FSMC), integral SMC (ITSMC), and fast non-singular FSMC (NFSMC) [16–18]. To further enhance the controller's functionality, fractional-order control (FOC) has been integrated with SMC [19–21]. Researchers have found Fo control, around for three hundred years, is formulated with calculus of a non-integer-order and can be applied in a wide range of scientific fields [22–25]. A significant number of fractional-order SMC (FoSMC) methods have also been developed and implemented in various Euler–Lagrange systems [5,26].

The initial conditions of a non-linear system have a considerable effect on the time required to achieve convergence in a finite-time [27]. A fixed-time control strategy is an alternative that may be used to precisely calculate the convergence time notwithstanding the initial values [28]. Several different finite-time FoSMC algorithms have been utilized with Euler–Lagrange systems. These methods consider the presence of uncertainties and disturbances in addition to other factors. For non-linear robotic systems, a dependable finite-time FoSMC with time delay control was developed. In addition, a finite-time FoSMC was utilized to make an estimate of the unknowable dynamics of the non-linear robot by utilizing an adaptive controller for the estimation of unknown dynamics. This was performed to obtain an accurate prediction of the behaviour of the non-linear robot [29]. In the face of uncertainty, a class of high-order FoSMC was used to construct a strong control method [30].

Each of the preceding studies focused on compensating the bounds of bounded uncertain dynamics using a finite-time sliding mode technique. Avoiding non-singularity, providing robustness against uncertain dynamics and external disturbances, and guaranteeing that the convergence rate is not reliant on the initial circumstances are the primary benefits of using FoFtNTSM control. This research found that the fractional-order fixed-time non-singular TSM scheme has not thoroughly been looked into, and that only a few of works provide fraction-order-based FtNTSM control. As a result, this study analyses the fixed-time technique for externally perturbed Euler–Lagrange systems. Thus, research was carried out to develop a fixed-time fractional-order non-singular TSM (FoFtNTSM) for an externally perturbed uncertain Euler–Lagrange system.

The most important findings of this research can be broken down into these main points:

- The features of a fixed-time non-singular TSM are used to make a sliding surface with good tracking, low chatter in the control inputs, and a fast rate of convergence.
- The performance of the fixed-time nonsingular TSM control scheme for the uncertain Euler–Lagrange system under disturbances, is improved upon by employing the fractional-order method.
- The Lyapunov synthesis proves that the proposed FoFtNTSM method can be used to perform fixed-time stability analysis of the overall system.

The paper adopts the following structure: Section 2 discusses the literature of the related schemes. Section 3 presents the preliminaries. Section 4 discusses the system's stability as well as its control scheme and modelling. In Section 5, we present the numerical results and accompanying discussion of the proposed method. The final thoughts are provided in Section 6.

2. Related Work

Due to the fact that terminal SMC approaches for non-linear dynamics are exceptional in that they converge in a fixed amount of time, an increasing number of researchers have been concentrating on this subject over the past several years [22]. An uncertain microgyroscope system with disturbances was the focus of the authors' approach in [31], which presented a fixed-time FoSMC technique as a solution. The findings of the research were reported in [32], and one of the main takeaways was the development of a new synchronized fixed-time TSM technique for use with fractional chaotic systems. The authors in [33] suggested a non-singular fixed-time sliding mode method as a solution to the non-linear chaotic system. In order to better accommodate scattered quadrotors, an improved adaptive fractional-order rapid integral TSMC was devised [34]. The following strategy was suggested for the unmanned surface vessel: a fixed-time SMC control with compensation for unmodelled dynamics and unknown disturbances [35]. In [36], the authors presented a fast fixed-time super-twisting control for the Euler–Lagrange system with exponential SMC. Additionally, they used a high-order sliding mode finite-time observer to provide an estimation of the angular velocity and disturbances in the system. Another observer-based fixed-time robust control technique for the uncertain dynamics of robot manipulators was presented to obtain good position tracking and robustness [37]. A high-order super-twisting SMC approach using a fixed-time scheme was devised to deal with a piezoelectric nano-positioning stage [38]. In order to achieve control over a symmetrically chaotic supply chain system, a super-twisting fixed-time SMC constrained by a controlled input was developed [39].

One way to account for uncertain dynamics is to use a variety of fractional-order TSM approaches, including the ones provided, to create robust schemes for a wide range of linear and non-linear systems. Fractional-order control has been utilized to improve performance, and a finite-time fractional-order TSM (FTFOSMC) was used in a study to produce rapid responses and reduce chattering and singularity issues [5]. For applications involving uncertain robotic systems in the presence of actuator faults, [40] suggested an adaptive control-based fractional fixed-time SMC scheme. A separate study guaranteed the trajectory tracking performance of an underactuated autonomous underwater vehicle using disturbance observer-based fractional-order sliding mode control [41]. Adaptive fixed-time control was used to create a fractional-order SMC for a non-linear 3-DOF robot with unknown uncertain dynamics [42].

3. Preliminaries

Definition 1. *Many applications of fractional calculus based on the Riemann–Liouville (RL) formulation were discovered in the literature [43–45]. The following equations give the RL fractional differentiation as well as the integration of the η_{th}-order for the $f(t)$ function with constant a [10]*

$$\begin{aligned} {}_aI_t^{\eta} f(t) &= \frac{1}{\Gamma(\eta)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\eta}} d\tau \\ {}_aD_t^{\eta} f(t) &= \frac{d^{\eta} f(t)}{dt^{\eta}} = \frac{1}{\Gamma(1-\eta)} \frac{d}{dt} \int_a^t \frac{f(\tau)}{(t-\tau)^{\eta}} d\tau \end{aligned} \tag{1}$$

where $n-1 < \eta < n$, $n \in \mathbb{N}$. Furthermore, $\Gamma(\cdot)$ is the gamma function defined as

$$\Gamma(\eta) = \int_0^{\infty} e^{-t} t^{\eta-1} dt$$

whereas the fractional integral and derivative are represented by the symbols I and D, respectively.

Property 1. *The formula that we use for the RL derivative is as follows [46]:*

$${}_aD_t^{1-\eta}\left({}_aD_t^{\eta} f(t)\right) = \dot{f}(t) \tag{2}$$

Property 2. *The Riemann–Liouville derivative conforms to the following equality [46]:*

$${}_aD_t^{\eta}\left({}_aD_t^{-\eta} f(t)\right) = f(t) \tag{3}$$

The definition of RL fractional derivative and integral is commonly used in control systems due to the application of its properties, and these properties are helpful to derive and compute the closed system and stability analysis.

Lemma 1. *If there is a continuous radially bounded function $V(x)$ then the fixed-time convergence requires the Lyapunov analysis to meet the following conditions [36]:*

i. $V(x) = 0 \Leftrightarrow x = 0$, ii. $\dot{V}(x) \leq -\lambda_1 V^{\gamma_1}(x) - \lambda_2 V(x)^{\gamma_2}$ with λ_1, $\lambda_2 > 0$, $0 < \gamma_1 < 1$ and $\gamma_2 > 1$. Once this happens, we may say the system is fixed-time stable and determine how long it will take to converge using the formula

$$T \leq \frac{1}{\lambda_1(1-\gamma_1)} + \frac{1}{\lambda_2(\gamma_2 - 1)} \tag{4}$$

Lemma 2. *For $\delta_1, \delta_2, \delta_3, ..., \delta_n > 0$, the following inequalities exist [47]*

$$\begin{array}{ll} \sum_{i=1}^{n} |\delta_i|^{1+r} \geq \left(\sum_{i=1}^{n} |\delta_i|^2 \right)^{\frac{1+r}{2}}, & for \;\; 0 < r < 1 \\ \sum_{i=1}^{n} |\delta_i|^{r} \geq n^{1-r} \left(\sum_{i=1}^{n} |\delta_i| \right)^{r}, & for \;\; r > 1 \end{array} \tag{5}$$

4. Fixed-Time Fractional Sliding Mode Control

This section includes a description of the dynamics of Euler–Lagrange systems, continues with an examination of a fractional-order non-singular sliding manifold, and then follows up with the development of a proposed FoFtNTSM scheme. Furthermore, Lyapunov theorem analysis is employed to evaluate the stability of the suggested FoFtNTSM.

Here, we provide the dynamic equation for an Euler–Lagrange system [10]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau(t) + d(t) \tag{6}$$

where $q \in \mathbb{R}^n$ is the angular position, $\dot{q} \in \mathbb{R}^n$ is the angular velocity and $\ddot{q} \in \mathbb{R}^n$ is the angular acceleration. $M(q) \in \mathbb{R}^{n \times n}$ represents the inertia matrix and follows the condition that $\underline{M}(M(q)) \leq \|M(q)\| \leq \overline{M}(M(q))$ with $\underline{M}$ and $\overline{M} > 0$ as the minimum and maximum eigenvalues of $M(q)$, respectively. $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ are the Coriolis and centripetal forces, and $G(q) \in \mathbb{R}^n$ is the gravitational force. $d(t) \in \mathbb{R}^n$ is the external disturbances, and $\tau(t) \in \mathbb{R}^n$ is the joint control input.

The dynamics Equation (6) can be rewritten as

$$\ddot{q} = M^{-1}(q)\tau(t) + M^{-1}(q)d(t) - M^{-1}(q)[C(q,\dot{q})\dot{q} + G(q)] \tag{7}$$

Using (6), the tracking error can be computed by the expression $\bar{q} = q - q_d$ as follows

$$\ddot{\bar{q}} = M^{-1}(q)\tau(t) + L(q,t) + N(\dot{q},q,t) \tag{8}$$

where $N(\dot{q}, q, t) = -\ddot{q}_d - M^{-1}(q)[C(q,\dot{q})\dot{q} + G(q)]$ is the known system's dynamics. $L(q,t) = M^{-1}(q)d(t)$ is the lumped disturbances. q_d is the desired angular position and q is the actual angular position.

Assumption 1. *Equation (9), that establishes a bounded condition of uncertain dynamics, is given here*

$$\|L(q,t)\| \leq \zeta \tag{9}$$

where ζ is the positive constant.

To achieve the Euler–Lagrange system's desired fixed-time tracking performance, a fractional-order sliding surface is proposed here

$$s(t) = D^{\beta}e(t) + k_1 D^{\beta-1}\lceil e(t) \rfloor^{\psi_1} + k_2 D^{\beta-1}\lceil e(t) \rfloor^{\psi_2} \tag{10}$$

where $e(t) = \dot{\bar{q}}(t) + \bar{k}_1\lceil \bar{q}(t) \rfloor^{\psi_{11}} + \bar{k}_2\lceil \bar{q}(t) \rfloor^{\psi_{22}}$, $s(t) \in \mathbb{R}^n$ represents the sliding surface, $\lceil \bullet \rfloor = |\bullet|^{\bar{\alpha}} sign(\bullet)$, $k_1, \bar{k}_1 > 0$ and $k_2, \bar{k}_2 > 0$. The fractional-order parameter β has a range

$0 < \beta < 1$, while ψ_1, ψ_2, ψ_{11} and ψ_{22} have ranges $0 < \psi_1, \psi_{11} < 1$ and $\psi_2, \psi_{22} > 1$, respectively.

By taking the time derivative of the sliding surface (10), it can be calculated as

$$\dot{s}(t) = D^{\beta}\dot{e}(t) + k_1 D^{\beta}\lceil e(t)\rfloor^{\psi_1} + k_2 D^{\beta}\lceil e(t)\rfloor^{\psi_2} \tag{11}$$

By substituting (8) into (11), one can obtain

$$\dot{s}(t) = D^{\beta}\begin{bmatrix} M^{-1}(q)\tau(t) + L(q,t) + N(\dot{q},q,t) \\ +\bar{k}_1\psi_{11}|\bar{q}(t)|^{\psi_{11}-1}\dot{\bar{q}}(t) + \bar{k}_2\psi_{22}|\bar{q}(t)|^{\psi_{22}-1}\dot{\bar{q}}(t) \end{bmatrix} + k_1 D^{\beta}\lceil e(t)\rfloor^{\psi_1} + k_2 D^{\beta}\lceil e(t)\rfloor^{\psi_2} \tag{12}$$

Now that the sliding manifold design is complete, the proposed FoFtNTSM approach for an uncertain Euler–Lagrange system can be developed to achieve the desired robust performance in the face of external disturbances. The FoFtNTSM control law τ can be designed as follows with the objective of controlling the uncertain non-linear Euler–Lagrange system under bounded disturbances:

$$\tau(t) = -M(q)\begin{bmatrix} \zeta + N(\dot{q},q,t) + \bar{k}_1\psi_{11}|\bar{q}(t)|^{\psi_{11}-1}\dot{\bar{q}}(t) + \bar{k}_2\psi_{22}|\bar{q}(t)|^{\psi_{22}-1}\dot{\bar{q}}(t) \\ +k_1\lceil e(t)\rfloor^{\psi_1} + k_2\lceil e(t)\rfloor^{\psi_2} + D^{-\beta}\left[K_1\lceil s(t)\rfloor^{\omega_1} + K_2\lceil s(t)\rfloor^{\omega_2}\right] \end{bmatrix} \tag{13}$$

where $|\bar{q}(t)|^{\psi_{11}-1} = 0 \ \text{ if } \ \bar{q}(t) = 0$. $K_1 > 0$ and $K_2 > 0$ are positive constants. ω_1 and ω_2 are positive constants as $0 < \omega_1 < 1$ and $\omega_2 > 1$, respectively. The overall system model is depicted in Figure 1.

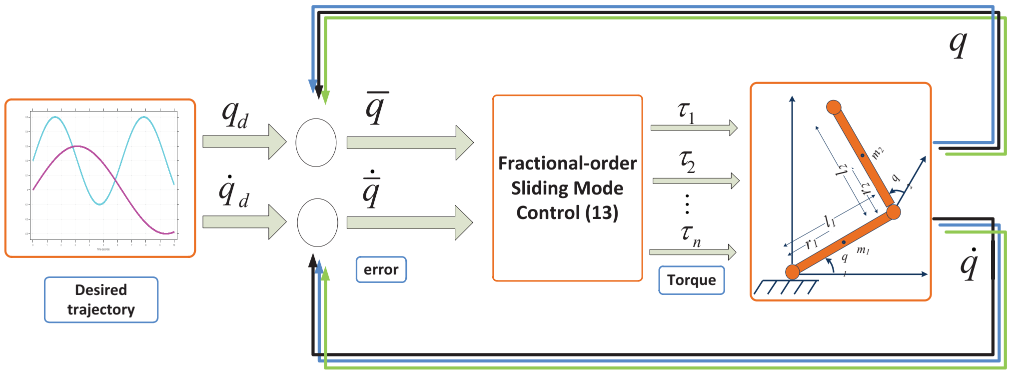

Figure 1. FoFtNTSM proposed model.

Substitution of (13) in (12), one can obtain

$$\dot{s}(t) = D^{\beta}\begin{bmatrix} -\zeta - N(\dot{q},q,t) \\ -\bar{k}_1\psi_{11}|\bar{q}(t)|^{\psi_{11}-1}\dot{\bar{q}}(t) - \bar{k}_2\psi_{22}|\bar{q}(t)|^{\psi_{22}-1}\dot{\bar{q}}(t) \\ -k_1\lceil e(t)\rfloor^{\psi_1} - k_2\lceil e(t)\rfloor^{\psi_2} \\ -D^{-\beta}(K_1\lceil s(t)\rfloor^{\omega_1} + K_2\lceil s(t)\rfloor^{\omega_2}) \\ +L(q,t) + N(\dot{q},q,t) \\ +\bar{k}_1\psi_{11}|\bar{q}(t)|^{\psi_{11}-1}\dot{\bar{q}}(t) + \bar{k}_2\psi_{22}|\bar{q}(t)|^{\psi_{22}-1}\dot{\bar{q}}(t) \end{bmatrix} + k_1 D^{\beta}\lceil e(t)\rfloor^{\psi_1} + k_2 D^{\beta}\lceil e(t)\rfloor^{\psi_2} \tag{14}$$

By simplifying (14), one can have

$$\dot{s}(t) = D^{\beta}\begin{bmatrix} -\zeta + L(q,t) \\ -k_1\lceil e(t)\rfloor^{\psi_1} - k_2\lceil e(t)\rfloor^{\psi_2} \\ -D^{-\beta}(K_1\lceil s(t)\rfloor^{\omega_1} + K_2\lceil s(t)\rfloor^{\omega_2}) \end{bmatrix} + k_1 D^{\beta}\lceil e(t)\rfloor^{\psi_1} + k_2 D^{\beta}\lceil e(t)\rfloor^{\psi_2} \tag{15}$$

$$\dot{s}(t) = D^{\beta}\begin{bmatrix} -\zeta + L(q,t) \\ -D^{-\beta}\left(K_1\lceil s(t)\rfloor^{\omega_1} + K_2\lceil s(t)\rfloor^{\omega_2}\right) \end{bmatrix} \tag{16}$$

Now the Lyapunov analyses is performed to determine the stability of the closed-loop system in the following theorems.

Theorem 1. *Considering the Euler–Lagrange system described in (6), the sliding manifold proposed in (10), and the FoFtNTSM controller developed in (13), it is achievable for an uncertain dynamic system under bounded condition (9) for the angular position to converge in a fixed amount of time.*

Proof. The selected Lyapunov candidate in the following expression is given as

$$V_1(t) = 0.5\sum_{i=1}^{n} s_i^2(t) \tag{17}$$

The derivative $\dot{V}_1(t)$ can be obtained as

$$\dot{V}_1(t) = \sum_{i=1}^{n} s_i(t)\dot{s}_i(t) \tag{18}$$

$$\dot{V}_1(t) = \sum_{i=1}^{n} s_i(t)\left\{D^{\beta}\begin{bmatrix} -\zeta + L_i(q,t) \\ -D^{-\beta}\left(K_1\lceil s_i(t)\rfloor^{\omega_1} + K_2\lceil s_i(t)\rfloor^{\omega_2}\right) \end{bmatrix}\right\} \tag{19}$$

Simplification of (19) can be written as

$$\dot{V}_1(t) = \sum_{i=1}^{n} s_i(t)\left\{-\left(K_1\lceil s_i(t)\rfloor^{\omega_1} + K_2\lceil s_i(t)\rfloor^{\omega_2}\right) + D^{\beta}(-\zeta + L_i(q,t))\right\} \tag{20}$$

Using the condition given in (9), one can solve (20) as

$$\dot{V}_1(t) \le -\sum_{i=1}^{n} s_i(t)\left\{K_1\lceil s_i(t)\rfloor^{\omega_1} + K_2\lceil s_i(t)\rfloor^{\omega_2}\right\} \tag{21}$$

$$\begin{aligned} \dot{V}_1(t) &\le -K_1\sum_{i=1}^{n}|s_i(t)|^{\omega_1+1} - K_2\sum_{i=1}^{n}|s_i(t)|^{\omega_2+1} \\ &\le -K_1\sum_{i=1}^{n}\left(|s_i(t)|^2\right)^{\frac{\omega_1+1}{2}} - K_2\sum_{i=1}^{n}\left(|s_i(t)|^2\right)^{\frac{\omega_2+1}{2}} \end{aligned} \tag{22}$$

According to Lemma 2, one obtains

$$\dot{V}_1(t) \le -K_1\left(\sum_{i=1}^{n}|s_i(t)|^2\right)^{\frac{\omega_1+1}{2}} - K_2 n^{\frac{1-\omega_2}{2}}\left(\sum_{i=1}^{n}|s_i(t)|^2\right)^{\frac{\omega_2+1}{2}} \tag{23}$$

$$\begin{aligned} \dot{V}_1(t) &\le -K_1(2V_1)^{\frac{\omega_1+1}{2}} - K_2 n^{\frac{1-\omega_2}{2}}(2V_1)^{\frac{\omega_2+1}{2}} \\ &\le -2^{\frac{\omega_1+1}{2}}K_1 V_1^{\frac{\omega_1+1}{2}} - 2^{\frac{\omega_2+1}{2}}K_2 n^{\frac{1-\omega_2}{2}} V_1^{\frac{\omega_2+1}{2}} \end{aligned} \tag{24}$$

As a result of this, the system's states arrive to $s(t)$ in a fixed-time. In light of Lemma 1, the following formula may be used to determine the fixed settling time:

$$T_1 = \frac{1}{2^{\frac{\omega_1+1}{2}}K_1\left(1 - \frac{\omega_1+1}{2}\right)} + \frac{1}{2^{\frac{\omega_2+1}{2}}n^{\frac{1-\omega_2}{2}}K_2\left(\frac{\omega_2+1}{2} - 1\right)} \tag{25}$$

Once the system's states are on the sliding manifold, the following equation holds $s(t) = 0$, obtaining

$$D^{\beta}e(t) = -k_1 D^{\beta-1}\lceil e(t)\rfloor^{\psi_1} - k_2 D^{\beta-1}\lceil e(t)\rfloor^{\psi_2} \tag{26}$$

□

Theorem 2. *The dynamics of the sliding mode (26) are stable, and the trajectories of its states tend to zero in a fixed-time.*

Choosing the Lyapunov function given by

$$V_2(t) = 0.5\sum_{i=1}^{n} e_i^2(t) \tag{27}$$

The $\dot{V}_2(t)$ can be derived as

$$\dot{V}_2(t) = \sum_{i=1}^{n} e_i(t)\dot{e}_i(t) \tag{28}$$

According to fractional-order Property 1, (28) can be rewritten as

$$\dot{V}_2(t) = \sum_{i=1}^{n} e_i(t)D^{1-\beta}(D^{\beta}e_i(t)) \tag{29}$$

By substitution of (26) into (29), one can obtain

$$\dot{V}_2(t) = \sum_{i=1}^{n} e_i(t)D^{1-\beta}\left[-k_1 D^{\beta-1}\lceil e_i(t)\rfloor^{\psi_1} - k_2 D^{\beta-1}\lceil e_i(t)\rfloor^{\psi_2}\right] \tag{30}$$

Using to fractional-order Property 2, (30) can be expressed as

$$\dot{V}_2(t) = -\sum_{i=1}^{n} e_i(t)\left[k_1\lceil e_i(t)\rfloor^{\psi_1} + k_2\lceil e_i(t)\rfloor^{\psi_2}\right] \tag{31}$$

$$\begin{aligned}\dot{V}_2(t) &= -k_1\sum_{i=1}^{n}|e_i(t)|^{\psi_1+1} - k_2\sum_{i=1}^{n}|e_i(t)|^{\psi_2+1} \\ &= -k_1\sum_{i=1}^{n}\left(|e_i(t)|^2\right)^{\frac{\psi_1+1}{2}} - k_2\sum_{i=1}^{n}\left(|e_i(t)|^2\right)^{\frac{\psi_2+1}{2}}\end{aligned} \tag{32}$$

According to Lemma 2, one obtains

$$\dot{V}_2(t) \le -k_1\left(\sum_{i=1}^{n}|e_i(t)|^2\right)^{\frac{\psi_1+1}{2}} - k_2 n^{\frac{1-\psi_2}{2}}\left(\sum_{i=1}^{n}|e_i(t)|^2\right)^{\frac{\psi_2+1}{2}} \tag{33}$$

$$\begin{aligned}&\dot{V}_2(t) \le -k_1(2V_2(t))^{\frac{\psi_1+1}{2}} - k_2 n^{\frac{1-\psi_2}{2}}(2V_2(t))^{\frac{\psi_2+1}{2}} \\ &\le -2^{\frac{\psi_1+1}{2}}k_1 V_2(t)^{\frac{\psi_1+1}{2}} - 2^{\frac{\psi_2+1}{2}}k_2 n^{\frac{1-\psi_2}{2}}V_2(t)^{\frac{\psi_2+1}{2}}\end{aligned} \tag{34}$$

According to Theorem 2, the states must eventually approach zero. We prove in the following that the convergence to zero takes place in a fixed amount of time

$$T_2 = \frac{1}{2^{\frac{\psi_1+1}{2}}k_1\left(1-\frac{\psi_1+1}{2}\right)} + \frac{1}{2^{\frac{\psi_2+1}{2}}n^{\frac{1-\psi_2}{2}}k_2\left(\frac{\psi_2+1}{2}-1\right)} \tag{35}$$

The total settling time can be obtained using the relation $T = T_1 + T_2 + T_3$, where T_3 can be obtained from the equation $\dot{\bar{q}}(t) = -\bar{k}_1\lceil\bar{q}(t)\rfloor^{\psi_{11}} - \bar{k}_2\lceil\bar{q}(t)\rfloor^{\psi_{22}}$ when $e(t) = 0$.

Remark 1. *The suggested FoFtNTSM approach uses fractional-order sliding surface (10) and robust SMC control scheme (13) to suppress the tracking error to zero in a fixed-time, even if the dynamics of the uncertain Euler–Lagrange system (6) are affected by external disturbances. This is because the technique provided here makes use of a sort of fixed-time fractional-order sliding mode control, a feature that allows for higher performance.*

Remark 2. *According to Lemma 1, the choice made using the parameters* k_1*,* k_2*,* K_1 *and* K_2 *can significantly affect the fixed-time T. When a significant value is assigned to any of these parameters, the rate of convergence will therefore vary as a direct consequence.*

5. Results and Discussions

A non-linear robotic manipulator is utilized to realize the Euler–Lagrange system, demonstrating its simulation performance, and validating the proposed FoFtNTSM method. To deal with the external disturbances, a 2-DOF robotic manipulator is used. Detailed simulations demonstrating the FoFxNTSM's robust performance in the presence of noise and other external disturbances are provided. Simulations in MATLAB/Simulink are used to illustrate the findings of this study. The mathematical equation and detailed dynamics of 2-DOF robotic manipulators are described, along with their model parameters (see Table 1), desired trajectories, and external disturbances [36]

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, C(q,\dot{q}) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, G(q) = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix},$$

$$\tau(t) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, q_d = \begin{bmatrix} 0.3\sin(t) + 0.2 \\ 0.3\sin(0.5t) \end{bmatrix}, d(t) = \begin{bmatrix} 2\sin(t) + 0.5\sin(10t) \\ \cos(2t) + 0.5\sin(10t) \end{bmatrix}.$$

where $M_{11} = m_1 r_1^2 + m_1(r_1^2 + l_1^2) + 2\cos(q_2) m_2 l_1 r_2 + J_1 + J_2$, $M_{12} = m_2 r_2^2 + \cos(q_2) m_2 r_2 l_1 + J_2$, $M_{21} = M_{12}$, $M_{22} = m_2 r_2^2 + J_2$, $C_1 = -\sin(q_2) m_2 r_2 l_1 \dot{q}_1 \dot{q}_2 - \sin(q_2) m_2 r_2 l_1 (\dot{q}_1 + \dot{q}_2)\dot{q}_2$, $C_2 = \sin(q_2) m_2 r_2 l_1 \dot{q}_1 \dot{q}_1$, $G_1 = \cos(q_1)(m_1 r_1 + m_2 l_1) g + \cos(q_1 + q_2) m_2 r_2 g$, $G_2 = \cos(q_1 + q_2) m_2 r_2 g$.

Table 1. 2-DOF robot parameters.

Parameter	Description	Value
m_1	mass of link 2	0.4 kg
m_2	mass of link 2	1.2 kg
r_1	centroid length of joint 1	0.5 m
r_2	centroid length of joint 2	0.85 m
l_1	length of the link l	1 m
l_2	length of the link 2	1 m
J_1	moment of inertia 1	5 kg·m^2
J_2	moment of inertia 2	5 kg·m^2
g	gravitational constant	9.8 m/s^2

5.1. Scenario-1: Simulation and Comparison of the Proposed Method and FTSMC

This subsection presents the implementation of the suggested FoFtNTSM controller in the robotic manipulator. The fractional-order value β is selected using a trial-and-error method. The desired trajectories quickly approach the suitable value of β = 0.9. Moreover, the appropriate parameters of the proposed control scheme are given in Table 2. When comparing simulations, fractional-order sliding mode control (FTSMC) [48] is used to further prove the effectiveness of the planned method. The required parameters of the compared FTSMC scheme are selected fairly. The compared results of the proposed method and FTSMC to the dynamics of 2-DOF robotic manipulators are shown in Figures 2–4. Position accuracy, tracking error and control inputs without chattering are all depicted here.

Table 2. Proposed scheme parameters.

Parameter	Value
k_1	28
k_2	5
$\bar{k}_1$	5
$\bar{k}_2$	6
ψ_1	0.7
ψ_2	1.01
ψ_{11}	0.8
ψ_{22}	1.01
β	0.9
K_1	0.1
K_2	0.1
ω_1	0.9
ω_2	1.01
$q_1(0)$	0.3
$q_2(0)$	0.1

From the simulations presented in Figures 2–4, the presented FoFtNTSM performs better than existing methods displaying low tracking errors, fast convergence, and smooth control inputs. Excellent and robust tracking performances are obtained using the fractional-order and SMC schemes.

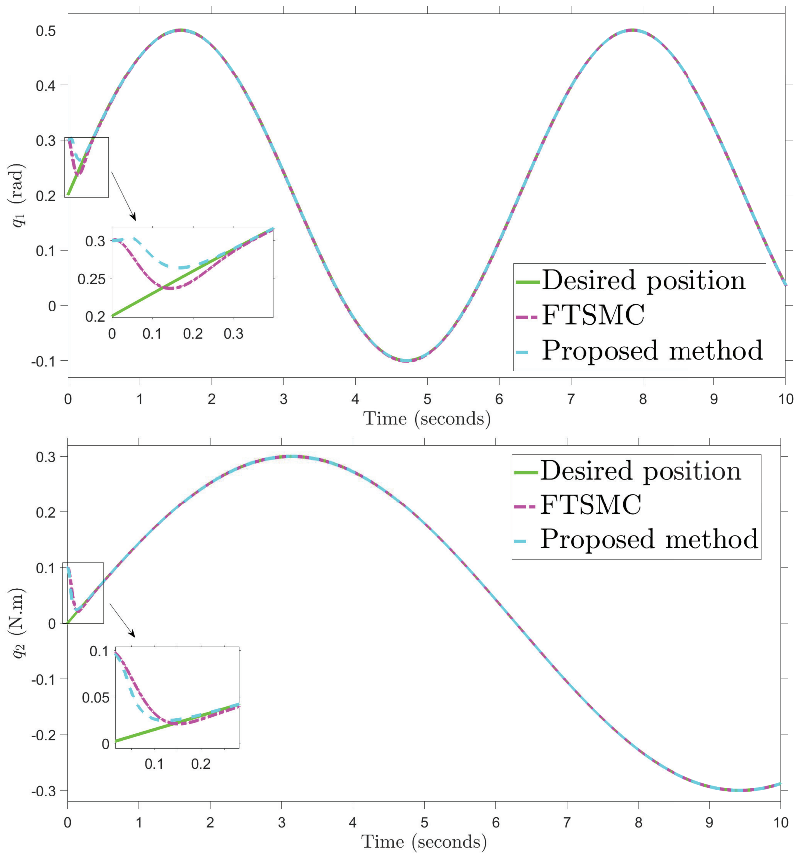

Figure 2. Position tracking.

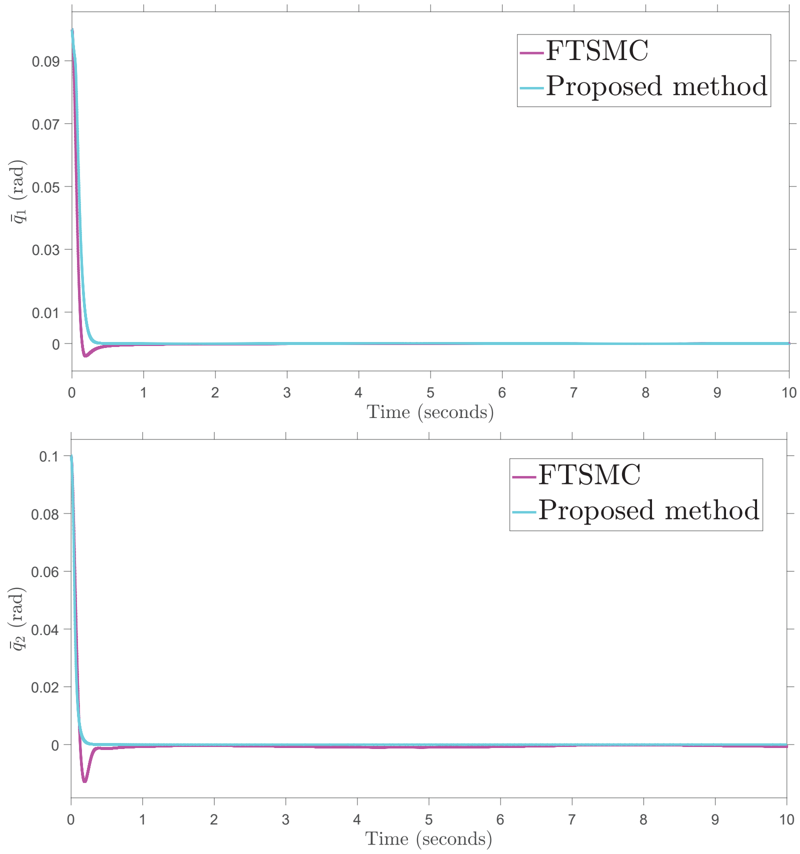

Figure 3. Tracking error.

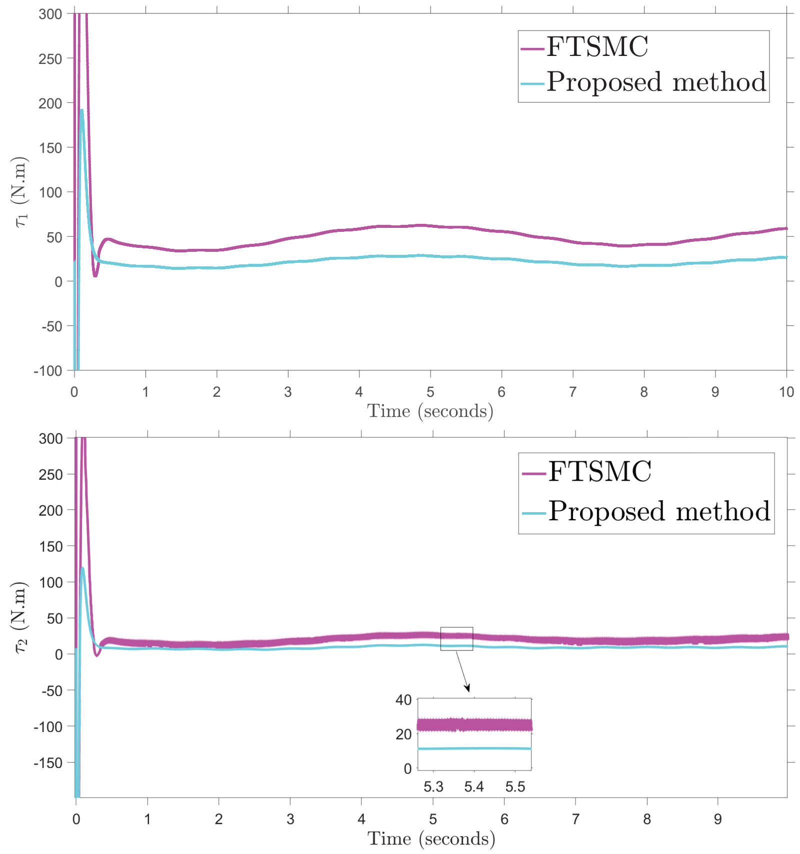

Figure 4. Control input.

5.2. Scenario-2: Comparison under Measurement Noise

This paragraph explains how the suggested FoFtNTSM approach can be used to control the dynamics of a 2-DOF robotic manipulator in the presence of random measurement noise and bounded external disturbances. This is conducted to ensure the robot is able to perform its function well. Figures 5–7 show the simulation results comparing the FoFtNTSM and FTSMC approaches in the presence of disturbances and measurement noise, with the aim of validating the effectiveness of the proposed approach with regard to tracking the desired trajectory, minimizing tracking error, and imposing chatter-free control torque.

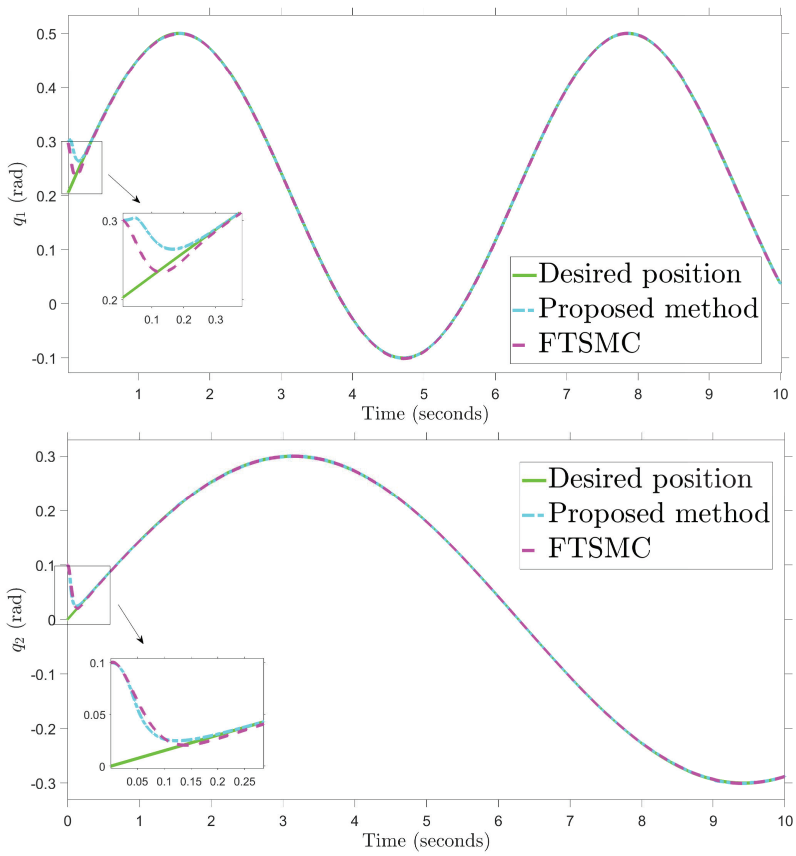

Figure 5. Position tracking with noise.

From the results presented in Figures 5–7, the FoFtNTSM obtains improved position tracking, quick convergence, and non-chatter control inputs despite the presence of perturbed dynamics in the existence of measurement noise, while the FTSMC has chattering problems in the control inputs. This is the case despite the fact that the stud shows perturbed and noisy dynamics are present.

Subsequently, the proposed scheme's simulated results are critically evaluated. Therefore, a brief examination of the controller parameter limits is presented. The proposed gain values of the controller, as well as stability analyses, are discussed in light of these boundaries. By comparing Figures 2 and 3, it is easy to see that the proposed scheme reduces the angular position error while simultaneously minimizing the necessary convergence time. Figure 4 also displays the control performance, demonstrating that the proposed method provides a chatter-free and therefore most applicable control input. As a result, the performance of the Euler–Lagrange system under measurement noise is superior in terms of position tracking and control torque.

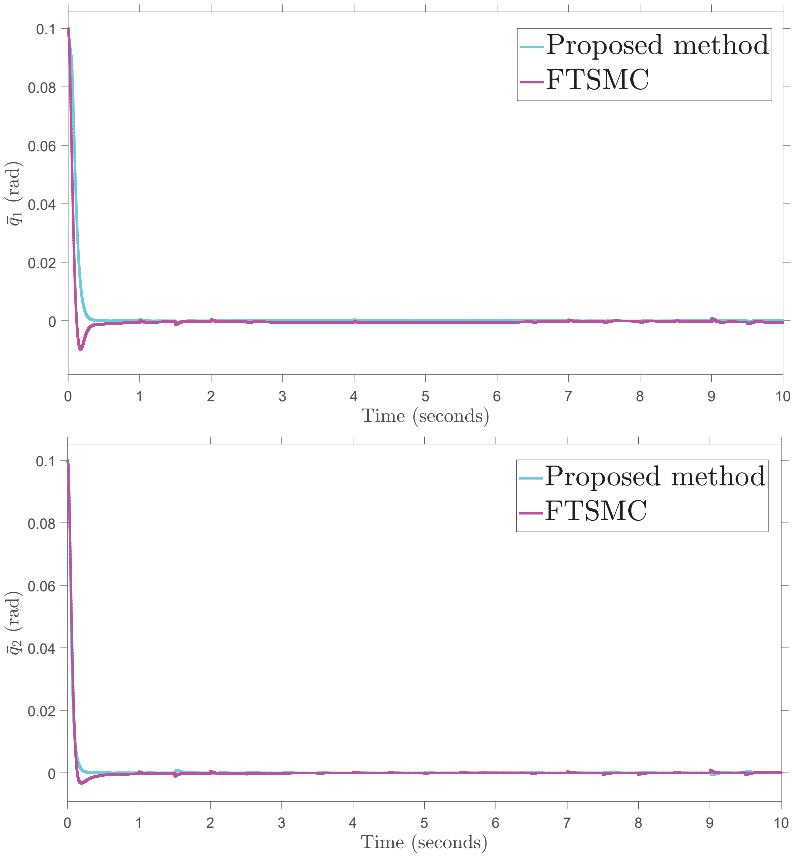

Figure 6. Tracking error with noise.

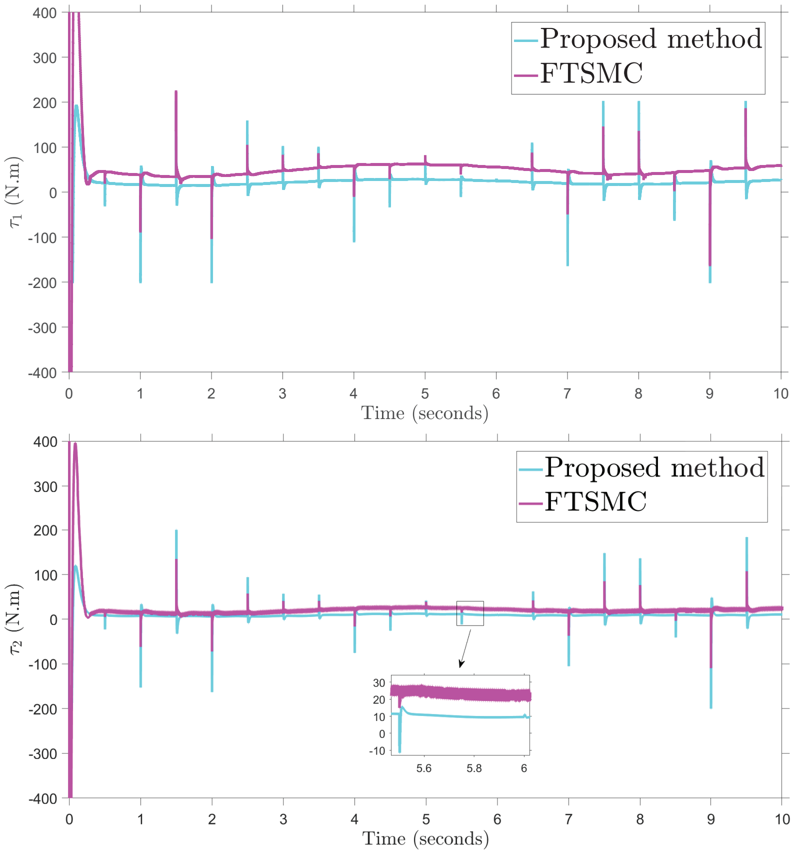

Figure 7. Control input with noise.

Remark 3. *The parameters used for the proposed method are the ones that fall within the range* $k_1, k_2 >, \bar{k}_1, \bar{k}_2 > 0$, $\psi_1, \psi_{11} \in (0, 1)$, $\psi_2, \psi_{22} > 1$, $0 < \beta < 1$, $K_1, K_2 > 0$, $\omega_1 \in (0, 1)$ *and* $\omega_2 > 1$. *Therefore, if these issues are ignored, the proposed scheme will remain unaffected, and the system will lose its fixed-time stability. Considering* T, *it is plainly clear that* k_i, $\bar{k}_i$ *and* K_i *have an inverse relationship; this is in addition to the fact that* k_i, $\bar{k}_i$ *and* K_i *have a direct relationship with* $\tau(t)$ *in (13). Therefore, adjusting* k_i, $\bar{k}_i$ *and* K_i *to the appropriate values is necessary to achieve error convergence within a fixed settling time and overall system stability. As a result, the suitable tracking performance and fixed-time stability of the system can be achieved using these values. The parameter values can be chosen adequately because it is known where they fall within their ranges. Because of this, the procedure for selecting the appropriate value is made possible.*

6. Conclusions

A FoFtNTSM has been proposed as a possible solution in an effort to achieve good tracking results for uncertain Euler–Lagrange systems in the presence of external disturbances and measurement noise. To account for the uncertainty of dynamics in the presence of external disturbances and noise, the proposed scheme employs a robust design. The FoFtNTSM converges in a fixed amount of time and achieves the desired tracking performance as a direct result of utilizing the proposed method. This was conducted to demonstrate that the designed method is effective. A 2-DOF robotic manipulator was utilized in one of the applications of the Euler–Lagrange system. The findings demonstrated that the proposed FoFtNTSM method outperforms the FTSMC method in terms of tracking error and its convergence, and the ability to mitigate disturbance and noise.

In addition, the investigation of non-smooth non-linearities may be included in this field of research on the Euler–Lagrange system. The work that was initially envisioned would need to be substantially expanded in order to accommodate this.

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